INFORMATION THEORY AND MUSIC

01

Introduction

Music as a syntactic sequence

In this presentation, we will delve into the intersection of information theory and the musical field. Initially, information theory has been about optimizing communication channels. Over time, its principles, having broad universal allure, it have branched into diverse fields, music included.

This presentation about applying Information Theory to explore various synthetic and analytic applications in music while considering music's artistic nuances.

Before we move forward however it will be important to note that we will talk about music in a ***syntactic*** sense not a semantic one.

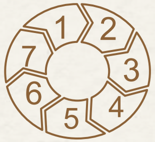
Music here is just a collection of symbols that have are pleasing to hear. It doesn't matter what one is supposed to feel while listening to or what is the meaning that the piece is supposed to convey.

02

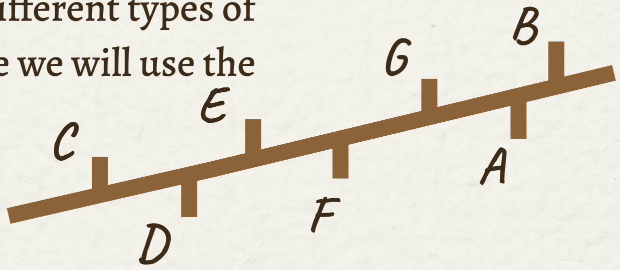
Background Information

The musical sign system

The note set in any kind of music is typically based on the heptatonic scale.



There are many different types of scales but for ease we will use the C major scale



This scale forms the foundation of Western music, and you can think of each song as a collection of those tones.

One could then think of song a number of segments (m) each segment has a number of notes (n) where each part sets a precedent for the other

03

Entropy in Music

Music as a stochastic source

In music the first segment or note is considered a reference and the next ones are those which actually carry information.

While the goal in communication systems is always to increase information. Things are a little different in our case.

The goal in music is actually to hit a certain point where we have just enough information to make the piece interesting but not so much so it becomes overwhelming. Redundancy after all provides a quantitative measure of order.

Redundancy in the signal from a transmitter to receiver serves to combat errors introduced by noise in the channel.

Noise changes the signal from a more probable to a less probable, and from a more certain to a less certain, state; the information-content of the message is increased.

The paradox that noise should increase the information-content of a message is resolved by distinguishing between desirable and undesirable information.

Redundancy provides data about the characteristics of the source. for example, if the final chord of a Beethoven symphony can easily be predicted, it still gives information about the structure of Beethoven's composition as a stochastic source; there is no reason to leave off the last chord because it is redundant.

For our purposes the entropy of music is calculated as this:

$$H= \sum p_i \log(p_i)$$

And the redundancy of music is calculated as this:

$$R=1-(H/\log(n))$$

As someone who composes music. I could do the following:

1- propose a simple arbitrary initial sequence
2-re-iterate that sequence multiple time and modify slightly each time in a way that maintains a redundancy of 60%

If composers are Markov sources,then the formulae for redundancy measure the degree of order or structuration in their works. Making the entropy of their song as a parameter of their style

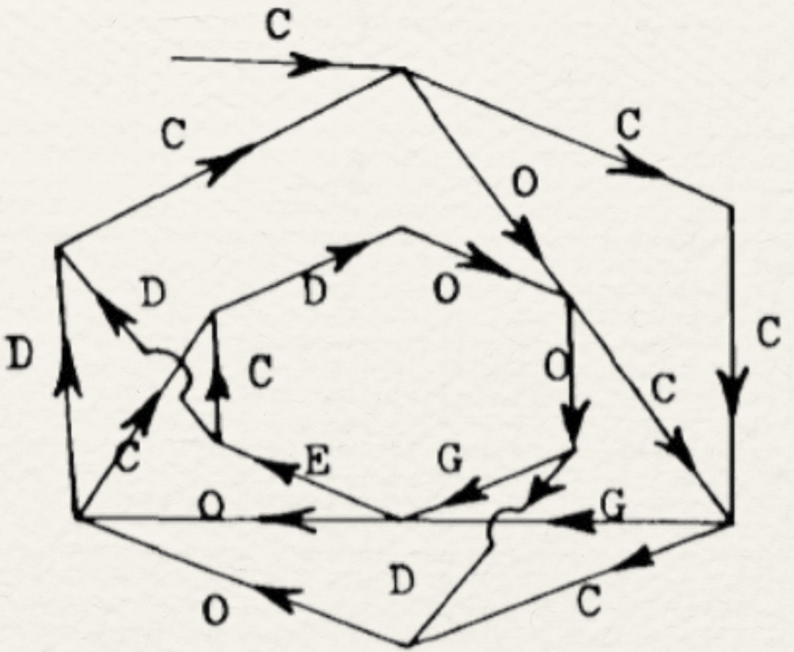
"Styles in music are basically complex systems of probability relationships in which the meaning of any term or series of terms depends upon its relationships with all other terms possible within the style system."

—Meyer

04

Banal Tone-Maker

Analytic-synthetic application of music


C
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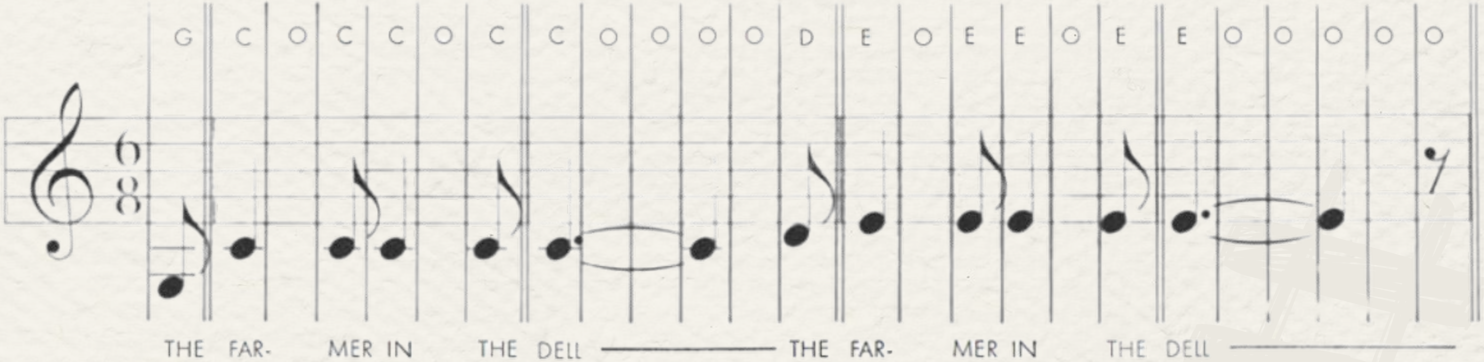

G C O C C O C C O O O O D E O E E O E E O O O O O
THE FAR- MER IN THE DELL ——— THE FAR- MER IN THE DELL ———

- Let us conduct statistical analysis of familiar nursery tunes to better understand the underlying mathematical and information-theoretic properties of simple melodies.

- Select 39 nursery tunes from a songbook and coded each tune as a series of symbols representing the notes. All tunes were transposed to the key of C and notes were represented as C, D, E, F, G, A, B or O for rest.

  Ex: G/COCCOC/COOOOD/ EOEEOE/ EOOOOO/GOOGOA/GOECOD/EOEDOD/COOOO.

- Count the frequency of each note across all tunes. Hence, the probability was calculated as:

C	D	E	F	G	A	B	O
0.163	0.112	0.132	0.066	0.149	0.045	0.036	0.297

- As expected, notes like G that are harmonically related to the key note C by simple ratios had higher probabilities than notes like B with more complex ratios.

- Calculate the entropy, or average information per note with assumption that all notes are equally probable:

$$H(max) = \log_{10} 8 = 0.903$$

- The actual entropy = 0.821, indicating a redundancy of 9%.

However, this did not fully capture note relationships

Hence, transition probabilities between note pairs were counted in a matrix.

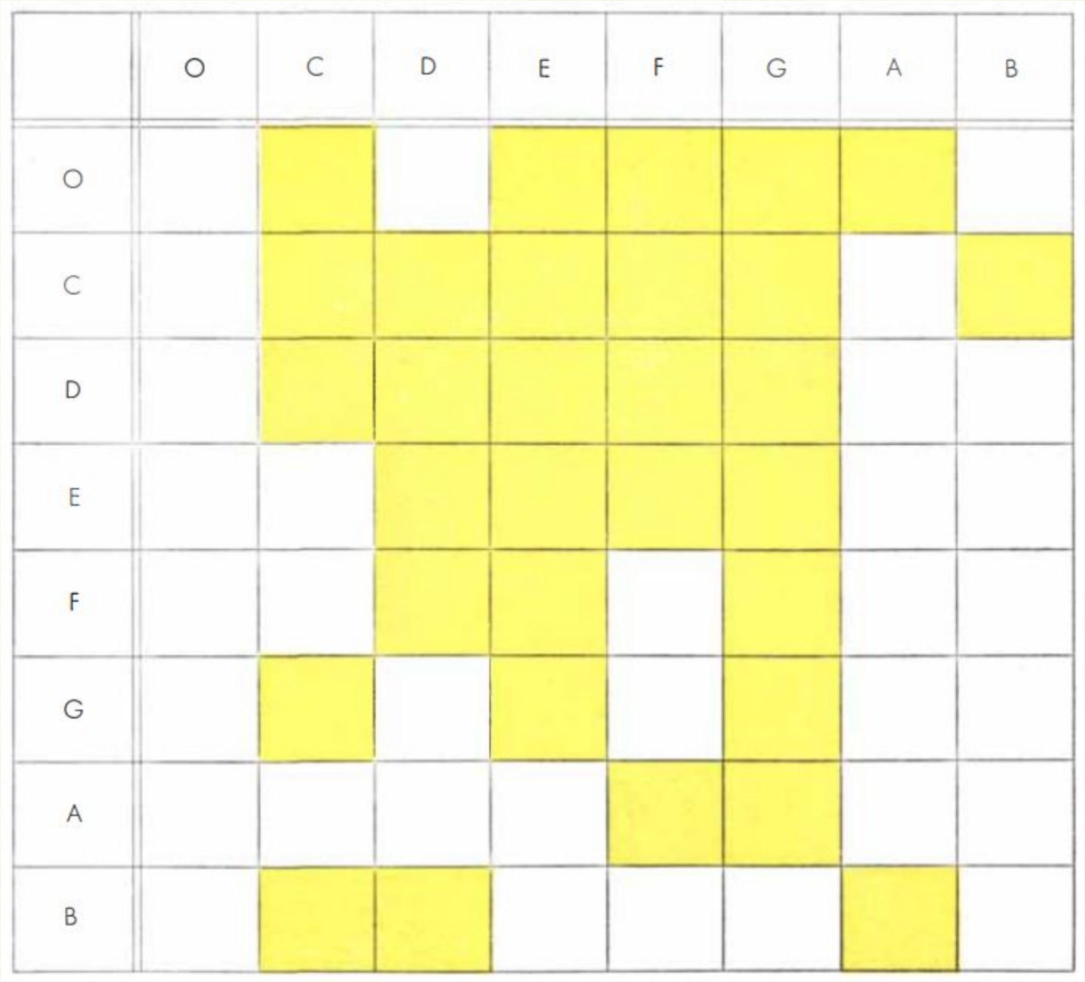
TRANSITION PROBABILITIES

Show how frequently any note follows any other in the 39 nursery tunes

	O	C	D	E	F	G	A	B
O	0.38	0.17	0.10	0.10	0.06	0.13	0.03	0.02
C	0.36	0.23	0.13	0.07	0.02	0.10	0.03	0.07
D	0.26	0.20	0.21	0.19	0.03	0.06	0.01	0.05
E	0.22	0.15	0.18	0.16	0.16	0.12	0.01	0.00
F	0.15	0.00	0.14	0.35	0.14	0.20	0.01	0.01
G	0.29	0.14	0.00	0.16	0.06	0.26	0.08	0.00
A	0.17	0.05	0.07	0.00	0.02	0.36	0.15	0.17
B	0.18	0.30	0.12	0.01	0.01	0.08	0.21	0.08

TRANSITION PATTERN

Shows the transition from the last note of a measure to the first note of the next.



TRANSITION	C-O	C-C	C-D	C-E	C-F	C-G	C-A	C-B
PROBABILITY "p"	0.36	0.23	0.13	0.07	0.02	0.10	0.03	0.07
APPROXIMATE "p"	4/12	3/12	2/12	1/12	0	1/12	0	1/12

O	O	O	O
C	C	C	D
D	E	G	B

Matlab Implementation

Computer generated tunes

References

- https://www.scientificamerican.com/issue/sa/1956/02-01/
- https://www.youtube.com/watch?v=RENk9PK06AQ
- https://lab.rockefeller.edu/cohenje/assets/file/003InformationTheoryMusicBehavScience1962.pdf

Thank You!