Notes

- Filtering should ensure that we do not distort the signal, ex: remove high frequency components that affect the signal shape.
- Sampling theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.
- The sampling rate Fs which is equals to 2 Fmax is called Nyquist rate
- Low Frequencies: Slow small change in values :
  - signal → smooth
  - image -> blurring
- High Frequencies: Sudden noticeable change in values :
  - signal → includes sharp peaks
  - image → includes sharp edges
- Smoothing of signals and images preserves low frequencies Smoothing → Moving average or low pass filters
- Sharpening of signals and images preserves high frequencies Sharpening → Derivative or high pass filters
- to concentrate on a specific frequency band and neglect the others. Band pass filters can be used.

DC Component

- removing the shift and allow the signal to oscillate back on the X axis
- time Domain → Subtract the mean of the signal from it.
- Frequency domain → remove the first harmonic X(0).

Quantization

- Non linear quantization is used to alleviate this problem. Goal is to keep quantization error fixed for all sample values.
- The quantization levels follow a logarithmic curve. Smaller Δ's at lower amplitudes and larger Δ's at higher amplitudes

DFT

- the DFT is periodic with period N. this is the cyclical property of the DFT. The values of DFT are repetitive.
- the amplitude spectrum distribution of N-point DFT is symmetrical about harmonic N/2.
- the phase function being odd exhibits anti-symmetry about harmonic N/2
- The harmonic N/2 represents F_{max} the maximum frequency present in the signal.

IDFT

- discrete transformation from the frequency to the time domain

1. Functional representation, such as

$$x(n) = \begin{cases} 1, & \text{for } n = 1, 3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

2. Tabular representation, such as

n	$\cdots$	-2	-1	0	1	2	3	4	5	$\ldots$
$x(n)$	$\cdots$	0	0	0	1	4	1	0	0	$\ldots$

3. Sequence representation

An infinite-duration signal or sequence with the time origin ($n = 0$) indicated by the symbol ↑ is represented as

$$x(n) = \{\ldots 0, 0, \underset{\uparrow}{1}, 4, 1, 0, 0, \ldots\}$$

- Unit Sample (Impulse) Sequence δ(n) = 1 -> n=0 else 0
- Unit step Signal δ(n) = 1 -> n>=0 else 0
- Unit Ramp Signal δ(n) = n -> n>=0 else 0
- A signal x(n) is periodic with period N (N > 0) if and only if x(n + N) = x(n) for all n else the signal is non-periodic and is called aperiodic signal
- A real valued signal x(n) is called symmetric (even) if x(-n) = x(n)
- a signal x(n) is called anti-symmetric (odd) if x(-n) = -x(n)
- if the signal is in real time, it is not possible to advance the signal in time.
- Static(memory less) : if the output at any instant n depends at most on input sample at the same time, but no past or future samples of input. Any other case the system is said to be dynamic or have memory
- time invariant or shift invariant if and only if the input-output characteristics don't change with time
- if the output y(n, k) ≠ y(n - k), even for one value of k, the system is time variant.
- causal if the output [y(n)] of the system at any time n depends only on the present and past inputs But does not depend on future inputs

Cross Correlation

- Positive large results indicate strong correlation between signals, while negative results indicate negative correlation which means an increase in one signal is associated with a decrease in the other signal and vice versa.
- Small results which tend toward zero indicate that the two signals are almost independent.
- are useful for determining the time delay between two signals.
- are useful for determining the similarity of two signals.
- can be used as a criterium for template matching

auto-correlation

- can be used to detect repeats or periodicity in a signal or if it is pure random
- it can identify signals that are embedded in noise. Auto-Correlation

DCT

- used in lossy data compression applications, such as the JPEG image format.

- makes it quite suitable for compression is its high degree of "spectral compaction;" at a qualitative level, a signal's DCT representation tends to have more of its energy concentrated in a small number of coefficients when compared to other transforms like the DFT.
- a compression algorithm; if you can approximately represent the original (time- or spatial-domain) signal using a relatively small set of DCT coefficients, then you can reduce your data storage requirement by only storing the DCT outputs that contain significant amounts of energy

The Z transform

- The z-transform is the most general concept for the transformation of discrete-time series.
- The Laplace transform is the more general concept for the transformation of analog or continuous time processes
- Laplace transform allows you to transform a differential equation, and its corresponding initial and boundary value problems, into a space in which the equation can be solved by ordinary algebra
- The Z transform can do the same but in the discrete domain.
- Use of z-transform permits simple algebraic manipulation
- Fourier transform is special case from z-transform when r = 1
- Zeros -> values make z be 0
- Poles -> values make z be infinity
- If poles in unit circle (1 = نق) not on it , system is stable

Digital filter

- changes the wave-shape, amplitude-frequency and /or phase-frequency characteristics of a signal in a desired manner.
- improve the quality of a signal (for example, to remove or reduce noise) or to extract information from signals.
- Used at very low frequency
- Can work on a wide range of frequency
- when the bk are set to zero, the equation reduces to FIR equation (convolution sum)
- FIR is linear Phase but IIR is nonlinear Phase
- FIR is always stable
- Limited number of bits in fir is less severe than IIR
- FIR required more coefficient for sharp cutoff filters than IIR

FIR

- The phase delay (is the amount of time delay each frequency component of the signal suffers in going through the filter) or group delay of the filter (the average time delay the composite signal suffers at each frequency)
    - provides a useful measure of how the filter modifies the phase characteristics of the signal

    - Mathematically, the phase delay is the negative of the phase angle divided by frequency whereas the group delay is the negative of the derivative of the phase with respect to frequency
- type 1 is the most versatile of the all, type 3 and 4 are suitable for differentiators and Hilbert transformers, because of the 90 phase shift that each can provide.
- The lowpass filter, removes all frequencies greater than the cut off frequency of the filter
- The high-pass filter removes all frequencies less than the cut off frequency of the filter.

IIR

- IIR filters are used when sharp cutoff and high throughput are the important requirements.
- the IIR filter can become unstable and it causes non linear phase shift.
- analog filters can be readily transformed into equivalent digital IIR filter

Wavelet

- stationary signals : frequency content do not change in time
- Non-Stationary (chirp) signal : frequency constantly changes in time
- some portion of a non-stationary signal is stationary
- W(t) infinitely long: W=∞ → STFT turns into FT, providing excellent frequency localization, but no time information.
- W(t) infinitely short: W=δ (t) → gives the time signal back, with a phase factor, providing excellent time localization but no frequency information.
- Wide window → good frequency resolution, poor time resolution.
- Narrow window → good time resolution, poor frequency resolution.
- Frequency resolution: How well two spectral components can be separated from each other in the transform domain
- Time resolution: How well two spikes in time can be separated from each other in the transform domain.
- MRA is designed to give good time resolution and poor frequency resolution at high frequencies and good frequency resolution and poor time resolution at low frequencies.
- ψ(t) is the transforming function, and it is called the mother wavelet.
- the mother wavelet is a prototype for generating the other window functions
- low frequencies(high scales) correspond to a global information of a signal (that usually spans the entire signal
- high frequencies (low scales) correspond to a detailed information of a hidden pattern in the signal that usually lasts a relatively short time
- The first and simplest wavelet in the world of wavelet transform is Haar wavelet
- Daubechies wavelets family is the most popular family of wavelets
- Biorthogonal family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction.
- Approximations: low frequency components of the signal obtained by low-pass filtering
- Details: high frequency components obtained by high-pass filtering
- to reduce the number of obtained samples after filtering the signal -> down-sampling
- The second level decomposition is obtained by low pass and high pass filtering of the level one approximation of the original signal
- select a suitable number of levels based on the nature of the signal.

- h[n] is low-pass filter and g[n] is high -pass filter.
- The wavelet decomposition consists of filtering and downsampling while wavelet reconstruction involves up-sampling (adding zeros) and then filtering the details and approximations which come from the decomposition process
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