

MATH 494 Homework 1

Nada El Arabi

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Abstract

In this paper we examine the application of Dynamic Mode Decomposition to the separation of the background and foreground of short videos. Through the use of SVD, proper low-rank truncation of high dimensional matrices and eigenvalue distribution analysis we achieve the separation of the videos into both parts and uncover in which ways changing certain variables such as the rank of truncation and the number of modes selected can affect results.

1 Introduction and Overview

1.1 Brief introduction to Dimensionality Reduction

The soaring size of data systems is a salient problem in any industry in which data is involved. In particular, high-dimensional data is difficult to work with and analyze given the high computation costs associated. Thankfully, it is often the case that these very high dimensional data sets can be greatly reduced in rank to their most important components and be made much easier to process while retaining their essential substance. The particular dimensionality reduction algorithm of interest in this paper is a linear one: Dynamic Mode Decomposition.

1.2 Problem overview and the Role of Dimensionality Reduction

Our goal is to successfully separate two short videos into their background and foreground component. By reshaping each frame of the video into tall columns of pixel values, we may take the singular value decomposition of the data matrix which contains the frames. We must then decide which reduced matrix rank would be appropriate to keep. We then proceed to analyze the frequency of oscillation of the eigenvalues of our step matrix A to determine an appropriate separation threshold to construct the background and foreground. Finally, some processing is necessary to extract positive, real valued matrices which constitute can then be converted back into the desired frames.

2 Theoretical Background

As we have seen in our textbook, there are essential conditions for dynamic mode decomposition to be applicable.

Suppose matrices $X_n, Y_n \in \mathbb{C}^{M \times N}$ are snapshots of data in time.

3 Algorithm Implementation and Development

4 Computational Results

5 Summary and Conclusions

Add your summary and conclusions here.

Appendix A MATLAB Functions

Appendix B MATLAB Code