

5-2023

(3 Hours)

[Total Marks: 80]

- N.B.: 1) Question No. 1 is Compulsory.
 2) Answer any THREE questions from Q.2 to Q.6.
 3) Figures to the right indicate full marks.

- Q.1 (a) Evaluate $\int_0^{1+i} (y + ix^2) dz$ along the parabola $y = x^2$. (5)
 (b) If $\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational then find the values of a, b, c (5)
 (c) A continuous random variable has pdf $f(x) = ke^{-x}, 0 \leq x < \infty$. Determine k , mean, variance. (5)
 (d)

x	3	5	4	6	2
y	3	4	5	2	6

 (5)

Calculate the Karl Pearson's coefficient of correlation.

- Q.2 (a) The following are the marks scored by students in two tests in a subject. Calculate Spearman's rank correlation coefficient of from the following data. (6)

Marks in Test 1	18	20	34	52	12
Marks in Test 2	39	23	35	18	46

(6)

- (b) Find the MGF of a random variable X whose p.m.f is given by

x	0	1	2	3
P(x)	1/18	1/9	5/18	10/18

. Hence find mean and Variance of X .

- (c) Obtain all possible Laurent's series expansion of (8)

$f(z) = \frac{z}{(z-1)(z-2)}$ about $z = 0$.

- Q.3 (a) Three urns are there containing white and black balls; first urn has 3 white and 2 black balls, second urn has 2 white and 3 black balls and third urn has 4 white and 1 black balls. Without any biasing one urn is chosen from that one ball is chosen randomly which was white. What is probability that it came from the third urn? (6)

- (b) Fit the second degree polynomial for following data (6)

X	1	2	3	4	5	6	7
Y	-5	-2	5	16	31	50	73

- (c) Prove that $\vec{F} = (2xy + z)i + (x^2 + 2yz^3)j + (3y^2z^2 + x)k$ is (8)

conservative. Find scalar potential of \vec{F} . Hence find the work done of moving particle from $(1,2,0)$ to $(2,2,1)$.

- Q.4 (a) Using Green's Theorem evaluate $\int_c (x^2 - y)dx + (y^2 + x)dy$ and c is closed curve of the region bounded by $y = 4$ and $y = x^2$. (6)
- (b) Samples of two types of electric bulbs were tested for length of life and following data were obtained (6)

	Type I	Type II
Number of samples	8	7
Mean of samples(in hour)	1134	1024
Standard Deviation(in hour)	35	40

Test at 5% level of significance whether the difference in the sample means is significant.

- (c) In a normal Distribution, 30% of students scored below 35 and 10% scored above 60. Find the mean and standard deviation. (8)
- Q.5 (a) The standard deviation from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regard as drawn from normal population with same standard deviation? ($F_{(8,12)}(0.025) = 3.51$, $F_{(12,8)}(0.025) = 4.20$) (6)
- (b) Use Gauss's Divergence Theorem to evaluate $\iint_S \vec{N} \cdot \vec{F} ds$, where $\vec{F} = 4xi - 2y^2j + z^2k$ and S is region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 4$. (6)
- (c) Obtain both Line of regressions for the data given below (8)

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

Also find X for Y = 70.

- Q.6 (a) Evaluate $\int_c \frac{z+3}{(2z^2+3z-2)}dz$, where c is the circle $|z-i|=2$. (6)
- (b) The following data relate to marks obtained by 11 students in 2 tests, one held at the beginning of the year and the other at the end of the year after intensive coaching: (6)

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

- (c) The following table gives the number of accidents in a district during a week. Apply χ^2 test to find whether the accidents are uniformly distributed over the week. (8)

Day	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
No. of days	13	12	11	9	15	10	14
