Paper / Subject Code: 50291 / Engineering Mathematics - III

DSE Sem III (R-2019 C scheme) "ECS" May '23

24/5/2023

Time: 3 hour **Max. Marks: 80**

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

			Marks
Q1	**Attempt All questions**		
a)	If $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 1 \end{bmatrix}$ then find the eigen values of A^3		5
b)	Find Laplace transform of $f(t) = te^t \cos 2t$		5
c)	Find the Fourier Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$		5
d)	Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic.		5
Q2			
a)	A vector field $\bar{F}$ is given by $\bar{F} = (ysinz - sinx)i + (xsinz + 2yz)j + (xycosz + y^2)k$ Prove that $\bar{F}$ is irrotational.		6
b)	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$		6
c)	Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ satisfies Laplace's equation, also find analytic function.		8
Q3			
a)	If $\bar{F} = xye^{2z}i + xy^2coszj + x^2cosxyk$ find div$\bar{F}$ and curl$\bar{F}$		6
b)	Find an analytic function whose real part is $u = y^3 - 3x^2y$. Also find the corresponding imaginary part.		6
c)	Show that the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable and hence find the transforming matrix and diagonal matrix.		8

Q4

a) Find $\nabla\phi$ at point (1, -2, -1), where $\phi = 4xz^2 + x^2yz$ 6

b) Evaluate $\int_0^\infty e^{-2t}\sin^3 t\ dt$, using Laplace transforms 6

c) Using Partial Fraction method find $L^{-1}\left[\frac{s}{(s^2+1)\ (s^2+4)\ (s^2+9)}\right]$ 8

Q5

a) Find $L\left\{t\ \sqrt{1+sint}\ \right\}$ 6

b) Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by

$$\bar{F}(x,y,z) = y\ \hat{\imath} + (zcos(yz) + x)\ \hat{\jmath} + (y\cos(yz))\ \hat{k}$$ 6

Show that $\bar{F}$ is conservative.

c) Find the Fourier Series for $f(x)$ in $(-\pi,\pi)$where 8

$$f(x) = 1 + \frac{2x}{\pi} \qquad -\pi \le x \le 0$$

$$= 1 - \frac{2x}{\pi} \qquad 0 \le x \le \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ \ldots\ldots\ldots$

Q6

a) Obtain Fourier series expansion of $f(x) = 9 - x^2$ in $(0,\ 2\pi)$ 6

b) Find Eigen values and Eigen vectors of 6

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$$

c)

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{(s+a)}{(s+b)}}\right)\right\}$ 4

ii) Find $L^{-1}\left\{\frac{1}{s^2-2s+5}\right\}$ 4