

SE Sem III (R-2019 C. Scheme) "Computer" Jan-2023 (DSE)

25/1/2023

Max. Marks : 80

(Time: 03 hours)

- N.B. : (1) Question No. 1 is compulsory
(2) Attempt any three questions from Q. 2 to Q. 6
(3) Figures to the right indicate full marks

Q. 1 (a) Find the Laplace Transform of $e^{2t} + 4t^3 - \sin 2t \cos 3t$ 05

(b) Find the Fourier series of $f(x) = x$, $-\pi < x < \pi$ 05

(c) Calculate Spearman's coefficient of rank correlation from the following data 05

| | | | | | |
|----|-----|-----|-----|-----|-----|
| X: | 12 | 17 | 22 | 27 | 32 |
| Y: | 113 | 119 | 117 | 115 | 121 |

(d) Find the constants a, b, c, d, e
if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic 05

Q.2 (a) Determine whether the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{y}{x}$ is analytic and if so, find its derivative. 06

(b) A random variable X has the following probability distribution 06

| | | | | | | | |
|--------|---|----|----|----|----|-----|-----|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |

Find (i) k, (ii) $P(X < 4)$ (iii) $P(3 < X \leq 6)$

(c) Evaluate $\int_0^\infty e^{-2t} t \cos t dt$ 08

Q.3 (a) Find the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$, $-\pi < x < \pi$ 06

(b) A continuous random variable has probability density function 06

$$f(x) = k(x - x^2); 0 \leq x \leq 1$$

Find (i) k, (ii) mean, (iii) variance

(c) Find the inverse Laplace transform of $\frac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$ 08

Q.4(a) Find the Laplace Transform of $f(t)$, 06

where $f(t) = \cos t$, for $0 < t < \pi$ and $f(t) = \sin t$, for $t > \pi$

(b) Calculate the Karl Pearson's coefficient of correlation from the following data 06

| | | | | | | | | |
|----|----|----|----|----|----|----|----|----|
| X: | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| Y: | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

- (c) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$

08

- Q.5 (a) Find the inverse Laplace transform of $\frac{s}{(2s+1)^2}$

06

- (b) Find the Laplace transform of $t \left(\frac{\sin t}{e^t} \right)^2$

06

- (c) Find the lines of regression for the following data

| | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|
| X: | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
| Y: | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

08

- Q.6 (a) Find the mean and the variance of the following distribution

| | | | | |
|--------|-----|-----|-----|-----|
| X | 1 | 3 | 4 | 5 |
| P(X=x) | 0.4 | 0.1 | 0.2 | 0.3 |

06

- (b) Find the inverse Laplace transform of $\log \left(1 + \frac{a^2}{s^2} \right)$

06

- (c) Find the analytic function $f(z) = u + iv$ whose imaginary part is $v = x^2 - y^2 + \frac{x}{x^2 + y^2}$

08
