SE Sem III (R-2019) "AI & DS" Nov - Dec 2022

23/11/2022

(3 Hours) [Total Marks : 80

N.B.
1) **Q.1 is compulsory.**
2) Solve any 3 questions out of remaining 5 questions.
3) Assumptions made should be clearly stated.
4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

a) Prove using Mathematical Induction that n^3+2n is divisible by 3 for all $n >= 1$ **5**

b) Explain the following terms with suitable example: **5**
i) Partition set
ii) Power set.

c) State the Pigeonhole principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9. **5**

d) Consider the function $f(x) = 2x-3$. Find a formula for the composition functions **5**
i) $f^2 = f \circ f$
ii) $f^3 = f \circ f \circ f$

e) Explain the bipartite graph with suitable example. **5**

Q.2

a) What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where $A=\{1, 2, 3, 4, 5\}$ & $R=\{(x,y) \mid x-y = \pm 1\}$ **10**

b) What is a ring? Let $A= \{0, 1, 2, 3, 4, 5, 6, 7\}$. Determine whether a set A with addition modulo 8 & multiplication modulo 8 is a commutative ring? Justify your answer. **10**

Q.3

a) A survey in 1986 asked households whether they had a VCR, a CD player or cable TV. 40 had a VCR. 60 had a CD player; and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player? **8**

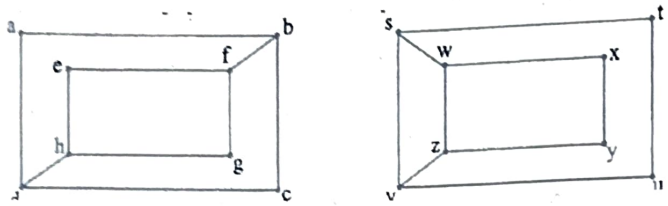
b) Find the complete solution of a recurrence relation **6**
$a_n + 2a_{n-1} = n+ 3$ for $n \geq 1$ and with $a_0 = 3$

c) Obtain CNF & DNF for the following expression: **6**
$p \longleftrightarrow (\sim p \vee \sim q)$

Q.4

a) What is a group? Let A={3, 6 ,9, 12} 10
i) Prepare the composition table w.r.t. the operation of multiplication modulo 15.
ii) Whether it is an abelian group? Justify your answer.
iii) Find the inverses of all the elements.
iv) Whether it is a cyclic group?

b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. 10



Q.5

a) Let X={1, 2, 3, 6, 24, 36} & R ={(x,y) ∈ R | x divides y} 10
i) Write the pairs in a relation set R.
ii) Construct the Hasse diagram.
iii) What are the Maximal and Minimal elements?
iv) Mention Chains and Ant chains from above set.
v) Is this poset a lattice?

b) Define the term bijective function. 5

Let $f : R \to (7/5) \to R - \left(\frac{2}{5}\right)$ be defined by $f(x) = \frac{2x-3}{5x-7}$.

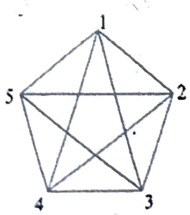
Whether a function is bijective? Justify your answer.

c) Define minimum hamming distance. Consider e : $B^3 \to B^6$. Find the code words generated by the parity check matrix H given below. 5

$$H = \begin{matrix} 1\,1\,1 \\ 1\,1\,0 \\ 0\,1\,1 \\ 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{matrix}$$

Q.6

a) Define with example Euler path, Euler circuit, Hamiltonian path, and Hamiltonian circuit. Determine if the following diagram has Euler circuit and Hamiltonian circuit. Mention the path/circuit. 6



b) Let p denote the statement 'The food is good',
q denote the statement 'The service is good' &
r denote the statement 'The rating is 3 star.'
Write the following statements in a symbolic form- **8**

i) Either food is good or service is good or both.
ii) The food is good but service is not good.
iii) If both food & service are good then the rating is 3 star.
iv) It is not true that a 3 star rating always means good food & good service.

c) Find out the incidence matrix of following graphs. **6**