Total Marks: 80

N.B.
1) Q.1 is compulsory.
2) Solve any 3 questions out of remaining 5 questions.
3) Assumptions made should be clearly stated.
4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

a) Prove using Mathematical Induction that $2 + 5 + 8 + \ldots\ldots\ldots +(3n-1) = n(3n+1)/2$ **5**

b) Explain the term poset. Consider a set D_{165}. Find the elements of this set & draw the hasse diagram for this poset.

    5

c) How many strings of length 7 either begin with 2 zeros or end with 3 ones?   **5**

d) Explain the term partition set with suitable example.   **5**

e) State the Pigeonhole principle and show that If there are 10 marbles in the jar & you have a jar filled with red, green, and blue marbles, you'll always have at least two marbles of the same colour.

    5

Q.2
a)

    Let A={0, 1 ,2, 3, 4, 5}   **10**
    i) Explain the term group.
    ii) Prepare the composition table for the above set w.r.t. the operation of addition modulo 6.
    iii) Determine whether it is a group.
    iv)Whether elements of set A are invertible? If yes, then find the inverses of these elements.
    v) Determine whether it is a cyclic group.

b

    Let A = {a_1, a_2, a_3, a_4, a_5} and let R be a relation on A whose matrix is :   **10**

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

    Find out transitive closure of R using Warshall's algorithm.

Q.3
a) A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.

38552   Page 1 of 3

Determine the number of computer programmers that are not proficient in any of these three languages.
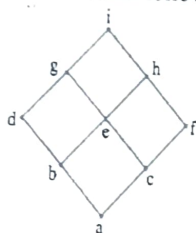
b) Explain the terms Conjunctive & Disjunctive Normal Form with suitable examples.

c)

Determine the sequence b_n whose recurrence relation is $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$.

Q.4

a) What is a lattice? Determine whether following hasse diagram represents a lattice.   6

b)   6

Consider (3, 8) an encoding function $e : B^3 \rightarrow B^8$ defined as

$e\,(000) = 00000000$
$e\,(001) = 10111000$
$e\,(010) = 00101101$
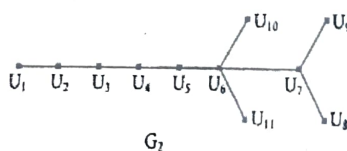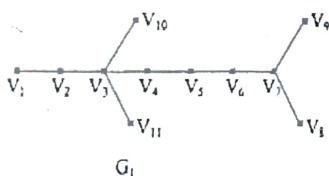$e\,(011) = 10010101$
$e\,(100) = 10100100$
$e\,(101) = 10001001$
$e\,(110) = 00011100$
$e\,(111) = 00110001$

How many errors can 'e' detect & correct?

c) What are the necessary conditions for the isomorphism between 2 graphs? Determine   8
whether following 2 graphs are isomorphic.

Q.5

a) If the addition & multiplication modulo 10 is defined on a set of integers
A={0, 2, 4, 6, 8}. Then determine whether this algebraic system is a ring.   8

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(AI & DS)

Sem III

A function $f : R \to R$ is defined by $f(x) = x^2$
   i) injective   ii) surjective   iii) bijective   6
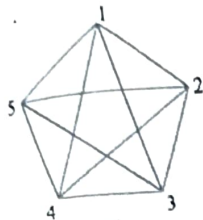Is it

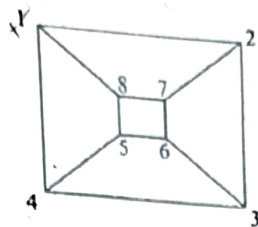Define the terms Euler path & a circuit .
Determine whether following graphs have Euler path or a circuit.   6

a)   b)

a) Explain the following terms with suitable example (any 4)   8
   i) Hamiltonian path & circuit
   ii) Bipartite graph
   iii) Adjacency matrix
   iv) Equivalence relation
   v) Cartesian product

Solve the following using the laws of logic   4
$p \lor q \lor (\sim p \land \sim q \land r) \leftrightarrow p \lor q \lor r$

          8

$f : R \to R$ is defined by $f(x) = x^3$
$g : R \to R$ is defined by $g(x) = 4x^2 + 1$
$h : R \to R$ is defined by $h(x) = 7x - 2$
Find the rule defining
   i) fog   ii) gof
   iii) (goh)of   iv) go(hof)

_____