Paper / Subject Code: 58651 / Engineering Mathematics - I

FE Sem I (C Scheme R-2019) "All Branches" July 2023

04/07/2023

(Time: 3 hours) **Max.Marks:80**

N.B
(1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Use of Statistical Tables permitted
(4) Figures to the right indicate full marks.

1 a) Solve the equation $7\cosh x + 8\sinh x = 1$, for real values of x. **5**

b) Find α, β, γ when $A = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal. **5**

c) If $u = 3(ax + by + cz)^2 - \left(x^2 + y^2 + z^2\right)$ and $a^2 + b^2 + c^2 = 1$ **5**

show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$

d) Find nth derivative of $y = \dfrac{x}{x^2 + a^2}$ **5**

2 a) *If* $\cos\alpha + 2\cos\beta + 3\cos\gamma = \sin\alpha + 2\sin\beta + 3\sin\gamma = 0$ *then prove that* $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma)$ **6**

b) If $v = \left(x^2 - y^2\right) f(xy)$, show that $\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} = \left(x^4 - y^4\right) f''(xy)$ **6**

c) If $y = e^{m\cos^{-1} x}$, then prove that **8**

$\left(1 - x^2\right) y_{n+2} - (2n+1) x\, y_{n+1} - \left(n^2 + m^2\right) y_n = 0$. Find $y_n(0)$

3 a) Prove that $\sinh^{-1}(\tan x) = \log\left[\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)\right]$ **6**

b) Verify Euler's theorem for $u = \left(\dfrac{x^2 + y^2}{x + y}\right)$ **6**

c) Examine the consistency of the system of equations **8**
$2x - y - z = 2, \quad x + 2y + z = 2, \quad 4x - 7y - 5z = 2$ and solve then if found consistent.

4 a) Find the real values of λ for which the system has non-zero solutions. 6

$$x+2y+3z=\lambda x, \quad 3x+y+2z=\lambda y, \quad 2x+3y+z=\lambda z$$

b) Find the product of all the values of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{3/4}$ 6

c) If $u=\sin^{-1}\left[\left(x^2+y^2\right)^{1/5}\right]$ then show that 8

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left(2\tan^2 u-3\right)$$

5 a) Using De Moivre's theorem, express $\frac{\sin 7\theta}{\sin\theta}$ in powers of $\sin\theta$ 6

b) If $xyz=8$ find the values of x,y,z for which $u=\frac{5xyz}{x+2y+4z}$ is maximum. 6

c) Considering only principle value, if $(1+i\tan\alpha)^{(1+i\tan\beta)}$ is real prove that its value is $\sec\alpha^{\sec^2\beta}$ 8

6 a) Reduce to normal form and find its rank $A=\begin{bmatrix}3 & 2 & 5 & 7 & 12\\ 1 & 1 & 2 & 3 & 5\\ 3 & 3 & 6 & 9 & 15\end{bmatrix}$ 6

b) Find the extreme value of $u=x^3+xy^2+21x-2y^2-12x^2$ 6

c) Show that $\tan^{-1}\left(\frac{x+iy}{x-iy}\right)=\frac{\pi}{4}+\frac{i}{2}\log\left(\frac{x+y}{x-y}\right)$ 8
