

University of Mumbai  
Examination Summer 2022

Program: Computer Engineering

Curriculum Scheme: Rev2019

Examination: SE Semester III

07/06/2022

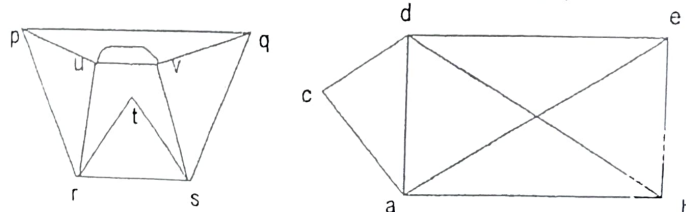
Course Code: CSC302 and Course Name: Discrete Structures & Graph Theory

Time: 2 hours 30 minutes

Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks
1.	Let the set A is {1, 2, 3} and B is {2, 3, 4}. Then the set $A - B$ is
Option A:	{1, -4}
Option B:	{1, 2, 3}
Option C:	{1}
Option D:	{2, 3}
2.	Let R be a relation on the set A of positive integers. Determine the property of relation R, if $(x, y) \in R$ where $R = \{(x, y) \mid xy \geq 1\}$
Option A:	Anti symmetric
Option B:	Transitive
Option C:	Symmetric
Option D:	Equivalence relation
3.	The statement $(\sim Q \leftrightarrow R) \wedge \sim R$ is true when?
Option A:	Q: True R: False
Option B:	Q: True R: True
Option C:	Q: False R: True
Option D:	Q: False R: False
4.	How many two-digit numbers can be made from the digits 1 to 9 if repetition is allowed?
Option A:	9
Option B:	18
Option C:	81
Option D:	99
5.	Let P (x) denote the statement " $x > 5$ ." Which of these have truth value true?
Option A:	P (0)
Option B:	P (1)
Option C:	P (2)
Option D:	P (9)
6.	How many binary relations are there on a set S with 5 distinct elements?
Option A:	$2^5$
Option B:	$2^{25}$

Option C:	$2^{10}$
Option D:	$2^{15}$
7.	The inverse of function $f(x) = x^3 + 2$ is _____
Option A:	$f^{-1}(y) = (y - 2)^{1/2}$
Option B:	$f^{-1}(y) = (y)^{1/3}$
Option C:	$f^{-1}(y) = (y - 2)^{1/3}$
Option D:	$f^{-1}(y) = (y - 2)$
8.	When is a graph said to be bipartite?
Option A:	If it can be divided into two independent sets A and B such that each edge connects a vertex from A to B
Option B:	If the graph is disconnected
Option C:	If the graph has at least $n/2$ vertices whose degree is greater than $n/2$
Option D:	If the graph is connected and it has odd number of vertices
9.	An algebraic structure _____ is called a semigroup.
Option A:	$(Q, +, *)$
Option B:	$(P, *)$
Option C:	$(P, *, -)$
Option D:	$(+, *)$
10.	Condition for monoid is _____
Option A:	$(a+e)=a$
Option B:	$(a*e)=(a+e)$
Option C:	$a=(a*(a+e))$
Option D:	$(a*e)=(e*a)=a$

<b>Q2</b> <b>(20</b> <b>Marks</b> <b>Each)</b>	
<b>A</b>	<b>Solve any Two</b> <span style="float: right;"><b>5 marks each</b></span>
i.	Prove that $8^n - 3^n$ is a multiple of 5 by mathematical induction, $n \geq 1$
ii.	What is a distributed lattice? Draw the hasse diagram of $D_{1001}$ . Whether it is a distributive lattice? Find the inverses of all elements of $D_{1001}$ .
iii.	Determine the Eulerian and Hamiltonian path, if exists, in the following graphs:
	
<b>B</b>	<b>Solve any One</b> <span style="float: right;"><b>10 marks each</b></span>

i.	What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where $A = \{a, b, c, d, e, f\}$ & $R = \{(a, b), (b, c), (c, e), (e, f), (e, b)\}$
ii.	Let $f(x) = x + 2$ , $g(x) = x - 2$ and $h(x) = 3x$ for all $x \in R$ . (R is the set of real number). Find i) $f \circ g \circ h$ ii) $h \circ g \circ f$ iii) $f \circ f \circ f$

<b>Q3</b> <b>(20 Marks Each)</b>	
<b>A</b>	<b>Solve any Two</b> <span style="float: right;"><b>5 marks each</b></span>
i.	Let R be the following equivalence relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ : $R = \{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)\}$ Find the partitions of A induced by R, i.e., find the equivalence classes of R.
ii.	Find truth table for the following expression & determine whether it is a tautology: $(\sim P \wedge (Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \leftrightarrow R$
iii.	In an auditorium, the chairs are to be numbered with an alphabet followed by a positive integer not exceeding 60. Find the maximum no. of chairs that can be placed in the auditorium.
<b>B</b>	<b>Solve any One</b> <span style="float: right;"><b>10 marks each</b></span>
i.	Let $(x_1 \wedge x_2) \vee (x_1 \wedge x_3) \vee (x_2 \wedge x_3)$ be the Boolean expression. Write E ( $x_1, x_2, x_3$ ) in a Disjunctive & Conjunctive Normal Form.
ii.	Define minimum hamming distance. Find the code words generated by the parity check matrix H given below. $H = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<b>Q4</b> <b>(20 Marks Each)</b>	
<b>A</b>	<b>Solve any Two</b> <span style="float: right;"><b>5 marks each</b></span>
i.	If 5 points are taken in a square of side 2 units, show that at least 2 of them are no more than $\sqrt{2}$ units apart.
ii.	Consider (3,8) encoding function $e : B^3 \rightarrow B^8$ defined by $e(000) = 00000000$ $e(100) = 10100100$ $e(001) = 10111000$ $e(101) = 10001001$ $e(010) = 00101101$ $e(110) = 00011100$

	$e(011) = 10010101$ $e(111) = 00110001$
	and let $d$ be the $(8,3)$ maximum likelihood decoding function associated with $e$ . How many errors can $(e, d)$ correct?
iii.	Find the generating functions for the following sequences: a. $0, 0, 0, 1, 2, 3, 4, 5, 6, 7, \dots$ b. $6, -6, 6, -6, 6, -6, \dots$
B	<b>Solve any One</b>
i.	Define the term bijective function. Show that the mapping $f: \mathbb{R} \rightarrow \mathbb{R}$ given by i) $f(x) = 4x-3$ & ii) $f(x) = 4x+7$ is bijective.
ii.	Explain the following terms with suitable example: a) Incidence matrix b) Hamiltonian path c) Partition set d) Principle of inclusion & exclusion e) Commutative ring

10 marks each