FE SemI (R-2019 C Scheme) "All Branches" Dec'2023

14/12/2023

(Time: 3 hours) Max.Marks:80

N.B (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Figures to the right indicate full marks.

1 (a) If $\tan(\alpha + i\beta) = x + iy$ then show that $\tanh 2\beta = \dfrac{2y}{1+x^2+y^2}$ 5

(b) If $z=\sin^{-1}(x-y)$, $x=3t$, $y=4t^3$ prove that $\dfrac{dz}{dt}=\dfrac{3}{\sqrt{1-t^2}}$ 5

(c) If $y=x^2\sin x$, prove that 5

$$y_n=(x^2-n^2+n)\sin\left(x+\frac{n\pi}{2}\right)-2nx\cos\left(x+\frac{n\pi}{2}\right)$$

(d) Find the real root of the equation $x^3-2x-5=0$ by Newton-Raphson method, correct to three places of decimals. 5

2 (a) Find k such that the following system of equations has 6
(1) Unique solution (2) many solutions (3) no solution.

$$kx+y+z=1, x+ky+z=1, x+y+kz=1$$

(b) Solve $x^6+1=0$ using De Moivre's theorem. 6

(c) Solve by Gauss-Seidel method with an accuracy of 0.0001 8

$$5x+y-z=10; \quad 2x+4y+z=14; \quad x+y+8z=20$$

(5 iterations only)

3 (a) Solve the equations $x_1+x_2-x_3+x_4=0$ 6

$$x_1-x_2+2x_3-x_4=0 \quad 3x_1+x_2+x_4=0$$

(b) Prove that $\left[\dfrac{1+\sin\left(\frac{\pi}{8}\right)+i\cos\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)-i\cos\left(\frac{\pi}{8}\right)}\right]^8=-1$ 6

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(c) If $u=\frac{x^2y^2}{x^2+y^2}+\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ using Euler's theorem. 8

4 (a) Prove that $A-A^{\theta}$ is skew Hermitian where 6

$$A=\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$$

(b) Find the extreme values of $f(x,y)=xy(3-x-y)$ 6

(c) Show that $\frac{\sin 6\theta}{\sin 2\theta}=16\cos^4\theta-16\cos^2\theta+3$ 8

5 (a) Expand in powers of x using Maclaurin's series and find the values of a, b, c where $\log\sec x=ax^2+b\frac{x^4}{4}+c.\frac{x^6}{6}+\cdots\cdots$ 6

(b) If u=f $(e^{x-y},e^{y-z},e^{z-x})$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ 6

(c) If $\tan(\theta+i\varphi)=\tan\alpha+i\sec\alpha$,then show that 8

$$e^{2\varphi}=\cot\frac{\alpha}{2}, 2\theta=n\pi+\frac{\pi}{2}+\alpha.$$

6 (a) Find non-singular matrices P and Q such that PAQ is in the normal form of A. Hence find rank of A where $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$ 6

(b) Show that $\log(-\log i)=\log\frac{\pi}{2}-i\frac{\pi}{2}$ 6

(c) If $y=\frac{x}{x^2+a^2}$ prove that $y_n=\frac{(-1)^n n!\sin^{(n+1)}\theta}{a^{(n+1)}}\cos(n+1)$ where $\theta=\tan^{-1}\left(\frac{a}{x}\right)$ 8
