

- N.B. (1) Question No. 1 is compulsory.  
 (2) Answer any three questions from Q.2 to Q.6.  
 (3) Use of Statistical Tables permitted.  
 (4) Figures to the right indicate full marks

Q1 a) If  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ , then find the Eigen values of  $4A^{-1} + A^3 + I$  [5]

b) Evaluate  $\int_C |z| dz$ , where C is the left half of unit circle  $|z|=1$  from  $z=-i$  to  $z=i$ . [5]

c) Maximise  $z = x_1 + 3x_2 + 3x_3$  [5]  
 Subject to  $x_1 + 2x_2 + 3x_3 = 4$   
 $2x_1 + 3x_2 + 5x_3 = 7$ .

Find all the basic solutions to the above problem. Which of them are basic feasible, non-degenerate, infeasible basic and optimal solution.

d) Tests made on breaking strength of 10 pieces of a metal wire gave the following results  
 578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 in kgs. [5]

Test if the breaking strength of the metal wire can be assumed to be 577 kg ?

Q2 (a) Using Cauchy's residue theorem evaluate  $\int_C \frac{(z+4)^2}{z^4+5z^3+6z^2} dz$ , Where c is  $|z|=1$ . [6]

(b) Find  $Z\{f(k) * g(k)\}$  if  $f(k) = 4^k U(k)$ ,  $g(k) = 5^k U(k)$ . [6]

(c) Solve the following L.P.P by Simplex Method [8]

Maximise  $z = 3x_1 + 2x_2 + 5x_3$   
 Subject to  $x_1 + 2x_2 + x_3 \leq 430$   
 $3x_1 + 2x_3 \leq 460$   
 $x_1 + 4x_2 \leq 420$   
 $x_1, x_2, x_3 \geq 0$

Q3 a) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory? [6]

(Given that Critical value of chi-square 3 d. f and 5% L.O.S is 7.81 )

b) Obtain Taylor's and Laurent's series expansion of  $f(z) = \frac{z-1}{z^2-2z-3}$  [6]

c) Use the method of Lagrange's multipliers to solve the following N.L.P.P [8]

Optimize  $z = 6x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to  $4x_1 + 3x_2 = 16,$

$3x_1 + 5x_2 = 15$

$x_1, x_2 \geq 0$  [6]

Q4a) fit a Poisson distribution to the following data

No. of deaths	0	1	2	3	4
Frequencies	123	59	14	3	1

b) Find the inverse Z-transform of  $\frac{1}{(z-2)(z-3)}$ , if ROC is (i)  $|z| < 2$  (ii)  $2 < |z| < 3$  [6]

c) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalizable. Find the transforming matrix and [8]

the diagonal matrix.

Q5a) Using the method of Lagrange's multipliers to solve the following N.L.P.P [6]

Optimize  $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to  $x_1 + x_2 = 4,$

$x_1, x_2 \geq 0.$  [6]

b) Verify Cayley- Hamilton Theorem for the matrix  $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$  [6]

c) Solve by the dual Simplex Method [8]

Minimise  $z = 6x_1 + x_2$

Subject to  $2x_1 + x_2 \geq 3,$

$x_1 - x_2 \geq 0,$

$x_1, x_2 \geq 0$

Q6a) Find the Z-transform of  $f\{k\} = \begin{cases} b^k, & k < 0 \\ a^k, & k \geq 0 \end{cases}$  [6]

b) The income of a group of 10,000 persons were found to be normally distributed with mean Rs.520 and standard deviation Rs.60. Find the lowest income of the richest 500. [6]

c) Using Kuhn Tucker conditions, solve the following NLPP [8]

Maximise  $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to  $2x_1 + x_2 - 5 \leq 0$

$x_1, x_2 \geq 0$