Paper / Subject Code: 51121 / Engineering Mathematics-III

SE Sem III (R-2019 C scheme) "ECS" May'2023
24/5/2023

(Time : 3 Hours)

Note: **1). Question 1 is compulsory.** **[Total marks: 80**

2) Attempt any 3 questions from Question 2 to Question 6

Q1 **Attempt All questions**

		Marks
A	If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the eigen values for the matrix $A^3 + 5A + 8I + A^{-1}$	5
B	Find Laplace transform of $f(t) = te^{-t}\sin(4t)$	5
C	Find the Fourier Series Expansion $f(x) = x$,where $x \in (-\pi, \pi)$	5
D	Determine the constant a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic.	5

Q2

A	Using Green's theorem in a plane to evaluate the line integral $\oint_C (xy^2 - y)dx + (x + y^2)dy$ Where C is the triangle with vertices at (0,0), (2,0) and (2,2) and it is traversed in anticlockwise direction	6
B	Find the matrix $A_{2\times 2}$ whose eigen values are 4 and 1 and their corresponding eigen vectors are $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$	6
C	Find the analytic function $f(z) = u + iv$ such that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ when $f\left(\frac{\pi}{2}\right) = 0$	8

Q3

A	Find the direction derivative of $\phi(x, y, z) = \sin(xy) + e^{3xz}$ in the direction of the vector $v = \boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ at the point $P = \left(1, \frac{\pi}{4}, 1\right)$	6
B	Find an analytic function $f(z)$ whose real part is given $u(x, y) = x^3 - 3xy^2 + 2x + y$	6
C	Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} \frac{37}{60} & \frac{17}{60} & \frac{17}{60} \\ \frac{1}{5} & \frac{7}{10} & \frac{1}{5} \\ \frac{1}{12} & -\frac{1}{12} & \frac{5}{12} \end{bmatrix}$ And show that it is diagonalizable matrix and find its transforming matrix and the diagonal form	8

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Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ 6
Where $\bar{F} = (x - y - z)i + (y - z - x)j + (z - x - y)k$ over the paraboloid $x^2 + y^2 = 4 - z, z \geq 0$

B Find the orthogonal trajectories of family of curves given by $x^3y - xy^3 = c$ 6

C Using Convolution theorem, find the inverse Laplace transform of 8

$$\phi(s) = \frac{s+1}{(s^2+2s+2)(s^2+2s+5)}$$

Q5

A Evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt$, using Laplace transforms 6

B Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by 6

$$\bar{F}(x, y, z) = y\,\hat{\imath} + (z\cos(yz) + x)\,\hat{\jmath} + (y\cos(yz))\,\hat{k}$$

Show that $\bar{F}$ is conservative and find its scalar potential.

C Find the Fourier Series for $f(x)$ in $(0,2\pi)$where 8

$$f(x) = \begin{cases} x & ,0 < x \leq \pi \\ 2\pi - x & ,\pi \leq x < 2\pi \end{cases}$$

Hence deduce that

$$\sum_{n \in Odd\ natural\ numbers} \frac{1}{n^4} = \frac{\pi^4}{96}$$

Q6

A Obtain half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$, 6
Hence show that

$$\frac{\pi^3}{32} = 1 - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots$$

B Using Cayley Hamilton theorem find 6

$$A^6 - 12A^5 + 30A^4 + 72A^3 - 207A^2 - 110A + 330I$$

Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

C

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{s^2+a^2}{s^2}}\right)\right\}$ 4

ii) Find $L^{-1}\left\{\frac{s-1}{(2s+1)^2}\right\}$ 4