SE   Sem III   (R-2019)   " C Scheme"   AI. & DS

21/11/2023

**(Time: 3 Hours )**                           **Max. Marks: 80**

N.B.  (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of   $\frac{\cos 2t \sin t}{e^t}$                           [ 5 ]

(b) Find k such that $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$   is analytic           [ 5 ]

(c) Calculate the Spearman's rank correlation coefficient R                          [ 5 ]
X   :  10 ,  12 ,  18 ,  18 ,  15 ,  40 .
Y   :  12 ,  18 ,  25 ,  25 ,  50 ,  25 .

(d) Find the inverse Laplace transform of   $\log \left( \frac{s^2 + a^2}{s^2 + b^2} \right)$ .                    [ 5 ]

Q2.

(a) A continuous random variable has probability density function
$$f(x) = k(x - x^2), \qquad 0 \leq x \leq 1.$$
$$f(x) = 0 \qquad\qquad\qquad \text{otherwise}$$
Find k , mean and variance.                           [ 6 ]

(b) Find the Laplace transform of   $e^{-3t} \int_0^t u \sin 3u \, du.$                    [ 6 ]

(c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$
Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots \dots$                    [ 8 ]

Q3.

(a) If the imaginary part of the analytic function w $= u + i v = f(z)$ is
$V = x^2 - y^2 + \frac{x}{x^2 + y^2}$ , then show that $u = -2xy + \frac{y}{x^2 + y^2}$ .           [ 6 ]

(b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$                    [ 6 ]

(c) Fit a second-degree parabolic curve  and estimate y when x = 10
X         :  1 ,   2 ,   3 ,   4 ,    5 ,    6 ,    7 ,    8 ,  9 ,
y         :  2 ,   6 ,   7 ,   8 ,   10 ,   11 ,   11 ,   10 ,  9 .                    [ 8 ]

Q4.

(a)  Obtain the  Fourier series to represent $f(x) = x^3$ in $(-\pi, \pi)$.           [ 6 ]

(b)  Find ( i ) the  equation of  the lines of Regression  ( ii ) coefficient of correlation for
the following data
X:     65 ,   66 ,   67 ,   67 ,   68 ,   69 , 70 , 72 .
Y:     67 ,    68 ,   65 ,   66 ,   72 ,   72 , 69 , 71 .                    [ 6 ]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t} \, dt = \frac{\pi}{8}.$                    [ 8 ]

37856                           **Page 1 of 2**

Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$.  [6]

(b) Find the moment generating function of the distribution

| X | : | -2 | 3 | 1 |
|---|---|----|---|---|
| P X = x ) | : | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{6}$ |

hence find first four central moments .  [6]

(c) Obtain the half range cosine series of $f(x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4}$ .......  [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2^2)^2}\right]$  [6]

(b) The probability density function of a random variable X is

| X | : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|----|----|-----|--------|--------|--------|
| P ( X =x ) : | | k | 2k | 3k | $k^2$ | $k^2 + k$ | $2 k^2$ | $4k^2$ |

Find k , $p(X < 5)$, $P(X > 5)$  [6]

.(c) If  $v = 3x^2 y + 6xy - y^3$ ,show that v is harmonic function

And find the corresponding analytic function .  [8]

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