SE
Sem IY (C Scheme) R-2019   " Mechanical" Dec'22

8/12/2022

(3 Hours)   [Total Marks: 80]

N.B. 1) Question No. **1** is Compulsory.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) What is the value of $\int_{0}^{1+i} \left(x - y + ix^2 \right) dz$ along the line from $z = 0$ to $z = 1 + i$ (5)

(b) Find a and b such that $\vec{F} = (axy + z^3)i + x^2j + bz^2xk$ is irrotational   (5)

(c) A random variable X has probability mass function p(x) =kx^3 ; x=1,2,3,4 then   (5)
find the value of k , mean, variance.

(d) Find the probability that at most 4 defective bulbs will be found in a box of 200   (5)
bulbs if it is known that 2% of the bulbs are defective.

Q.2 (a) Find the rank correlation coefficient between X and Y ;   (6)

X	17	13	15	16	6	11	14	9	7	12
Y	36	46	35	24	12	18	27	22	2	8

(b) A random variable has the MGF $M_X(t) = \frac{3}{3-t}$. Find mean and   (6)
Variance of X.

(c) Obtain Laurent's series expansions of f(x) = $\frac{z-1}{z^2-2z-3}$;$|z| > 3$.   (8)

Q.3 (a) A coin is tossed. If it turns up heads two balls are drawn from urn A otherwise   (6)
two ballsare drawn from urn B. Urn A contains 3 black and 5 white balls. Urn B
contains 7 blackand one white ball. What is the probability that urn A was used,
given that both ballsdrawn are black?

(b) Fit a straight line $y = a + bx$ into the given data.   (6)

x:	10	20	30	40	50
y:	22	23	27	28	30

(c) Prove that$\bar{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ is   (8)
irrotational. Find scalar potential of F . Hence find the work done of moving
particle from $(1,0,2)$ to $(0,1,1)$.

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(a) Using Green's Theorem evaluate $\int_c (xy + y^2)dx + x^2 dy$ and c is closed curve **(6)**
of the region bounded by $y = x$ and $y = x^2$.

(b) A machinist is expected to make engine parts with axle diameter of 1.75 cm. A **(6)**
random sample of 10 parts shows a mean diameter of 1.85 cm, with a S.D of
0.1 cm. Based on this sample, would you say that the work of the machinist is
inferior?

(c) A random variable X follows a normal distribution with mean 14 and standard **(8)**
deviation 2.5 find (1)P[X<8]   (2) P[X>18]   (3) P[12<X<15] Given:
Area between z=0 and z=2.4 is 0.4918 ; Area between z=0 and z=1.6 is
0.4452 ; Area between z=0 and z=0.8 is 0.2882 ; Area between z=0 and z=0.4
is 0.1554.

Q.5 (a) The standard deviation from two random samples of sizes 9 and 13 are 1.99 **(6)**
and 1.9. Can the samples be regard as drawn from normal population with
same standard deviation?$(F_{(8,12)}(0.025) = 3.51, F_{(12,8)}(0.025) = 4.20)$

(b) Use Gauss's Divergence Theorem to evaluate $\iint_S \overline{N}.\overline{F}\ ds$, where **(6)**
$\overline{F} = 4xi - 2y^2 j + z^2 k$ and S is region bounded by $x^2 + y^2 = 4, z = 0, z = $
4.

(c) Obtain both Line of regressions for the data given below **(8)**
Given $\sum X = 250$; $\sum Y = 300$; $\sum XY = 7900$;$\sum X^2 = 6500$; $\sum Y^2 = 10000$ and
n =10 (in usual notation)

Q.6 (a) Evaluate Value of$\int_c \frac{sin2z\ dz}{(z + \pi/3)^4}dz$is where C:$|z| = 2$ **(6)**

(b) The following data find the correlation coefficient to marks obtained by 11 **(6)**
students in 2 tests, one held at the beginning of the year and the other at the end
of the year after intensive coaching:

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

(c) A die was thrown 132 times and the following frequencies were observed. **(8)**

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased at 5% level of significance.

(Given: Table value of χ^2 at 5% level of significance and 5 degree of freedom
is 11.07)

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