SE Sem III "Computer" (R-2019) C Scheme Nov. 2023

21/11/2023

(Time: 3 Hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks

Q1.

(a) Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ [5]

(b) Find k such that $f(z) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{kx}{y}$ is analytic [5]

(c) Calculate the Spearman's rank correlation coefficient R [5]

X : 10, 12, 18, 18, 15, 40.

Y : 12, 18, 25, 25, 50, 25.

(d) Find the inverse Laplace transform of $\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$. [5]

Q2.

(a) A continuous random variable has probability density function

$f(x) = k(x - x^2)$, $0 \le x \le 1$.
$f(x) = 0$ otherwise

Find k, mean and variance. [6]

(b) Find the Laplace transform of $e^{-3t}\int_0^t u \sin 3u\, du$. [6]

(c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots\ldots$ [8]

Q3.

(a) If the imaginary part of the analytic function $w = u + iv = f(z)$ is $V = x^2 - y^2 + \frac{x}{x^2+y^2}$, then show that $u = -2xy + \frac{y}{x^2+y^2}$. [6]

(b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$ [6]

(c) Fit a second-degree parabolic curve and estimate y when x = 10

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

[8]

Q4.

(a) Obtain the Fourier series to represent $f(x) = x^3$ in $(-\pi, \pi)$. [6]

(b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for the following data

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	66	72	72	69	71

[6]

(c) Prove that $\int_0^\infty e^{-\sqrt{2}t}\frac{\sin t \sin ht}{t}\, dt = \frac{\pi}{8}$. [8]

D13AF8121463317BCID1013329BDF368

Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$. [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P X = x)	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments. [6]

(c) Obtain the half range cosine series of $f(x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} \ldots\ldots$ [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2^2)^2}\right]$ [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P (X = x)	:	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

Find k , p (X < 5), P (X > 5) [6]

.(c) If $v = 3x^2y + 6xy - y^3$,show that v is harmonic function

And find the corresponding analytic function. [8]

D13AF8121463317BC1D1013329BDF368