SE Sem III (C Scheme) R-2019 (DSE) Jan 2023
"AI & DS"

5/1/2023

Max. Marks : 80
(Time: 03 hours)

N.B. : (1) Question No.1 is **compulsory**
(2) Attempt **any three**questions from Q.2 to Q.6
(3) Figures to the right indicate full marks

Q.1 (a) Find the Laplace Transform of $e^{2t} + 4t^3 - sin2tcos3t$ 05

(b) Find the Fourier series of $f(x) = x, \quad -\pi < x < \pi$ 05

(c) Calculate Spearman's coefficient of rank correlation from the following data 05

X:	12	17	22	27	32
Y:	113	119	117	115	121

(d) Find the constants a, b, c, d, e
if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic 05

Q.2 (a) Determine whether the function $f(z) = \frac{1}{2}log(x^2 + y^2) + itan^{-1}\frac{y}{x}$ is analytic and if so, find its derivative. 06

(b) A random variable X has the following probability distribution 06

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

Find (i) k, (ii)$P(X < 4)$ (iii)$P(3 < X \leq 6)$

(c) Evaluate $\int_0^\infty e^{-2t}\, t\, costdt$ 08

Q.3 (a) Find the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}, \quad -\pi < x < \pi$ 06

(b) A continuous random variable has probability density function 06
$f(x) = k(x - x^2); \quad 0 \leq x \leq 1$
Find (i) k, (ii) mean, (iii) variance

(c) Find the inverse Laplace transform of $\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$ 08

Q.4(a) Find the Laplace Transform of $f(t)$, 06
where $f(t) = cost, \ for\ 0 < t < \pi \ and\ f(t) = sint, \ for\ t > \pi$

(b) Calculate the Karl Pearson's coefficient of correlation from the following data 06

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(c) Find the Fourier series of $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$ 08

Q.5 (a) Find the inverse Laplace transform of $\frac{s}{(2s+1)^2}$ 06

(b) Find the Laplace transform of $t\left(\frac{sint}{e^t}\right)^2$ 06

(c) Find the lines of regression for the following data 08

X:	78	36	98	25	75	82	90	62	65	39
Y:	84	51	91	60	68	62	86	58	53	47

Q.6 (a) Find the mean and the variance of the following distribution 06

X	1	3	4	5
P(X=x)	0.4	0.1	0.2	0.3

(b) Find the inverse Laplace transform of $log\left(1+\frac{a^2}{s^2}\right)$ 06

(c) Find the analytic function f(z) = u +iv whose imaginary part is 08

$v = x^2 - y^2 + \frac{x}{x^2+y^2}$
