FE Sem I (R-2019 'C' Scheme) "All Branches" Feb 2023

20/12/2022

(Time: 3 hours) Max.Marks:80

N.B (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Use of Statistical Tables permitted
(4) Figures to the right indicate full marks.

1 a) Prove that $\operatorname{sech}^{-1}(\sin\theta) = \log\left(\cot\frac{\theta}{2}\right)$ 5

b) If $z = x^y + y^x$ then prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ 5

c) If α, β are the roots of the quadratic equation $x^2 - 2\sqrt{3}\,x + 4 = 0$, find the value of $\alpha^3 + \beta^3$ 5

d) Test the consistency and if possible solve 5
$2x - 3y + 7z = 5,\ 3x + y - 3z = 13,\ 2x + 19y - 47z = 32$

2 a) Is $A = \begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ a unitary matrix? 6

b) Find the nth derivative of $y = \frac{4x}{(x-1)^2(x+1)}$ 6

c) If $u = \frac{x^4 + y^4}{x^2 y^2}$ then find the value of 8
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ at $x = 1$ and $y = 2$

3 a) Prove that $\log(1 + \cos 2\theta + i\sin 2\theta) = \log(2\cos\theta) + i\theta$ 6

b) Solve $x^7 + x^4 + i(x^3 + 1) = 0$ using De Moivre's theorem 6

c) Discuss for all values of k for which the system of equations has a non-trivial solution 8
$2x + 3ky + (3k+4)z = 0,$
$x + (k+4)y + (4k+2)z = 0,\ x + 2(k+1)y + (3k+4)z = 0$

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4 a) If $u = \log(r)$ and $r = x^3 + y^3 - x^2 y - xy^2$ then show that 6

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$

b) Find two non-singular matrices P and Q such that PAQ is in 6

the normal form where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$

c) Prove that $\tan^{-1}(e^{i\theta}) = \frac{n\pi}{2} + \frac{\pi}{4} - \frac{i}{2}\log\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ 8

5 a) Considering principal value, express in the form $a + ib$ the 6

quantity $(\sqrt{i})^{\sqrt{i}}$

b) Prove that $\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ 6

c) If $y = e^{a\sin^{-1}x}$, then Prove that 8

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ Also find $y_n(0)$

6 a) If $u = \frac{1}{r}$, $r = \sqrt{x^2 + y^2 + z^2}$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ 6

b) If $\frac{3}{x} + \frac{4}{y} + \frac{5}{z} = 6$ find the values of x, y, z such that $x + y + z$ is minimum 6

c) Prove that every Skew-Hermitian matrix can be expressed in the form B+iC, where B is real Skew-Symmetric and C is real Symmetric matrix and express the matrix 8

$A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as B+iC where B is real Skew-symmetric matrix and C is real Symmetric matrix

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