SE Sem III (OSE) "R-2019, C Scheme" Computer Jan' 2023

27/01/2023

Time: 3 Hours **Marks: 80**

N.B. : (1) Question Number 1 is compulsory

(2) Solve any three questions from the remaining questions

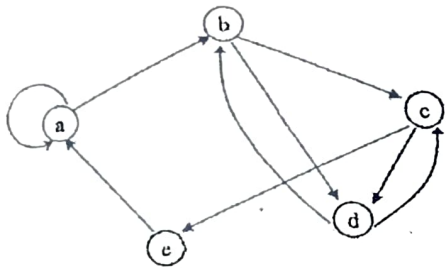
(3) Make suitable assumptions if needed

(4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 Solve any four of the following questions.

a) Prove using Mathematical Induction that $1+5+9+ \ldots\ldots + (4n-3) = n(2n-1)$ **5**

b) Find the relation set & relation matrix for the following digraph. Determine in degree & out degree of each vertex. **5**



c) State the pigeon hole principle. If 30 people are assembled in a room, then show that of them must have their birthday on the same day of a week. **5**

d) Explain the following terms with suitable example: **5**
i) Eulerian graph
ii) Quantifier

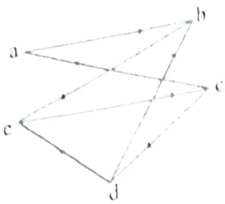
e) What is a partial order relation? Determine the hasse diagram for following relation **5**
A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (3, 4), (4, 4)}

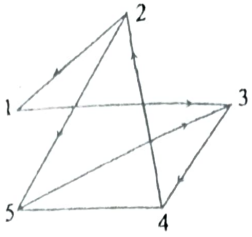
Q.2

a) What is a transitive closure? Let A={a1, a2, a3, a4, a5}. Find the transitive closure of R using Warshall's algorithm where relation matrix M_R is given as follows- **10**

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. **10**





8

Q.3

a) Among the integers 1 to 300,
i) How many of them are not divisible by 3 nor by 5 nor by 7?
ii) How many of them are divisible only by 3?

b) There are 6 Communication Skills books, 8 Engg. Mathematics books, 10 books on C Programming. How many ways can be used to choose 2 books of different categories from them? **6**

c) What is a partition set? Determine whether each of the following is a partition. Justify your answer. **6**

Let X = {1, 2, 3, ... 8, 9}. Determine whether or not each of following is a partition
(a) [{1, 3, 6, }, {2, 8}, {5, 7, 9}]
(b) [{2, 4, 5, 8}, {1, 9}, {3, 6, 7}]
(c) [{1, 5, 7}, {2, 4, 8, 9}, {3, 5, 6}]
(d) [{1, 2, 7}, {3, 5}, {4, 6, 8, 9}, {3, 5}]

10

Q.4

a) What is a group? Let A={5, 10 ,15, 20}
i) Prepare the composition table w.r.t. the operation of multiplication modulo 25.
ii) Whether it is an abelian group? Justify your answer.
iii) Find the inverses of all the elements.
iv) Whether it is a cyclic group?

b) What is a ring? Let A= {0, 1, 2, 3, 4, 5}. Determine whether a set A with addition modulo 6 & multiplication modulo 6 is a commutative ring? Justify your answer. **10**

Q.5

a) Define a lattice. Prove that in a distributive lattice the complement of any element is unique. Determine whether D_{105} is a distributive lattice. Find the complements of all elements. **8**

b) Define the term bijective function. Let f: R—> R be a function defined by f(x) = 2x-3. Determine whether it is a bijective function. **6**

c) Draw the graph G corresponding to each adjacency matrix 6

(a) $A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

Q.6

a) 8

Let A = {2, 5, 9, 13, 16}
R = {(2, 5), (2, 13), (16, 5), (16, 13), (9, 13), (5, 16)}
S = {(2, 9), (2, 13), (5, 13), (9, 16), (5, 16)}
Compute (i)S^{-1} (ii) $(R \cup S) \cap S^{-1}$ (iii) $\bar{R} \cap S$ (iv) $\bar{R}$

b) What is a planer graph? A connected planer graph has 8 vertices having degrees 2,2,2,3,3,3,4,4. How many edges are there in this graph? 6

c) Write the following statements in a symbolic form using quantifiers. Assume a suitable data wherever applicable. 6

i) All students have taken a course in mathematics.
ii) There is a girl student in a class who is also a sports person.
iii) Some students are intelligent, but not hardworking.