Detecting Properties of Quantum Channels

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1. Introduction

This project focuses on practical implementation and simulation of a novel method for detecting inherent properties of Quantum Channels (QCs), first introduced in [1]. By representing QCs as density matrices through the Choi-Jamiokovki isomorphism and subjecting them to specifically designed Hermitian operators known as Witness Operators, the method reveals specific QC properties. These properties include aspects such as **Entanglement Breaking** and **Separability** of randomly applied unitary operations. The outcomes have significance for the optimal design and performance of quantum communication and computation systems. The method's practicality is highlighted by its compatibility with present-day quantum processors, allowing us to experimentally verify its effectiveness on IBMQ's manila processor.

1.1 Quantum Channels and Their Representations

Quantum Channels are characterized by being **Completely Positive** (CP) and **Trace Preserving** (TP) maps. This implies that they maintain positivity and trace even when applied to subsystems of larger quantum systems. These maps, known as **superoperators**, capture the evolution of quantum states and operators under the influence of quantum processes.

1.1.1 Kraus Form

Quantum Channels are often represented in their Kraus form:

$$\mathscr{M}[\rho] = \sum_k A_k \rho A_k^\dagger$$

Where Kraus operators A_k, fulfill the **trace preserving** constraint: $\sum_k A_k A_k^\dagger = I$.

1.1.2 Choi Form

The Choi-Jamiołkowski isomorphism serves as a significant bridge between density matrices and Quantum Channels (QCs). It establishes a mapping that facilitates the transition between the two representations:

$$\mathscr{M} \Leftrightarrow C_{\mathscr{M}} = (\mathscr{M} \otimes I)[|\alpha\rangle\langle\alpha|]$$

In this map, $\mathscr{M}$ represents a Quantum Channel, and $C_{\mathscr{M}}$ is its corresponding Choi matrix. The Choi matrix is generated by applying the QC $\mathscr{M}$ to one part of a maximally entangled state $|\alpha\rangle$ while leaving the other part unchanged through the identity operator I. **This correspondence enables insights to be gained about the properties and behaviors of Quantum Channels through the analysis of their corresponding Choi state**.

1.2 From Quantum Tomography to Witness Operators

Quantum tomography provides a comprehensive reconstruction of QCs through extensive measurements, requiring a large number of state copies and measurement settings. **This work covers efficient methods to detect QC properties, eliminating the need for full quantum process tomography.**

2. Single Qubit Depolarising Channel

The Depolarising Channel in it's Choi representation:

$$\rho_p = (1 - \frac{4}{3}p)\,|\alpha\rangle\langle\alpha| + \frac{p}{3}I$$

The appropriate Witness Operator for detecting Entanglement Breaking (EB):

$$W_{EB} = I \otimes I + X \otimes X + Y \otimes Y + Z \otimes Z$$

Where X, Y, Z are Pauli operators.

W_{EB} designed for getting **negative values** for: $\langle W_{EB}\rangle = Tr[W_{EB}\rho_p]$ only for none Entanglement Breaking channels.

3. Separable Random Unitary

Separable Random Unitary (SRU) channel:

$$\mathscr{U}[\rho] = \sum_k p_k U_k \rho U_k^\dagger$$

This kind of maps includes several simple but interesting models of noisy QCs.
Choi representation:

$$|U\rangle = (U \otimes I)\,|\alpha\rangle$$

Witness Operators for detecting SRU Channel:

$$W_{CNOT} = \alpha_{SRU}^2 I - C_U$$

α_{SRU} is the maximum overlap with the nearest biseparable state in the set S_{SRU}:

$$\alpha_{SRU}^2 = \max_{\mathscr{M}_{SRU}} \langle U|\, C_{\mathscr{M}_{SRU}}\, |U\rangle = \max_{U_A,U_B} |\langle U_A \otimes U_B | U\rangle|^2$$

3.1 Using CNOT gate to detect none-SRU

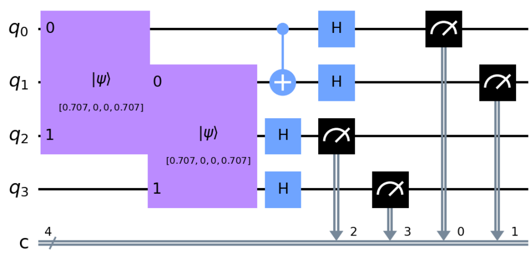
$$|U\rangle = |CNOT\rangle = \frac{1}{\sqrt{2}}[|00\rangle_{AC}\,|\alpha\rangle_{BD} + |11\rangle_{AC}\,|\psi^+\rangle_{BD}]$$

Witness Operators for detecting SRU Channel:

$$\begin{aligned} W_{CNOT} = \frac{1}{64}[31 \cdot IIII - IXIX - XXXI - XIXX \\ - ZZIZ + ZYIY + YYXZ + YZXY \\ - ZIZI - ZXZX + YXYI + YIYX \\ - IZZZ + IYZY + XYYZ + XZYY] \end{aligned}$$

Reducing the measurement count from 16 to 9 using classical processing of the set: $\{XXXX, ZZZZ, ZYZY, YXYX, YYXZ, YZXY, ZXZX, XYYZ, XZYY\}$

Example for implementation of $XXXX$:



Statistical analysis provides significant evidence that $|CNOT\rangle$ is not separable, **highlighting the method's ability to detect the entangling behavior of noise.**

4. Conclusions

- First implementation of the techniques on **scalable superconducting qubit hardware.**
- The techniques can be **adjusted** and **expanded** to assess EB or separability of quantum gates or channels, even in presence of noise [2].
- In contrast to conventional channel tomography, which requires various input states, this approach **transforms the channel itself into a state.** This eliminates the need for probing with different inputs, **simplifying** the process and **reducing resource consumption.**
- The method has a wide range of applications, such as **validating noise models** in quantum processors and **designing quantum communication systems for optimal performance**, among others.

References

[1] C. Macchiavello and M. Rossi. Quantum channel detection. *Phys. Rev. A*, 88:042335, Oct 2013.
[2] Adeline Orieux, Linda Sansoni, Mauro Persechino, Paolo Mataloni, Matteo Rossi, and Chiara Macchiavello. Experimental detection of quantum channels. *Phys. Rev. Lett.*, 111:220501, Nov 2013.