תרגיל 1

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$f(x) = \sin x$

$f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$

$f^{(6)}(x) = -\sin x$

$f^{(7)}(x) = -\cos x$

$f^{(6)}(0) = 0$

$$P_5(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$$P_6(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{0}{6!}x^6 = P_5(x)$$

$$f(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + R_6(x)$$

$$|R_6(x)| = \frac{|\cos c|}{7!}x^7 < \frac{x^7}{7!} < \frac{1}{7! \cdot 100^7} = 1.984 \cdot 10^{-18}$$

$f(x) = e^{2x}$ $\quad f(0) = 1$

$f'(x) = 2e^{2x}$ $\quad f'(0) = 2$

$f''(x) = 2^2 e^{2x}$ $\quad f''(0) = 2^2$

$f'''(x) = 2^3 e^{2x}$ $\quad f'''(0) = 2^3$

$\vdots$

$f^{(n)}(x) = 2^n e^{2x}$ $\quad f^{(n)}(0) = 2^n$

$$P_n(x) = 1 + 2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \ldots + \frac{(2x)^n}{n!}$$

$f(x) = (x+1)\ e^x$ $\qquad f(1) = 2e$ $\qquad x=1$ $\qquad .c$

$f'(x) = e^x + (x+1)e^x = (x+2)e^x$ $\qquad f'(1) = 3e$

$f''(x) = e^x + (x+2)e^x = (x+3)e^x$ $\qquad f''(1) = 4e$

$\vdots$

$f^{(n)}(x) = (x+n+1)e^x$ $\qquad f^{(n)}(1) = (n+2)e$

$$P_n(x) = 2e + 3e\cdot(x-1) + \frac{4e}{2!}(x-1)^2 + \ldots + \frac{(n+2)e}{n!}(x-1)^n$$