Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Units & Dimensions - Part 1

1. Fundamental Units
2. Supplementary Units
3. Must Know / Practice Dimensional Formulae
4. Principle of Homogeneity
5. Conversion of Units
6. Dimension in Terms of other Physical Quantity
7. KEY Points (Dimensionless Qty)

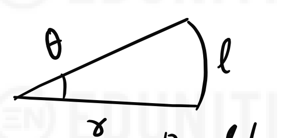
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. FUNDAMENTAL UNITS

	Quantities	SI UNIT	C.G.S	SYMBOL
1.	LENGTH	m	cm	L
2.	MASS	kg	g	M
3.	TIME	s	s	T
4.	TEMP°	K		θ
5.	CURRENT	A		A
6.	INTENSITY	cd		cd
7.	AMOUNT	mol		mol

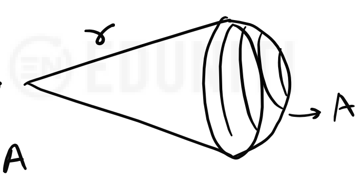
2. SUPPLEMENTARY UNITS

(i) ANGLE ⟶ Radian



$\theta = l/r$

(ii) SOLID ANGLE ⟶ Steradian



$\Omega = \frac{A}{r^2}$

3. MUST KNOW DIMENSIONAL FORMULAE

	Physical Qty	DIMENSIONAL FORMULAE
1	FORCE	MLT^{-2}
2	ENERGY	ML^2T^{-2}
3	ε_0	$M^{-1}L^{-3}T^4A^2$
4	μ_0	$MLT^{-2}A^{-2}$

Ex: Steffan boltzmann Const. (σ)

$$\frac{dQ}{dt} = \sigma e A T^4 \Rightarrow \sigma = \frac{Q}{t \times A \times T^4}$$

$$[\sigma] = \frac{ML^2T^{-2}}{T \times L^2 \times \theta^4} = MT^{-3}\theta^{-4}$$

↳ e is dimensionless.

NOTE: You may practice to find Dimension of some constants like:

(a) Gravitational const., $G = M^{-1}L^3T^{-2}$

(b) Gas const., $R = ML^2T^{-2}\theta^{-1}$

(c) Boltzmann's Const., $K_B = ML^2T^{-2}\theta^{-1}$

(d) Planck const., $h = ML^2T^{-1}$

(e) Rydberg's const., $R_y = L^{-1}$

(f) Magnetising field, $H = \frac{B}{\mu_0} = \frac{\mu_0 i/2r}{\mu_0} = \frac{i}{r} \Rightarrow AL^{-1}$

4. PRINCIPLE OF HOMOGENEITY

(i) If $a = b + c$

$\Rightarrow [a] = [b] = [c]$

Ex: $S = ut + \frac{1}{2}at^2$

$[s] = [ut] = [at^2]$

5. CONVERSION OF UNITS

$Q = nu$

amount of physical qty. ← Q; n → numeric value; u → units (can be SI, CGS or Given in Question)

Ex: Find how many POISE (C.G.S unit of viscosity) is equal to 1 POISEUILLE (in SI)

Soln: $F = 6\pi\eta r v \Rightarrow [\eta] = \frac{[F]}{[r][v]} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$

$$n_1u_1 = n_2u_2$$

$$\Rightarrow n_1 \times (g\,cm^{-1}s^{-1}) = 1 \times (kg\,m^{-1}s^{-1})$$

$$\Rightarrow n_1 = \frac{kg\,m^{-1}s^{-1}}{g\,cm^{-1}s^{-1}} = \frac{10^3 g \times 10^{-2} cm^{-1} \times s^{-1}}{g\,cm^{-1}\,s^{-1}}$$

$$= \boxed{10}$$

6. DIMENSION IN TERMS OF OTHER PHYSICAL QUANTITY

 # Ex: Expression for time in terms of G (Gravitational const.), h (Planck const.) and c (speed of light) is proportional to ?

Solⁿ: $t \propto G^a h^b c^c \Rightarrow [t] = [G]^a [h]^b [c]^c$

$$\Rightarrow [M^0L^0T^1] = [M^{-1}L^3T^{-2}]^a [ML^2T^{-1}]^b [LT^{-1}]^c$$

$$\Rightarrow [M^0L^0T^1] = [M^{-a+b} L^{3a+2b+c} T^{-2a-b-c}]$$

$\therefore 0 = -a+b, \quad 0 = 3a+2b+c, \quad 1 = -2a-b-c$

$\Rightarrow a = b = 1/2, \quad c = -5/2$

$\therefore t \propto G^{1/2} h^{1/2} c^{-5/2}$ or $\boxed{t \propto \sqrt{\dfrac{Gh}{c^5}}}$

7. KEY POINTS (dimensionless qty)

(i) All trigonometric ratios ($\sin\theta$, $\cos\theta$ etc.)

(ii) Angle, θ ($\sin\frac{ab}{c}$, here $[ab] = [c]$)

(iii) Exponential functions, $e^x \rightarrow x$ must be dimensionless

($e^{-t/RC} \rightarrow [t] = [RC]$)

(iv) Reynolds number ($R_e = \frac{\rho V D}{\eta}$), Dielectric const. (K) Refractive index and many more.

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Error Analysis - Part 2

1. Absolute Error, Relative Error & Percentage Error
2. Combination of Errors
3. Vernier caliper and Screw Gauge

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Absolute Error, Relative Error & Percentage Error

(Error = True Value – Measured Value)

STEP 1: $R_{avg} = \dfrac{R_1 + R_2 + \dots + R_n}{n}$ (we take R_{avg} as True Value)

STEP 2: Absolute Error $\Delta R_1 = R_1 - R_{avg}$

$\Delta R_2 = R_2 - R_{avg}$

$\dots \quad \dots \quad \dots$

$\Delta R_n = R_n - R_{avg}$

(take only magnitude)

STEP 3: Mean Absolute Error

$$\Delta R_{avg} = \frac{|\Delta R_1| + |\Delta R_2| + \dots + |\Delta R_n|}{n}$$

Final Reading $= R_{avg} \pm \Delta R_{avg}$

Rel Error: $\dfrac{\Delta R_{avg}}{R_{avg}}$

% Error: $\dfrac{\Delta R_{avg}}{R_{avg}} \times 100$

2. Combination of Errors

↳ error in measurement is very small

(a) Sum or difference

$l = 4.1 \pm 0.1$ cm, $b = 3.3 \pm 0.1$ cm

$s = l + b = 7.4 \pm 0.2$ cm

$s = l - b = 0.8 \pm 0.2$ cm

(b) Product or Division

$P = d\dfrac{x^a y^b}{z^c}$ { Error is Δx, Δy & Δz. Find Error in P

$\Rightarrow \ln P = a\ln x + b\ln y - c\ln z + \ln d$

$\Rightarrow \dfrac{dP}{P} = a\dfrac{dx}{x} + b\dfrac{dy}{y} - c\dfrac{dz}{z} + 0$

To find Max error in P

$$\boxed{\frac{\Delta P}{P} = a\frac{\Delta x}{x} + b\frac{\Delta y}{y} + c\frac{\Delta z}{z}}$$

($\dfrac{\Delta x}{x}$ is rel error in x

3. Vernier Caliper & Screw Gauge

↳ Must refer video for this from our youtube channel "EDUNITI".

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

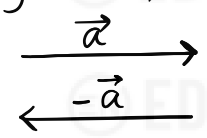
Topics to cover in Vectors - Part 3

1. Basic properties of vectors
2. Addition of vectors
3. Subtraction of vectors
4. Resolution of vectors in 2-D & 3-D
5. Dot product (scalar product)
6. Cross product (vector product)

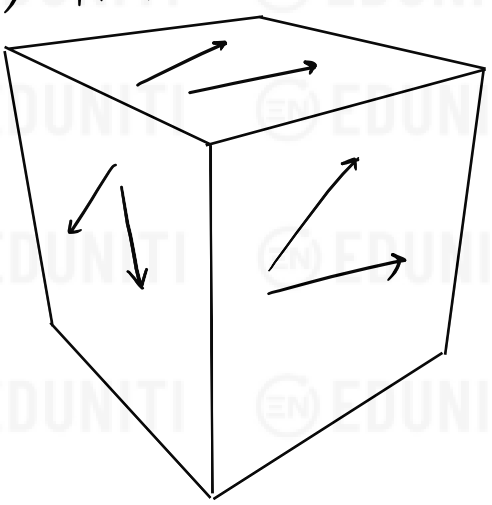
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Basic Properties of Vectors

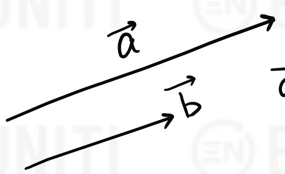
(a) Negative Vectors



(b) Coplanar Vectors



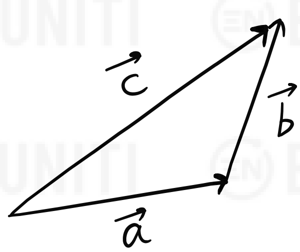
(C) Collinear Vectors



$\vec{a} = \lambda \vec{b}$

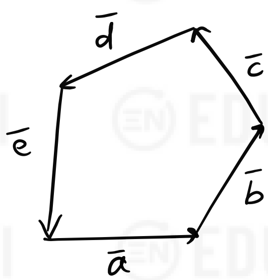
2. Addition of Vectors

Triangle Law



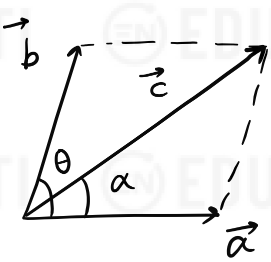
$\vec{a} + \vec{b} = \vec{c}$

Polygon Law (an extension of triangle law)



$\bar{a} + \bar{b} + \bar{c} + \bar{d} + \bar{e} = 0$

Parallelogram law



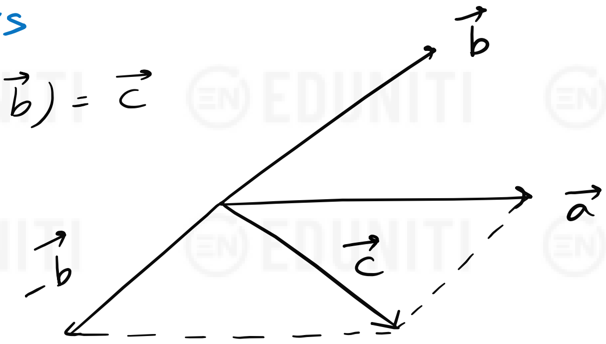
$c = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$\tan\alpha = \dfrac{b\sin\theta}{a + b\cos\theta}$

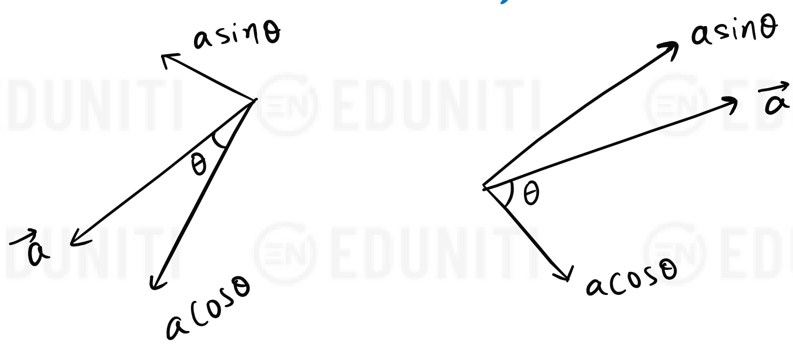
Vectors – Part 3

3. Subtraction of Vectors

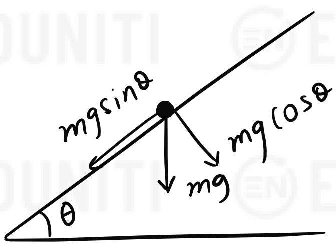
$\vec{a} - \vec{b} = c \Rightarrow \vec{a} + (-\vec{b}) = \vec{c}$



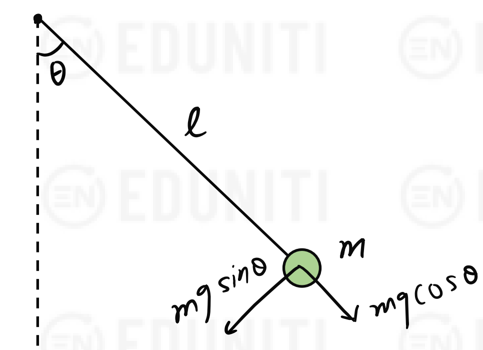
4. Resolution of Vectors (why resolve and how decide axis)



(i)

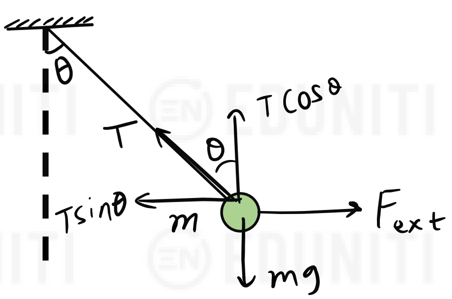


(ii)



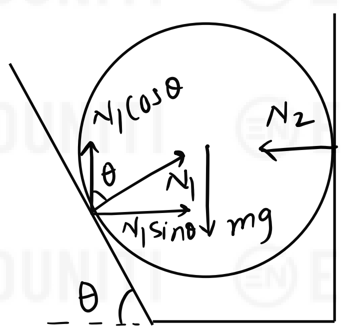
If particle is in accelerated state or tends to move.

(iii)



NOTE: These forces makes closed polygon.

(iv)

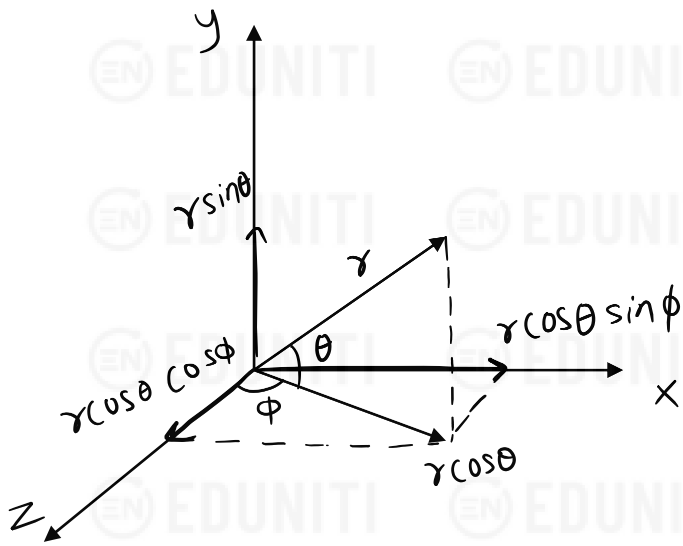


If Particle is in Equilibrium

Vectors – Part 3

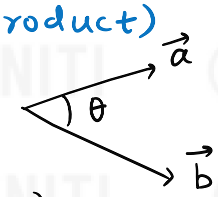
Resolution of vectors (3 dimension)

$$\vec{r} = r\cos\theta\sin\phi\,\hat{i} + r\sin\theta\,\hat{j} + r\cos\theta\cos\phi\,\hat{k}$$



5. Dot product (scalar product)

↳ $\vec{a}\cdot\vec{b} = ab\cos\theta$



↳ $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (commutative law)

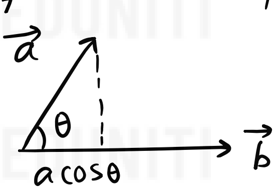
↳ $\bar{a}\cdot\bar{a} = a^2$

It's application:

(i) If $\bar{a}\cdot\bar{b} = 0 \Rightarrow$ Vectors are orthogonal (90°)

(ii) Find angle betⁿ two vectors, $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{ab}$

(iii) Finding projection of one vector on another:



$\bar{a}\cdot\bar{b} = ab\cos\theta$

$\therefore a\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{b}$

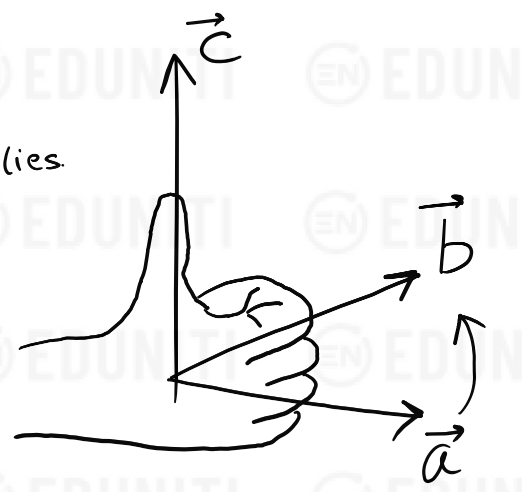
6. Cross product (Vector product)

↳ $\vec{a}\times\vec{b} = \vec{c}$, $\vec{c} = ab\sin\theta\,\hat{n}$

(i) $\vec{c}$ is $\perp$ to plane in which $\vec{a}$ & $\vec{b}$ lies.

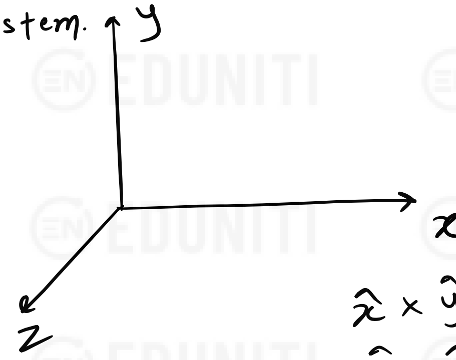
(ii) $\bar{a}\times\bar{b} \neq \bar{b}\times\bar{a}$ $(\bar{a}\times\bar{b} = -\bar{b}\times\bar{a})$

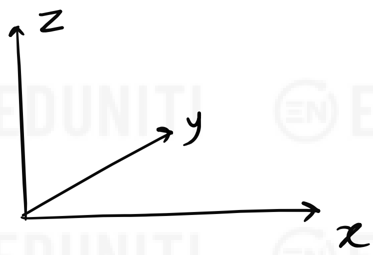
(iii) $\bar{a}\times\bar{a} = 0$



Vectors – Part 3

Co-ordinate system.





$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}$$

Cross Product (Vector Product)

Ex: $\vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$

find $\vec{c}$

$$\vec{c} = \begin{array}{ccc} + & - & + \\ \end{array}$$
$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 1 & -3 & 4 \end{vmatrix}$$

$$\vec{c} = \hat{i}(12+15) - \hat{j}(8-5) + \hat{k}(-6-3)$$
$$= 17\hat{i} - 3\hat{j} - 9\hat{k}$$

Space to add concepts learnt from PYQs if any

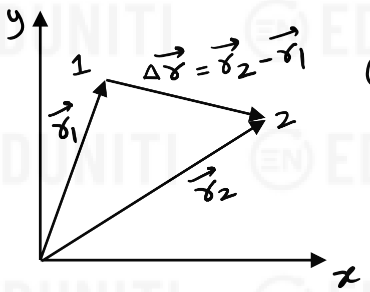
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Motion in 1D – PART 1

1. Average velocity & acceleration
2. Instantaneous velocity & acceleration (Calculus based)
3. Uniform acceleration (equations of motion)
4. Important points under motion under gravity
5. Basics of graphs
6. Types of graphs in 1D motion (x-t, v-t, a-t)

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Average Velocity & acceleration



(a) $\vec{v}_{av} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \dfrac{\Delta\vec{r}}{\Delta t}$ or If $v = f(t)$ is given $v_{av} = \dfrac{\int_{t_1}^{t_2} v\,dt}{t_2 - t_1}$

(b) $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \dfrac{\Delta\vec{v}}{\Delta t}$ or If $a = f(t)$ is given $a_{av} = \dfrac{\int_{t_1}^{t_2} a\cdot dt}{t_2 - t_1}$

2. Instantaneous v & a (calculus based)

$x = f(t)$
$v = dx/dt$
$a = \dfrac{dv}{dt}$

$v = f(t)$
$= f(x)$

$a = dv/dt$
$= v\dfrac{dv}{dx}$

$\dfrac{dx}{dt} = f(t)$
$\Rightarrow \int dx = \int f(t)\,dt$

$\dfrac{dx}{dt} = f(x)$
$\Rightarrow \int \dfrac{dx}{f(x)} = \int dt$

$a = f(t)$
$= f(v)$
$= f(x)$
$= \dfrac{dv}{dt}$ or $v\dfrac{dv}{dx}$

Ex: $v\dfrac{dv}{dx} = f(x)$
$\Rightarrow \int v\,dv = \int f(x)\,dx$

3. Uniform Acceleration (a = constant)

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 = u^2 + 2as$

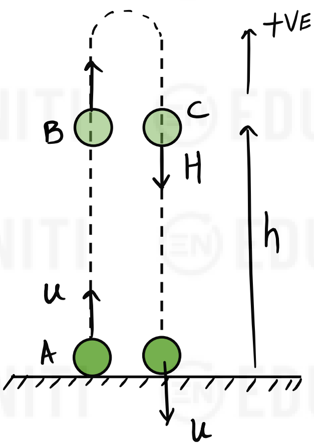
(iv) $s_n = u + \frac{a}{2}(2n-1)$

here v, u, a & s are vectors. so be careful about signs.

If $u=0$, $t = \frac{v}{a}$, $t = \sqrt{\frac{2s}{a}}$, $s = \frac{v^2}{2a}$

$\boxed{s = vt - \frac{1}{2}at^2}$

4. Imp Points in Motion under Gravity



(i) time of ascent = time of decent = $u/g = t$

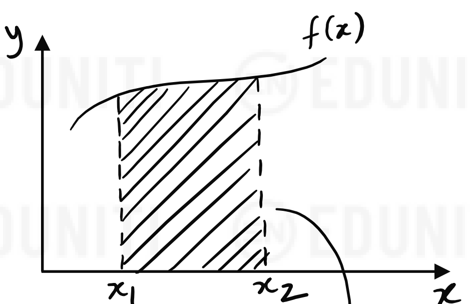
$\therefore$ Total time, $T = 2u/g$

(ii) $H = \frac{u^2}{2g}$ (iii) $t = \sqrt{\frac{2H}{g}}$

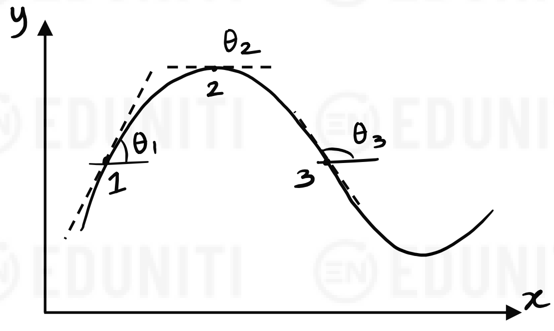
(iii) Particle is at "h" twice in its journey ($h<H$):

$h = ut - \frac{1}{2}gt^2 \longrightarrow$ solving this $\rightarrow t_{AB}$, t_{AC} $(t_{AC} > t_{AB})$

5. Graphs Basics



Area under Curve $= \int_{x_1}^{x_2} f(x).dx$

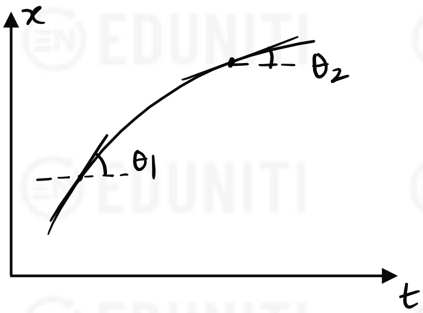


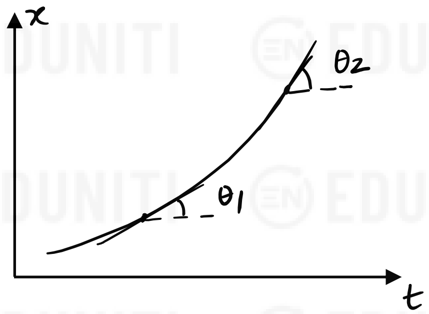
↳ slope, $\frac{dy}{dx} = \tan\theta$

1: $\tan\theta_1$ +ve 2: Zero 3: $\tan\theta$ -ve

6. Types of Graphs in 1-d Motion

(i) x vs t :





slope, $\frac{dx}{dt} = v$

↳ with t slope ↓ ⇒ v ↓

∴ Retarding Motion

↳ with t slope↑ ⇒ v↑

∴ Accelerated Motion

(ii) v vs t

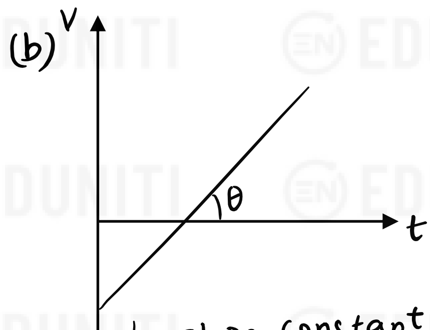
→ slope, $\frac{dv}{dt} = a$

→ area under curve $= \int v\,dt$ = Displacement

(a)

v const. ⇒ $a = 0$

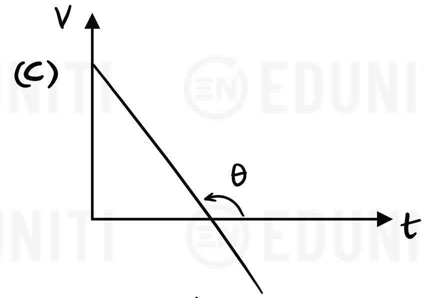
(b)



↳ slope constant and v↑

⇒ a const & +ve

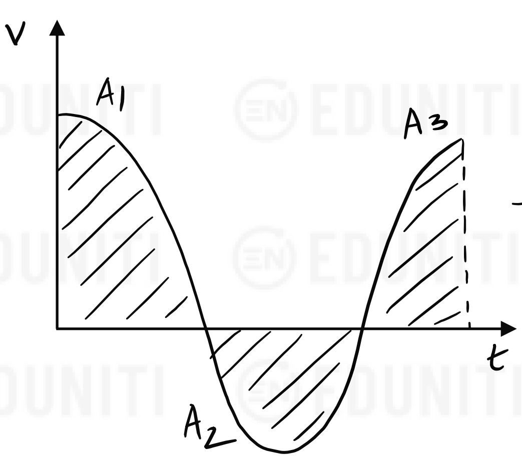
(c)



↳ slope const. and v↓

⇒ a const & −ve

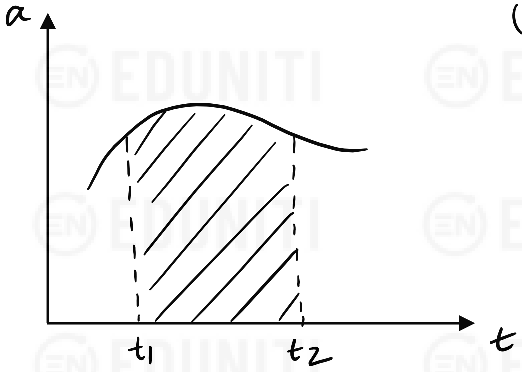
(retardation)



→ Displacement $= A_1 - A_2 + A_3$

→ Distance $= A_1 + A_2 + A_3$

(iii) a vs t



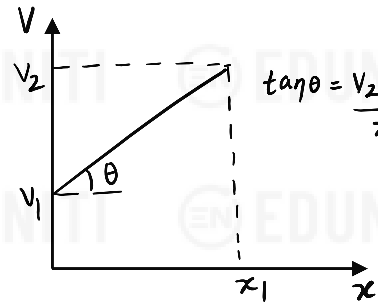
Area = ΔV = $V - u$

(if area is below t axis, take it as −ve)

(iv) V vs x

↳ slope, $\frac{dv}{dx} = \frac{a}{V}$

Ex 7.



$\tan\theta = \frac{V_2 - V_1}{x_1}$

$y = mx + c$

$\Rightarrow \boxed{V = x\tan\theta + V_1}$

↳ do $v\frac{dv}{dx}$ to get $a = f(x)$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Motion in 2D – PART 2

1. Motion in a Plane
2. Projectile Motion (Ground to Ground)
   a) Standard equation
   b) Condition for same range and max range
   c) Analysis of projectile at any time t
   d) Equation of trajectory
3. Projectile fired from a tower
4. Projectile motion on a inclined plane
5. Projectile from a moving trolley

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Motion in a Plane

↳ 2D motion is Vector Sum of two independent 1D motion (along x & y)

If $\vec{a}$ is constant :

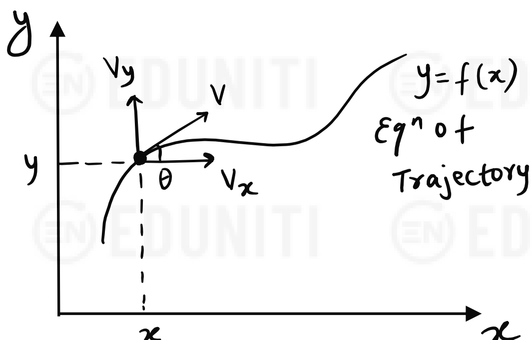
$V_x = u_x + a_x t$

$V_y = u_y + a_y t$

$x = u_x t + \frac{1}{2} a_x t^2$

$y = u_y t + \frac{1}{2} a_y t^2$

$\vec{r} = x\hat{i} + y\hat{j}$



$\vec{V} = V_x\hat{i} + V_y\hat{j}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

2. Projectile Motion (Ground to Ground)

2a. Standard eqⁿ :

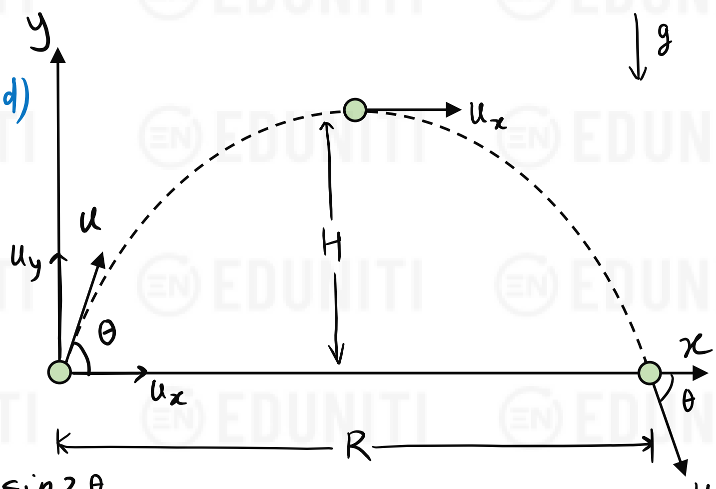
(i) Time of Flight

$$T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

(ii) $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$

(iii) Range, $R = u_x \cdot T$

$$= \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}$$



(iv) $\because a_x = 0 \Rightarrow u_x = u\cos\theta$ is const.

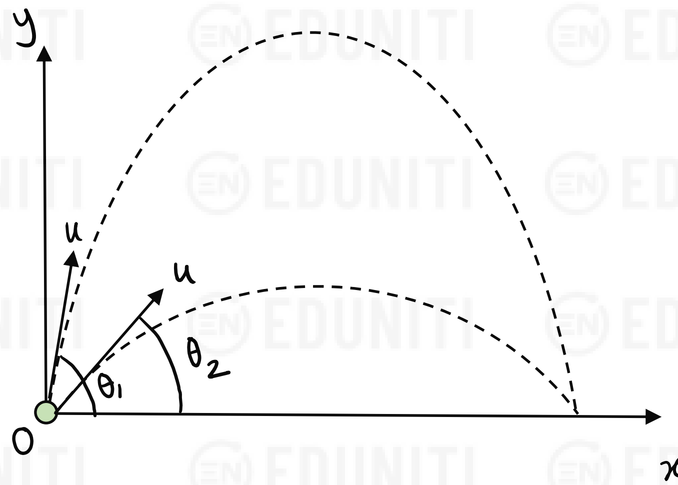
2b. Condition for R_{max} & Same Range :

(i) $R = \dfrac{u^2 \sin 2\theta}{g}$

$\therefore$ For $\theta = 45^\circ$, $R_{max} = \dfrac{u^2}{g}$

(ii) If $\theta_1 + \theta_2 = 90^\circ$

$\Rightarrow R_1 = R_2$



2c. Analysis of Projectile at any time t

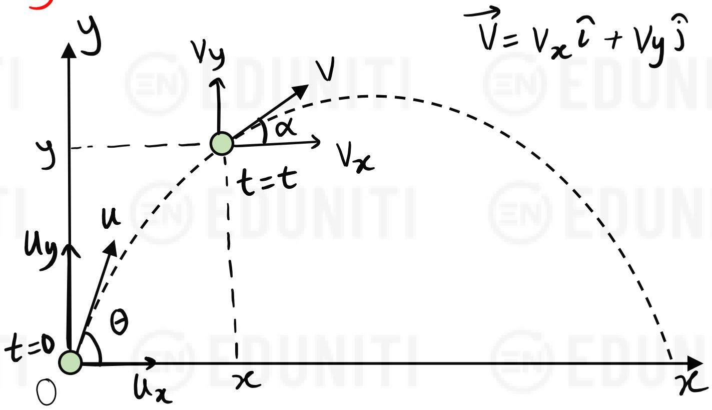
(i) $V_x = u_x$, $V_y = u_y - gt$

$V = \sqrt{V_x^2 + V_y^2}$, $\alpha = \tan^{-1}\left(\dfrac{V_y}{V_x}\right)$

(ii) $x = u_x t$, $y = u_y t - \frac{1}{2} g t^2$

$\vec{r} = x\hat{i} + y\hat{j}$, $r = \sqrt{x^2 + y^2}$

$\vec{V} = V_x \hat{i} + V_y \hat{j}$



$\left.\begin{array}{l} u_x = u\cos\theta, \; u_y = u\sin\theta \\ a_x = 0, \; a_y = -g \end{array}\right\}$ Given

2d. Eqⁿ of Trajectory

Using $x = u\cos\theta . t$ and

$y = u\sin\theta . t - \frac{1}{2} g t^2$

(i) $y = x\tan\theta - \dfrac{g x^2}{2u^2 \cos^2\theta}$

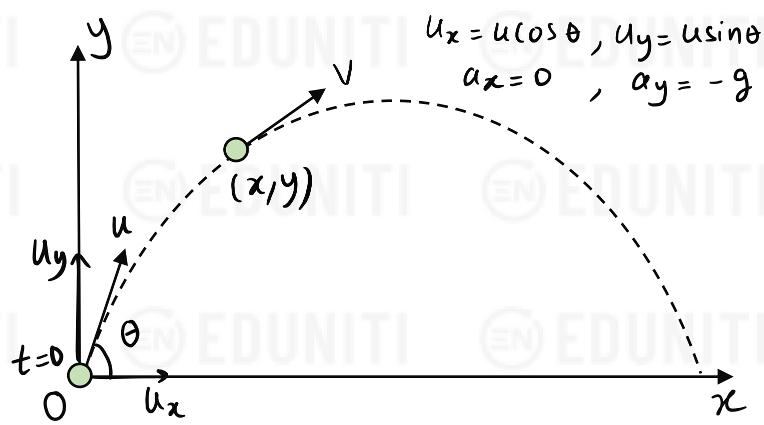
$(y = ax - bx^2) \rightarrow$ Quadratic Eqⁿ

NOTE

Range, $R = \dfrac{a}{b}$

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$a_x = 0$, $a_y = -g$



(ii) $y = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

R → Range

3. Projectile Fired from a Tower

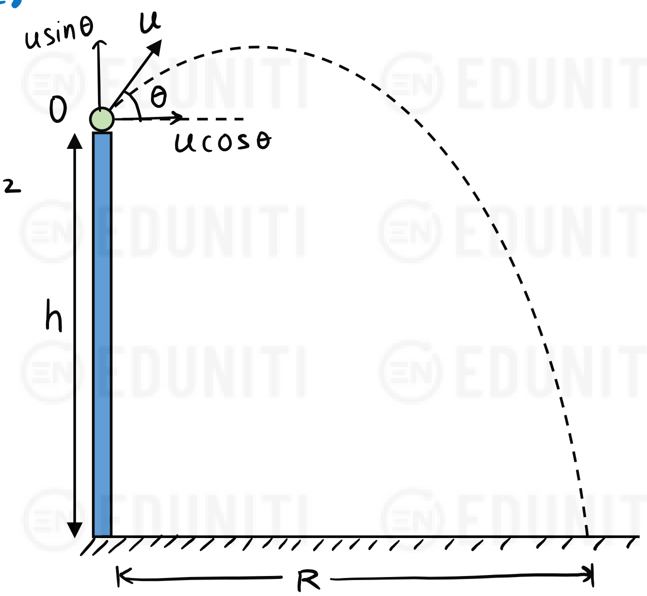
3a. At Some angle

(i) To find T (write eq^n along y)

$S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -h = u\sin\theta . T - \frac{1}{2} g T^2$

↳ Find T

(ii) $R = u\cos\theta \cdot T$



3b. Projected horizontally

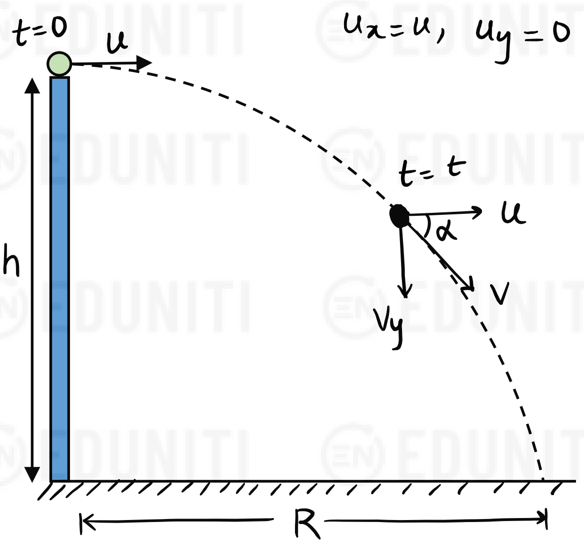
(i) Time of Flight, $T = \sqrt{\frac{2h}{g}}$

(ii) Range, $R = u.T = u\sqrt{\frac{2h}{g}}$

(iii) $V_y = gt$ $\therefore \alpha = \tan^{-1}\left(\frac{V_y}{u}\right)$

and

$V = \sqrt{u^2 + V_y^2}$

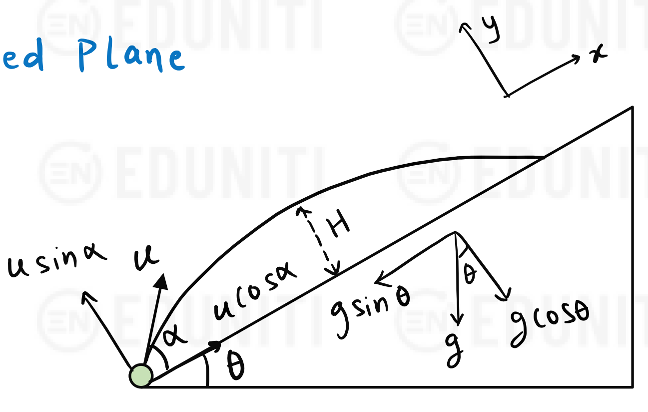


4. Projectile Motion on Inclined Plane

4a. Up the Incline

(i) $T = \frac{2u_y}{a_y} = \frac{2u\sin\alpha}{g\cos\theta}$

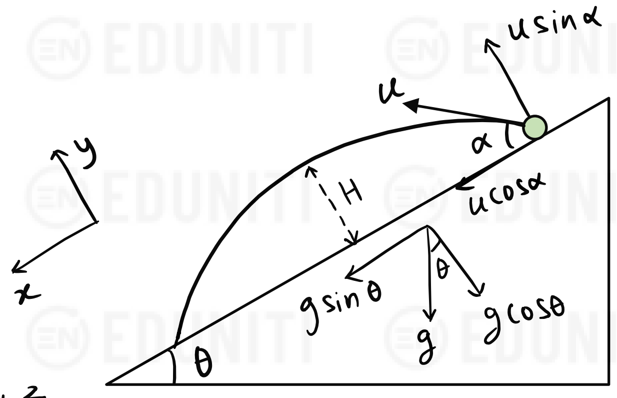
(ii) $H = \frac{u_y^2}{2a_y} = \frac{u^2\sin^2\alpha}{2g\cos\theta}$



(iii) For Range, $x = u_x t + \frac{1}{2} a_x t^2$

$\Rightarrow R = u\cos\alpha \cdot T - \frac{1}{2} g\sin\theta . T^2$

4b. Down the Incline



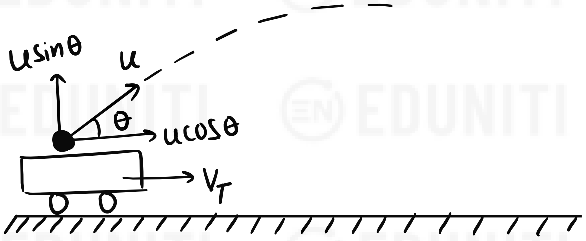
(i) $T = \dfrac{2u_y}{a_y} = \dfrac{2u\sin\alpha}{g\cos\theta}$

(ii) $H = \dfrac{u_y^2}{2a_y} = \dfrac{u^2\sin^2\alpha}{2g\cos\theta}$

(iii) For Range, $x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow R = u\cos\alpha \cdot T + \frac{1}{2}g\sin\theta . T^2$

5. Projectile from moving Trolley



Concept: W.r.t ground
$u_x = u\cos\theta + V_T$
$u_y = u\sin\theta$

(i) $T = \dfrac{2u\sin\theta}{g}$

(ii) $H = \dfrac{u^2\sin^2\theta}{2g}$

(iii) $R = u_x . T = (u\cos\theta + V_T) . \dfrac{2u\sin\theta}{g}$

↳ To find R_{max}, $\dfrac{dR}{d\theta} = 0$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Relative Motion – PART 3

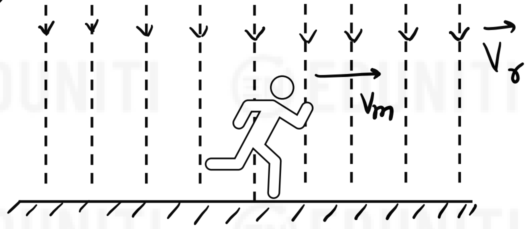
1. Rain – Man
2. River – Swimmer (basic terms)
3. River Swimmer Case A – Least Time
4. River Swimmer Case B – Shortest Path (zero drift)
5. River Swimmer Case C – Shortest path (non zero drift)

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

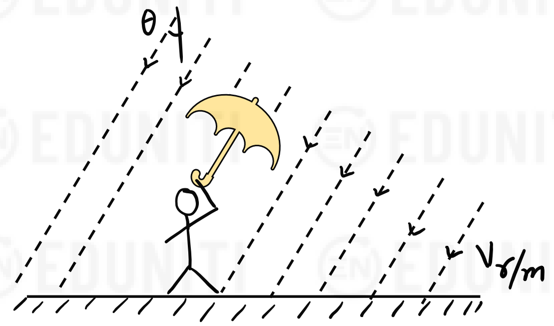
1. Rain- Man

Man Running

(A)



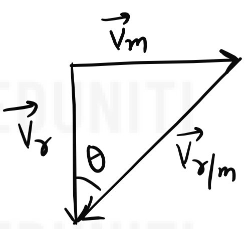
Ground Frame

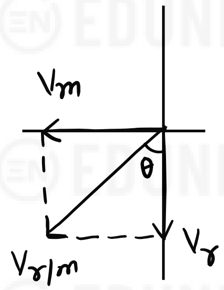


Man's Frame

Another way:

$\vec{V}_r = \vec{V}_m + \vec{V}_{r/m}$



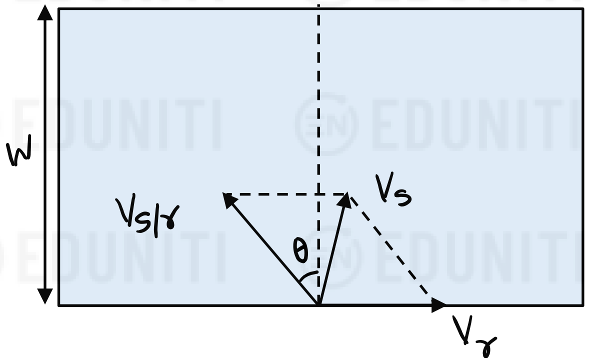


(i) $\tan\theta = V_m / V_r$

(ii) $V_{r/m} = \sqrt{V_m^2 + V_r^2}$

$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$

2. River- Swimmer (basis terms)

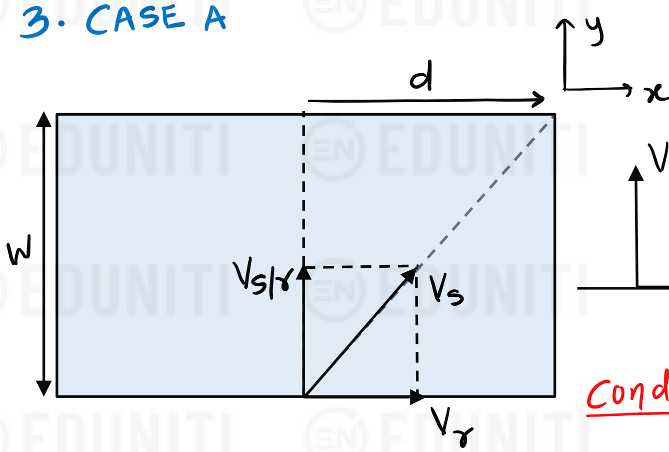


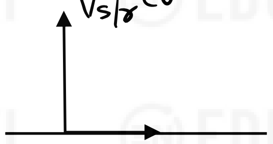
$\vec{V}_r$: velocity of river w.r.t Ground

$\vec{V}_{s/r}$: velocity of swimmer w.r.t river (vel of swimmer in still river)

$\vec{V}_s$: velocity of swimmer w.r.t Ground

3. CASE A





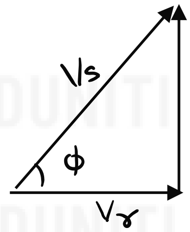
$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta}\right)$

$V_r - V_{s/r}\sin\theta$

Condition to cross in minimum time:

For t_{min}, $\cos\theta = 1 \Rightarrow \theta = 0° \Rightarrow V_{s/r}$ should be $\perp$ to V_r

$$\boxed{t_{min} = \frac{W}{V_{s/r}}}$$

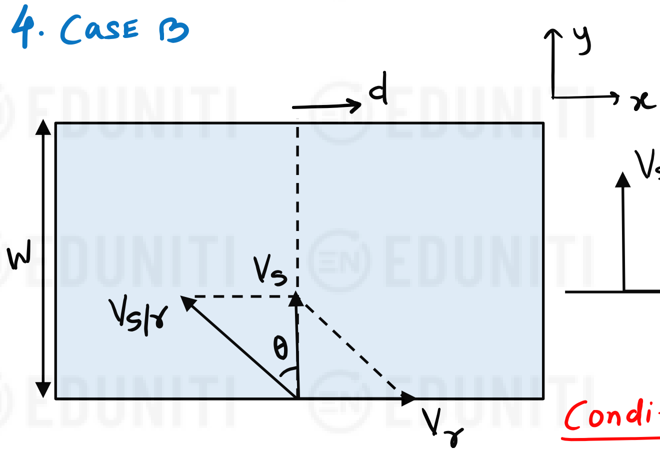


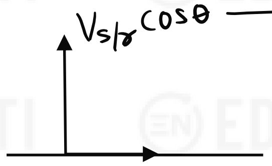
$\vec{V_s} = \vec{V_{s/r}} + \vec{V_r}$

$V_s = \sqrt{V_{s/r}^2 + V_r^2}$

$\tan\phi = \dfrac{V_{s/r}}{V_r}$

4. Case B





$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \dfrac{W}{V_{s/r}\cos\theta}\right)$

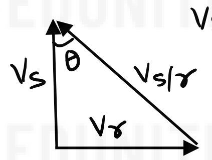
$V_r - V_{s/r}\sin\theta \longrightarrow$ Causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

Condition for drift to be zero (shortest path)

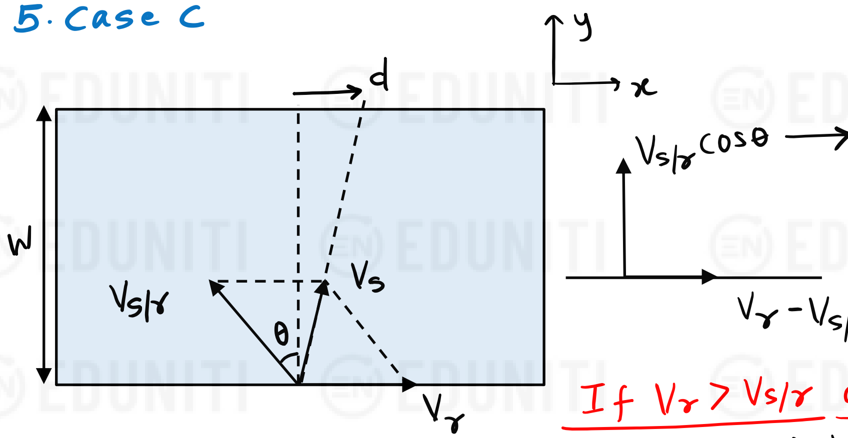
For $d = 0$, $V_r - V_{s/r}\sin\theta = 0 \Rightarrow V_r = V_{s/r}\sin\theta$

also, $\sin\theta = \dfrac{V_r}{V_{s/r}}$ $\because \sin\theta \leq 1 \Rightarrow \dfrac{V_r}{V_{s/r}} \leq 1 \Rightarrow \boxed{V_r \leq V_{s/r}}$



$V_s = \sqrt{V_{s/r}^2 - V_r^2}$

5. Case C



$V_{s/r}\cos\theta \longrightarrow$ Responsible for crossing river $\left(t = \frac{W}{V_{s/r}\cos\theta}\right)$

$V_r - V_{s/r}\sin\theta \longrightarrow$ causes drift

$d = (V_r - V_{s/r}\sin\theta)\cdot t$

If $V_r > V_{s/r}$ d can never be zero :

so, $d = (V_r - V_{s/r}\sin\theta)\cdot\frac{W}{V_{s/r}\cos\theta}$ ∴ Condⁿ for d_{min}?

Differentiate w.r.t θ and equate to zero.

We get : $\sin\theta = \frac{V_{s/r}}{V_r} \Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{V_{s/r}}{V_r}\right)}$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

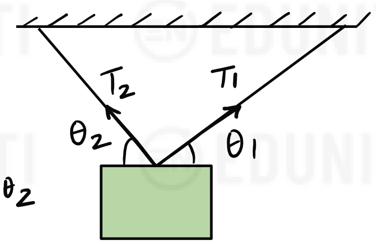
Topics to cover in NLM – PART 1

1. Static Equilibrium
2. Pulley Block Systems
3. Concept of Weighing Machine
4. Concept of Spring Balance
5. Spring Cutting & Combination
6. Pseudo Force
7. Variable Force

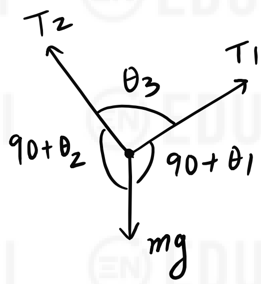
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

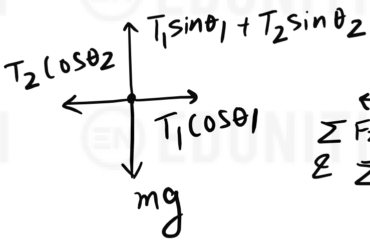
1. Static Equilibrium

↳ $\Sigma \vec{F} = 0$ (body at rest)



$\theta_3 = 180 - \theta_1 - \theta_2$





$\Sigma F_x = 0$ & $\Sigma F_y = 0$ → Lami's theorem

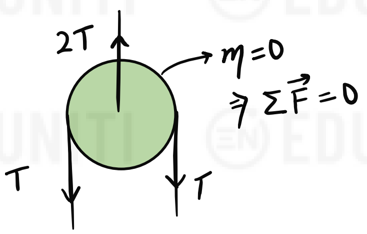
$$\frac{mg}{\sin\theta_3} = \frac{T_1}{\sin(90+\theta_2)} = \frac{T_2}{\sin(90+\theta_1)}$$

$$T_1\sin\theta_1 + T_2\sin\theta_2 = mg$$

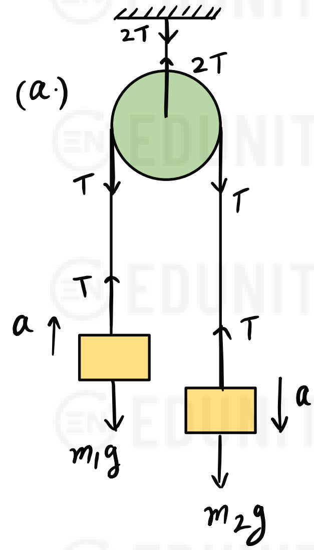
$$T_1\cos\theta_1 = T_2\cos\theta_2$$

2. Pulley-Block Systems

↳ Pulley is massless & smooth



(a.)



$\Sigma F = ma$

↓

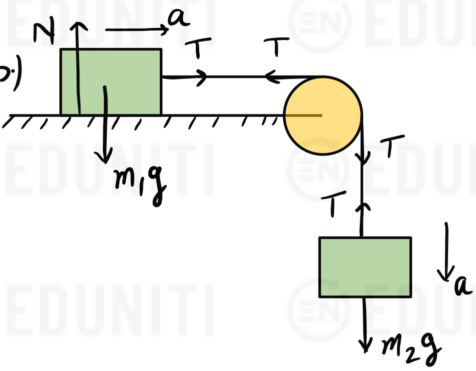
$m_2g - T = m_2a$ — (i)

$T - m_1g = m_1a$ — (ii)

$\therefore$ # $T = \dfrac{2m_1m_2g}{m_1+m_2}$

$a = \left(\dfrac{m_2-m_1}{m_1+m_2}\right)g$

(b.)

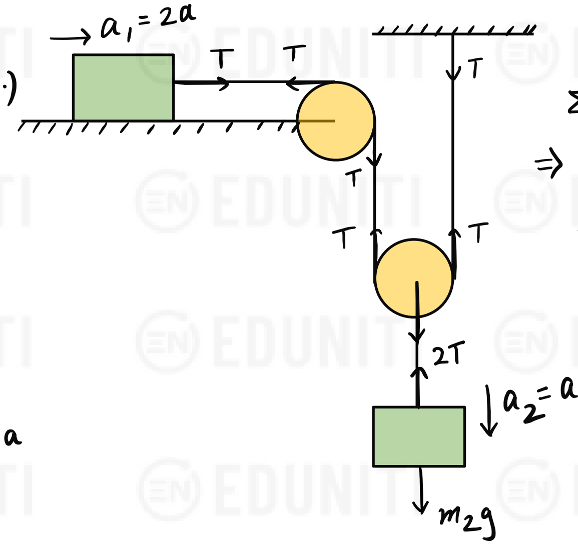


$m_2g - T = m_2a$

$T = m_1a$

$N = m_1g$

(c.)



$\Sigma \vec{T}\cdot\vec{a} = 0$

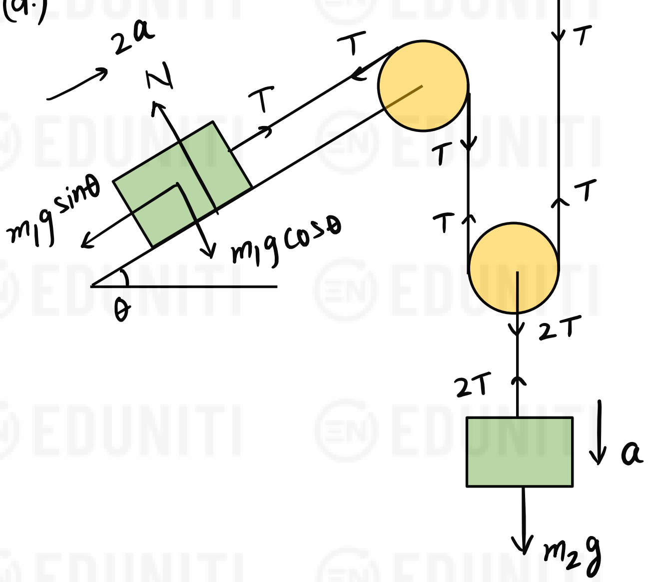
$\Rightarrow Ta_1 - 2Ta_2 = 0$

$\therefore a_1 = 2a_2$

$m_2g - 2T = m_2a$ — (i)

$T = m_1 \times 2a$ — (ii)

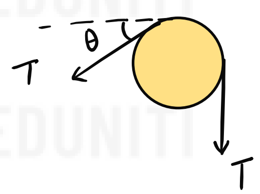
(d.)



$m_2g - 2T = m_2a$ -- (i)

$T - m_1g\sin\theta = m_1 \times 2a$ —(ii)

$N = m_1g\cos\theta$ -- (iii)

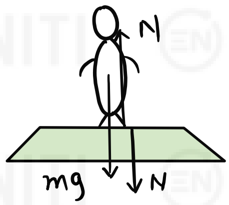


clamp force

$= \sqrt{T^2 + T^2 + 2T^2\cos\left(\frac{\pi}{2} - \theta\right)}$

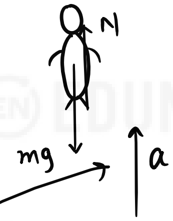
3. Concept of Weighing Machine

↳ Measures Normal reaction acting on surface of m/c.



$N = mg$

$\text{Reading} = \frac{N}{g} = m$

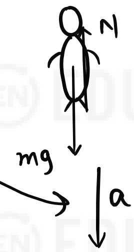


$N - mg = ma$

$\Rightarrow N = m(g+a)$

$\text{Reading} = N/g$

$= m\left(1+\frac{a}{g}\right)$



$mg - N = ma$

$\Rightarrow N = m(g-a)$

$\text{Reading} = N/g$

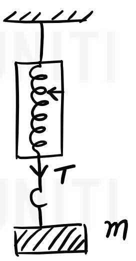
$= m\left(1-\frac{a}{g}\right)$

If $N=0$

$\Rightarrow$ weightlessness → Ex: free fall

4. Concept of Spring Balance (massless spring)

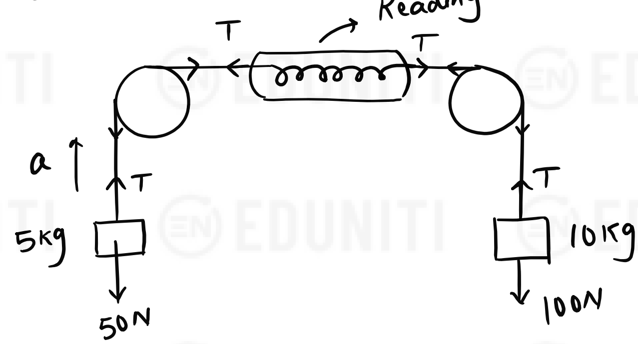
↳ Measures Tension in string attached to it.



$T = mg$

$\text{Reading} = T/g$

$= m$



$100 - T = 10a$ — (i)

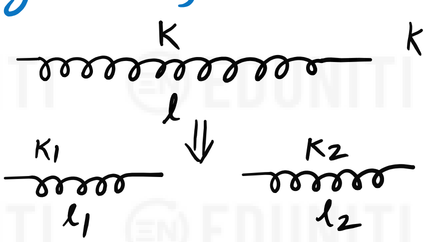
$T - 50 = 5a$ — (ii)

$\therefore T = 200/3 \ N$

$\text{Reading} = \frac{20}{3}$ kg

5. Spring cutting & Combination

Cut



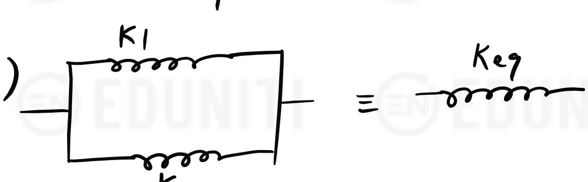
$K \cdot l = \text{const.}$

$K_1 = \frac{Kl}{l_1}$ $\qquad K_2 = \frac{Kl}{l_2}$

Combination

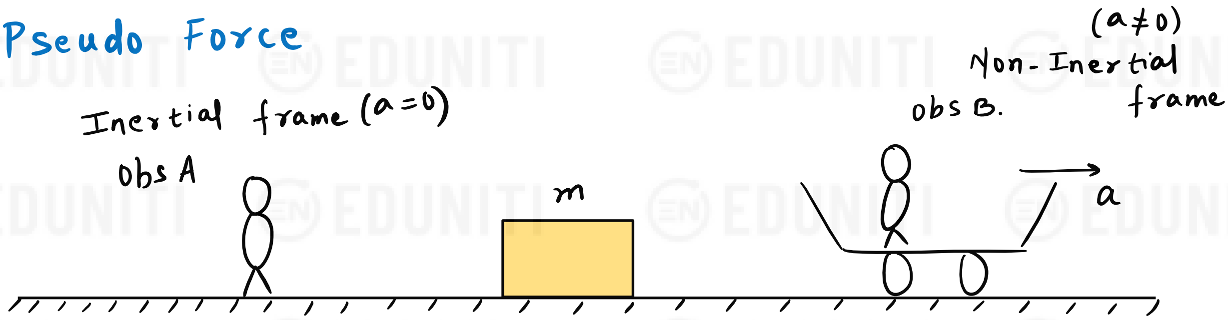
(i) 

$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$

(ii) 

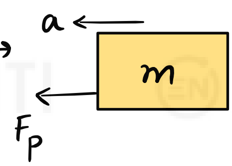
$K_{eq} = K_1 + K_2$

6. Pseudo Force



Obs A. $\rightarrow$ Body is at rest

obs B. $\rightarrow$

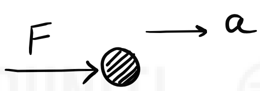


$\because$ accelerated, thus force acting on it must be

$F_p = ma$

$\hookrightarrow$ false force

7. Variable Force



$F(t)$: $\quad F(t) = m\dfrac{dv}{dt} \Rightarrow \displaystyle\int_0^t F(t)\,dt = \int_u^v m\,dv$

$F(x)$: $\quad F(x) = mv\dfrac{dv}{dx} \Rightarrow \displaystyle\int_0^x F(x)\,dx = \int_u^v mv\,dv$

$F(v)$: $\quad F(v) = mv\dfrac{dv}{dx} \Rightarrow \displaystyle\int_0^x dx = \int_u^v \frac{mv}{F(v)}\,dv$

$\quad F(v) = m\dfrac{dv}{dt} \Rightarrow \displaystyle\int_0^t dt = \int_u^v m\frac{dv}{F(v)}$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Friction – PART 2

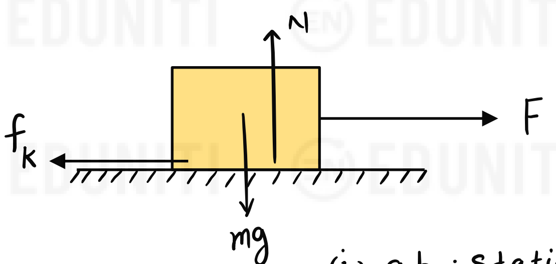
1. Variation of Friction with External Force
2. Direction of Friction (*learn through FBD*)
3. Pushing vs Pulling
4. Minimum force to start slipping
5. Contact Force
6. Angle of Repose
7. Two Block System
8. Friction on Chain

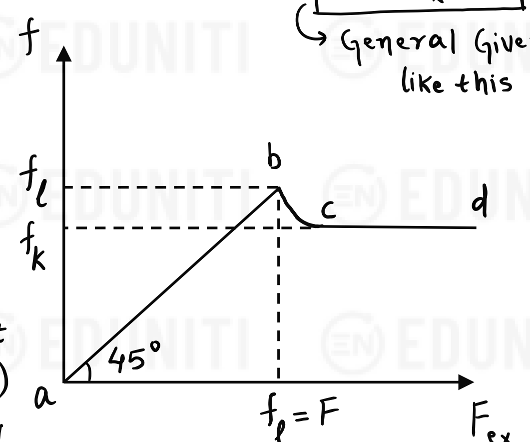
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Variation of Friction with External Force

$\boxed{\mu_s = \mu_k = \mu}$

General Given like this





(i) ab : Static friction (block is at rest w.r.t ground)

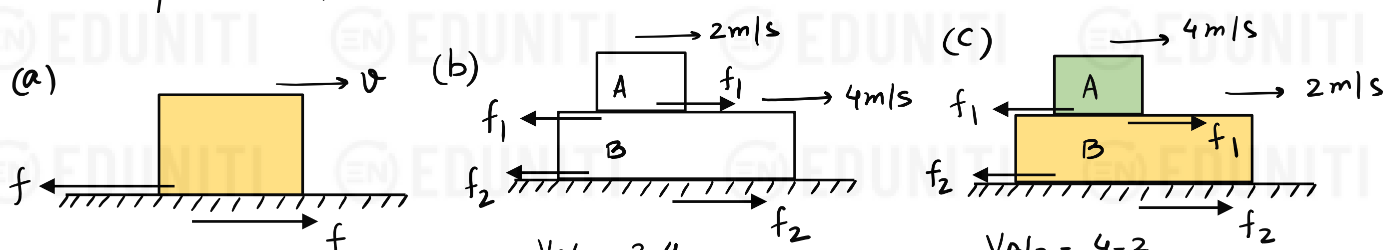
(ii) At b : $f_{s\,max}$ or f_l

(iii) cd : Kinetic friction ($f_k < f_{s\,max}$)

(iv) $f_{s\,max} = \mu_s N$, $f_k = \mu_k N$

2. Direction of friction

↳ friction opposes the relative motion between two bodies

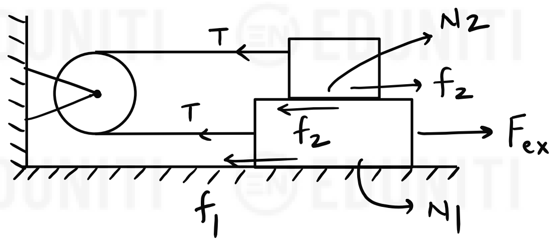


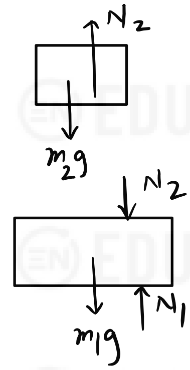
$V_{A/B} = 2 - 4 = -2$

$V_{A/B} = 4 - 2 = 2$

Friction – Part 2

(d)





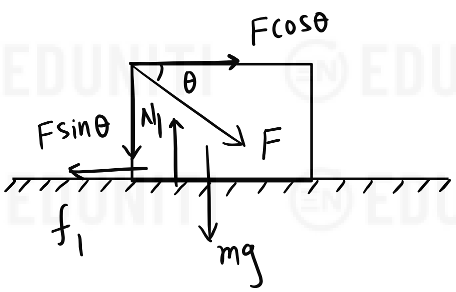
$N_2 = m_2g$

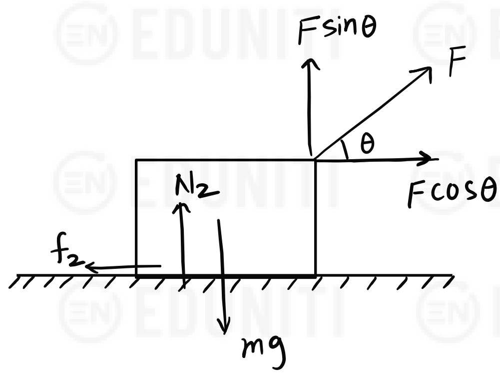
$N_1 = m_1g + N_2$
$= m_1g + m_2g$
$= (m_1+m_2)g$

Assuming slipping at all places :

$f_1 = \mu_1 N_1 \quad , \quad f_2 = \mu_2 N_2$

3. Pushing vs Pulling





$N_1 = mg + F\sin\theta$

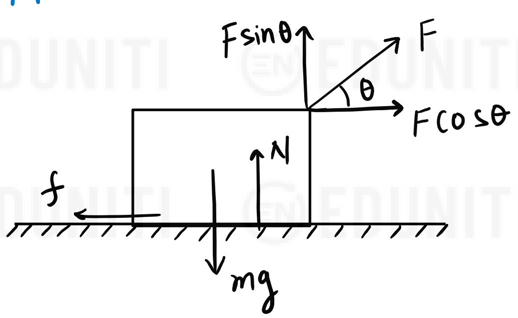
$f_1 = \mu N_1$

$N_2 = mg - F\sin\theta$

To slide, friction to over come

$f_2 = \mu N_2$

4. Minimum Force to Start slipping (condⁿ on θ)



To just start slipping :

$F\cos\theta = f = \mu N$

$\Rightarrow F\cos\theta = \mu(mg - F\sin\theta)$

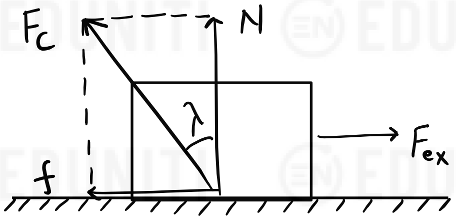
$\Rightarrow F = \dfrac{\mu mg}{\cos\theta + \mu\sin\theta}$

To Minimize F, $\cos\theta + \mu\sin\theta \rightarrow$ Maximized

$\Rightarrow \dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0 \Rightarrow \boxed{\tan\theta = \mu}$

$\therefore \cos\theta = \dfrac{1}{\sqrt{\mu^2+1}} \quad , \quad \sin\theta = \dfrac{\mu}{\sqrt{\mu^2+1}} \Rightarrow \boxed{F_{min} = \dfrac{\mu mg}{\sqrt{\mu^2+1}}}$

5. Contact Force (F_c)



$$F_c = \sqrt{N^2 + f^2}$$

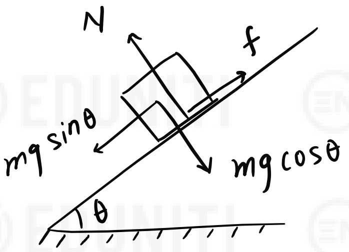
$F_{c,min} = N \quad \{f = 0\}$

$F_{c,max} = \sqrt{N^2 + \mu^2 N^2} = N\sqrt{1+\mu^2} \quad \{\mu = \mu_s$

$N \leq F_c \leq N\sqrt{1+\mu^2}$

6. Angle of Repose

↳ Minimum angle for which body slips due to its weight



Block starts to just slip if:

$mg\sin\theta = f_{s\,max}$

$\Rightarrow mg\sin\theta = \mu N \Rightarrow mg\sin\theta = \mu\, mg\cos\theta$

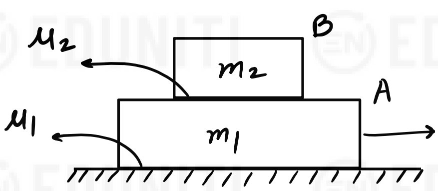
$\therefore \tan\theta = \mu$ or $\boxed{\theta = \tan^{-1}\mu}$

(i) If angle is $< \tan^{-1}\mu \Rightarrow$ No slipping

(ii) If angle is $> \tan^{-1}\mu \Rightarrow$ slipping

$a = g\sin\theta - \mu g\cos\theta$

7. Two Block System (Block over block)



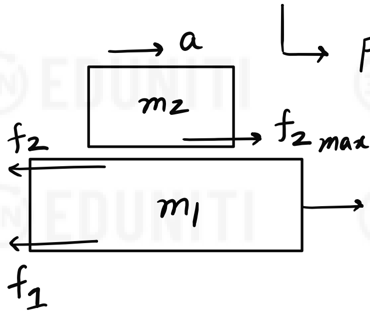
(i) Block B will never move before A

(ii) Min F to move A $= \mu_1(m_1+m_2)g = f_1$

(iii) On further ↑ F upto a F_{max}

m_2 & m_1 moves together (m_2 not slipping on m_1)

↳ $F_{max} = ?$

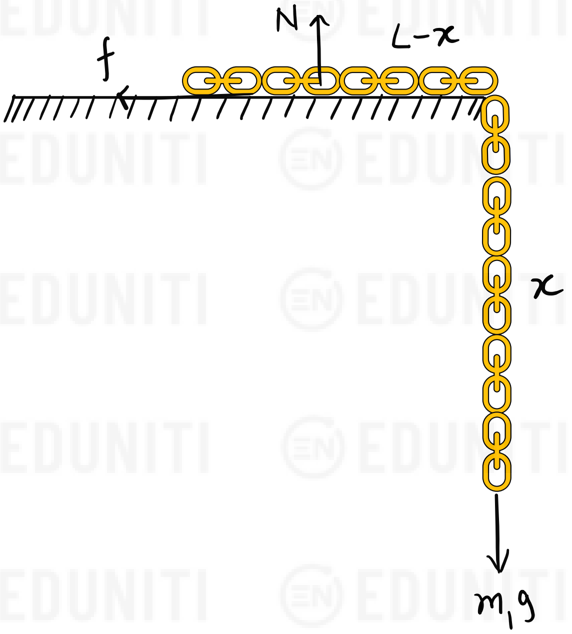


$f_{2\,max} = \mu_2 m_2 g$

$\therefore a = \frac{\mu_2 m_2 g}{m_2} = \mu_2 g$

$\Rightarrow \boxed{F_{max} - f_1 = (m_1+m_2)\mu_2 g}$

8. Friction on chain



$m_1 \rightarrow$ hanging mass

For Maximum hanging length without slipping:

$$m_1 g = f_{max}$$

$$\Rightarrow \frac{m}{L} x g = \mu \underbrace{\frac{m}{L}(L-x) g}_{N}$$

$$\Rightarrow x = \mu (L-x)$$

$$\therefore \boxed{x = \frac{\mu L}{1+\mu}}$$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Work Energy Power

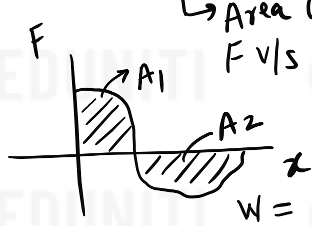
1. Work done
2. Work Energy Theorem
3. Conservation of Mechanical Energy
4. Cases where Work done is zero
5. Potential Energy
6. How to write Change in P.E.
7. Conservative Force
8. Power

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Work done

Work

- F is constant → $\vec{F}\cdot\vec{S}$ or $\vec{F}\cdot(\vec{r}_2-\vec{r}_1)$
- $F(x)$ → $\int_{x_1}^{x_2} F(x)\,dx$
  - ↳ Area under F v/s x graph
- $F(t)$ → (1) $a(t) = \frac{F(t)}{m}$
  - ↓ INTEGRATE
  - $v = f(t)$
  - $\Rightarrow dx = f(t)\,dt$
  - $W = \int_{t_1}^{t_2} m\,a(t) \times f(t)\,dt$



$W = A_1 - A_2$

2. Work - Energy Theorem

↳ Net work done = change in Kinetic energy

$$W_{net} = K_f - K_i \Rightarrow W_{net} = \Delta K$$

↳ Work can be due to various forces:

↳ Conservative force (gravitational, electrostatic, spring force)

↳ Non-conservative (friction, viscous force)

↳ External agent (person applying forces)

↳ Tension, Normal

$W_{net} = K_f - K_i \Rightarrow W_{net} = \Delta K$

$\hookrightarrow W_c + W_{NC} + W_{ext} = \Delta K$

$\Rightarrow -\Delta U + W_{NC} + W_{ext} = \Delta K$

$\Rightarrow W_{NC} + W_{ext} = \underbrace{\Delta K + \Delta U}_{\text{Total change in Mechanical Energy}}$

3. Conservation of Energy

If on $W_{NC} = 0$ & $W_{ext} = 0$ (either they are absent or their work done is zero)

$\Rightarrow \boxed{0 = \Delta K + \Delta U}$ → Total change in Mech Energy is zero

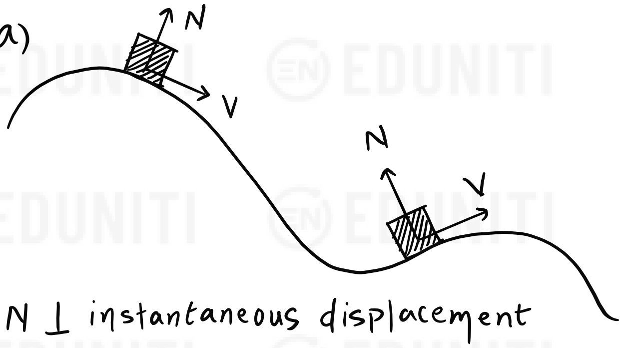
$\Rightarrow$ Energy conservation

or $0 = K_f - K_i + U_f - U_i$

$\Rightarrow \boxed{K_i + U_i = K_f + U_f}$

4. Sitiuation where $W = 0$

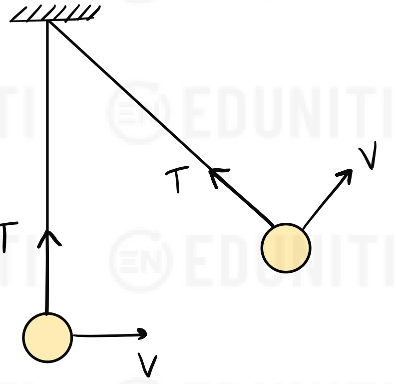
(a)



N ⊥ instantaneous displacement

$\Rightarrow W_N = 0$

(b)

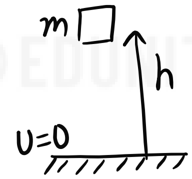


T ⊥ to instantaneous displacement of bob

$\Rightarrow W_T = 0$

5. POTENTIAL ENERGY

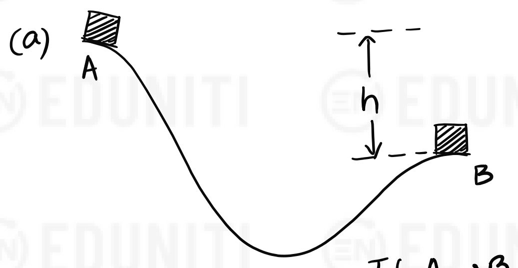
- Gravitational → $U = mgh$ (h is height of c.o.m)
  - → $U = -Gm_1m_2/r$



- SPRING → $U = \frac{1}{2}kx^2$
- Electrostatics → $U = Kq_1q_2/r$ (Put q_1 and q_2 with sign)
  - → Dipole, $U = -\vec{P}\cdot\vec{E}$
- Magnetism → magnet in B, $U = -\vec{M}\cdot\vec{B}$
- Elastic → $U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Vol.}$

6. How to write ΔU

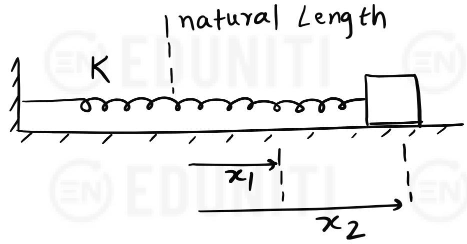
(a)



If A → B
$\Delta U = -mgh$

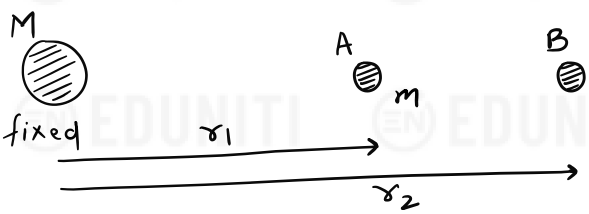
If B → A
$\Delta U = mgh$

(b)



From $x_1 \rightarrow x_2$
$\Delta U = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

(c)



$A \rightarrow B$, $\Delta U = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$

7. CONSERVATIVE FORCE $\left(F = -\frac{dU}{dx}\right)$

Equillibrium ⇒ $F = 0$ or $\frac{dU}{dx} = 0$

- Stable Eq.
  - # SHM can occur
  - # $\frac{d^2U}{dx^2} > 0$
- Unstable Eq
  - # No SHM
  - # $\frac{d^2U}{dx^2} < 0$
- Neutral Eq.
  - # NO SHM
  - # $\frac{d^2U}{dx^2} = 0$

8. POWER (unit : Watt)

$P_{av} = \frac{\Delta W}{\Delta t}$

$P_{inst} = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt}$

$= \vec{F} \cdot \vec{V}$

$= FV\cos\theta$

Space to add concepts learnt from PYQs if any

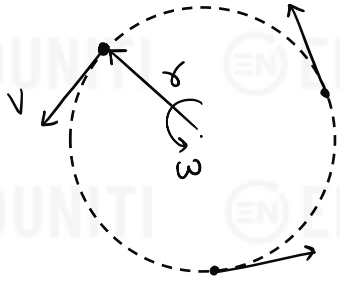
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Circular Motion

1. Kinematics of Circular Motion
2. Radius of Curvature
3. Relative Angular Velocity
4. Uniform vs Non-Uniform Circular Motion
5. Non-Uniform C.M. ($\alpha = Const.$)
6. Non-Uniform C.M. ($\alpha \neq Const.$)
7. Centripetal Force
8. Conical Pendulum
9. Centrifugal Force
10. Dynamics of Vertical C.M.
11. Vertical C.M. of a Pendulum Bob (string)
12. Vertical C.M of a Light Rod Pendulum
13. Vertical C.M. Inside a Circular Track
14. Vertical C.M. Outside a Circular Track
15. Vertical C.M. in a Circular Tube
16. Banked Roads

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Kinematics of Circular Motion



(a) → Angular displacement, $\Delta\theta$ (in rad)

→ Angular velocity, ω (rad/s)

$\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$ $\omega_{inst} = \dfrac{d\theta}{dt}$

→ Angular accn, α (rad/s²) $= \dfrac{d\omega}{dt} = \omega\dfrac{d\omega}{d\theta}$

(b) Relation between V and ω : $V = \omega r$

(c) $\dfrac{dV}{dt} = r\dfrac{d\omega}{dt} \Rightarrow a_t = r\alpha$ { a_t changes speed of particle

(d) ∵ dirn changing, $a_n = \dfrac{V^2}{r} = \omega^2 r$ { normal or centripetal accn

2. Radius of Curvature

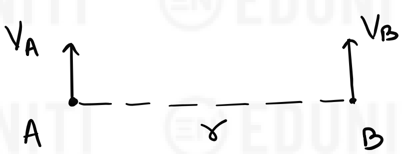
(a) $r_1 = v_1^2/a_{n1}$, $r_2 = v_2^2/a_{n2}$

↳ here a_{n1} & a_{n2} are accⁿ ⊥ to v_1 & v_2

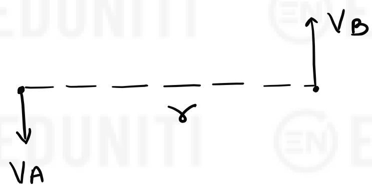
(b) At (x_1, y_1), radius of curvature is

$$r_c = \left. \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \right|_{x_1, y_1}$$

3. Relative Angular Velocity

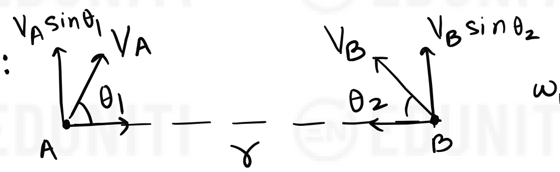


$$\omega_{BA} = \frac{v_{rel}}{r} = \frac{v_B - v_A}{r}$$



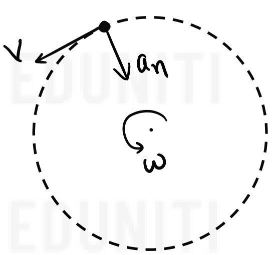
$$\omega_{BA} = \frac{v_{rel}}{r} = \frac{v_B + v_A}{r}$$

Note:



$$\omega_{BA} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$$

4. Uniform vs Non-uniform Circular Motion



Uniform

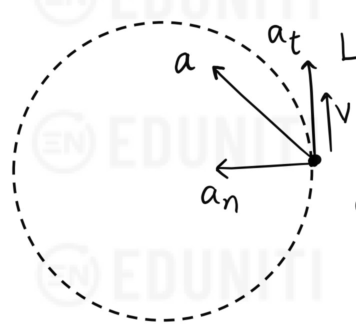
ω = const.

v is const.

$\alpha = 0$, $a_t = 0$

$a_n = \frac{v^2}{r} = \omega^2 r$

Non-Uniform



$\omega \neq$ const.

↳ speed & dirⁿ both changes

$a = \sqrt{a_t^2 + a_n^2}$, where $a_t = \alpha r$ and $a_n = \frac{v^2}{r}$

$\alpha \neq 0$

5. Non-Uniform CM (α = Const.)

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\Delta\theta_n = \omega_i + \frac{\alpha}{2}(2n-1)$

Note:

(i) If both ω_i and α are in same sense (CW or ACW), take α +ve.

(ii) If α opp sense of ω_i, α -ve (retardation, Ex- fan switching off)

(iii) Be carefull about units

ω in rpm $\longrightarrow$ ω in rad/s

Ex. 50 rpm $\longrightarrow$ $\frac{50 \times 2\pi}{60}$ rad/s

6. Non-Uniform CM ($\alpha \neq$ Const.)

↳ $\omega = d\theta/dt$, $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

Ex. $\alpha = 2t$ and particle starts from rest. At $t = 3s$ find a ? ($r = 2m$)

Sol^n: At $t = 3s$, $\alpha = 2 \times 3 = 6$ rad/s^2

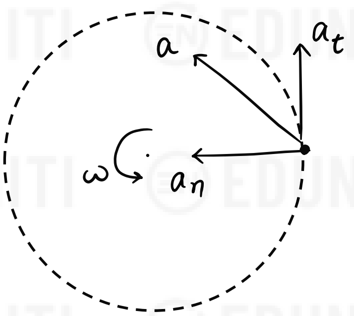
$a_t = \alpha r = 6 \times 2 = 12$ m/s^2

$\alpha = 2t \Rightarrow \frac{d\omega}{dt} = 2t$

$\Rightarrow \int_0^\omega d\omega = \int_0^3 2t\,dt \Rightarrow \omega = 9$ rad/s

$a_n = \omega^2 r = 81 \times 2 = 162$ m/s^2

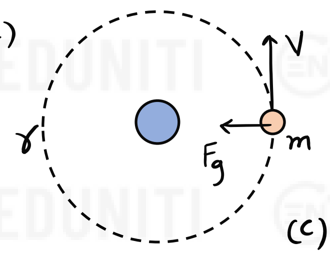
$\therefore a = \sqrt{12^2 + 162^2} \approx 162.4$ m/s^2



7. Centripetal Force

↳ net force towards centre of circular path, $F_{net} = m a_n = \frac{mv^2}{r}$

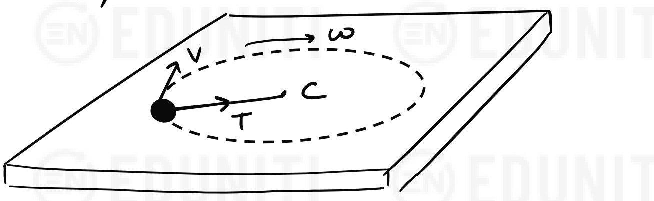
(a)



$F_g = \frac{mv^2}{r}$

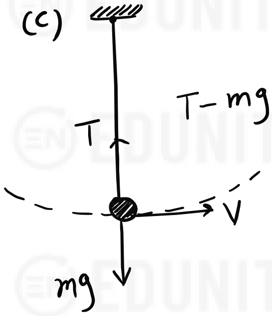
$\Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r}$

(b)



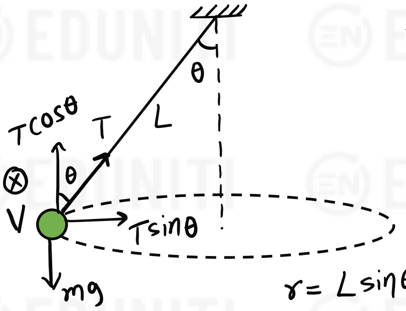
$T = \frac{mv^2}{\ell} = m\omega^2\ell$

(c)



$T - mg = \frac{mv^2}{\ell}$

8. Conical Pendulum



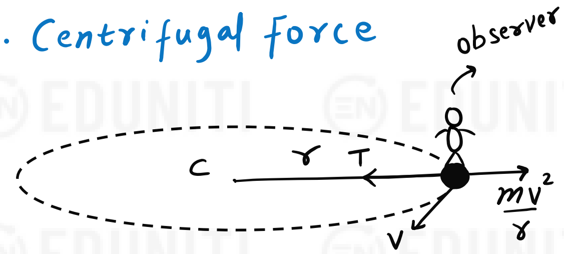
$r = L\sin\theta$

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$

9. Centrifugal force



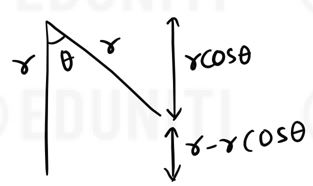
W.r.t obs → particle at rest

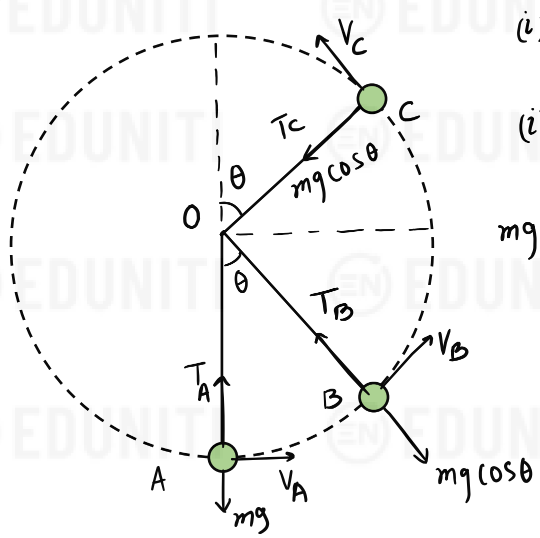
$\Rightarrow T = \frac{mv^2}{r}$

↳ here $\frac{mv^2}{r}$ is centrifugal force (which is pseudo force)

10. Dynamics of Vertical Circular Motion

↳ particle along circular path in vertical plane





(i) $T_A - mg = \frac{mV_A^2}{r}$

(ii) $T_B - mg\cos\theta = \frac{mV_B^2}{r}$

$mgr(1-\cos\theta) = \frac{1}{2}mV_A^2 - \frac{1}{2}mV_B^2$

(iii) $T_C + mg\cos\theta = \frac{mV_C^2}{r}$

$mgr(1+\cos\theta) = \frac{1}{2}mV_A^2 - \frac{1}{2}mV_C^2$

11. Vertical C.M. of a Pendulum Bob

(a) $u \leqslant \sqrt{2gl}$

Gain in P.E. = Loss in K.E

$\Rightarrow mgl(1-\cos\theta) = \frac{1}{2}mu^2$

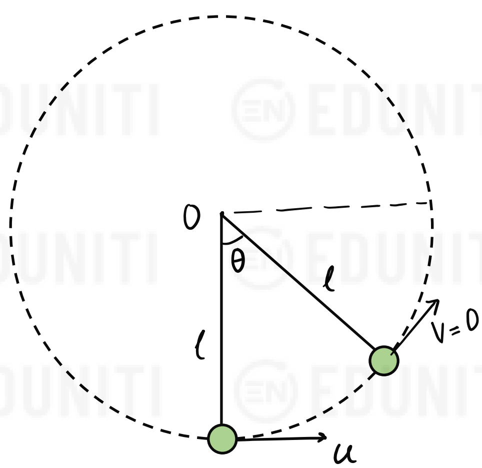
$\therefore \cos\theta = \frac{2gl - u^2}{2gl}$ ⟶ (i) $u = \sqrt{2gl}$

$\Rightarrow \theta = 90°$

(ii) use this if $u \leqslant \sqrt{2gl}$

Ex. $u = \sqrt{gl}$

$\Rightarrow \theta = 60°$

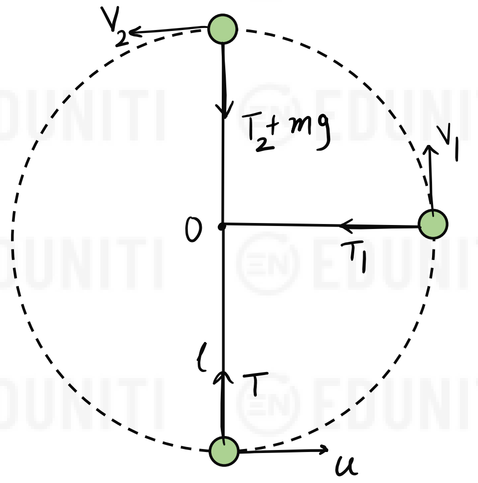


(b) $u \geq \sqrt{5gl}$

(i) bob does complete circle

(ii) If $u = \sqrt{5gl} \Rightarrow T_2 = 0$ but $V_2 \neq 0$

(iii) $T - T_1 = 3mg$ & $T - T_2 = 6mg$

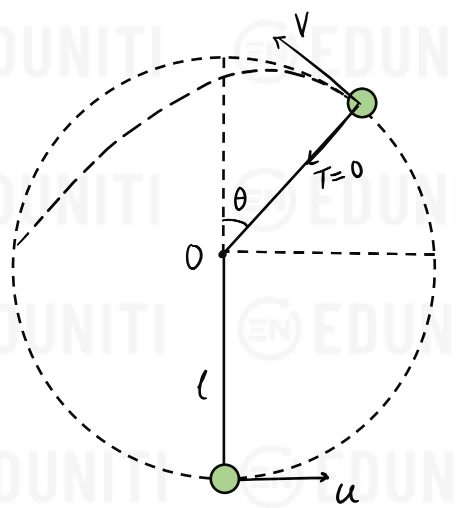


(c) $\sqrt{2gl} < u < \sqrt{5gl}$

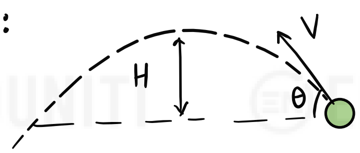
↳ rises above horizontal and somewhere T = 0 (string slacks) and bob does Projectile motion

(i) $\cos\theta = \dfrac{u^2 - 2gl}{3gl}$ & from Conserv. Energy

$V = \sqrt{u^2 - 2gl(1+\cos\theta)}$



NOTE :



$H = \dfrac{V^2 \sin^2\theta}{2g}$

12. Vertical C.M. of a Light Rod Pendulum

↳ Here there is no issue of T=0 as rod can't slack (massless rod)

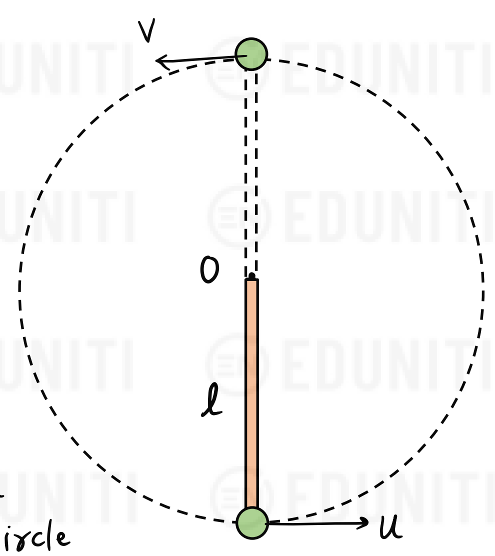
↳ for $u \geq \sqrt{4gl} \longrightarrow$ Complete Circle

NOTE:

(i) for $u = \sqrt{4gl}$

$V = 0$

↳ Due to inertia it completes circle



13. Vertical C.M. Inside a Circular Track

↳ All Cases are same as pendulum Bob with string.
(instead of T its N here)

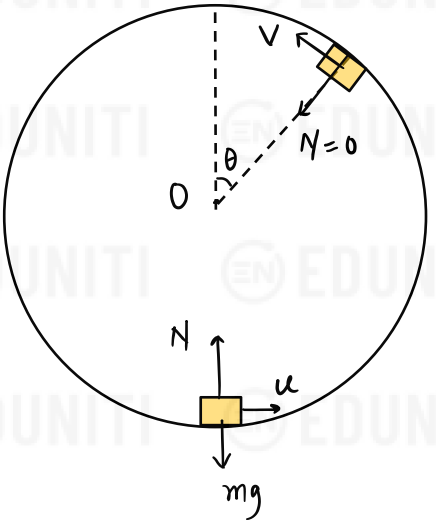
(i) $u \geq \sqrt{5gr} \longrightarrow$ complete circle

(ii) $\sqrt{2gr} < u < \sqrt{5gr} \longrightarrow$ Cross horizontal and at some θ with vertical $N = 0$

$$\cos\theta = \frac{u^2 - 2gr}{3gr} \quad \text{and}$$

$$v = \sqrt{u^2 - 2gr(1+\cos\theta)}$$

↳ use this θ & v for projectile motion



14. Vertical C.M. Outside a Circular Track

A → B : Loss in PE = Gain in KE

$$\Rightarrow mgr(1-\cos\theta) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\# \quad v = \sqrt{u^2 + 2gr(1-\cos\theta)}$$

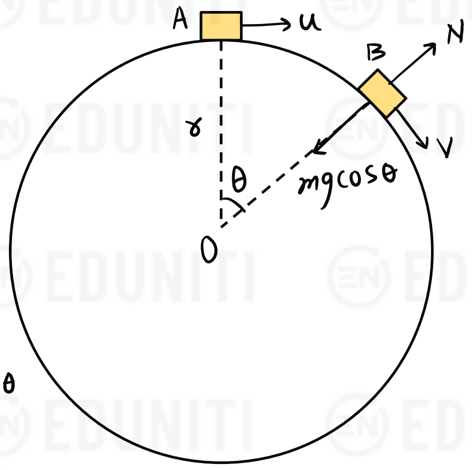
and, $mg\cos\theta - N = \frac{mv^2}{r}$

$\longrightarrow$ Contact Loose : $(N=0)$

$$v^2 = rg\cos\theta$$

$$\Rightarrow u^2 + 2gr(1-\cos\theta) = rg\cos\theta$$

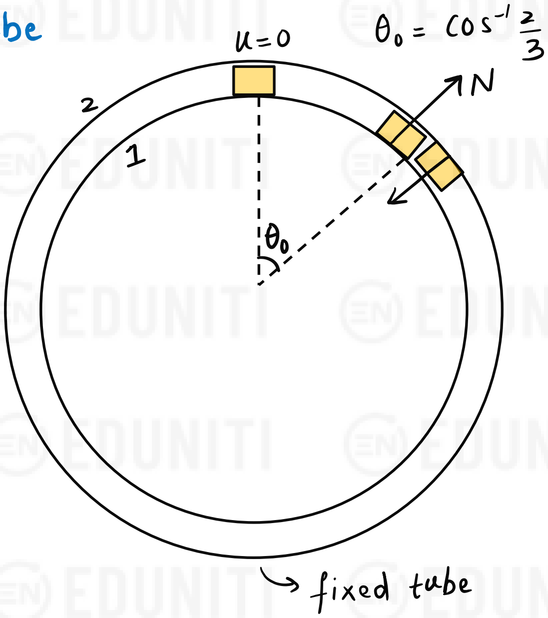
$$\# \quad \cos\theta = \frac{u^2 + 2gr}{3gr}$$



15. Vertical C.M. Inside a Circular Tube

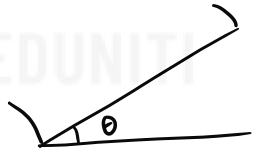
↳ till $\theta < \cos^{-1}\frac{2}{3}$, block on 1.
(N on block radially outwards)

↳ for $\theta > \cos^{-1}\frac{2}{3}$, block on 2
(N on block radially inwards)



16. Banked and Horizontal Road

1. For no slipping $\sqrt{Rg\left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right)} \leq v \leq \sqrt{Rg\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)}$



2. If $v=\sqrt{Rg\tan\theta}$, $f=0$ (car can turn on slippery road + no wear & tear of tyres)

3. If $v>\sqrt{Rg\tan\theta}$, f acts down the slope
   & $v<\sqrt{Rg\tan\theta}$, f acts up the slope

4. For horizontal circular turn,
   $V \leq \sqrt{\mu R g}$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

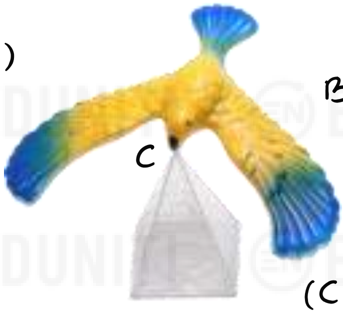
Topics to cover in COM – PART 1

1. Understanding of COM
2. COM of Multiparticle System
3. COM of Two Particle System
4. COM of Continuous Mass Distribution
5. COM of Standard Bodies (*uniform mass density*)
6. COM of Bodies (*Non-uniform mass density*)
7. COM of Combined Structure
8. COM in Cavity Problems (*2 Methods*)
9. Displacement of COM
10. Velocity & Acceleration of COM

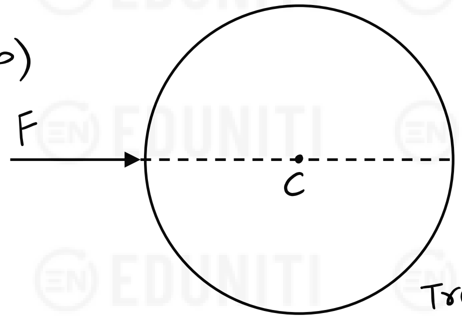
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Understanding of COM

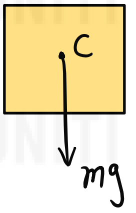
(a)



(b)

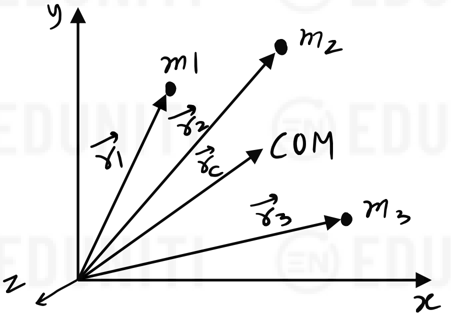


(C)



body lies in uniform gravitational field

2. COM of Multiparticle System



$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

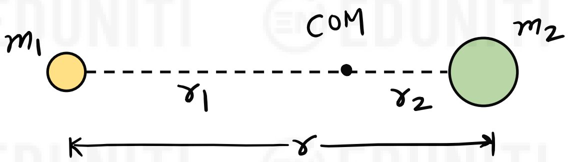
$$x_c = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$y_c = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$z_c = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1 + m_2 + m_3}$$

PART 1 - COM

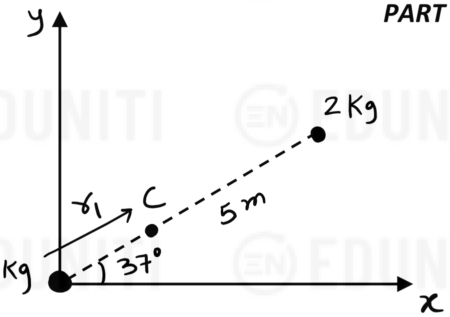
3. COM of 2-Particle System



(i) $r_1 = \dfrac{m_2 r}{m_1+m_2}$, $r_2 = \dfrac{m_1 r}{m_1+m_2}$

(ii) $m_1 r_1 = m_2 r_2$

Ex:



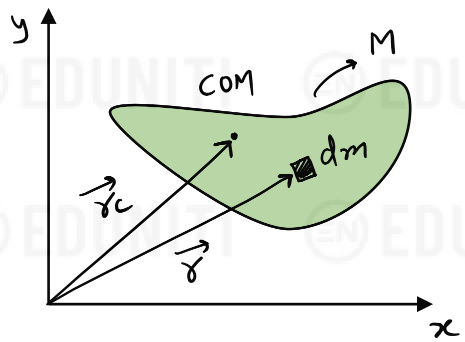
Find COM co-ordinates ?

Sol^n: $r_1 = \dfrac{2\times 5}{2+4} = \dfrac{10}{6} = \dfrac{5}{3}\,m$

$\therefore \ \dfrac{5}{3}\cos 37°, \ \dfrac{5}{3}\sin 37°$

or $\boxed{\dfrac{4}{3}, 1}$

4. COM of a continuous Mass Distribution



$$\vec{r}_c = \frac{\int \vec{r}\,dm}{\int dm} = \frac{1}{M}\int \vec{r}\,dm$$

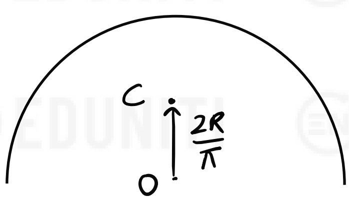
$$x_c = \frac{\int x\,dm}{\int dm}$$

$$y_c = \frac{\int y\,dm}{\int dm}$$

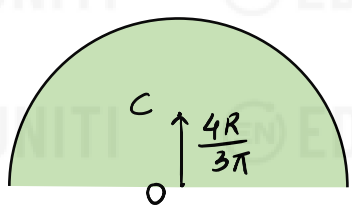
$$z_c = \frac{\int z\,dm}{\int dm}$$

5. COM of Standard bodies (uniform mass distribution)

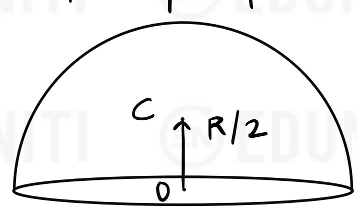
1. Semicircular Ring



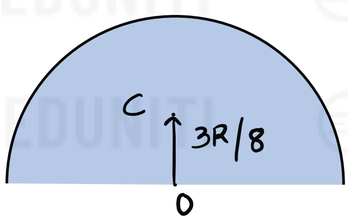
2. Semicircular Disc



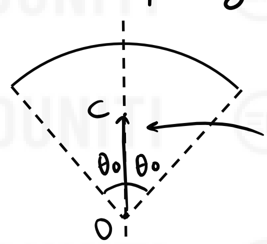
3. Hollow Hemisphere



4. Solid Hemisphere

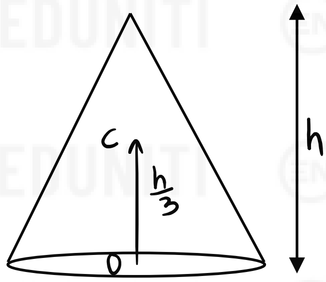


5. Arc of Ring

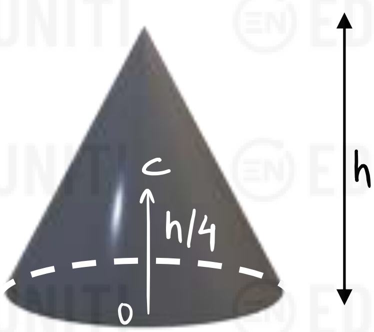


$OC = \dfrac{R\sin\theta_0}{\theta_0}$

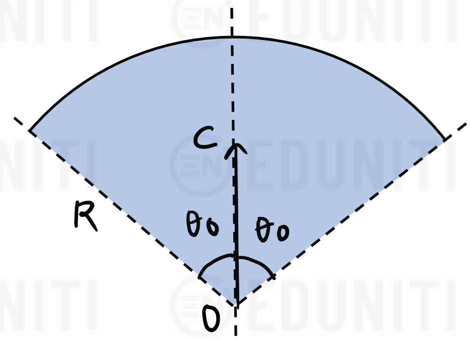
6. Hollow cone



7. Solid cone



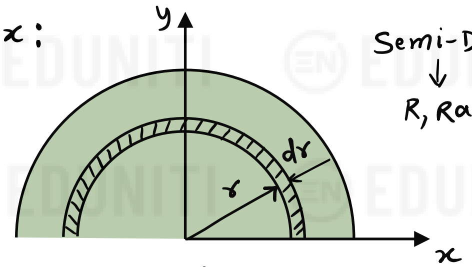
8. Sector of a Disc



$$OC = \frac{2R}{3}\,\frac{\sin\theta_0}{\theta_0}$$

6. COM of body (non-uniform mass distribution)

Ex:



Semi-Disc ↓ R, Radius

$\sigma = 2r\ (kg/m^2)$

Sol^n. COM lies on y axis

$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$dm = \sigma(r) \times dA = 2r \times \pi r\,dr = 2\pi r^2\,dr$$

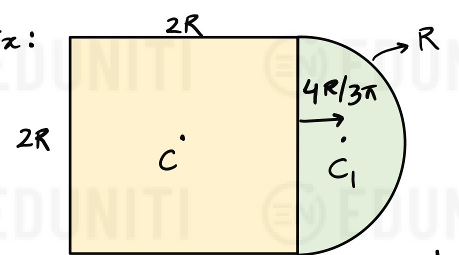
$$y = \frac{2r}{\pi} \text{ (com of dm)}$$

$$\Rightarrow y_{cm} = \frac{\int_0^R \frac{2r}{\pi} \times 2\pi r^2\,dr}{\int_0^R 2\pi r^2\,dr}$$

$$= \frac{4 r^4/4 \Big|_0^R}{2\pi \frac{r^3}{3}\Big|_0^R} = \boxed{\frac{3R}{2\pi}}$$

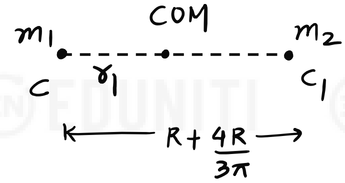
7. COM of Combined Structure

Ex:



Both are same material. Find COM from C.

Sol^n:



$\sigma = m/A$

$m = \sigma A$

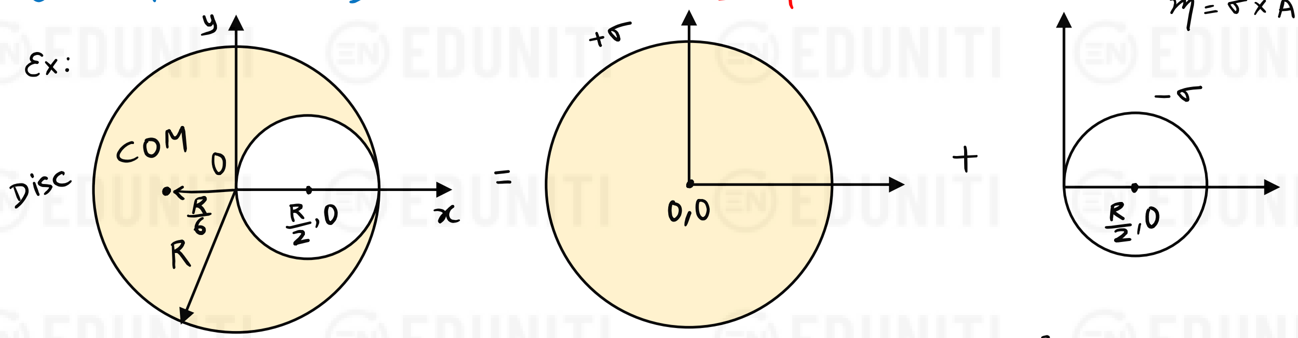
$$r_1 = \frac{m_2\left(R + \frac{4R}{3\pi}\right)}{m_1 + m_2} = \frac{\sigma \frac{\pi R^2}{2}\left(R + \frac{4R}{3\pi}\right)}{\sigma 4R^2 + \sigma \frac{\pi R^2}{2}}$$

$$= \frac{R(3\pi + 4)}{3(8 + \pi)} \text{ Ans.}$$

8. CoM in Cavity Problems

1st-Method

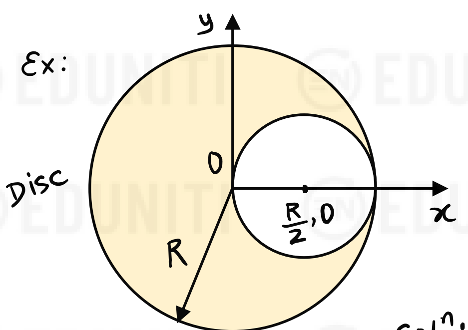
Ex:



$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{(\sigma\pi R^2 \times 0) + \left(-\sigma\pi\frac{R^2}{4}\times\frac{R}{2}\right)}{\sigma\pi R^2 - \sigma\pi\frac{R^2}{4}} = -\frac{R}{6}$$

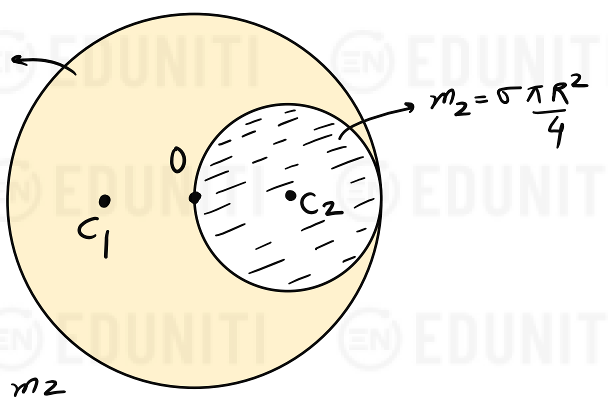
2nd- Method

Ex:

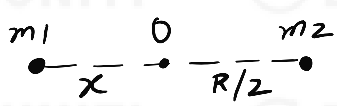


$\sigma\left(\pi R^2 - \frac{\pi R^2}{4}\right) = m_1$

$\frac{\sigma 3\pi R^2}{4}$



Sol^n:



$$m_1 x = m_2\frac{R}{2} \Rightarrow \frac{\sigma 3\pi R^2}{4}\cdot x = \frac{\sigma\pi R^2}{4}\times\frac{R}{2}$$

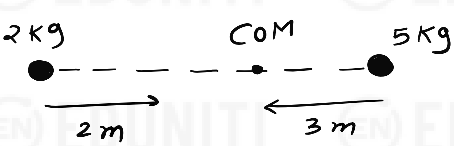
$$\Rightarrow \boxed{x = \frac{R}{6}}$$

9. Displacement of COM

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots\cdots}{m_1+m_2+\cdots\cdots}$$

$$\Rightarrow \Delta\vec{r}_c = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + \cdots}{m_1+m_2+\cdots\cdots}$$

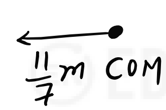
Ex:



Find displacement of COM.

Sol^n: $\Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1+m_2}$

$$= \frac{(2\times 2) + (5\times -3)}{2+5} = \frac{4-15}{7} = \boxed{\frac{-11}{7}m}$$



10. Velocity & Acceleration of COM
v_{cm} a_{cm}

$$\vec{r}_c = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1+m_2+\cdots} \xrightarrow{d\vec{r}_c/dt} \vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + \cdots}{\underbrace{m_1+m_2+\cdots}_{M}} \Rightarrow M\vec{V}_{cm} = \vec{P}_{net}$$

$\Big\downarrow d\vec{V}_{cm}/dt$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots}{\underbrace{m_1+m_2+\cdots}_{M}} \Rightarrow M\vec{a}_{cm} = \vec{F}_{net}$$

NOTE:

(i) $\vec{F}_{net} = \dfrac{d\vec{P}_{net}}{dt}$

(ii) $\vec{F}_{net} = 0 \Rightarrow \vec{P}_{net} = \text{Const.}$
$(a_{cm} = 0)$ $(\vec{V}_{cm} = \text{Const.})$

<u>Space to add concepts learnt from PYQs if any</u>

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Cons of P & Collision – PART 2

1. Understanding of Momentum Conservation
2. COM at rest question
3. Boy-Trolley Situation
4. Gun-Bullet Problem (*muzzle velocity*)
5. Explosion Concept
6. Explosion of Projectile
7. Bullet hitting Block / Bob
8. Block moving over another block
9. Two Block Spring System
10. Understanding of Head-on vs Oblique Collision
11. Concept of Impulse
12. Coefficient of Restitution
13. Collision with Floor
14. Head-on Collision
15. Oblique Collision
16. Mass-variation Concept

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Understanding of Momentum Conservation

$\vec{F}_{net} = M\vec{a}_{cm}$ & $\vec{P}_{net} = M\vec{V}_{cm}$

$\vec{F}_{net} = \dfrac{d\vec{P}_{net}}{dt}$ {M : system mass}

If $\vec{F}_{net} = 0$

$\Downarrow$

$\vec{P}_{net}$ = const.

$\Downarrow$

$\vec{V}_{cm}$ = const.

If $V_{cm} = 0$, it remain zero $\Rightarrow$ cm at rest although particles in system may move

If $V_{cm} \neq 0$ $\Rightarrow$ cm continues to move with const velocity.

2. COM at rest ($\vec{F}_{net} = 0$, $V_{cm} = 0$)

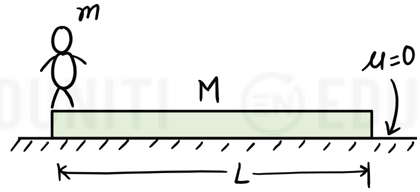
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$\Rightarrow 0 = m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + \cdots$$

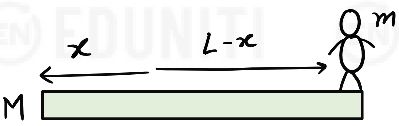
$$\{\Delta\vec{r}_{cm} = 0\}$$

here $\Delta\vec{r}_1, \Delta\vec{r}_2 \ldots$ are w.r.t. ground

Ex 1. If boy walks to other end of Plank, find distance moved by Plank.



Sol^n:

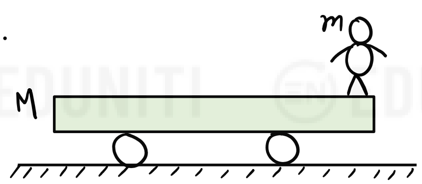


$$m_1\Delta x_1 + m_2\Delta x_2 = 0 \Rightarrow m(L-x) + M(-x) = 0$$

$$\therefore x = \frac{mL}{m+M} \quad \text{Ans}$$

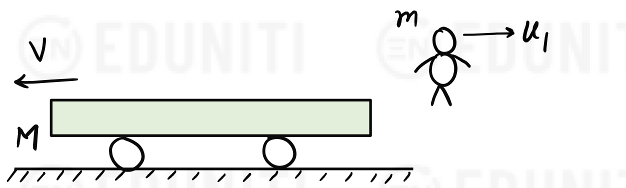
3. Boy-Trolley Sitiuation ($\vec{F}_{net} = 0$, $\vec{P}_{net} = \text{Const.}$)

Ex 2.



Initially all at rest. Boy Jumps with u (relative to trolley). Find trolley velocity.

Sol^n: NOTE – u is rel to trolley after jump



$$V_{b/T} = V_b - V_T \Rightarrow u = u_1 + V \Rightarrow u_1 = u - V$$

$$\because \vec{P}_i = 0 \Rightarrow \vec{P}_f = 0$$

$$\Rightarrow mu_1 = MV$$

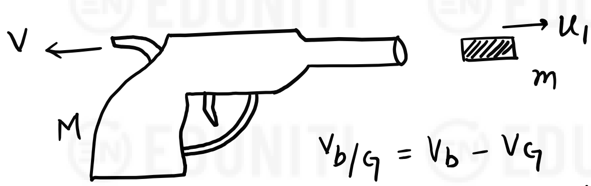
$$\Rightarrow m(u-V) = MV \Rightarrow V = \frac{mu}{m+M} \quad \text{Ans}$$

4. Gun-bullet (muzzle velocity)

Ex 3.

Muzzle velocity is u, find recoil speed of Gun.

Sol^n: Note – Muzzle vel. is vel of bullet w.r.t muzzle (end of Gun)



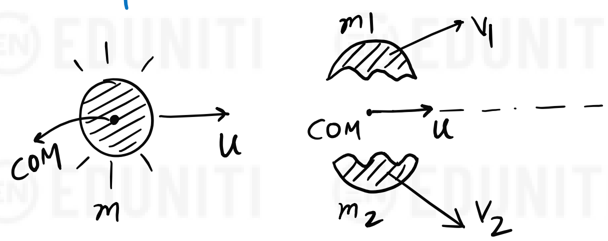
$$V_{b/G} = V_b - V_G$$

$$\Rightarrow u = u_1 + V \Rightarrow u_1 = u - V$$

$$\because \vec{P}_i = 0 \Rightarrow \vec{P}_f = 0 \Rightarrow MV = m(u-V)$$

$$\therefore V = \frac{mu}{m+M} \quad \text{Ans}$$

5. Explosion Concept



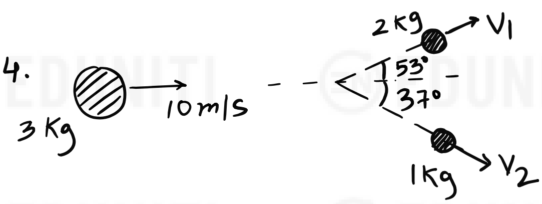
(i) Explosion due to internal forces,

$\because \sum \vec{F}_{ext} = 0 \Rightarrow \vec{P}_{sys} = \text{Const}$

$\Rightarrow \vec{V}_{cm} = \text{const}$

(ii) $m\vec{u} = m_1\vec{v}_1 + m_2\vec{v}_2$

Ex 4.



Find V_1 & V_2.

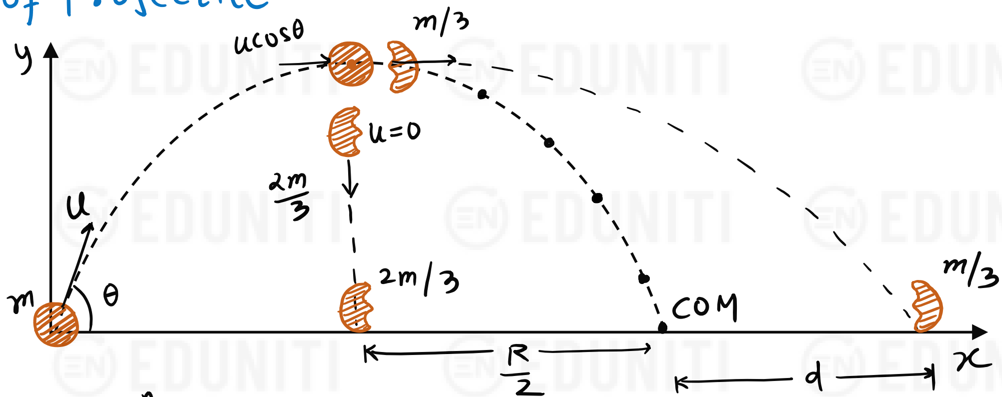
Sol^n: $\vec{P}_x = \text{Const} \Rightarrow 30 = 2V_1\cos 53^\circ + V_2\cos 37^\circ$ —(i)

$\vec{P}_y = \text{Const} \Rightarrow 0 = 2V_1\sin 53^\circ - V_2\sin 37^\circ$ —(ii)

$\therefore V_1 = 9\,m/s,\ V_2 = 24\,m/s$

6. Explosion of Projectile

Ex 5. Find distance of $\frac{m}{3}$ from origin.



Sol^n:

$\frac{2m}{3} \times \frac{R}{2} = \frac{m}{3} \times d$

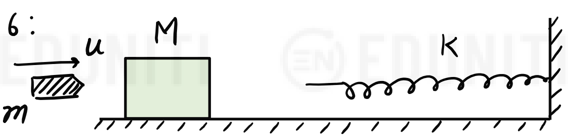
$\Rightarrow d = R$ $\therefore$ Answer $= d + R = 2R = 2 \times \frac{u^2 \sin 2\theta}{g}$

7. Bullet hitting Block/Bob

↳ many Questions are formed

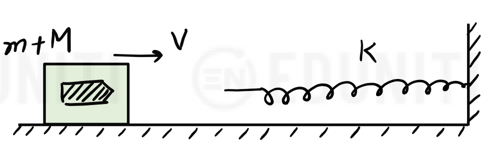
$\vec{P} = \text{Const}$ KE + P.E = Const.

Ex 6:



bullet gets embedded in block.
Find max compression in spring.

Sol^n:



$mu = (m+M)V \Rightarrow V = \frac{mu}{m+M}$

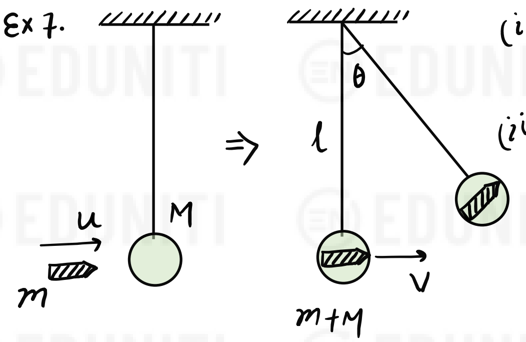
Loss in KE = Gain in PE

$\Rightarrow \frac{1}{2}(m+M)\left(\frac{mu}{m+M}\right)^2 = \frac{1}{2}Kx^2$

$\therefore x = \frac{mu}{\sqrt{K(m+M)}}$

PART 2 – Cons of P & Collison

Ex 7.



(i) $mu = (m+M)V \Rightarrow V = \frac{mu}{m+M}$

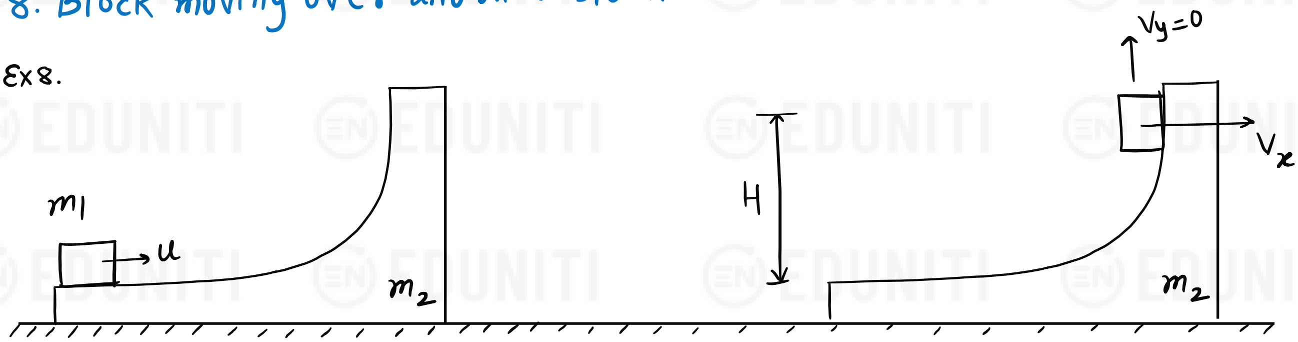
(ii) If $V \geq \sqrt{5gL} \longrightarrow$ full circle

If $V \leq \sqrt{2gL} \longrightarrow$ find θ (angular amplitude)

$\frac{1}{2}(m+M)V^2 = (m+M)g \times L(1-\cos\theta)$

8. Block moving over another block

Ex 8.



no friction any where

Let us analyse.

(i) $\vec{P}_x = \text{const} \Rightarrow m_1 u = (m_1+m_2)V_x$

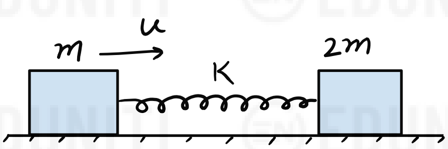
$\Rightarrow V_x = \frac{m_1 u}{m_1+m_2}$

(ii) Gain in PE = Loss in KE

$\Rightarrow m_1 g h = \frac{1}{2}m_1 u^2 - \frac{1}{2}(m_1+m_2)V_x^2$

9. Two Block – spring system

Ex 9.

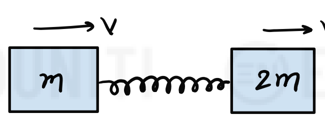


spring in natural length

Find max compression in spring.

1st Method

Solⁿ: At max comp. both block has same speed.

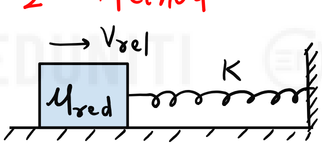


(i) $\vec{P} = \text{const} \Rightarrow mu = mV + 2mV$

$\Rightarrow V = u/3$

(ii) T.E = const $\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}mV^2 + \frac{1}{2}2mV^2 + \frac{1}{2}Kx^2 \Rightarrow x = u\sqrt{\frac{2m}{3K}}$

2nd Method



$\mu_{red} = \frac{m_1 m_2}{m_1+m_2}$

Loss in KE = Gain in PE $\Rightarrow \frac{1}{2}\mu_{red}V_{rel}^2 = \frac{1}{2}Kx^2$

$\Rightarrow \frac{1}{2}\left(\frac{m \times 2m}{3m}\right) \times u^2 = \frac{1}{2}Kx^2 \quad \therefore x = u\sqrt{\frac{2m}{3K}}$

10. Understanding of Head-on vs Oblique Collision

Head-on → LOM & LOI are same

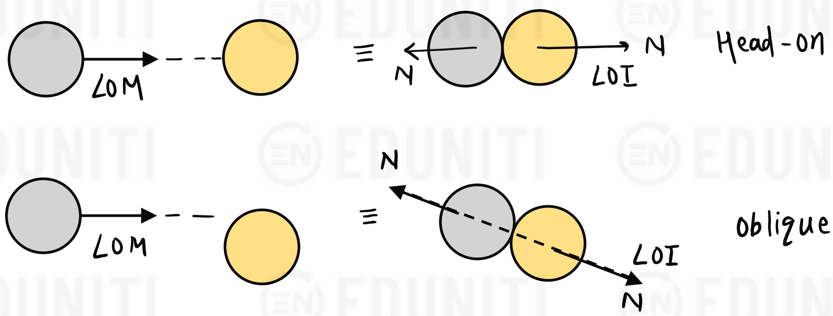
Oblique → LOM & LOI are different

LOM → Line of Motion

LOI → Line of Impact

(a)

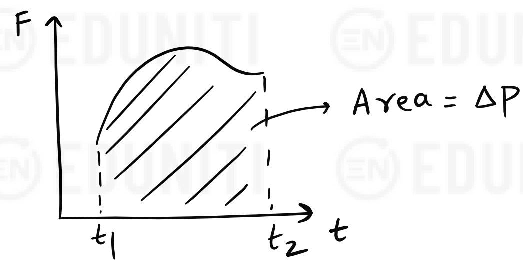
(b)



11. Concept of Impulse (J) ⟶ Change in P due to F ($t_1 \to t_2$)

$F = \frac{dp}{dt} \Rightarrow dp = F\,dt$

- $F = \text{Const.}$: $\Delta P = F\Delta t$
- $F = f(t)$: $\Delta P = \int_{t_1}^{t_2} F\,dt$ → J



(i) Impulsive force ⟶ large force acting for short time (they bring ΔP)

NOTE: During collision / impact mg is taken as non-impulsive (no Δp due to mg)

12. Coefficient of Restitution (e)

$e = \frac{\text{Vel of separation of pt. of contact}}{\text{Vel of approach of pt. of contact}}$ {Vel along Line of impact}

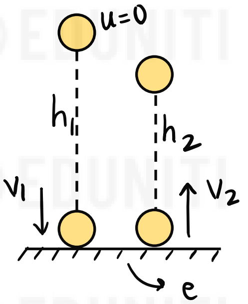
- Elastic: $e = 1$
- Partial elastic: $0 < e < 1$
- Inelastic: $e = 0$

} collision on basis of KE loss

(a) 4 m/s, 2 m/s ⇒ 3 m/s, 4 m/s $\quad e = \frac{4-3}{4-2} = \frac{1}{2}$

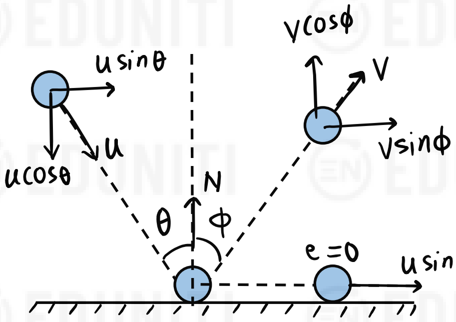
(b) 4 m/s, 1 m/s ⇒ 3 m/s, 5 m/s $\quad e = \frac{5-3}{4+1} = \frac{2}{5}$

13. Collision with Floor



(i) $e = \frac{v_2}{v_1} \Rightarrow v_2 = e v_1 \quad \{ v_1 = \sqrt{2gh_1} \}$

(ii) $h_2 = \frac{v_2^2}{2g} = \frac{e^2 v_1^2}{2g} = e^2 h_1$



$v\sin\phi = u\sin\theta$ — (1)
(as no Normal is $\perp$ to it)

(i) If $e=1 \Rightarrow v = u$ & $\phi = \theta$

(ii) If $0<e<1 \Rightarrow v\cos\phi = eu\cos\theta$ — (2)

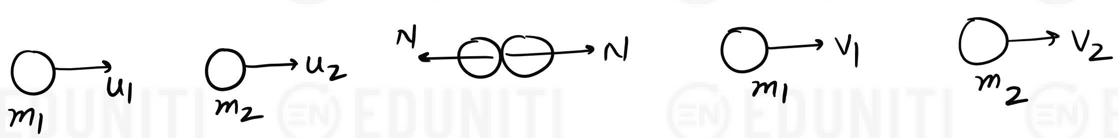
$(1)/(2) \Rightarrow \boxed{\tan\phi = \frac{\tan\theta}{e}}$

$(1)^2 + (2)^2 \Rightarrow \boxed{v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}}$

(iii) If $e=0 \Rightarrow v\cos\phi = 0$
Ball moves horizontally

(iv) Impulse, $\Delta p = m(v\cos\phi + u\cos\theta)$
$\{ J = 2mu\cos\theta, \ e=1 \}$

14. Head-on Collision



(i) $P_i = P_f \Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

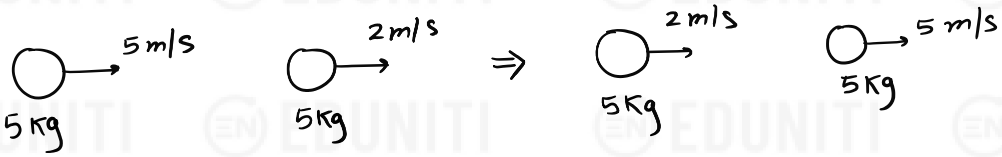
(ii) $e = \frac{v_2 - v_1}{u_1 - u_2}$

using these two eqⁿ find v_1 & v_2

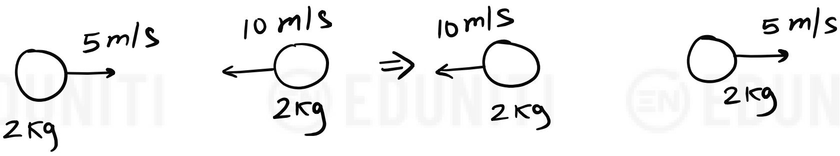
- $e=1$: KE = Const, $K_i = K_f$
- $0<e<1$: KE Loss, $K_i - K_f$
- $e=0$: KE Loss Max, $K_i - K_f$

Note: If $m_1 = m_2$ & $e = 1$, then after collision bodies exchange velocities

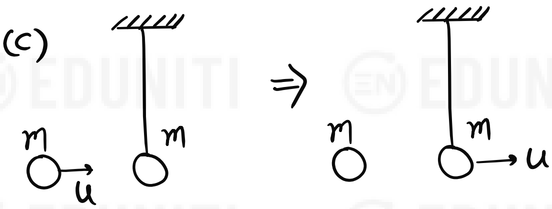
(a)



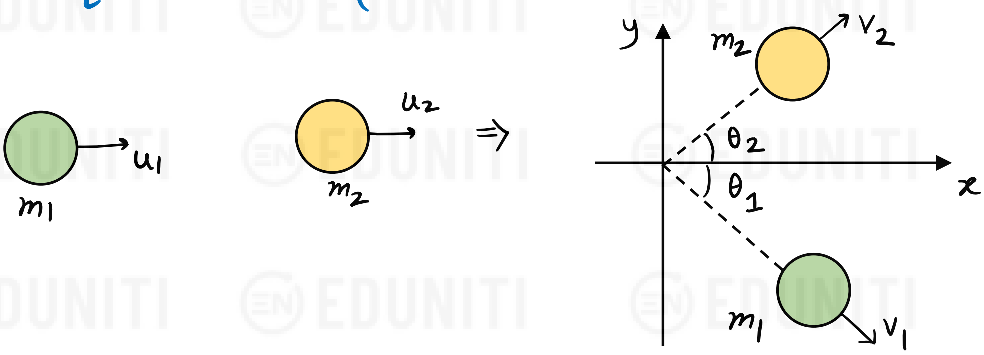
(b)



(c)



15. Oblique Collision (elastic, $e=1$) → 2D Collision



(i) $\vec{P}_x = \text{const} \Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$

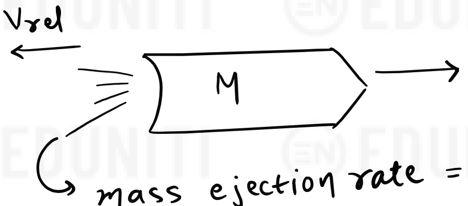
(ii) $\vec{P}_y = \text{const} \Rightarrow 0 = m_2 v_2 \sin\theta_2 - m_1 v_1 \sin\theta_1$

(iii) $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Note: If $m_1 = m_2$, $u_2 = 0$, elastic collision

$\Rightarrow \boxed{\theta_1 + \theta_2 = 90^\circ}$

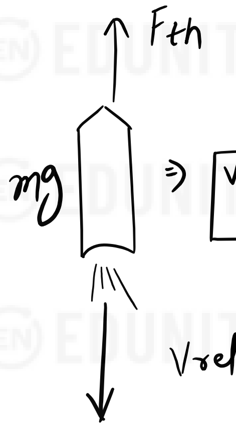
16. Mass - Variation



$F_{th} = V_{rel}\dfrac{dm}{dt}$

mass ejection rate $= \dfrac{dm}{dt}$ kg/s

Rocket Propulsion Equation



$\Rightarrow \boxed{v = u - gt + V_{rel}\ln\dfrac{m_i}{m_f}}$

<u>Space to add concepts learnt from PYQs if any</u>

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in ROTATIONAL MOTION – PART 1

1. Translatory vs Rotatory Motion
2. Moment of Inertia
3. Parallel & Perpendicular Axis Theorem
4. MOI of Standard Bodies (*uniform mass distribution*)
5. Addition or Subtraction of MOI
6. Radius of Gyration
7. Torque

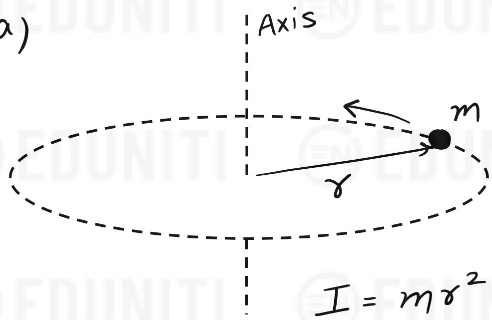
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Translatory vs Rotatory Motion

Refer animation shown in our revision video (Rotation-Part 1)

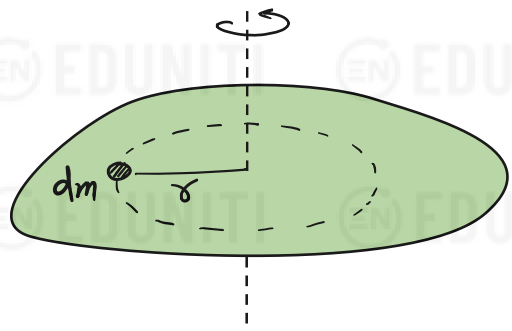
2. Moment of Inertia (I)

(a)



$I = mr^2$

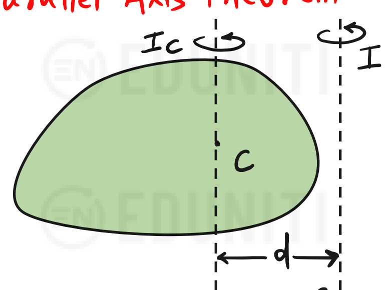
(b)



$dI = dm \cdot r^2 \Rightarrow I = \int dm \cdot r^2$

3. PARALLEL AND PERPENDICULAR AXIS THEOREM

Parallel Axis Theorem

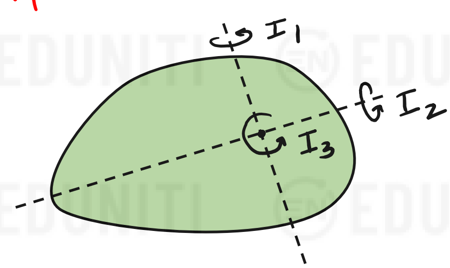


(a) $I = I_c + md^2$

(b) Applicable always

(c) I_c is MOI about an axis passing C.O.M.

Perpendicular Axis Theorem



(a) $I_3 = I_1 + I_2$

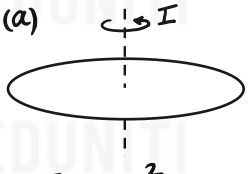
(b) Applicable for plane-Lamina

(c) I_1 and I_2 are about axis which are in the plane of body.

4. MOI of Standard bodies

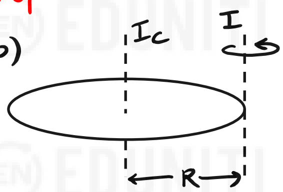
(i) MOI OF RING

(a)



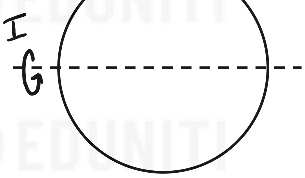
$I = MR^2$

(b)



$I = 2MR^2$

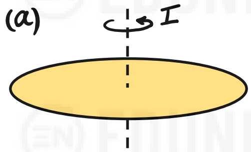
(c)



$I = MR^2/2$

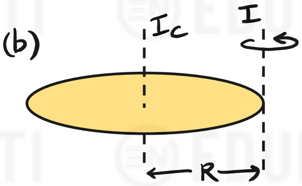
(ii) MOI OF DISC

(a)



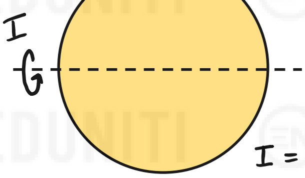
$I = MR^2/2$

(b)



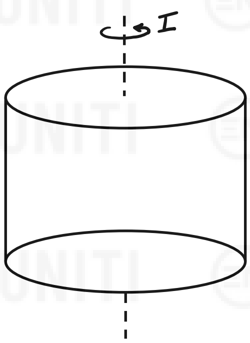
$I = \frac{3MR^2}{2}$

(c)



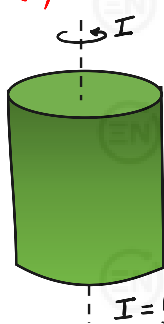
$I = \frac{MR^2}{4}$

(iii) MOI OF HOLLOW CYLINDER

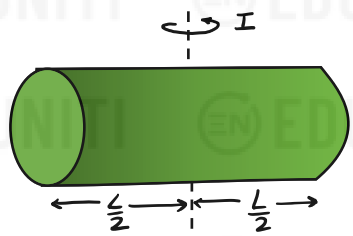


$I = MR^2$

(iv) MOI OF SOLID CYLINDER



$I = \frac{MR^2}{2}$



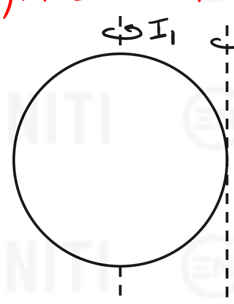
$I = \frac{MR^2}{4} + \frac{ML^2}{12}$

(v) MOI OF THIN ROD

$I_1 = ML^2/12$

$I_2 = ML^2/3$

(vi) MOI OF HOLLOW SPHERE



$I_1 = \frac{2}{3}MR^2$

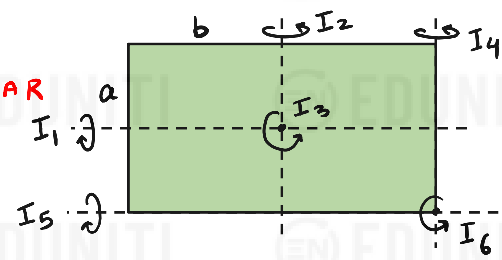
$I_2 = \frac{5}{3}MR^2$

(vii) MOI OF SOLID SPHERE

$I_1 = \frac{2}{5}MR^2$

$I_2 = \frac{7}{5}MR^2$

(viii) MOI OF RECTANGULAR SHEET



$I_1 = Ma^2/12$

$I_2 = Mb^2/12$

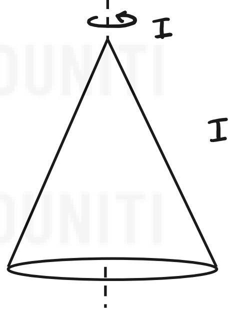
$I_3 = I_1 + I_2$

$I_4 = Mb^2/3$

$I_5 = Ma^2/3$

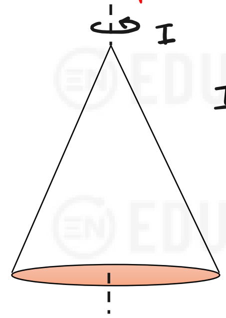
$I_6 = I_4 + I_5$

(ix) MOI OF HOLLOW CONE



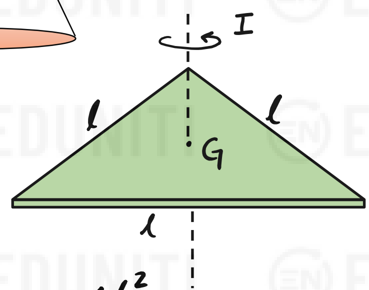
$I = \frac{MR^2}{2}$

(x) MOI OF SOLID CONE



$I = \frac{3MR^2}{10}$

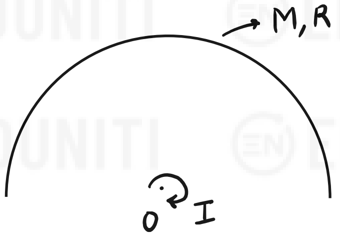
(xi) MOMENT OF INERTIA OF EQUILATERAL TRIANGLE SHEET (about Centroid)



$I = \frac{Ml^2}{12}$

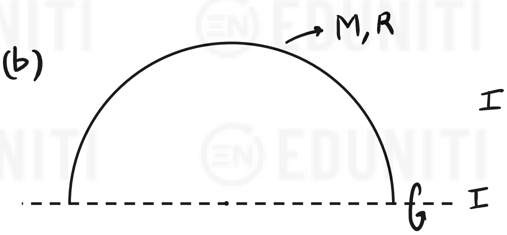
(xii) MOI OF SEMICIRCULAR RING

(a)



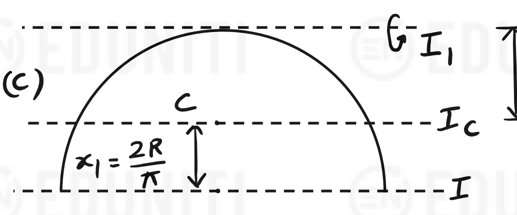
$I = MR^2$

(b)



$I = \frac{MR^2}{2}$

(c)



$R - \frac{2R}{\pi} = x_2$

Find I_1 ?

$I = I_c + Mx_1^2$ — (1)

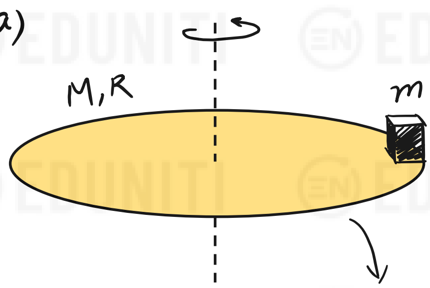
$I_1 = I_c + Mx_2^2$ — (2)

from (2) and (1) :

$I_1 = I + M(x_2^2 - x_1^2)$

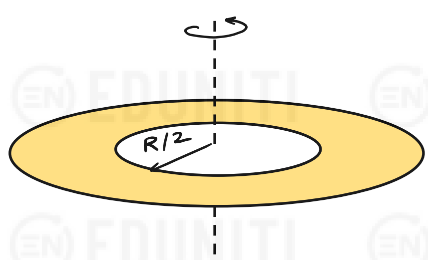
5. Addition or Subraction of MOI

(a)



$I = \frac{MR^2}{2} + mR^2$

(b)

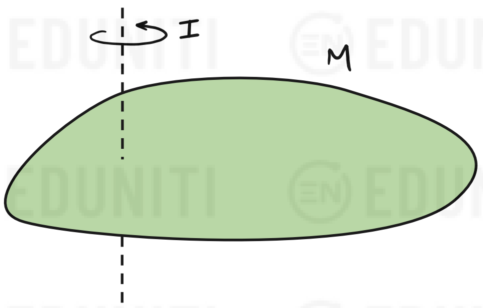


$I = \frac{MR^2}{2}$ — MOI of removed Disc

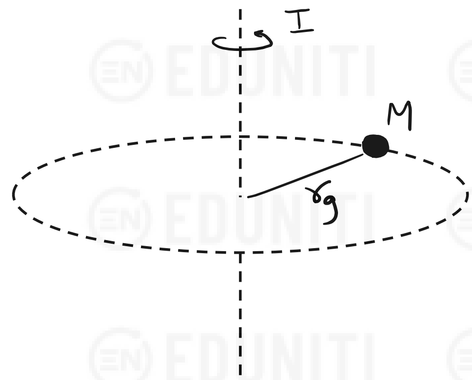
$= \frac{MR^2}{2} - \left(\frac{M}{\pi R^2} \times \frac{\pi R^2}{4}\right)\left(\frac{R}{2}\right)^2 \times \frac{1}{2}$

$= \frac{MR^2}{2} - \frac{MR^2}{32} = \frac{15}{32}MR^2$

6. Radius of Gyration



$\equiv$

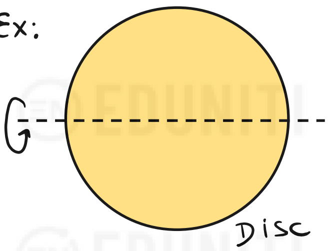


$I = M r_g^2$

$\Rightarrow r_g = \sqrt{\dfrac{I}{M}}$

Ex:

$\dfrac{MR^2}{4} = I$



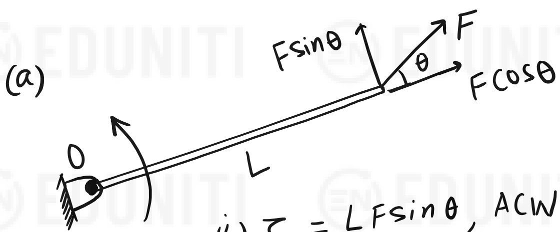
Disc

$r_g = \sqrt{\dfrac{MR^2/4}{M}}$

$= \dfrac{R}{2}$

7. Torque ($\vec{\tau} = \vec{r} \times \vec{F}$), N-m

(a)



(i) $\tau = LF\sin\theta$, ACW

(ii) $F\cos\theta$ passes through O, can't have rotational effect

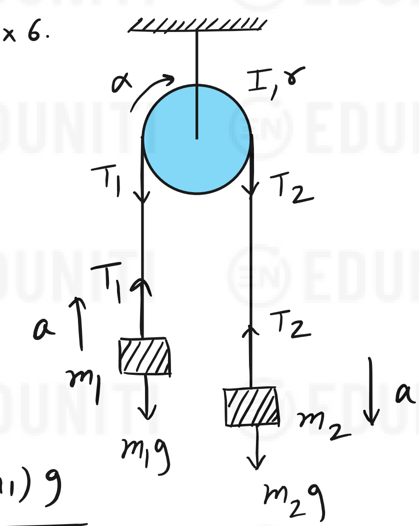
$\Sigma\vec{\tau} = I\vec{\alpha}$ → angular acc^n

Static Equillibrium Problems

$\Sigma\vec{\tau} = 0$ and $\Sigma\vec{F} = 0$

both ω and $V_{cm} = 0$

Ex 6.



Sol^n: $m_2 g - T_2 = m_2 a$ --(i)

$T_1 - m_1 g = m_1 a$ --(ii)

$T_2 r - T_1 r = I\alpha$

$\Rightarrow (T_2 - T_1) r = I \cdot \dfrac{a}{r}$

$T_2 - T_1 = \dfrac{Ia}{r^2}$ --(iii)

(i) + (ii) + (iii)

$(m_2 - m_1) g = a\left(m_1 + m_2 + \dfrac{I}{r^2}\right) \therefore a = \dfrac{(m_2 - m_1) g}{m_1 + m_2 + \dfrac{I}{r^2}}$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

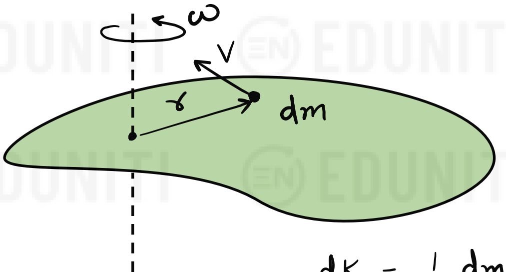
Topics to cover in ROTATIONAL MOTION – PART 2

1. K.E. in Rotational Motion
2. Angular Momentum
3. Conservation of Angular Momentum (*Impulse Equation*)
4. Standard Examples of L = Const.
5. Fixed & Free Collision
6. Hinge Reaction

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. KE in Rotational Motion

(a)



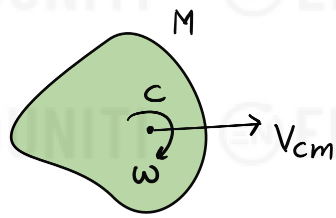
$$dK = \frac{1}{2}dm.V^2$$

$$= \frac{1}{2}dm.(\omega r)^2$$

$$\Rightarrow K = \frac{1}{2}\omega^2 \int dm.r^2$$

$$\therefore \boxed{K = \frac{1}{2}I\omega^2}$$

(b)

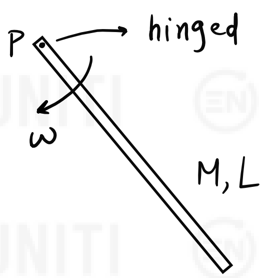


here Axis of rotation passes com

$$K = K_R + K_T$$

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}MV_{cm}^2$$

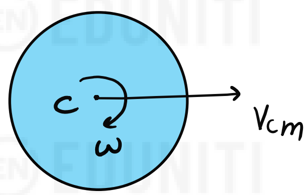
(i)



$$KE = \frac{1}{2}I_P\omega^2$$

$$= \frac{1}{2}.\frac{ML^2}{3}.\omega^2$$

(ii)

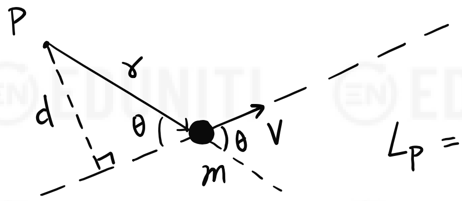


$I_c = kmr^2$, where k =
- 1 if Ring / Hollow cyl.
- $\frac{1}{2}$ if Disc / solid cyl.
- $\frac{2}{3}$ if Hollow sphere
- $\frac{2}{5}$ if solid sphere

$$KE = \frac{1}{2}I_c\omega^2 + \frac{1}{2}mV_{cm}^2$$

2. Angular Momentum (L)

(a) For point mass



$L_p = mvd$

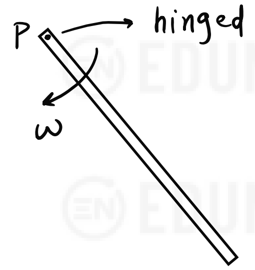
$\# \vec{L} = \vec{r} \times \vec{P}$
$= \vec{r} \times m\vec{v}$

$\therefore L = mv\underbrace{r\sin\theta}_{d}$

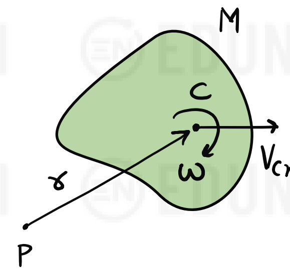
↳ for above fig: dir^n ⊙

(b) Rigid body

Pure rotation | rotation + translation



$L_p = I_p\omega$



$\vec{L}_p = I_{cm}\vec{\omega} + \vec{r} \times \vec{P}_{cm}$

3. Conservation of Angular Momentum

(a) $\tau = \dfrac{dL}{dt}$

If $\tau_{ext} = 0 \Rightarrow L$ is const. or conserved

{ similar to $F = \dfrac{dp}{dt}$ }

(b) If $\tau_{ext} \neq 0$

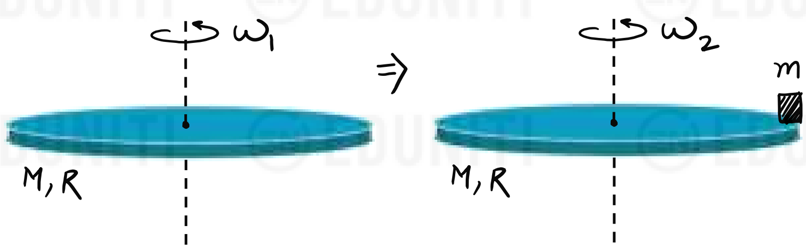
$\Rightarrow \tau\, dt = dL$

or $\boxed{\int \tau\, dt = \Delta L}$

↳ Angular impulse $= \Delta L$

4. Standard Examples of $L =$ const.

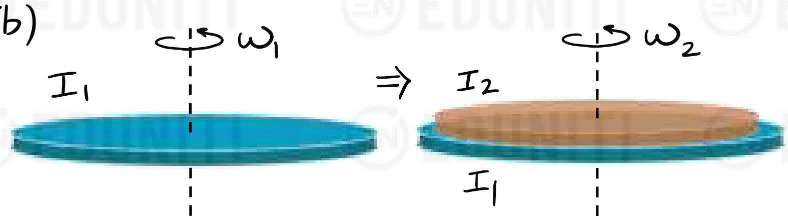
(a)



$L_i = L_f$

$\Rightarrow \dfrac{MR^2}{2}\omega_1 = \left(\dfrac{MR^2}{2} + mR^2\right)\omega_2$

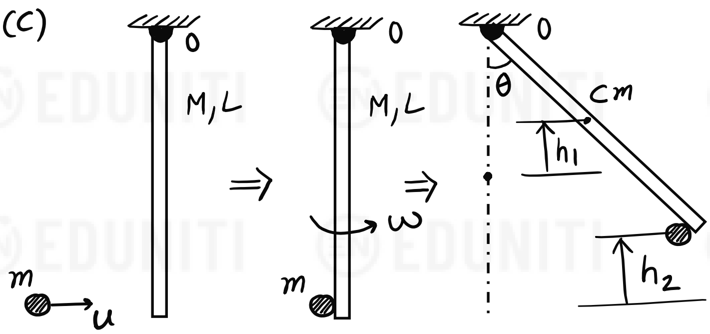
(b)



(i) $I_1\omega_1 = (I_1 + I_2)\omega_2$

(ii) Loss in KE $= \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}(I_1 + I_2)\omega_2^2$

(c)



$$muL = \left(\frac{ML^2}{3} + mL^2\right)\omega \quad \text{—(i)}$$

Gain in PE = Loss in KE

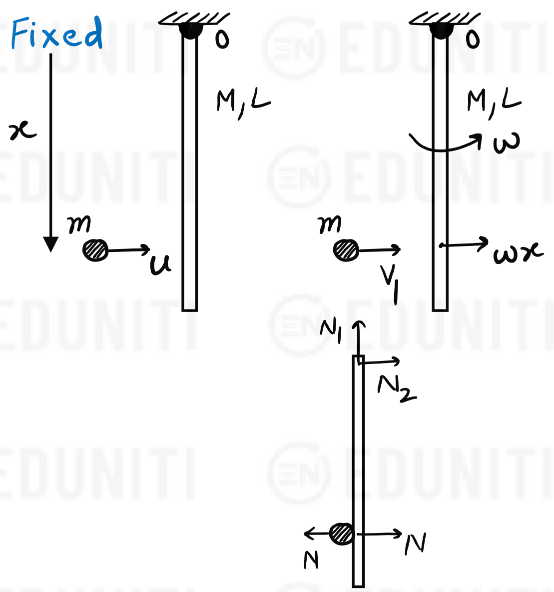
$$\Rightarrow Mg\underbrace{h_1}_{\frac{L}{2}(1-\cos\theta)} + mg\underbrace{h_2}_{L(1-\cos\theta)} = \frac{1}{2}\left(\frac{ML^2}{3} + mL^2\right)\omega^2$$

↳ ∵ τ_{net} about O is zero during impact

⇒ L is same just before & just after

5. Fixed & Free Collision (e = coefficient of restitution)

(a) Fixed



P is not conserved

$L_i = L_f$

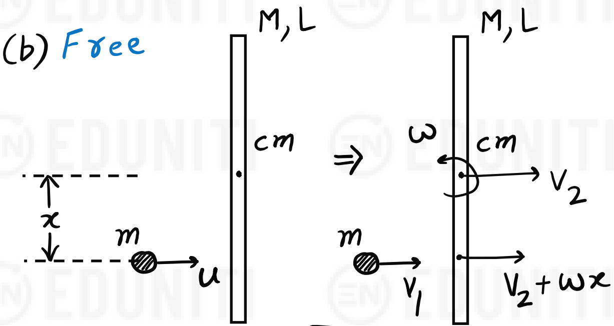
$$\Rightarrow mux = mv_1x + \frac{ML^2}{3}\omega \quad \text{—(i)}$$

$$e = \frac{\text{Vel of seperation}}{\text{Vel of approach}}$$

$$e = \frac{\omega x - v_1}{u} \quad \text{—(ii)}$$

Hence solve (i) & (ii) to get v_1 and ω

(b) Free



System on smooth Horizontal surface

(i) L is cons. about any point

About cm : $L_i = L_f$

$$\Rightarrow mux = mv_1x + \frac{ML^2}{12}\omega \quad \text{— ①}$$

(ii) P is cons. (As $F_{net} = 0$)

$$P_i = P_f \Rightarrow mu = mv_1 + Mv_2 \quad \text{— ②}$$

(iii) $$e = \frac{v_2 + \omega x - v_1}{u} \quad \text{— ③}$$

solving ①, ② & ③ we get v_1, v_2 and ω

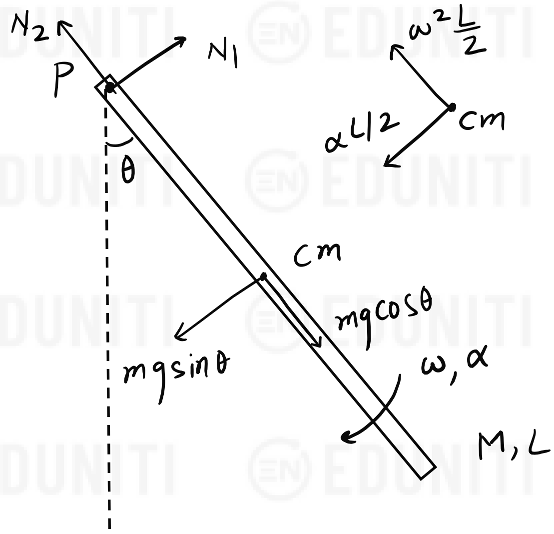
6. Hinge reaction

(i) Always take 2 ⊥ components of Hinge rkⁿ

↳ $mg\sin\theta - N_1 = m\alpha\frac{L}{2}$

↳ $N_2 - mg\cos\theta = m\omega^2\frac{L}{2}$

$\therefore$ $rk^n = \sqrt{N_1^2 + N_2^2}$



Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

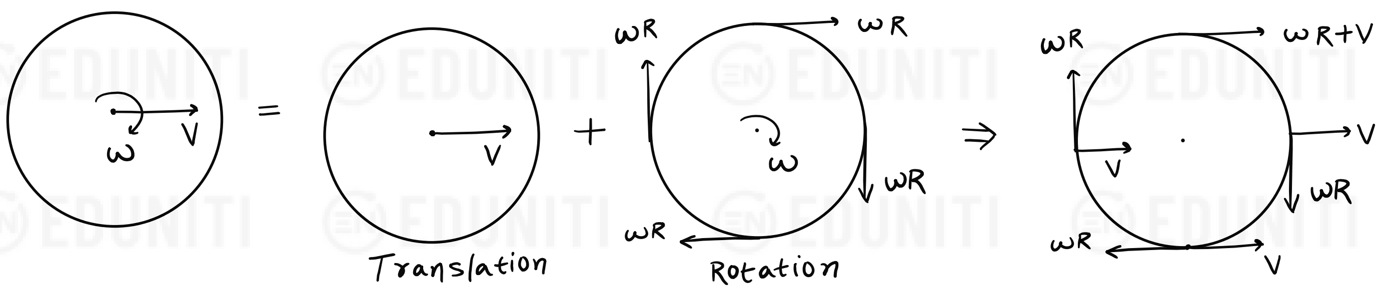
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in ROTATIONAL MOTION – PART 3

1. Velocity of a Point on Circumference (Trans + Rotational)
2. Acceleration of a Point of Circumference
3. Rolling Motion (*pure rolling & rolling with slipping*)
4. Total Kinetic Energy in Pure Rolling
5. External Force in Rolling
6. Energy Conservation (*pure rolling on Inclined*)
7. Acceleration & Friction (*Rolling on Inclined Plane*)
8. Toppling

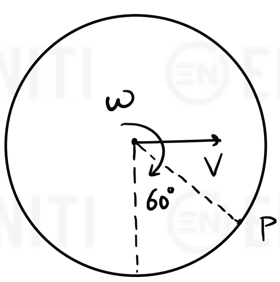
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Velocity of Point on Circumference (Trans + Rotational)

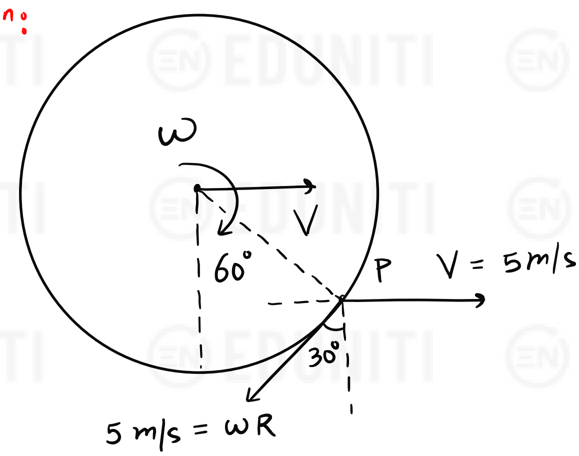


$V = 5\,m/s,\ \omega = 2.5\,rad/s,\ R = 2m$. find V_P ?

Ex 1.

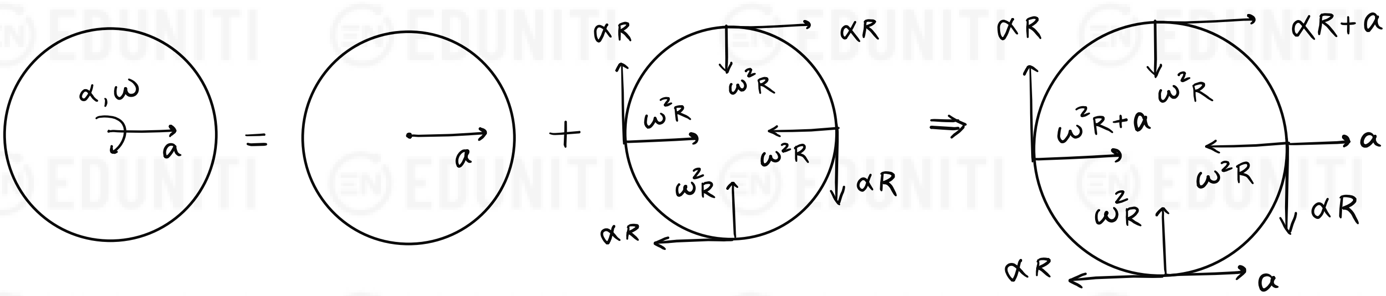


sol^n:



$$V_P = \sqrt{5^2 + 5^2 + 2\cdot 5^2 \underbrace{\cos 120^\circ}_{-\frac{1}{2}}} = \boxed{5\,m/s}$$

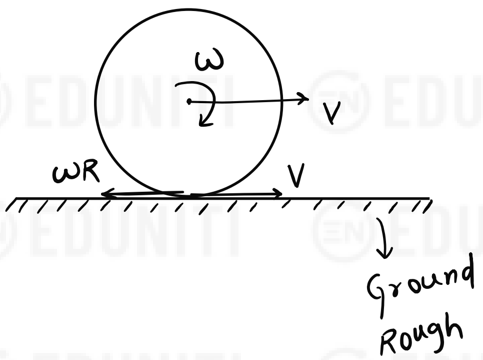
2. Acceleration of Point on circumference



3. Rolling Motion

(a) Pure rolling : No relative motion of Pt. of Contact or Pt. of Contacts have same Velocity.

$\because V_{ground} = 0 \Rightarrow V - \omega R = 0 \quad \therefore \boxed{V = \omega R}$



(i) $V = \omega R \quad (\therefore a = \alpha R)$

(ii) No friction force acting

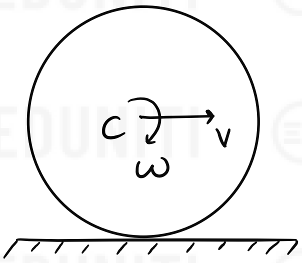
(b) Rolling with slipping : $V \neq \omega R$

↳ If $V > \omega R \Rightarrow f_K$ acts backward

↳ If $\omega R > V \Rightarrow f_K$ acts forward

NOTE: f_K until Pure rolling starts

4. KE in Pure Rolling $(V = \omega R)$



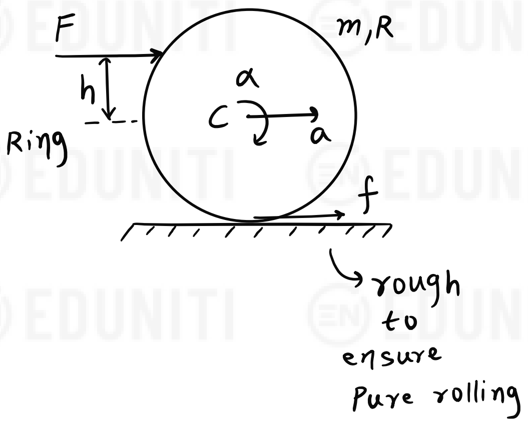
$I_C = k m R^2$

- 1 if Ring or Hollow Cylinder
- $\frac{1}{2}$ if Disc or solid Cylinder
- $\frac{2}{3}$ if Hollow sphere
- $\frac{2}{5}$ if solid sphere

$$KE = K_T + K_R = \frac{1}{2} m v^2 + \frac{1}{2} \cdot k m R^2 \cdot \left(\frac{V}{R}\right)^2$$

$$\therefore \boxed{K.E. = \frac{1}{2} m v^2 (1 + k)}$$

5. External Force in Rolling



NOTE: (i) ∵ Pure rolling ⇒ f is static

(ii) ∵ Pt. of Contact is at instantaneous rest ⇒ $W_f = 0$

$$F + f = ma \quad — ①$$

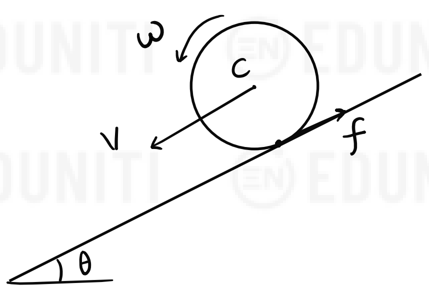
$$Fh - fR = mR^2 \cdot \alpha \qquad \left\{ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow Fh - fR = mR^2 \cdot \frac{a}{R} \quad — ②$$

Solve ① & ② to get a and f.

6. Energy Conservation (pure rolling on Inclined)

↳ $v = \omega R$ & $a = \alpha R$



NOTE: (i) $v \uparrow$ due to gravity

(ii) Thus ω must ↑ so that $v = \omega R$

(iii) Thus f acts (creating Torque about C)

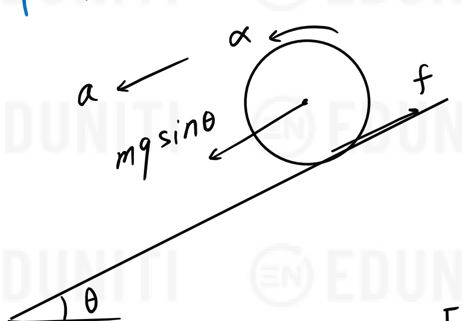
↓

Static ⇒ $W_f = 0$

⇓

Energy Cons. is applicable

7. Acceleration & Friction – Rolling on Inclined Plane



$$mg\sin\theta - f = ma \quad — ①$$

$$fR = kmR^2 \cdot \alpha \qquad \left\{ \alpha = \frac{a}{R} \right\}$$

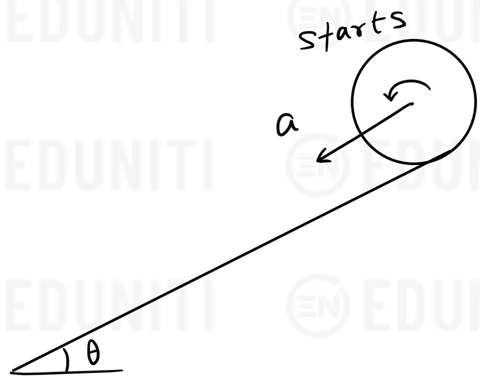
$$\Rightarrow fR = kmR^2 \cdot \frac{a}{R} \Rightarrow f = kma \quad — ②$$

From ① and ② :

$$\boxed{a = \frac{g\sin\theta}{1+k}} \quad \& \quad \boxed{f = \frac{kmg\sin\theta}{1+k}}$$

$a = \dfrac{g\sin\theta}{1+k}$ { Lower $k \Rightarrow$ higher a

Ring $k=1$ | Disc $k=\frac{1}{2}$ | H.sphere $k=\frac{2}{3}$ | S.sphere $k=\frac{2}{5}$

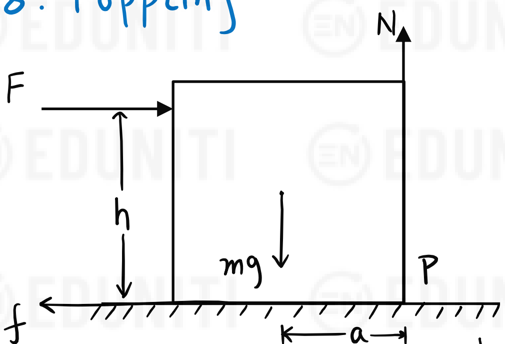


$\Rightarrow a_{S.S} > a_{disc} > a_{H.S} > a_{ring}$

$\Downarrow$

Solid sphere reaches 1^{st} if all starts together

8. Toppling



↪ friction enough to Prevent sliding

Condition for Toppling to begin

↪ at limiting case N passes pt. P

for toppling about P

$\tau_F > \tau_{mg}$

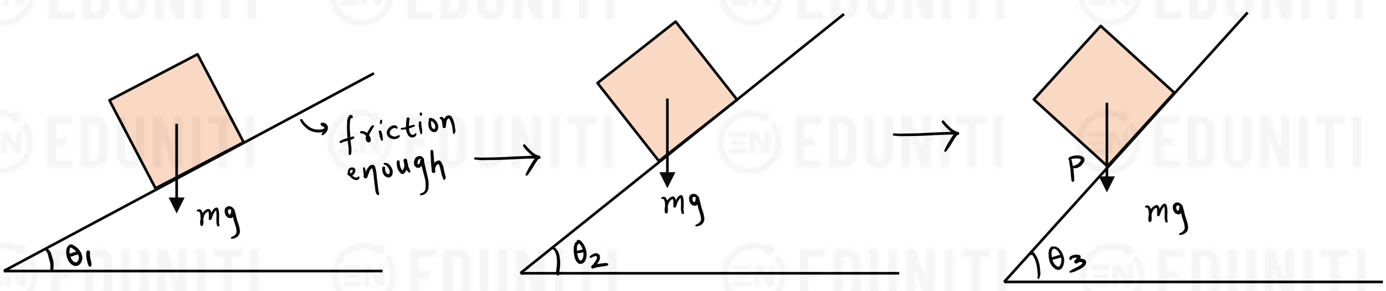
$\Rightarrow Fh > mg\,a$

$\Rightarrow F > mg\dfrac{a}{h}$

$\therefore \boxed{F_{min} = \dfrac{mga}{h}}$

Increase θ (Toppling on incline)

θ_3 is limiting angle.



<u>Space to add concepts learnt from PYQs if any</u>

Topics to cover in Properties of Solids | Elasticity

1. Elastic body vs Plastic body
2. Stress & Strain in elastic material *(example)*
3. Relation between Stress & Strain *(Hooke's Law)*
4. Modulus of Elasticity *(standard example)*
5. Analogy with springs
6. Potential Energy
7. Elongation due to Self weight
8. Poisson Ratio
9. Relation among Modulus of elasticity

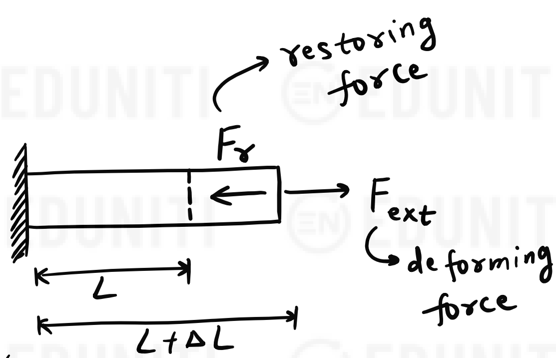
1. Elastic vs Plastic body

↳ body that regains orginal shape after removal of deforming force

Ex: steel

↳ body doesn't regain shape

Ex: dough, clay



↳ At equillibrium condition

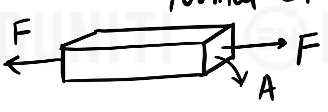
$F_{ext} = F_r$

2. Stress & strain (elastic material)

↳ due to restoring force

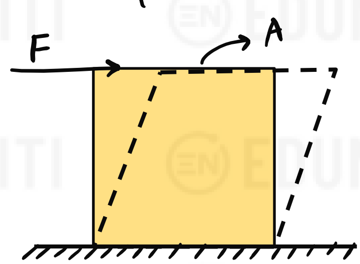
Stress

Normal stress



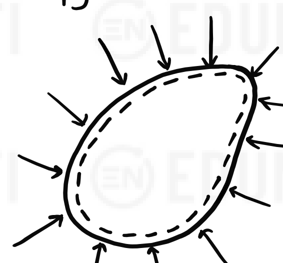
Stress $= \frac{F}{A}$

Shear stress



stress $= \frac{F}{A}$

Hydraulic stress

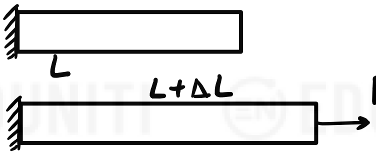


Here it is ΔP or P_{excess}

... continued

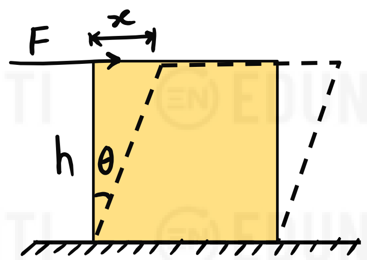
strain → Normal strain, Shear strain, Volume strain

Normal strain



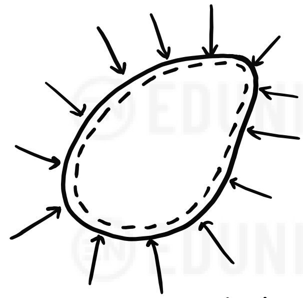
$\text{strain} = \frac{\Delta L}{L}$

Shear strain



$\text{strain} = \theta = \frac{x}{h}$

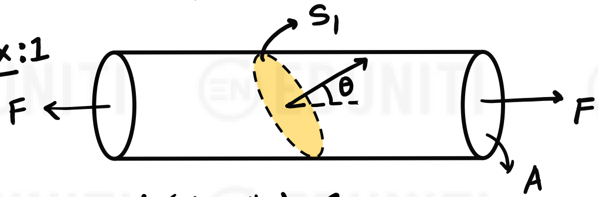
Volume strain



$\text{strain} = \frac{\Delta V}{V}$

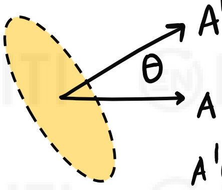
... continued

Ex:1



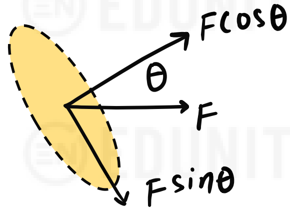
find Normal (tensile) & shear stress on S_1.

Solⁿ:



$A'\cos\theta = A$

$\Rightarrow A' = \frac{A}{\cos\theta}$



(i) Tensile stress

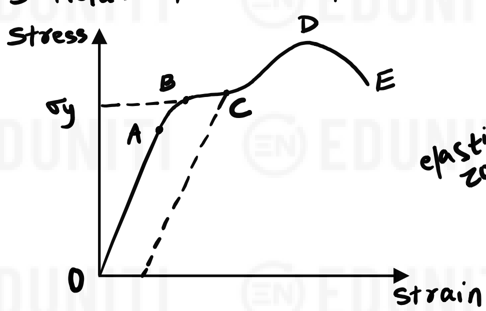
$= \frac{F\cos\theta}{A/\cos\theta}$

$= \frac{F\cos^2\theta}{A}$

(ii) shear stress

$= \frac{F\sin\theta}{A/\cos\theta} = \frac{F\sin 2\theta}{2A}$

3. Relation between Stress & strain (Hooke's Law)



elastic zone:

(i) 0 → A : Stress ∝ strain (Hooke's Law)

$\Rightarrow$ stress = k × strain

↳ Modulus of elasticity

(ii) A → B : non-Linear but still elastic

B point is elastic Limit

σ_y is yield strength

plastic zone:

(iii) B → D : strain ↑ rapidly and if deforming force removed, strain ≠ 0

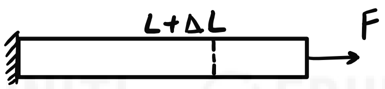
D pt. ultimate tensile strength

E → fracture pt.

NOTE: (a) Brittle if D & E are close and Ductile if D & E are far.

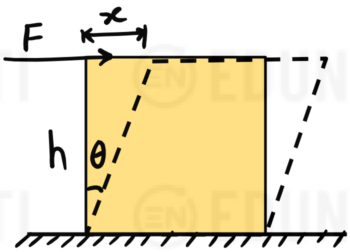
4. Modulus of Elasticity

Young's Modulus



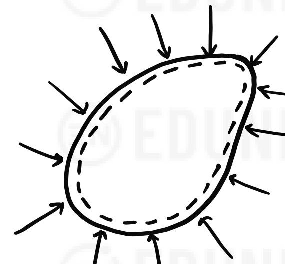
$$Y = \frac{F/A}{\Delta L/L}$$

Shear Modulus or Modulus of rigidity



$$G = \frac{F/A}{x/h}$$

Bulk Modulus

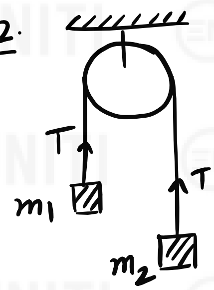


$$\beta = \frac{\Delta P}{-\Delta V/V} \text{ or } -V\frac{dP}{dV}$$

Compressibility, $K = \frac{1}{\beta}$

... continued

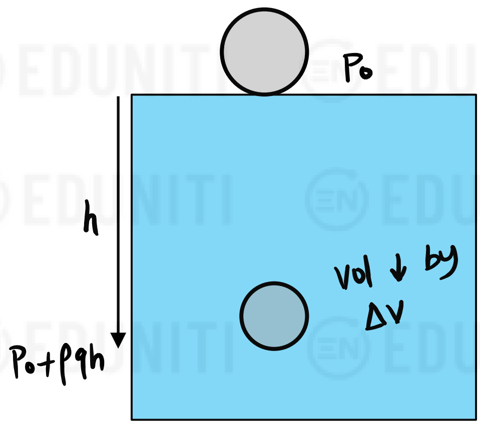
Ex2.



$\therefore \frac{T}{A} = Y \cdot \frac{\Delta L}{L}$

$T = \frac{2m_1m_2g}{m_1+m_2}$

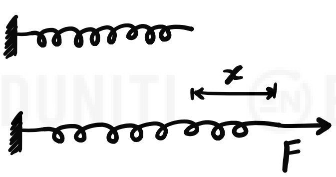
Ex3.



$\Delta P = \beta \times \frac{\Delta V}{V} \Rightarrow \Delta V = \frac{V}{\beta} \cdot \Delta P$

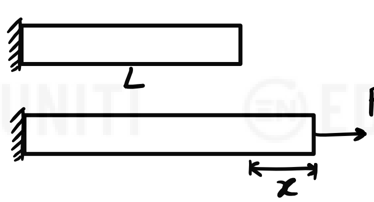
$\Rightarrow \Delta V = \frac{V}{\beta} \cdot \rho g h$

5. Analogy with springs



$F = kx$

$\equiv$

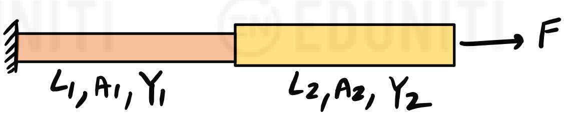


$\frac{F}{A} = Y \cdot \frac{x}{L} \Rightarrow F = \boxed{\frac{YA}{L}} x$

Equivalent force constant

$k = \frac{YA}{L}$

Ex4.



$K_1 = \frac{Y_1A_1}{L_1}$, $K_2 = \frac{Y_2A_2}{L_2}$ $\therefore K_{eq} = \frac{K_1K_2}{K_1+K_2}$

$\Rightarrow F = K_{eq} \cdot x$

6. Potential Energy

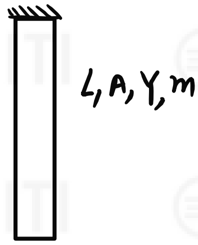
(a) Energy density, $u = \frac{1}{2} \times \text{stress} \times \text{strain}$
J/m^3

$\Rightarrow$ Energy, $U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$

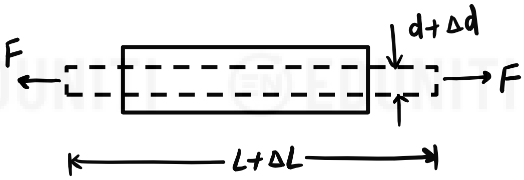
Note: $U = \frac{1}{2} K x^2 = \frac{1}{2} \frac{YA}{L} \cdot x^2$

7. Elongation due to Self weight

$x = \frac{MgL}{2YA}$



8. Poisson ratio, σ



$\sigma = -\frac{\Delta d/d}{\Delta L/L}$

9. Relation among Y, G & B

(a) $B = \frac{Y}{3(1-2\sigma)}$

(b) $G = \frac{Y}{2(1+\sigma)}$

from (a) & (b) $\rightarrow$ (c) $B = \frac{YG}{9G - 3Y}$ $\left\{ \begin{array}{l} \text{this Question} \\ \text{asked in} \\ \text{JEE 2021, Feb} \end{array} \right.$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID STATICS – PART 1

1. Introductory Points *(density, pressure)*
2. Pressure Variation *(vessel at rest)*
3. Free Surface *(vessel at rest, vessel accelerated linearly, vessel rotating)*
4. Pressure Variation *(vessel linearly accelerated, rotating)*
5. Pascal's Law *(hydraulic lift)*
6. Barometer
7. Force on side walls due to liquid
8. Torque on side walls
9. Archimedes' Principle *(Floatation, Apparent weight, center of buoyancy)*

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Introductory points

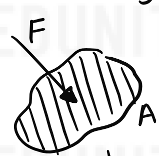
(a) Density, $\rho = m/V$ (kg/m^3 or g/cm^3)

(b) $\rho_{mix} = \dfrac{\text{total mass}}{\text{total volume}}$

(c) Relative density or specific gravity $= \rho/\rho_w$ at $4^\circ C$

Ex: Rel density of Hg = 13.6 $\Rightarrow \rho_{Hg} = 13.6 \times 1000$
$= 13600\ kg/m^3$
(or $13.6\ g/cc$)

(d) Pressure $= F_\perp / A$



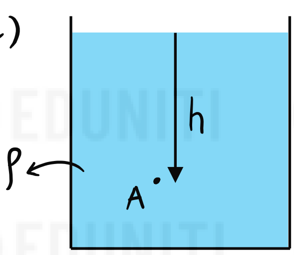
→ Pascal or Pa (N/m^2)

→ $P_0 = 1\,atm = 1.013 \times 10^5\ Pa$
$\approx 10^5\ Pa$

Pressure acts $\perp$ to surface

2. Pressure Variation (vessel at rest)

(a)

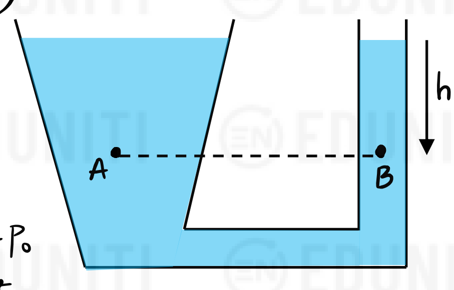


$P_A = \rho g h + P_0$

Absolute Pressure

Gauge pressure (P_g)

(b)

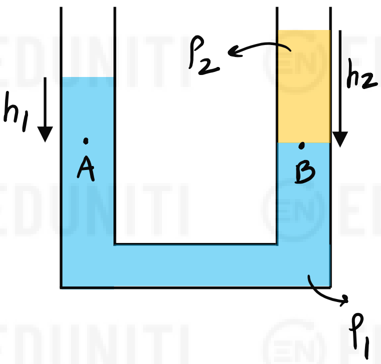


(i) $P_A = P_B = \rho g h + P_0$

(ii) Pressure at depth h is independent of shape of vessel

(iii) At same level, pressure is same (Same Liquid)

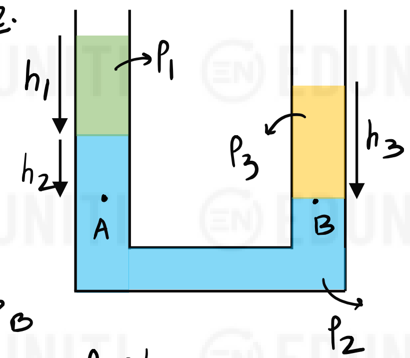
Ex1.



$P_A = P_B$

$\Rightarrow \rho_1 g h_1 + P_0 = \rho_2 g h_2 + P_0$

Ex2.



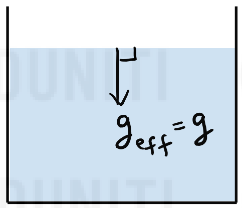
$P_A = P_B$

$\Rightarrow P_0 + \rho_1 g h_1 + \rho_2 g h_2 = P_0 + \rho_3 g h_3$

3. Free Surface

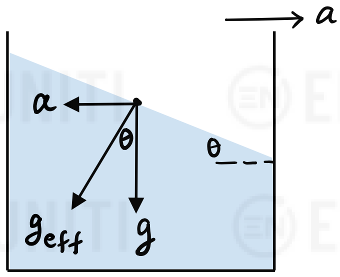
↳ free surface is always $\perp$ to g_{eff}

(a)



At rest

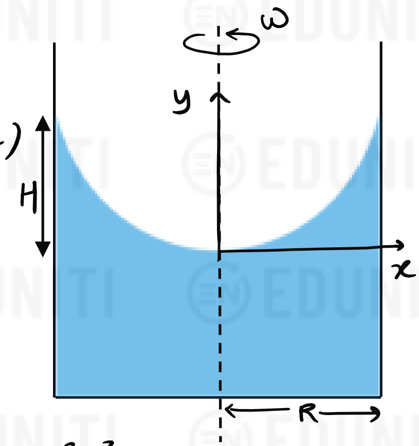
(b)



$g_{eff} = \sqrt{a^2 + g^2}$

$\tan\theta = \frac{a}{g}$

(c)



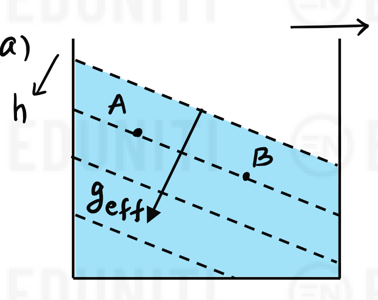
$y = \frac{\omega^2 x^2}{2g}$

↳ If $x = R$, $y = H$

$\therefore H = \frac{\omega^2 R^2}{2g}$

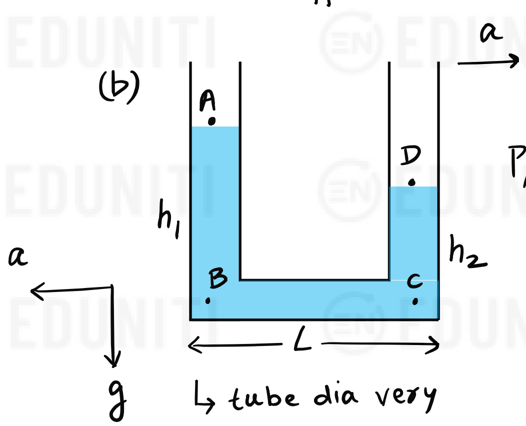
4. Pressure Variation (accelerated system)

(a)



$P_A = P_B = P_0 + \rho g_{eff} h$

(b)



↳ tube dia very small

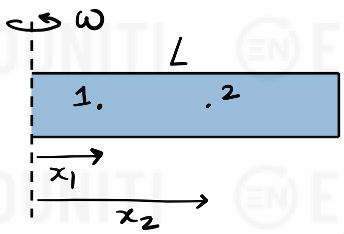
Write eqⁿ from A→D :

$P_A + \rho g h_1 - \rho a L - \rho g h_2 = P_D$

$P_A = P_D = P_0$

$\Rightarrow \boxed{h_1 - h_2 = \frac{aL}{g}}$

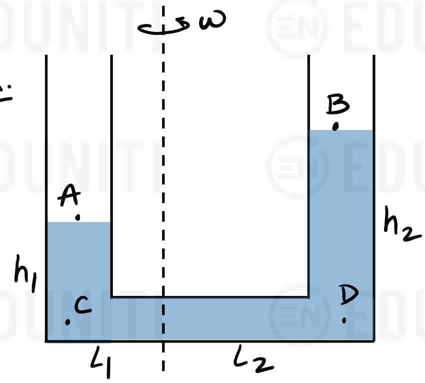
(c)



$$P_2 - P_1 = \frac{1}{2}\rho\omega^2(x_2^2 - x_1^2)$$

If $x_1 = 0$ i.e at axis
$P_2 - P_1 = \frac{1}{2}\rho\omega^2 x_2^2$
↳ As you from axis to Location 2, Press. increases by $\frac{1}{2}\rho\omega^2 x_2^2$

Ex.



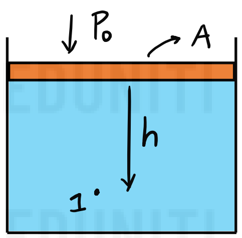
$$P_A + \rho g h_1 - \frac{1}{2}\rho\omega^2 L_1^2 + \frac{1}{2}\rho\omega^2 L_2^2 - \rho g h_2 = P_B$$

$(P_A = P_B = P_0)$

$$h_2 - h_1 = \frac{\omega^2}{2g}(L_2^2 - L_1^2)$$

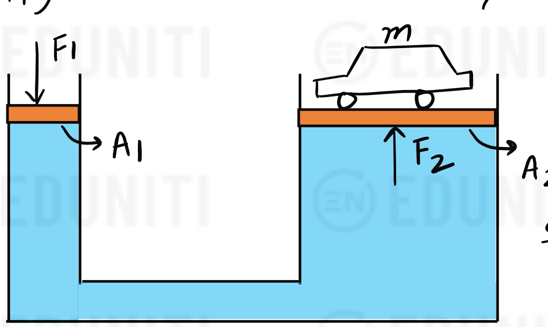
5. Pascal's Law

↳ External pressure gets distributed evenly in all direction



$$P_1 = \rho g h + \frac{mg}{A} + P_0$$

Hydraulic Lift



$$F_2 = PA_2 = \frac{F_1}{A_1} \cdot A_2$$

$$\Rightarrow F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

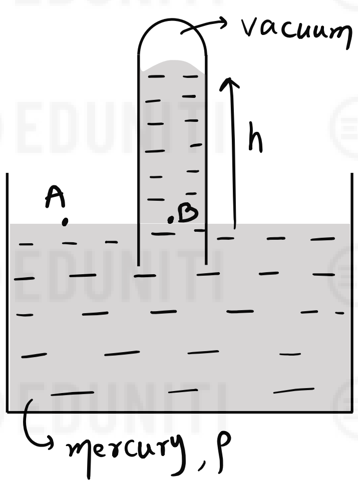
$\because A_2 \gg A_1 \Rightarrow F_2 \gg F_1$

Ex. $mg = F_2 = F_1 \frac{A_2}{A_1}$

$$\Rightarrow F_1 = \frac{A_1}{A_2} \times mg$$

6. Barometer

↳ setup to measure atmospheric pressure



$P_A = P_B$

$\Rightarrow P_0 = \rho g h$

If $P_0 = 1.013 \times 10^5$ Pa

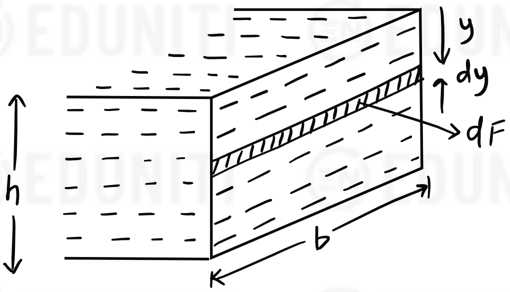
$$h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5}{13600 \times 9.8}$$

$= 0.76$ m or 760 mm

$\therefore$ 1 atm is also written as 760 mm of Hg.

NOTE: we don't use H_2O because $h \approx 10$m (not practical)

7. Force on side walls (due to liquid)



$$dF = P(y) \times dA = \rho g y \times b\,dy$$

$$\Rightarrow F = \rho g b \int_0^h y\,dy = \rho g b \times \frac{h^2}{2} = \frac{\rho g h}{2} \times bh$$

$$\therefore F = P_{av} \times \text{Area of wall}$$

$$P_{av} = \rho g \frac{h}{2} \text{ (Pressure at center)}$$

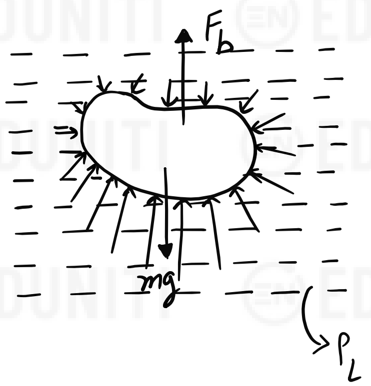
8. Torque on side walls

$$d\tau = dF \times (h-y) \Rightarrow d\tau = \rho g y \cdot b\,dy \cdot (h-y)$$

$$\Rightarrow \tau = \rho g b \int_0^h y(h-y)\,dy = \boxed{\frac{\rho g b h^3}{6}}$$

9. Archimedes' Principle / Floatation

(a)



Buoyant force, $F_b = \rho_L V g$ { V is volume of displaced liquid

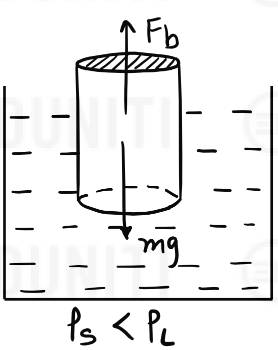
Body is solid (not hollow from inside) :

(i) Body sinks : $\rho_s > \rho_L$

(ii) Body Floats : $\rho_s \leq \rho_L$

- $\rho_s < \rho_L$: Some Portion is outside
- $\rho_s = \rho_L$: Completely Submerged

(b)



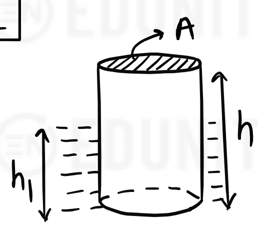
$\rho_s < \rho_L$

$$F_b = mg \Rightarrow \rho_L V_1 g = \rho_s V g \quad \begin{cases} V_1 : \text{displaced liquid vol.} \\ V : \text{Vol of body} \end{cases}$$

$$\Rightarrow \boxed{\frac{V_1}{V} = \frac{\rho_s}{\rho_L}} \rightarrow \text{Ex : Ice floating in water}$$

$$\frac{V_1}{V} = \frac{900}{1000} \Rightarrow \frac{V_1}{V} = 0.9$$

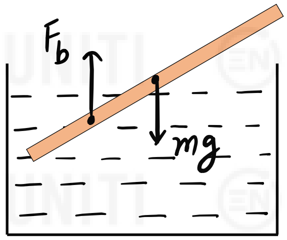
$\therefore$ 90% of ice is inside water



$$\frac{Ah_1}{Ah} = \frac{\rho_s}{\rho_L} \Rightarrow \boxed{\frac{h_1}{h} = \frac{\rho_s}{\rho_L}}$$

↳ valid for uniform cross-section.

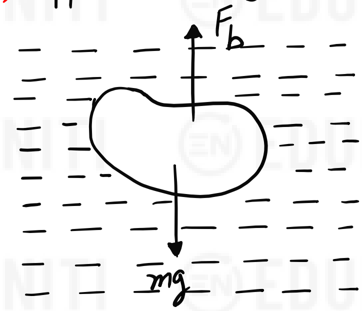
(c) Center of Buoyancy



(i) C.o.b is C.o.m of displaced Liquid

(ii) F_b passes through C.o.b

(d) Apparent weight



(i) $W_{app} = mg - F_b$ $\left\{ V = \frac{m}{\rho_s} \right.$

$\Rightarrow W_{app} = mg - \rho_L V g$

$= mg - \rho_L \cdot \frac{m}{\rho_s} g = mg\left(1 - \frac{\rho_L}{\rho_s}\right)$

$\therefore$ Reading $= m\left(1 - \frac{\rho_L}{\rho_s}\right)$

<u>Space to add concepts learnt from PYQs if any</u>

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID DYNAMICS – PART 2

1. Types of Flow *(Streamline, Turbulent)*
2. Reynolds Number
3. Ideal Fluid *(4 assumptions)*
4. Equation of Continuity
5. Bernoulli's Equation
6. Pressure-speed Relation *(horizontal flow)*
7. Speed of Efflux – Torricelli Law
8. Speed of Efflux *(important other case)*
9. Vessel emptying time *(y as a function of time)*
10. Thrust on Vessel *(liquid coming out)*
11. Venturimeter

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Streamline & Turbulent flow

(Laminar or steady flow)
↳ velocity of fluid at a point is always same

↳ random flow / mixing of flow
↳ vel of fluid at any point changes

2. Reynolds Number (R_e)

$R_e = \frac{\rho v d}{\eta}$ → Dimensionless qty.

ρ → density of liquid
v → speed
d → Pipe cross secⁿ dia
η → Coeff. of viscosity

$R_e < 1000$ → streamline flow
$R_e > 2000$ → Turbulent flow
$1000 < R_e < 2000$ → Unsteady flow
↳ As mentioned in NCERT

PART 2 –FLUID DYNAMICS

3. Ideal Fluid

(i) fluid is incompressible (ρ is const.)

(ii) non-viscous (no fluid friction)

(iii) streamline flow

(iv) no rotational effect

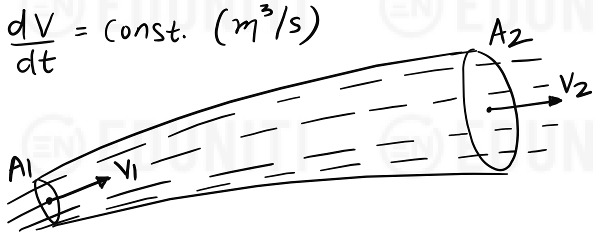
4. Eq^n of Continuity (incompressible)

↳ volume flow rate is constant, $\frac{dV}{dt}$ = const. (m^3/s)

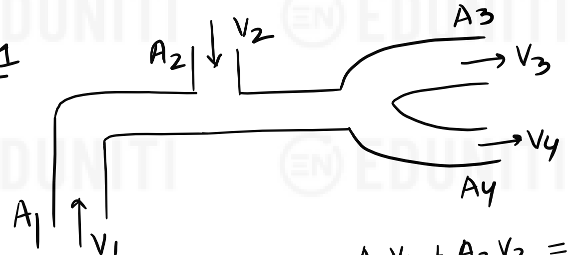
$\Rightarrow A\frac{dx}{dt} = \text{const.} \Rightarrow \boxed{Av = \text{const.}}$

$\Downarrow$

$A_1v_1 = A_2v_2$



Ex.1



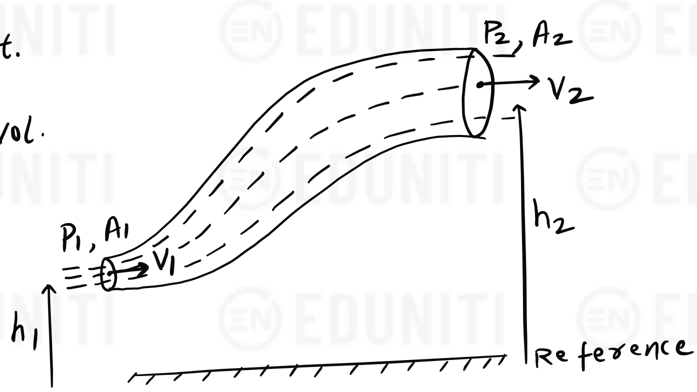
$A_1V_1 + A_2V_2 = A_3V_3 + A_4V_4$

5. Bernoulli's Eq^n

↳ based on conservation of mechanical energy to flow of Ideal fluid

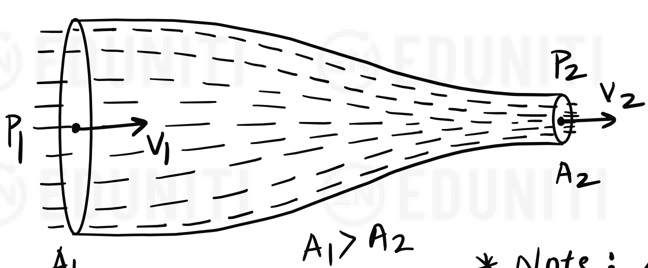
\# $P + \frac{1}{2}\rho v^2 + \rho g h = \text{const.}$

Pressure Energy/vol — K.E/vol — P.E/vol.



$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$

6. P- v Relation (horizontal flow)



$A_1 > A_2$

$\because$ flow is horizontal

$\Rightarrow P + \frac{1}{2}\rho V^2 = \text{const.}$

Also $A_1V_1 = A_2V_2$, so $V_1 < V_2$

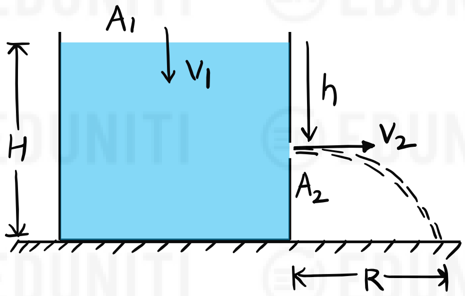
$\therefore \boxed{P_1 > P_2}$

* Note: Loss in pressure energy/vol. = gain in KE/vol.

(for horizontal flow)

7. Speed of Efflux : Toricellis Law

↳ speed of efflux from an open tank is identical to freely falling body.



$A_1 >> A_2$

$\Rightarrow V_2 >> V_1$ (speed of efflux)

→ Just to left and Right of hole :

Loss in Press. Energy /vol = Gain of KE/vol.

$$\Rightarrow (P_0 + \rho g h) - (P_0) = \frac{1}{2}\rho V_2^2 - \underbrace{\frac{1}{2}\rho V_1^2}_{\approx 0}$$

$$\Rightarrow \boxed{V_2 = \sqrt{2gh}}$$

(i) Range, $R = V_2 \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$

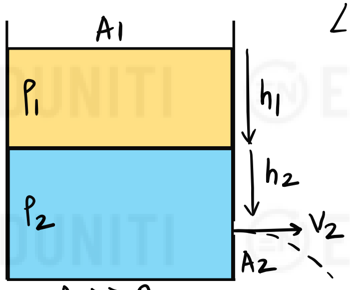
(ii) For R to be max:

$\frac{dR}{dh} = 0 \Rightarrow h = H/2$

and $R_{max} = H$

8. Speed of Efflux (other cases)

(a)



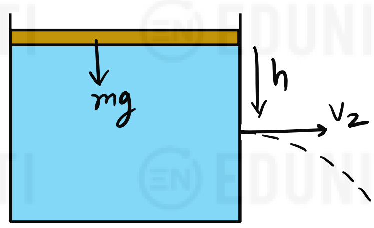
$A_1 >> A_2$

Loss in Press. Energy /vol = Gain of KE/vol.

$(P_0 + \rho_1 g h_1 + \rho_2 g h_2) - P_0 = \frac{1}{2}\rho_2 V_2^2$ { V_1 neglected

$\Rightarrow \rho_1 g h_1 + \rho_2 g h_2 = \frac{1}{2}\rho_2 V_2^2$

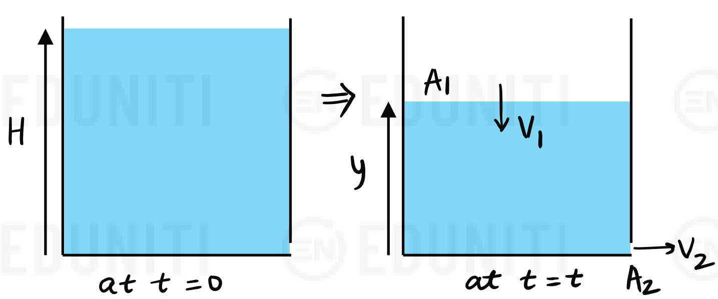
(b)



$\left(P_0 + \rho g h + \frac{mg}{A}\right) - P_0 = \frac{1}{2}\rho V_2^2$

$\Rightarrow \rho g h + \frac{mg}{A} = \frac{1}{2}\rho V_2^2$

9. Vessel Emptying time



$A_1 V_1 = A_2 V_2 \Rightarrow A_1\left(-\frac{dy}{dt}\right) = A_2\sqrt{2gy}$

$$\Rightarrow \int_H^y \frac{dy}{\sqrt{y}} = -\frac{A_2\sqrt{2g}}{A_1}\int_0^t dt$$

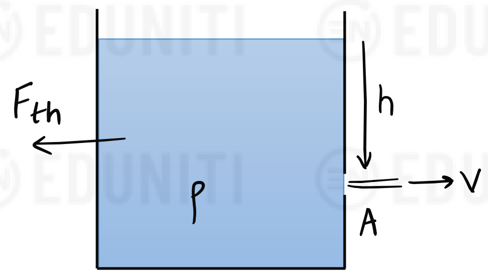
$$\Rightarrow 2(\sqrt{y} - \sqrt{H}) = -\frac{A_2}{A_1}\sqrt{2g}\cdot t$$

$$\therefore \boxed{t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{y}\right)}$$

↳ To empty $y = 0$ at $t = t_0$

$$\Rightarrow \boxed{t_0 = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}}$$

10. Thrust on Vessel (Liquid coming out)



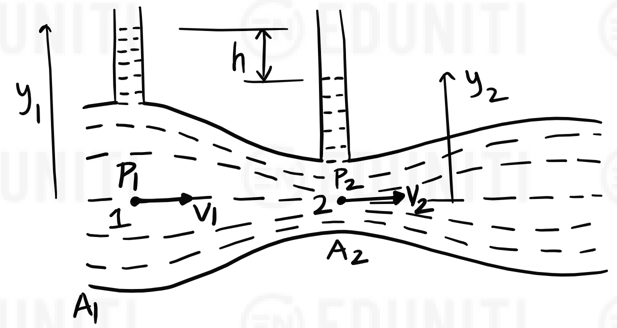
$$F_{th} = \rho A v^2$$

$$\Rightarrow F_{th} = \rho A \times 2gh$$

11. Venturimeter

↳ A device/setup to measure flow speed

(a) Setup 1 :



$A_1 > A_2$

$\Rightarrow V_1 < V_2$

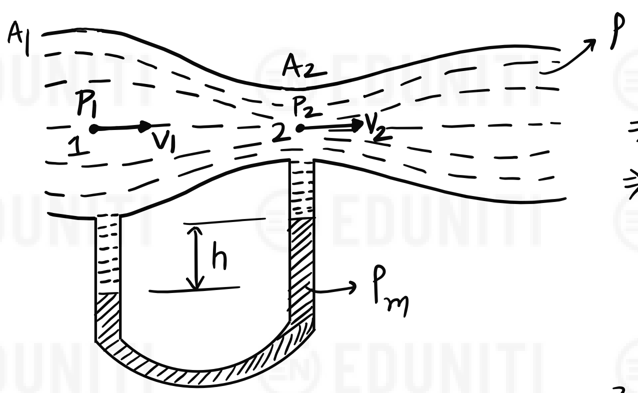
$\Rightarrow P_1 > P_2$

$V_2 = \dfrac{A_1 V_1}{A_2}$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 \Rightarrow \rho g y_1 - \rho g y_2 = \frac{1}{2}\rho\left(\frac{A_1 V_1}{A_2}\right)^2 - \frac{1}{2}\rho V_1^2$$

$$\Rightarrow \rho g h = \frac{1}{2}\rho V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \therefore \boxed{V_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

#(b) Setup 2 :



$A_1 > A_2$

$\Rightarrow V_1 < V_2$

$\Rightarrow P_1 > P_2$

$V_2 = \dfrac{A_1 V_1}{A_2}$

$$P_1 - P_2 = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 \Rightarrow \rho_m g h = \frac{1}{2}\rho\left(\frac{A_1 V_1}{A_2}\right)^2 - \frac{1}{2}\rho V_1^2$$

$$\therefore \boxed{V_1 = \sqrt{\frac{2\rho_m g h}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}}$$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID PROPERTIES – PART 3

1. Cohesive vs Adhesive Force
2. Understanding Surface Tension *(S = F/L)*
3. Lifting bodies *(ring, disc, plate etc.)*
4. Surface Energy *(S = E/A)*
5. Drop Coalesce & Bubble collapse
6. Excess Pressure *(Drop & Bubble)*
7. 2 Soap bubbles in contact *(Externally / Internally)*
8. Angle of contact
9. Capillary Action *(in tube as well as parallel plates)*
10. Insufficient tube length
11. Force to separate glass plates
12. Understanding Viscous Force
13. Poiseuille's Equation – Flow through narrow tube
14. Stokes' Law & Terminal velocity

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

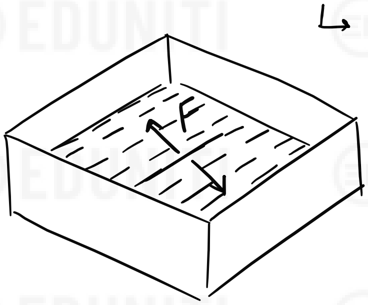
1. Cohesive vs Adhesive Force

↓ Force of attraction between molecules of same substance

↳ Force of attraction between molecules of different substance.

2. Understanding Surface Tension

↳ property of liquid where surface
(i) tries have minimum Area
(ii) Acts as stretched membrane

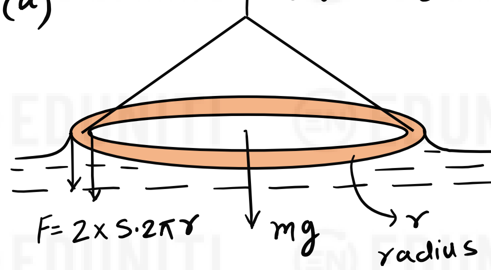


$\rightarrow S = \frac{F}{L}$ or $F = SL$

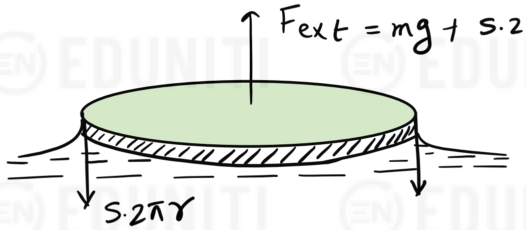
NOTE:
F acts perpendicular to line and tangential to free surface.

3. Lifting of bodies

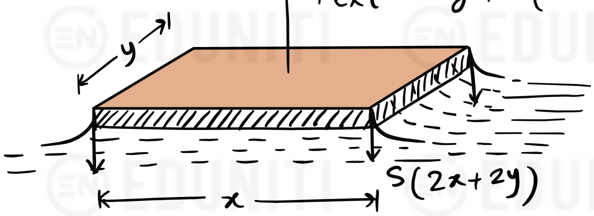
(a) $F_{ext} = mg + 2 \cdot S \cdot 2\pi r$



(b) $F_{ext} = mg + S \cdot 2\pi r$



(c) $F_{ext} = mg + S(2x+2y)$



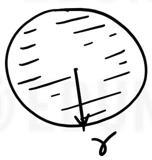
4. Surface Energy

↳ Due to intermolecular interaction, free surface of liquid has energy called "Surface Energy"

$S = \frac{E}{A}$ or $E = S \cdot A$

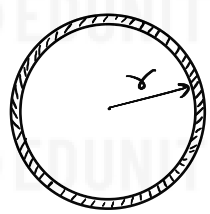
A : Surface Area
S : Surface Tension
E : Surface energy

Ex 1. Energy of drop



$E = S \cdot 4\pi r^2$

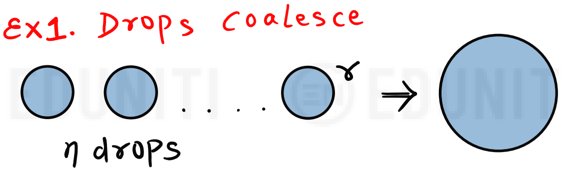
Ex 2. Energy of Bubble



$E = 2 \times S \cdot 4\pi r^2$

5. Standard Questions on Surface Energy

Ex 1. Drops Coalesce



∵ Volume is same

$\Rightarrow \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

or, $R = r n^{1/3}$

Here Area ↓ ⇒ Energy ↓

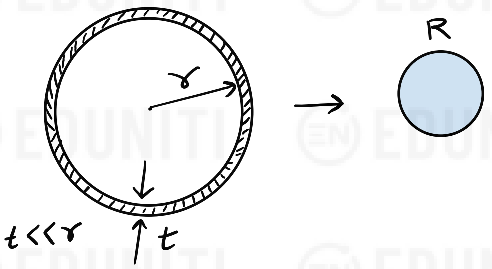
$E_i = n \cdot S \cdot 4\pi r^2$

$E_f = S \cdot 4\pi R^2 = S \cdot 4\pi r^2 \cdot n^{2/3}$

$\therefore E_i - E_f$ increases Temp° of Bigger drop by $\Delta\theta$

$\Rightarrow \boxed{E_i - E_f = m s \Delta\theta}$

Ex 2. Bubble Collapse



$$E_i = 2 \times S \cdot 4\pi r^2$$

$$E_f = S \cdot 4\pi R^2$$

$$= S \cdot 4\pi (3r^2 t)^{2/3}$$

$\because$ Vol of Liq. is same

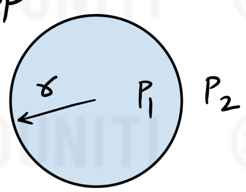
$$\Rightarrow 4\pi r^2 \times t = \frac{4}{3}\pi R^3$$

$$R = (3r^2 t)^{1/3}$$

$$\therefore E_i - E_f = ms\Delta\theta$$

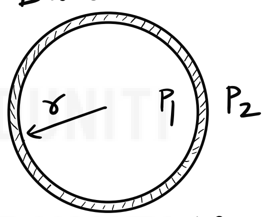
6. Excess Pressure in Liquid Drop and Bubble

Drop



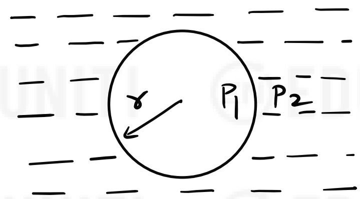
$$P_{ex} = P_1 - P_2 = \frac{2S}{r}$$

Bubble



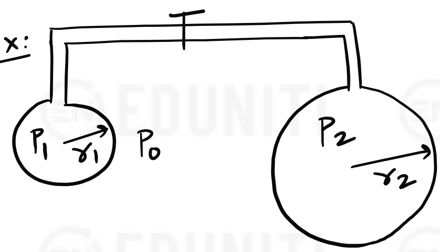
$$P_{ex} = P_1 - P_2 = \frac{4S}{r}$$

#NOTE: If the bubble is in Liquid (ex. Aquarium)



$$P_1 - P_2 = \frac{2S}{r}$$

Ex:

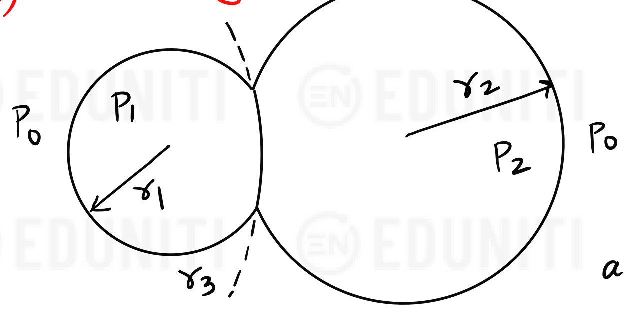


$$P_1 = \frac{4S}{r_1} + P_0$$

$$P_2 = \frac{4S}{r_2} + P_0$$

7. Two Soap bubbles in contact

(i) Externally



$$P_2 - P_0 = \frac{4S}{r_2} \quad \text{— (1)}$$

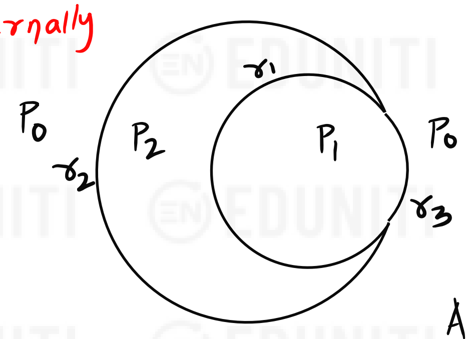
$$P_1 - P_0 = \frac{4S}{r_1} \quad \text{— (2)}$$

Subract (2) & (1) $\Rightarrow P_1 - P_2 = \frac{4S}{r_1} - \frac{4S}{r_2}$

also, $P_1 - P_2 = \frac{4S}{r_3}$

$$\therefore \frac{4S}{r_3} = \frac{4S}{r_1} - \frac{4S}{r_2} \Rightarrow \boxed{r_3 = \frac{r_1 r_2}{r_2 - r_1}}$$

(ii) Internally



$P_1 - P_2 = \frac{4S}{r_1}$ — (1)

$P_2 - P_0 = \frac{4S}{r_2}$ — (2)

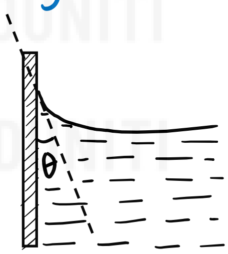
Adding (1) & (2) $\Rightarrow P_1 - P_0 = \frac{4S}{r_1} + \frac{4S}{r_2}$

Also, $P_1 - P_0 = \frac{4S}{r_3}$

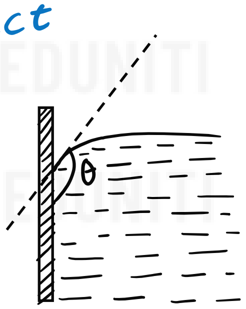
$\therefore \frac{4S}{r_3} = \frac{4S}{r_1} + \frac{4S}{r_2} \Rightarrow \boxed{r_3 = \frac{r_1 r_2}{r_1 + r_2}}$

r_3 will be smallest

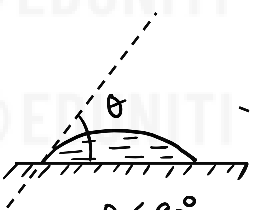
8. Angle of Contact



$\theta < 90^\circ$
ex: water

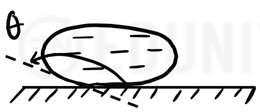


$\theta > 90^\circ$
Cohesive force dominates
ex: Hg



$\theta < 90^\circ$

↳ Liquid wets the surface
water

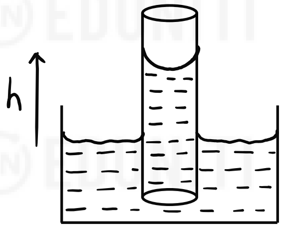


$\theta > 90^\circ$

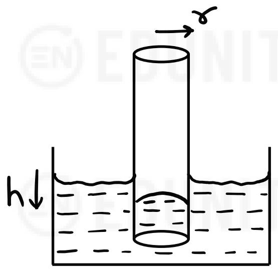
Note: Generally θ for distilled H_2O is 0°.

9. Capillary Action

(i) Thin tube



$\theta < 90^\circ$

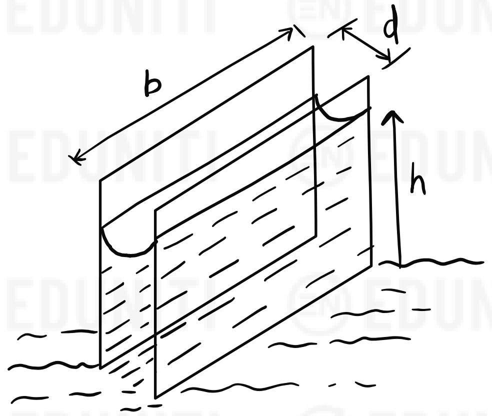


$\theta > 90^\circ$

$h = \frac{2S\cos\theta}{r\rho g}$ or $\frac{2S}{R\rho g}$

r is tube radius, R is meniscus radius

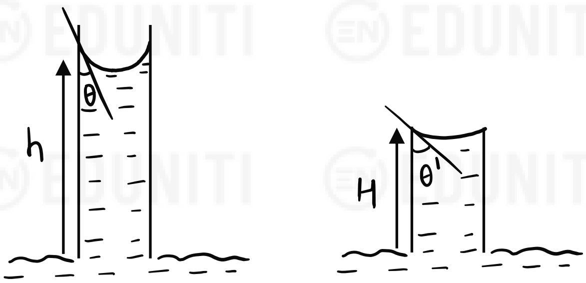
(ii)



$h = \frac{2S\cos\theta}{d\rho g}$

↳ Generally if its H_2O $\cos\theta = 1$.

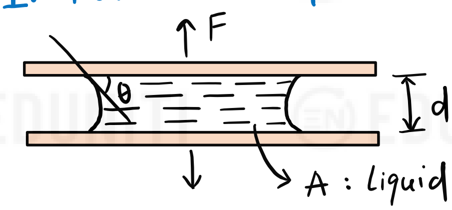
10. Insufficient Tube Length ($H<h$)



$h = \frac{2s\cos\theta}{\rho g}$ but $H<h$

↳ In this case liquid won't overflow rather it reaches top and adjusts contact angle to θ' $(\theta' > \theta)$

11. Force to separate Glass Plates

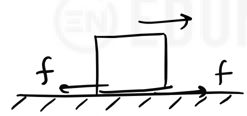


A : liquid cross-secn area

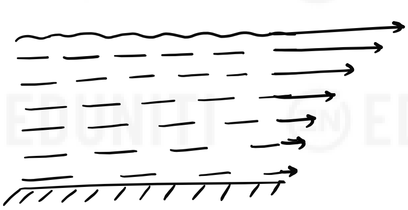
$F = \frac{2SA}{d}\cos\theta$

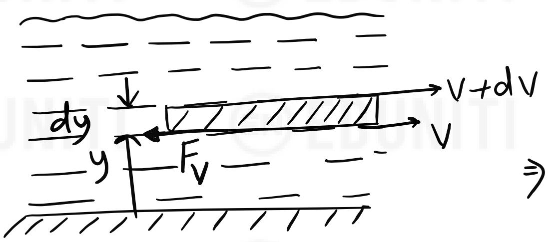
If its H_2O, $\cos\theta = 1$

12. Understanding Viscous Force



↳ Fluid opposes the relative motion between its different layers.





$F_V \propto A\frac{dv}{dy}$

$\Rightarrow F_V = -\eta A\frac{dv}{dy}$

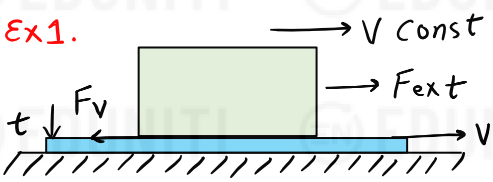
Coeff of Viscosity

(i) $[\eta] = ML^{-1}T^{-1}$

SI unit : Poiseuille, PI

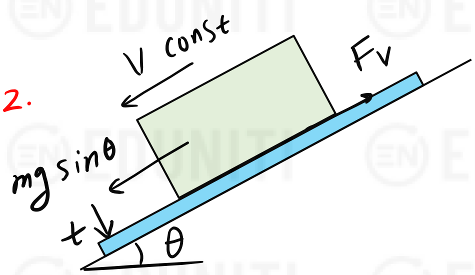
C.G.S : Poise 1 PI = 10 Poise

Ex1.



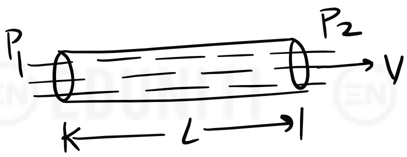
$F_{ext} = F_V = \eta A\frac{dv}{dy} = \eta A\frac{V}{t}$

Ex 2.



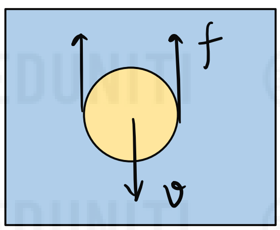
$mg\sin\theta = \eta A\frac{V}{t}$

13. Poiseuille's Eqⁿ – Flow through Narrow Tube



Flow rate, $Q = \dfrac{\pi r^4}{8\eta L}(P_1 - P_2)$
(m^3/s)

14. Stokes' Law & Terminal Velocity



$f = 6\pi\eta r v$

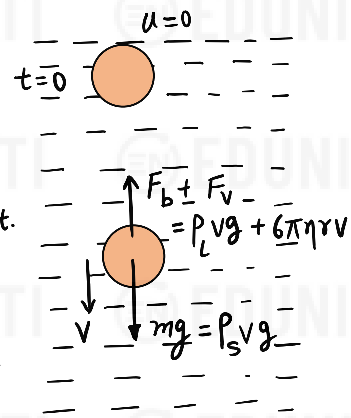
↳ As $v\uparrow$, $F_v\uparrow$
and at certain speed

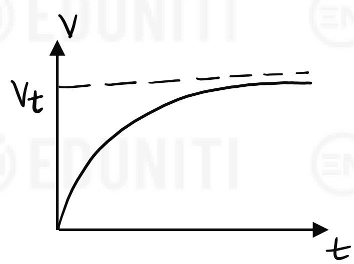
$F_b + F_v = mg$ $\Big\}\Rightarrow a = 0,\ v = \text{const.}$

Terminal Velocity

$\Rightarrow \rho_L \cdot \frac{4}{3}\pi r^3 g + 6\pi\eta r v_t = \rho_s \frac{4}{3}\pi r^3 g$

$\therefore \boxed{v_t = \dfrac{2r^2 g(\rho_s - \rho_L)}{9\eta}}$





<u>Space to add concepts learnt from PYQs if any</u>

Space to add concepts learnt from PYQs if any

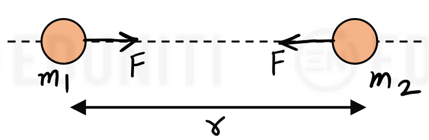
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in GRAVITATION

1. Law of Gravitation
2. Gravitational Field
3. g due to a Ring
4. g due to a long wire
5. g due to a thin spherical shell
6. g due to a solid sphere
7. Potential Energy
8. Gravitational Potential (point mass & ring)
9. Potential due to Spherical Shell
10. Potential due to Solid Sphere
11. Variation of g with height
12. Variation of g with depth
13. Variation of g with rotation
14. Escape Velocity
15. Orbital Velocity, Time Period, KE, PE & TE
16. Geostationary Satellite
17. Discussion on Elliptical Path
18. Speed at Perigee and Apogee
19. Projected velocity vs path
20. Kepler's Law
21. Double Mass System

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. LAW OF GRAVITATION

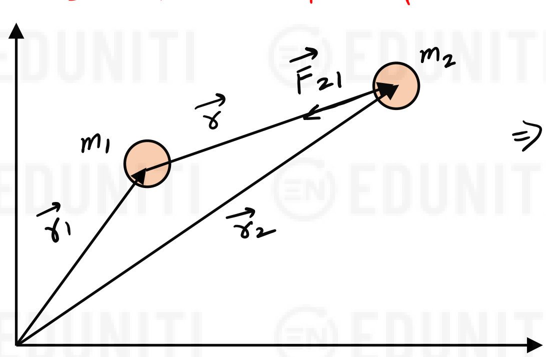


$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

G : Universal Gravitational constant
$6.67 \times 10^{-11}\ N m^2 / kg^2$

NOTE: 1. F acts along line joining masses.
2. Here m_1 and m_2 are point masses.

IN VECTOR FORM



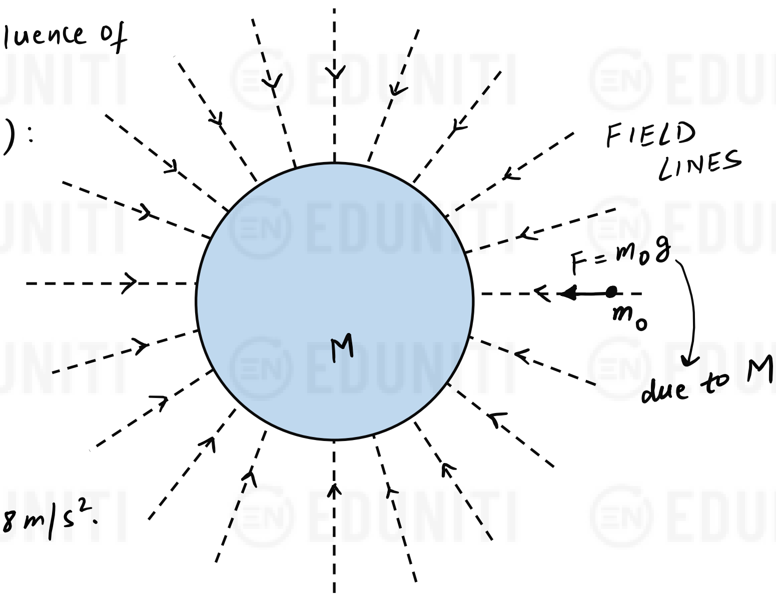
$$\Rightarrow \boxed{\vec{F}_{21} = -\frac{G m_1 m_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}}$$

2. GRAVITATIONAL FIELD

↳ A way of expressing influence of a mass.

Gravitational Field strength (g):

$m \; \overset{r}{\text{- - - -}} \; g = \dfrac{Gm}{r^2}$



NOTE:

1. g is also called "acceleration due to gravity"
2. At earth's surface its close to $g_0 = 9.8\ m/s^2$.

3. FOR A RING

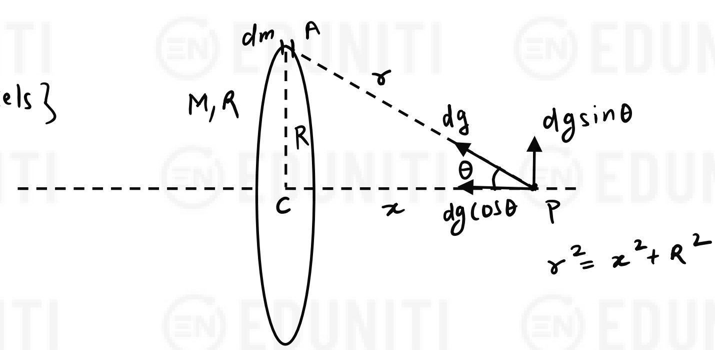
$\int dg \sin\theta = 0$ {vertical comp. cancels}

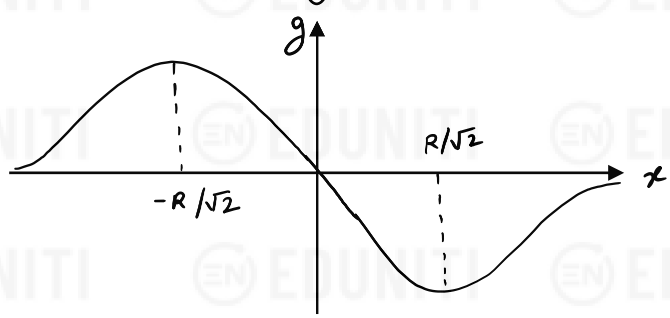
$$g(x) = \int dg \cos\theta$$

$$= \int \frac{G\,dm \cos\theta}{r^2} = \int \frac{G\,dm}{r^2} \times \frac{x}{r}$$

$$= \frac{Gx}{(x^2+R^2)^{3/2}} \int dm$$

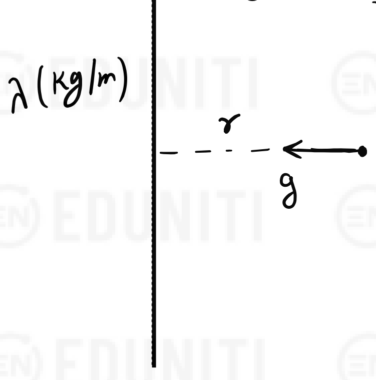
$$= \boxed{\frac{GMx}{(x^2+R^2)^{3/2}}}$$



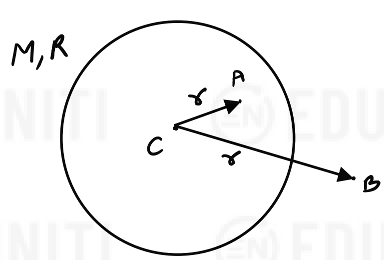


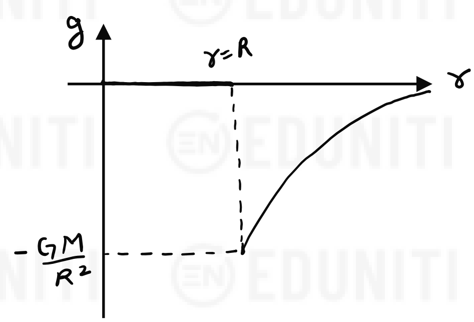
4. FOR A LONG WIRE

$$g = \frac{2G\lambda}{r}$$



5. FOR THIN SPHERICAL SHELL

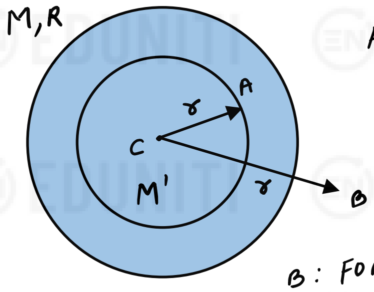




A : For $r < R$, $g = 0$

B : for $r \geq R$, $g = -\dfrac{GM}{r^2}$

6. FOR SOLID SPHERE



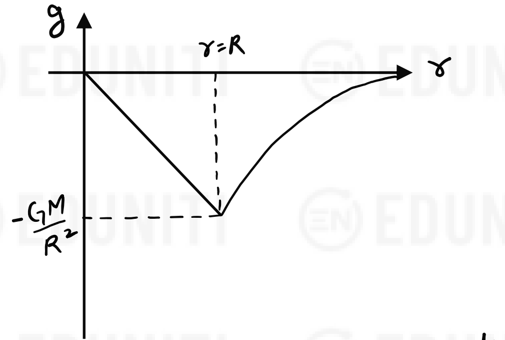
A: For $r < R$:

$M' = Mr^3/R^3$

$\Rightarrow g = -\frac{GM'}{r^2} = -\frac{GMr}{R^3}$

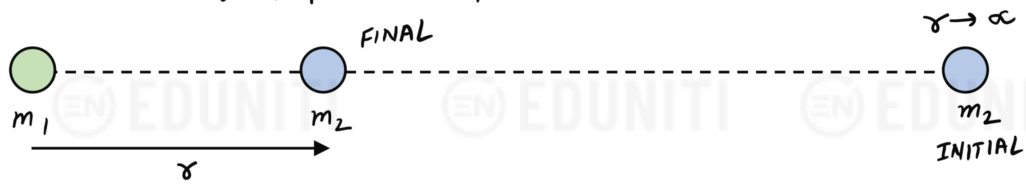
B: FOR $r \geq R$:

$g = -\frac{GM}{r^2}$



NOTE : Taking radially inwards as –VE direction

7. POTENTIAL ENERGY U : Work done to bring m_2 very slowly from ∞ to a separation r from m_1.



$$W = \int_{\infty}^{r} \frac{Gm_1m_2}{r^2}\,dr = \boxed{-\frac{Gm_1m_2}{r}}$$

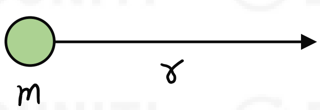
NOTE: At infinity, $U = 0$

Also, $W = \Delta U \Rightarrow W = U_f - U_i$

$\Rightarrow -\frac{Gm_1m_2}{r} = U_f - 0 \quad \therefore U_f = -\frac{Gm_1m_2}{r}$

8. GRAVITATIONAL POTENTIAL V

POINT MASS:



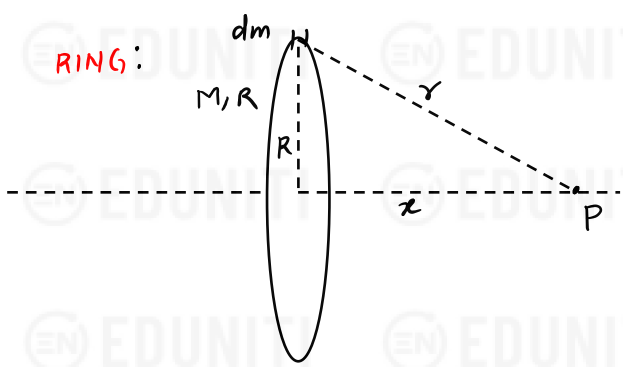
$V = -\frac{Gm}{r}$

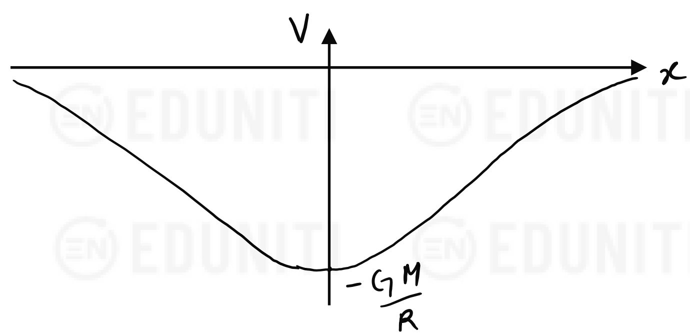
RING: $dV = -\frac{G\,dm}{r}$

$\Rightarrow V = -\frac{G}{r}\int dm$

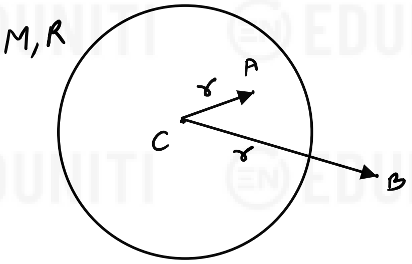
$$\boxed{V = \frac{-GM}{(R^2 + x^2)^{1/2}}}$$

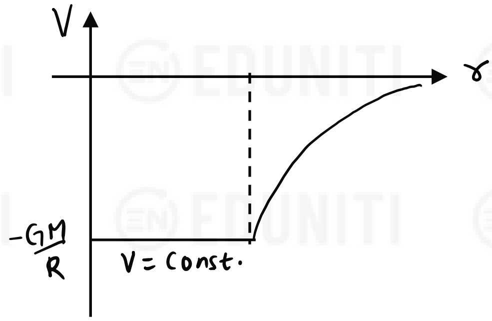
RING:





9. V DUE TO SPHERICAL SHELL

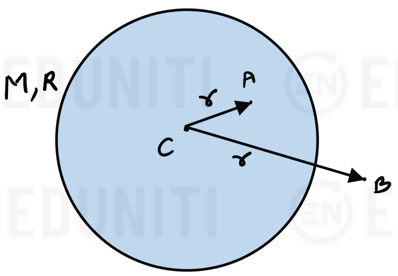


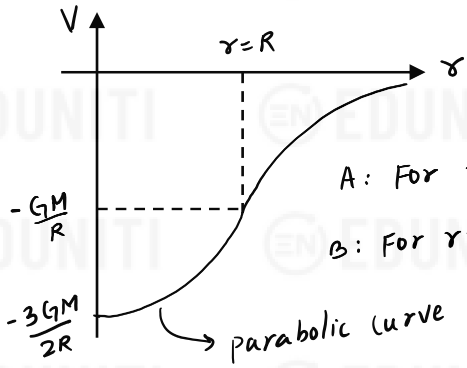


A: For $r < R$, $V = -GM/R$ B: For $r \geq R$, $V = -\frac{GM}{r}$

→ NOTE :
For an outside point, assume all mass to situated at centre.

10. V DUE TO SOLID SPHERE





A: For $r < R$, $V = -\frac{GM}{2R^3}(3R^2 - r^2)$

B: For $r \geq R$, $V = -GM/r$

→ NOTE :
For an outside point, assume all mass to situated at centre.

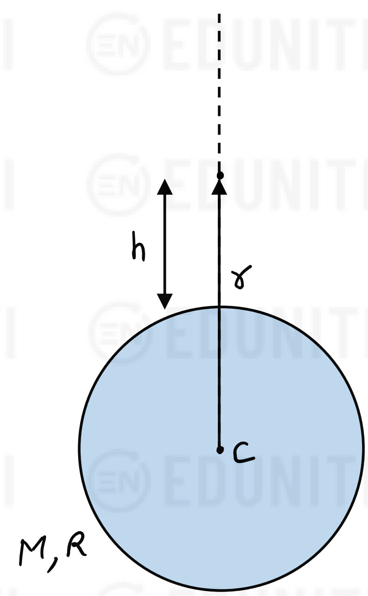
11. VARIATION OF g WITH HEIGHT

At height "h" :

$$g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1 + \frac{h}{R}\right)^{-2}, \quad g_0 = \frac{GM}{R^2}$$

If $\frac{h}{R} << 1$ or $h << R$

$$\Rightarrow \boxed{g = g_0\left(1 - \frac{2h}{R}\right)}$$



NOTE :
1. only if $h << R$ (for earth till $h \sim 400$ km)
use $g = g_0\left(1 - \frac{2h}{R}\right)$
2. If h is comparable with R, use
$$g = \frac{GM}{(R+h)^2} = \frac{g_0 R^2}{(R+h)^2}$$

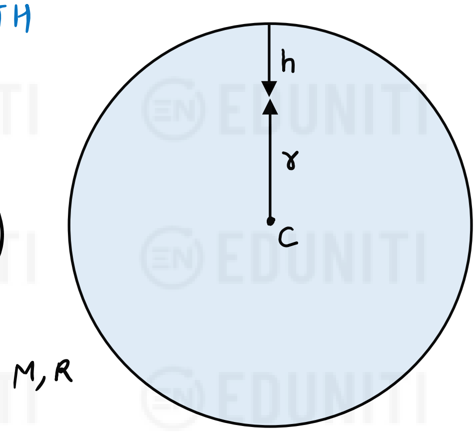
12. VARIATION OF g WITH DEPTH

At a depth "h" :

$g = \frac{GMr}{R^3}$, $r = R-h$

$\Rightarrow g = \frac{GM(R-h)}{R^3} = \frac{GM}{R^2}\left(\frac{R-h}{R}\right)$

$\therefore \boxed{g = g_0\left(1-\frac{h}{R}\right)}$



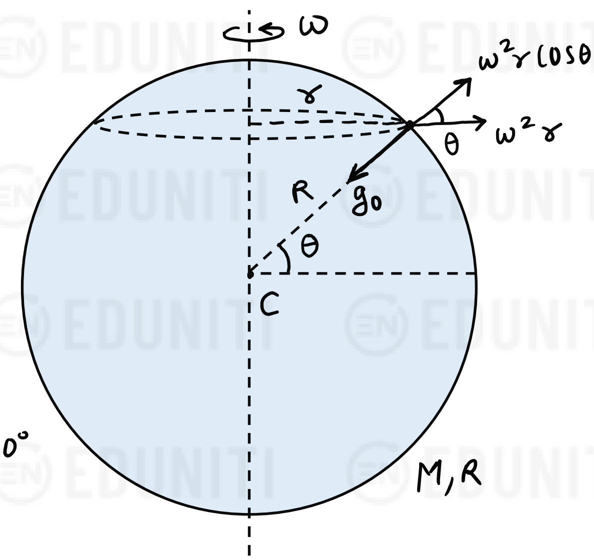
13. VARIATION OF g WITH ROTATION

$r = R\cos\theta$

$g_{effective}$ towards centre is

$g_{eff} = g_0 - \omega^2 r\cos\theta$

$= \boxed{g_0 - \omega^2 R\cos^2\theta}$



NOTE:

1. $\omega^2 R = 0.034$ rad/s (for Earth)
2. At equator, $g_{eff} = g_0 - \omega^2 R$, $\theta = 0°$
3. At poles, $g_{eff} = g_0$, $\theta = 90°$.

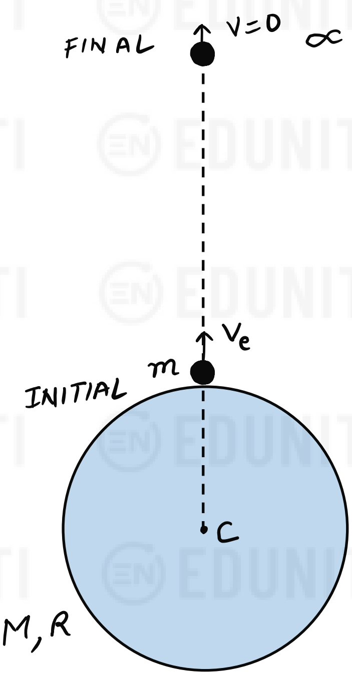
14. ESCAPE VELOCITY V_e

Minimum velocity V_e, so that particle escapes planet's gravitational pull.
(the distance where g of planet is zero is taken is ∞)

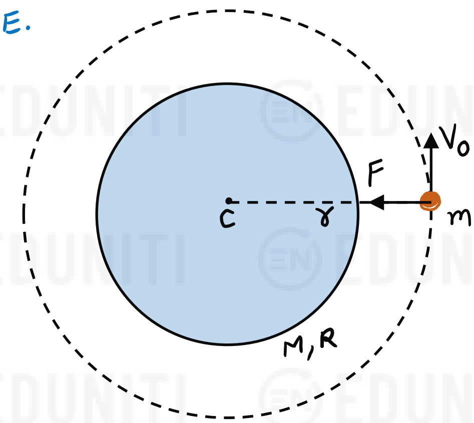
$K_i + U_i = K_f + U_f$

$\Rightarrow \frac{1}{2}mV_e^2 - \frac{GMm}{R} = 0 + 0$

$\therefore \boxed{V_e = \sqrt{\frac{2GM}{R}}} \approx 11.2$ Km/s at EARTH's surface.



15. ORBITAL VELOCITY $V_0, T, K, U, T.E., B.E.$



(a) $F = mV_0^2/r \Rightarrow \frac{GMm}{r^2} = \frac{mV_0^2}{r} \Rightarrow V_0 = \sqrt{\frac{GM}{r}}$

(b) $T = \frac{2\pi r}{V_0} = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$

(c) $K = \frac{1}{2} mV_0^2 = GMm/2r$

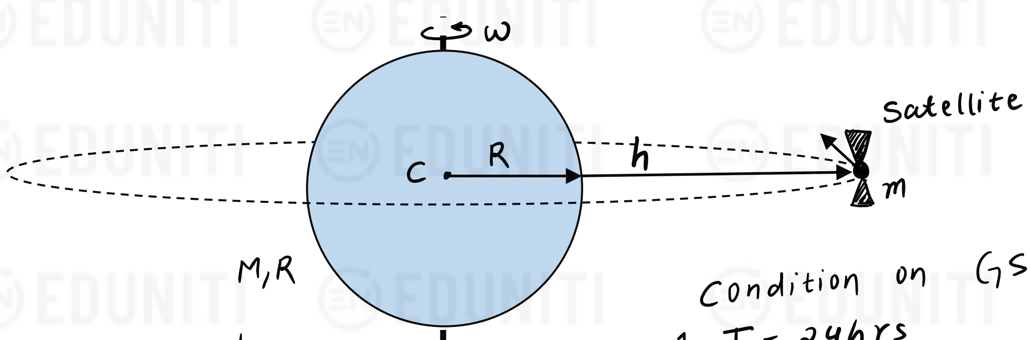
$U = -GMm/r$, $T.E = K + U = -\frac{GMm}{2r}$

$\therefore K = \left|\frac{U}{2}\right| = |T.E|$

(d) Binding Energy, $\boxed{B.E = -T.E} = \frac{GMm}{2r}$

↳ Energy required to be given so that "m" escapes.

16. GEOSTATIONARY SATELLITE



$T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$, $r = R + h$

$T = 24\,hrs$, $R = 6400\,Km$,

$M = 6 \times 10^{24}\,Kg$

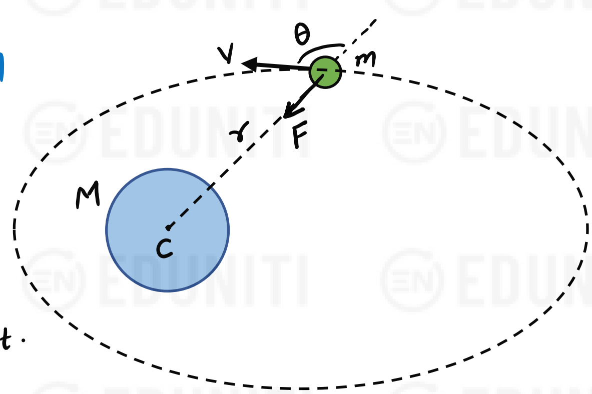
We get $\boxed{h \approx 36000\,Km}$

Condition on GSAT:

1. $T = 24\,hrs$
2. Must lie on equatorial plane
3. must revolve along the direction rotation of earth.

17. DISCUSSION ON ELLIPTICAL PATH

About τ, L, U+K



(1.) As "m" revolves, F always passes through C. Thus torque τ about C is zero.

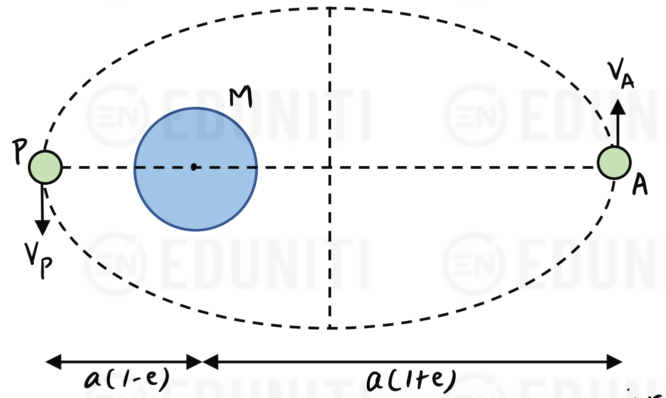
$\because \tau = 0 \Rightarrow L$ is constant.

or $mvr\sin\theta = const.$

(2.) $U + K = constant$

$\Rightarrow -\frac{GMm}{r} + \frac{1}{2}mv^2 = const.$

18. SPEED AT PERIGEE / APOGEE



a : Semi-major axis

e : eccentricity

$L_P = L_A \Rightarrow mV_P\, a(1-e) = mV_A\, a(1+e)$ ---(1)

$U_P + K_P = U_A + K_A$

$\Rightarrow -\dfrac{GMm}{a(1-e)} + \dfrac{1}{2}mV_P^2 = -\dfrac{GMm}{a(1+e)} + \dfrac{1}{2}mV_A^2$ --- (2)

solving (1) and (2), we get :

$$V_P = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}\ ,\quad V_A = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}$$

NOTE : Total energy of system is constant i.e. $-\dfrac{GMm}{2a}$.

19. PROJECTED VELOCITY V/S PATH

If :

(a) $V = 0 \Rightarrow$ path 1, straight line

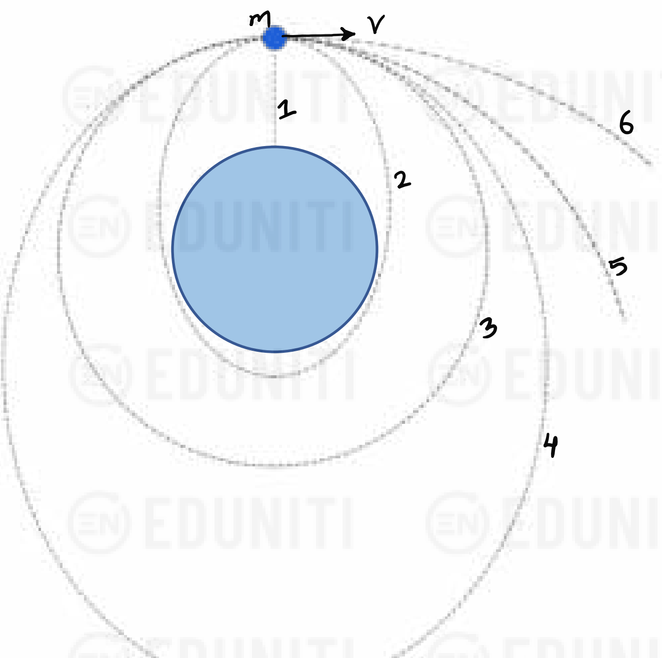
(b) $0 < V < V_0 \Rightarrow$ path 2, Elliptical

(c) $V = V_0 \Rightarrow$ path 3, Circular

(d) $V_0 < V < V_e \Rightarrow$ path 4, Elliptical

(e) $V = V_e \Rightarrow$ path 5, Parabola

(f) $V > V_e \Rightarrow$ path 6, Hyperbola

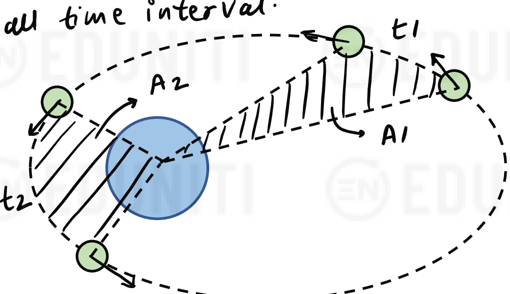


20. KEPLER'S LAW

1. Law of Orbit : Planet revolves around sun in an elliptical path with sun at one of the foci.

2. Law of Area : Line joining planet and sun sweeps equall Area in equall time interval.

$\boxed{\dfrac{dA}{dt} = \dfrac{L}{2m}} = \text{Constant.}$

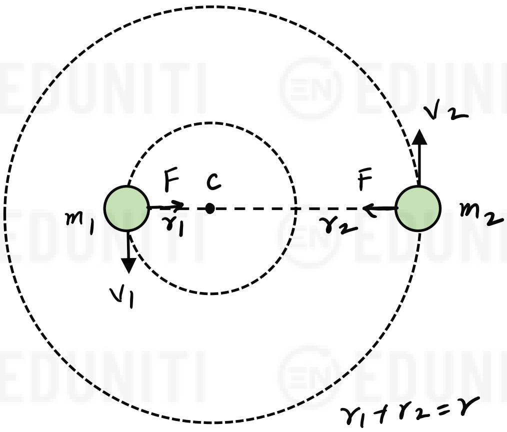


$\hookrightarrow \dfrac{A_1}{t_1} = \dfrac{A_2}{t_2}$

3. Law of Periods : $\boxed{T^2 \propto a^3}$

21. DOUBLE MASS SYSTEM

- Considering 2 Mass System, they revolve about their common centre of mass, "C".
- That point is called "BARYCENTER"



$r_1 + r_2 = r$

$r_1 = \frac{m_2 r}{m_1+m_2}$, $r_2 = \frac{m_1 r}{m_1+m_2}$

NOTE:

1. Both m_1 and m_2 has same Time Period T and ω, $\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}}$
2. $\frac{v_1}{v_2} = \frac{\omega r_1}{\omega r_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2}$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in SIMPLE HARMONIC MOTION

1. Kinematics and Energy of SHM
2. Steps to Find Time Period (Linear/Angular SHM)
3. Spring Block and Simple Pendulum
4. T of Spring Block (under constant force)
5. Combination of Spring
6. Spring Cut
7. Two Block System (Reduced Mass)
8. SHM of Block in a Liquid
9. SHM of Piston in Cylinder
10. SHM in Tunnel of Planet
11. SHM of Charge
12. Physical Pendulum
13. Torsional Pendulum
14. SHM of Dipole in Field (Electric/Magnetic Dipole)
15. Time Period of Simple Pendulum (Different Conditions)
16. Superposition of SHM
17. Damped Oscillation

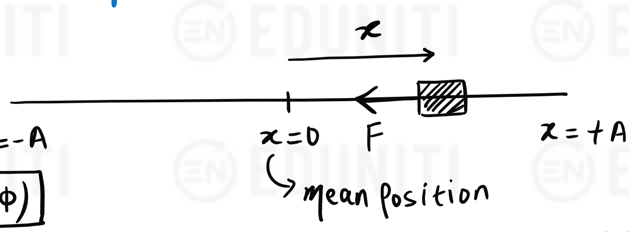
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. KINEMATICS/ENERGY OF SHM

$F \propto -x$

$\Rightarrow F = -kx$

$a = -\left(\frac{k}{m}\right)x$



$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow \boxed{x = A\sin(\omega t+\phi)}$

$\therefore \omega^2 = \frac{k}{m}$ or $T = 2\pi\sqrt{\frac{m}{k}}$

(a) $v(x) = \omega\sqrt{A^2-x^2}$ → $v_{max} = \pm\omega A,\ x=0$; $v_{min} = 0,\ x = \pm A$

$v(t) = A\omega\cos\omega t$

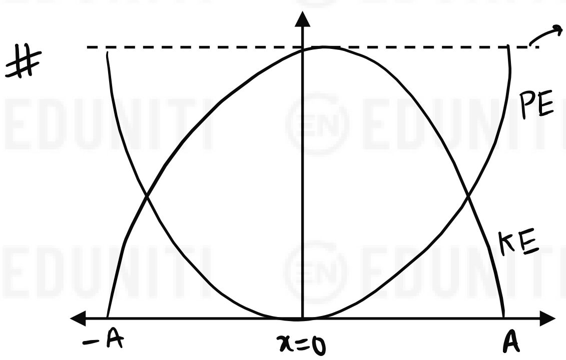
(b) $a(x) = -\omega^2 x$ → $a_{max} = \pm\omega^2 A,\ x = \mp A$; $a_{min} = 0,\ x = 0$

$a(t) = -\omega^2 A\sin\omega t$

(c) $K.E(x) = \frac{1}{2}m\omega^2(A^2-x^2)$, $PE(x) = \frac{1}{2}m\omega^2x^2$

$TE = KE + PE = \frac{1}{2}m\omega^2A^2$ constant.

#



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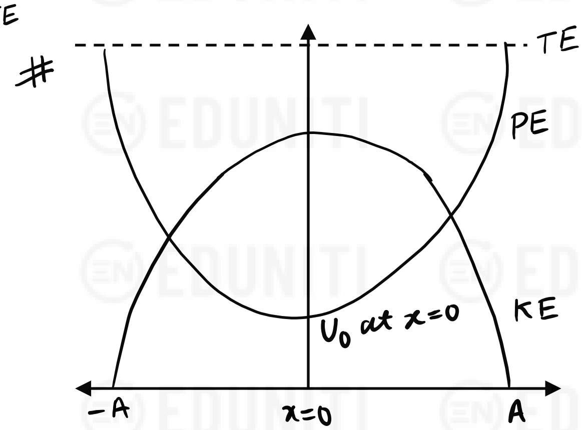
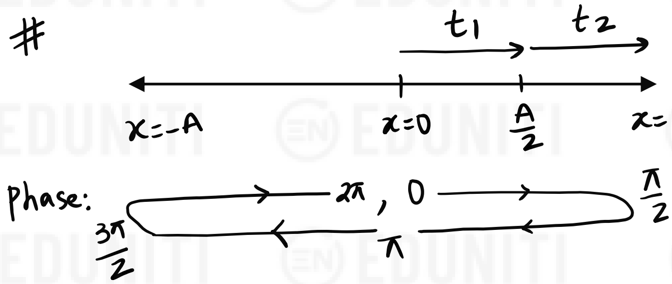


↳ At mean position system has PE.

#



(i) $t_{x=0 \to x=A} = T/4$

(ii) $x = A\sin\omega t$

$\Rightarrow \frac{A}{2} = A\sin\omega t_1$

$\Rightarrow \frac{1}{2} = \sin\omega t_1 \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T}t_1$

$\therefore \boxed{t_1 = \frac{T}{12}}$

(iii) $t_2 = \frac{T}{4} - t_1 = \boxed{\frac{T}{6}}$

2. STEPS TO FIND TIME PERIOD

LINEAR SHM

(1.) Give linear displacement of x from mean position

(2.) Find linear acceleration. $F = ma$

(3) you will get

$a = -\underbrace{(\text{some constant})}_{\omega^2} \times x$

$\therefore T = 2\pi/\omega$

ANGULAR SHM

(1.) Give angular displacement of θ (small) from mean position.

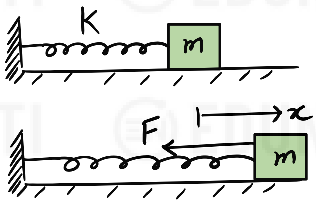
(2.) Find angular acceleration $\tau = I\alpha$

(3.) you will get

$\alpha = -\underbrace{(\text{some const.})}_{\omega^2} \times \theta$

$\therefore T = 2\pi/\omega$

3. SPRING BLOCK AND SIMPLE PENDULUM

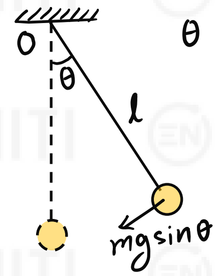


$F = -kx$

$\Rightarrow ma = -kx$

$\Rightarrow a = -\left(\frac{K}{m}\right)x$ ($\frac{K}{m} \to \omega^2$)

$\therefore T = 2\pi\sqrt{\frac{m}{K}}$



θ is very small

$\tau = -mg\sin\theta \times l$

$\Rightarrow I\alpha = -mgl \times \theta$

$\Rightarrow ml^2\alpha = -mgl \times \theta$

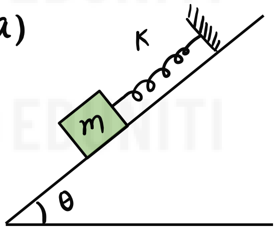
$\Rightarrow \alpha = -\left(\frac{g}{l}\right)\theta$ ($\frac{g}{l} \to \omega^2$)

$\therefore T = 2\pi\sqrt{\frac{l}{g}}$

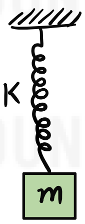
4. SPRING BLOCK (under presence of Constant force)

NOTE: Result remains same (only mean position changes)

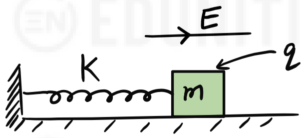
(a)



(b)



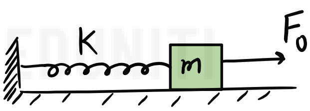
(c)



(i) In all cases

$T = 2\pi\sqrt{\frac{m}{K}}$

(d)



(ii) Mean position changes.

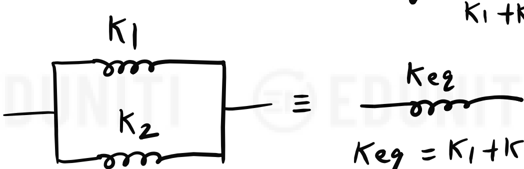
F_0 is external constant force

5. COMBINATION OF SPRING

(a)

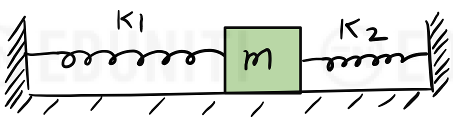
$K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$

(b)



$K_{eq} = K_1 + K_2$

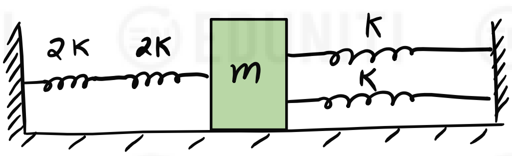
ex: 1



Here still K_1 and K_2 are in Parallel.

$T = 2\pi\sqrt{\frac{m}{K_1 + K_2}}$

ex: 2



$T = 2\pi\sqrt{\frac{m}{3K}}$

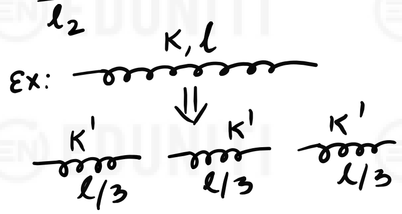
$K_{eq} = \frac{2K}{2} + 2K = 3K$

6. SPRING CUT ($K \cdot l = \text{const.}$)

$K l = K_1 l_1 = K_2 l_2$

$\therefore K_1 = \dfrac{K l}{l_1}$, $K_2 = \dfrac{K l}{l_2}$

Ex:

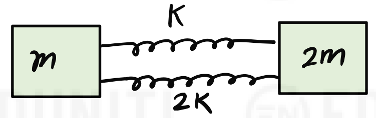


$\therefore$ Each $K' = 3K$

7. TWO BLOCK SYSTEM

(Reduced mass, $\mu_{red} = \dfrac{m_1 m_2}{m_1 + m_2}$)

Ex:



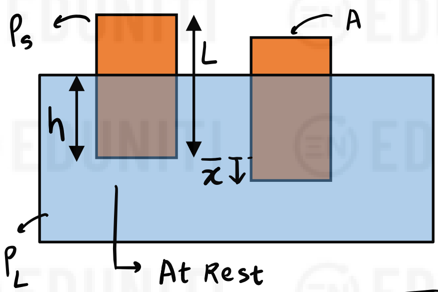
$T = 2\pi \sqrt{\dfrac{\mu_{red}}{K_{eq}}}$

$\mu_{red} = \dfrac{m \times 2m}{m + 2m} = \dfrac{2m}{3}$

$K_{eq} = K + 2K = 3K$

$\therefore T = 2\pi \sqrt{\dfrac{2m}{9K}}$

8. BLOCK IN LIQUID ($\rho_s < \rho_L$)



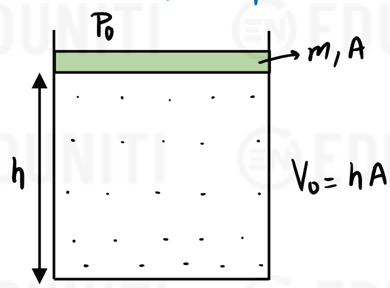
$T = 2\pi \sqrt{\dfrac{h}{g}}$ or $T = 2\pi \sqrt{\dfrac{m}{A \rho_L g}}$

A: Cross-section area

At equillibrium:

$\dfrac{\rho_s}{\rho_L} = \dfrac{h}{L}$

9. SHM OF PISTON IN CYLINDER



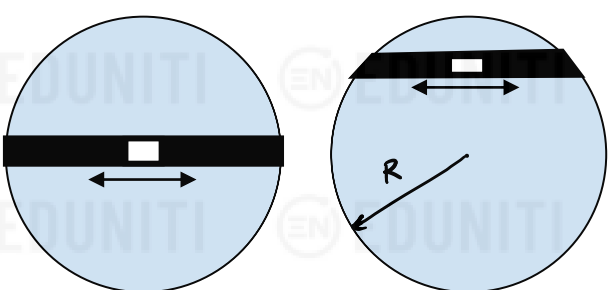
Piston given small displacement

Isothermal: $T = 2\pi \sqrt{\dfrac{m V_0}{P_0 A^2}}$

Adiabatic: $T = 2\pi \sqrt{\dfrac{m V_0}{\gamma P_0 A^2}}$

C_p / C_v

10. SHM IN TUNNEL IN A PLANET

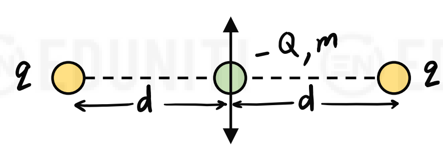


$T = 2\pi \sqrt{\dfrac{R}{g}}$

where $g = \dfrac{GM}{R^2}$

11. SHM OF CHARGE

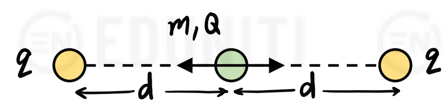
(a)



For small disturbance

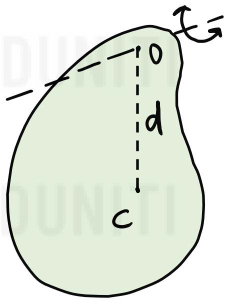
$T = 2\pi \sqrt{\dfrac{m d^3}{2KQq}}$, $K = \dfrac{1}{4\pi \varepsilon_0}$

(b)



$T = 2\pi \sqrt{\dfrac{m d^3}{4KQq}}$, $K = \dfrac{1}{4\pi \varepsilon_0}$

12. PHYSICAL PENDULUM

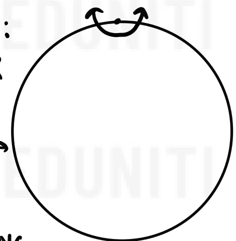


$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

I : MoI of body about O (Point of suspension)

d : Distance between O and C (Centre of mass)

Ex:



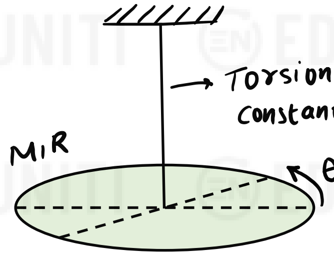
$I = MR^2 + MR^2 = 2MR^2$

$d = R$

$\therefore T = 2\pi\sqrt{\frac{2MR^2}{MgR}}$

$\Rightarrow T = 2\pi\sqrt{2R/g}$

13. TORSIONAL PENDULUM



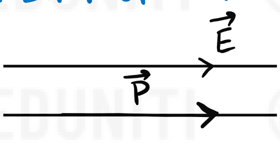
$\tau = -C\theta$

$\Rightarrow I\alpha = -C\theta$

$\therefore \alpha = -\underbrace{\left(\frac{C}{I}\right)}_{\omega^2}\theta$

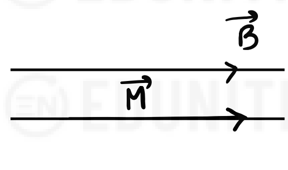
$\therefore T = 2\pi\sqrt{\frac{I}{C}}$

14. SHM OF DIPOLE IN FIELD (small angular displacement)



$$T = 2\pi\sqrt{\frac{I}{PE}}$$

I : Moment of Inertia

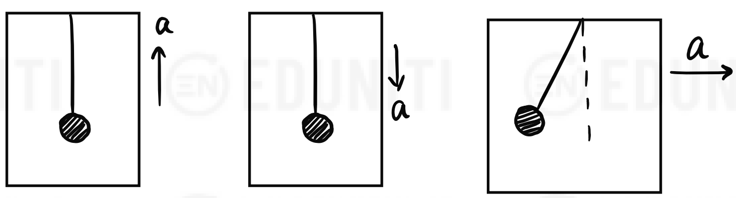


$$T = 2\pi\sqrt{\frac{I}{MB}}$$

I : Moment of Inertia

15. T OF SIMPLE PENDULUM (Different Condition)

$$\boxed{T = 2\pi\sqrt{\frac{L}{g_{eff}}}}$$



$g_{eff} = a + g$ $\quad$ $g_{eff} = g - a$ $\quad$ $g_{eff} = \sqrt{a^2 + g^2}$

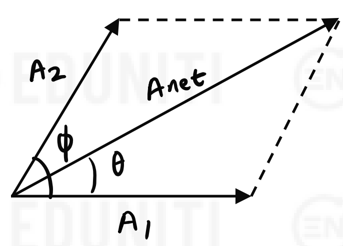
16. SUPERPOSITION OF SHM

$Y = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$

$= A_{net} \sin(\omega t + \theta)$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

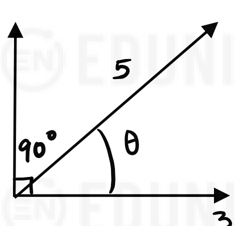
$\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right)$



Ex: $Y = 3\sin(\omega t + 30^\circ) + 4\sin(\omega t + 120^\circ)$

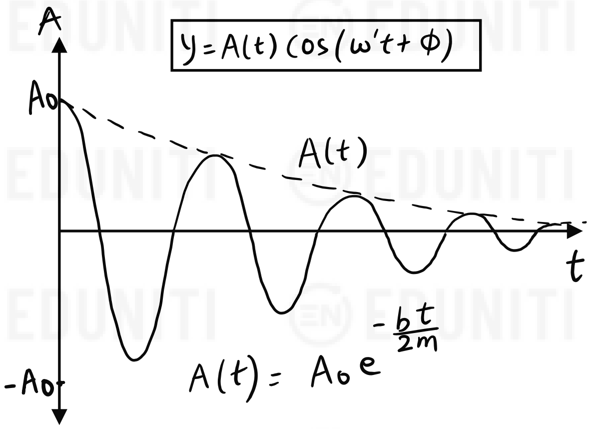
$\Rightarrow Y = 5\sin(\omega t + 30^\circ + \theta)$

where $\theta = \tan^{-1}\frac{4}{3}$



17. Damped Oscillation

$$\boxed{y = A(t)\cos(\omega' t + \phi)}$$



$$A(t) = A_0 e^{-\frac{bt}{2m}}$$

b: Damping Constant (*depends on medium*)

$$\boxed{y = A(t)\cos(\omega' t + \phi)}$$

$$\rightarrow A(t) = A_0 e^{-\frac{bt}{2m}}$$

$$\rightarrow \omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$$

<u>Space to add concepts learnt from PYQs if any</u>

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Thermal Properties – Part 1

1. Temperature Scales
2. Thermal Expansion
3. Effect of Temperature on density
4. Anomalous behavior of Water
5. Expansion in Cavities / Gaps etc.
6. Apparent Expansion of Liquid (*Hg rise in Thermometer*)
7. Effect of Temperature on Pendulum Clocks (*Second's Pendulum*)
8. Thermal Stress
9. Bimetallic Strip
10. Calorimetry – Heat Exchange
11. Phase Transformation (*Graph Understanding*)
12. Water Equivalent

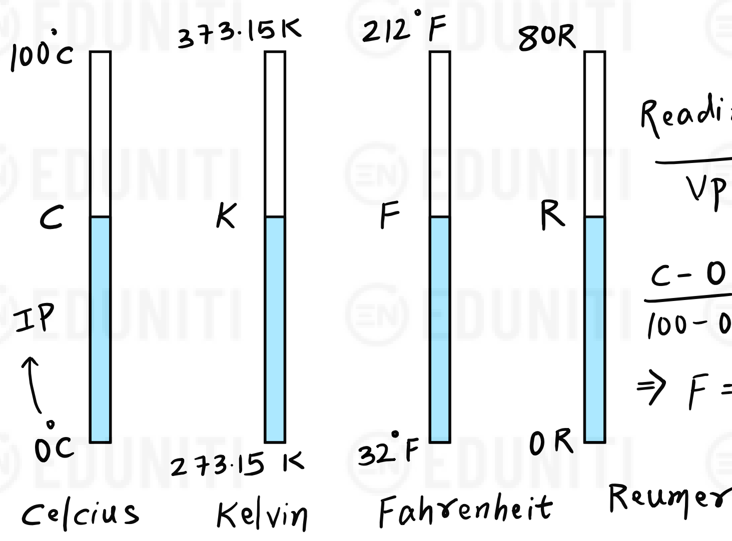
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Temperature Scales

VP

VP : Vapour point

IP : Ice point (water)



Conversion Relation :

$$\frac{\text{Reading} - IP}{VP - IP} = \text{Const.}$$

$$\frac{C-0}{100-0} = \frac{F-32}{212-32}$$

$$\Rightarrow F = \frac{9}{5}C + 32$$

2. Thermal Expansion

↳ distance between any two points ↑ due rise in temp.

Linear Expansion

$\Delta L = L_i \alpha \Delta T$

$\Rightarrow L_f = L_i(1+\alpha\Delta T)$

α → Coefficient of linear Expansion

Superficial or Areal Expansion

$\Delta A = A_i \beta \Delta T$

$\Rightarrow A_f = A_i(1+\beta\Delta T)$

$\beta = 2\alpha$

β → Coefficient of Area Expansion

Volume Expansion

$\Delta V = V_i \gamma \Delta T$

$\Rightarrow V_f = V_i(1+\gamma\Delta T)$

$\gamma = 3\alpha$

γ → Coefficient of Volume Expansion

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

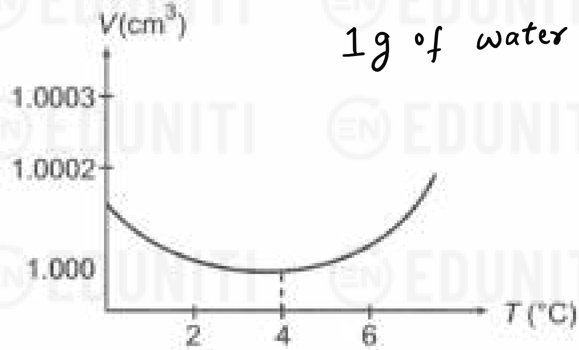
3. Effect of temperature on density

$$\rho_i = \frac{m}{V_i}, \quad \rho_f = \frac{m}{V_f} = \frac{m}{V_i(1+\gamma\Delta T)}$$

$$\Rightarrow \rho_f = \frac{\rho_i}{(1+\gamma\Delta T)}$$

If $\gamma\Delta T << 1 \Rightarrow \boxed{\rho_f = \rho_i(1-\gamma\Delta T)}$

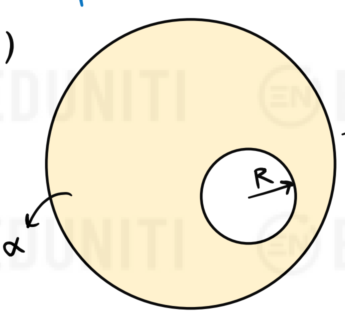
4. Anomalous Expansion of Water



↳ This anomalous behaviour causes ice to form first at the Surface of Lake in cold weather

5. Expansion in Cavities/ Gaps etc.

(i)



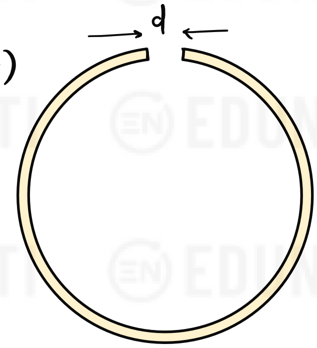
Disc, ΔT → $R_f = R(1+\alpha\Delta T)$

$A_f = \pi R^2(1+2\alpha\Delta T)$

Sphere, ΔT → $R_f = R(1+\alpha\Delta T)$

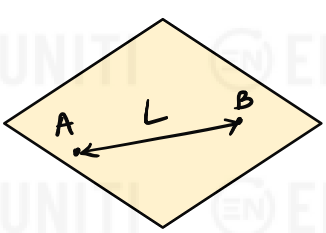
$V_f = \frac{4}{3}\pi R^3(1+3\alpha\Delta T)$

(ii)



$d_f = d(1+\alpha\Delta T)$

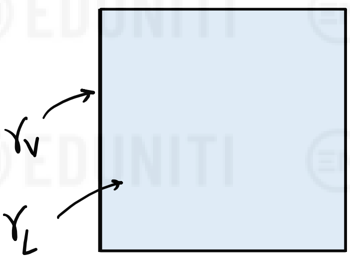
(iii)



$L_f = L(1+\alpha\Delta T)$

Part 1 – Thermal Properties

6. Apparent Expansion of Liquid



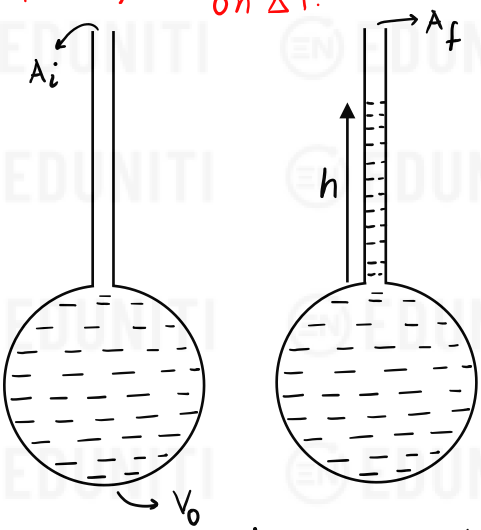
on increasing temp° $(\Delta T) \Rightarrow \Delta V_V = V_0 \gamma_V \Delta T$, $\Delta V_L = V_0 \gamma_L \Delta T$

$\therefore$ Apparent Expansion, $\Delta V_{app} = V_0 \underbrace{(\gamma_L - \gamma_V)}_{\gamma_{APP}} \Delta T$

γ_{APP}

- $\gamma_{APP} > 0$ $\Rightarrow \gamma_L > \gamma_V$ $\therefore$ Liquid overflow
- $\gamma_{APP} = 0$ $\Rightarrow \gamma_L = \gamma_V$ $\therefore$ Liquid level same
- $\gamma_{APP} < 0$ $\Rightarrow \gamma_L < \gamma_V$ $\therefore$ Liquid Level falls

Ex2. Mercury rise in thermometer on ΔT.



γ_L for Liquid
α for vessel

$\gamma_L > 3\alpha$

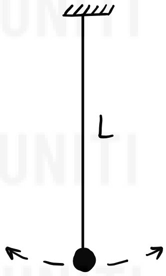
sol^n: App ↑ in vol of liquid,

$$\Delta V = V_0(\gamma_L - 3\alpha)\Delta T$$

$$\therefore \Delta V = h \times A_f$$

$$\Rightarrow h = \frac{\Delta V}{A_f} = \frac{V_0(\gamma_L - 3\alpha)\Delta T}{A_i(1 + 2\alpha\Delta T)}$$

7. Effect of Temperature on Pendulum Clocks

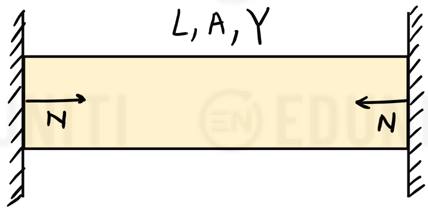


Second's pendulum → for 1 oscillation of Pendulum, clock shows 2 sec

$\Rightarrow$ T of Pendulum is also 2 sec $\left\{ T = 2\pi\sqrt{\frac{L}{g}} \right.$

- Temp ↑, $\Delta\theta$ $\Rightarrow L\uparrow \Rightarrow T\uparrow$ $\therefore$ clock becomes slow/Looses time
- Temp ↓, $\Delta\theta$ $\Rightarrow L\downarrow \Rightarrow T\downarrow$ $\therefore$ Fast/Gain time

8. Effect of ΔT on Rod between walls (thermal stress)



on $\uparrow$ temp° by ΔT, $\Delta L = L\alpha\Delta T$

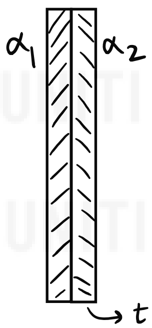
(ΔL is also compression by N)

$\therefore$ Thermal stress $= Y \times \frac{\Delta L}{L}$

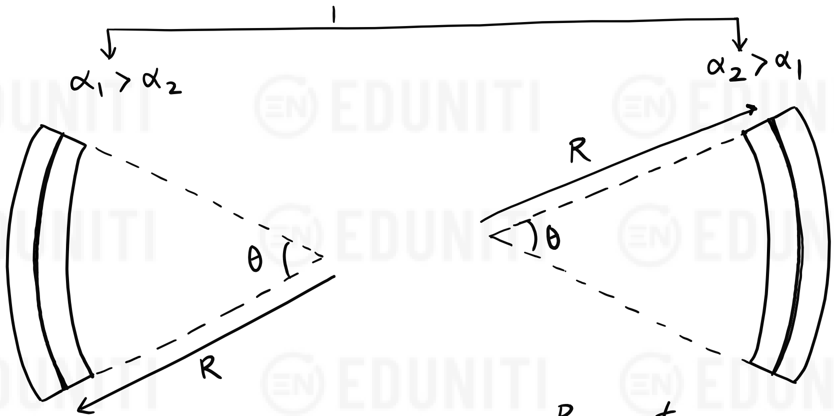
$= \boxed{Y\alpha\Delta T}$

and, $\boxed{N = AY\alpha\Delta T}$

9. Bimetallic strip



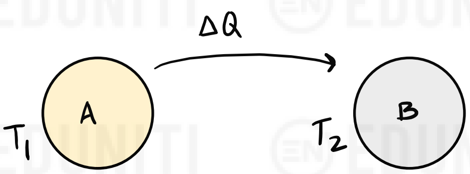
on heating by ΔT



$R = \frac{t}{(\alpha_1 - \alpha_2)\Delta T}$

$R = \frac{t}{(\alpha_2 - \alpha_1)\Delta T}$

10. Calorimetry - Heat Exchange



$T_1 > T_2$

Assuming no heat lost to surrounding:

(i) Heat loss by A = Heat Gain by B

$\Rightarrow m_A s_A (T_1 - T) = m_B s_B (T - T_2)$

↳ T is final temp°

↳ $s = \frac{\Delta Q}{m\Delta T}$, specific heat capacity

$J/kg°C$ or $cal/g°C$

↳ 1 cal = 4.2 J

NOTE :

Heat capacity, $C = ms$

$\therefore \Delta Q = C\Delta T$

11. Phase Transformation

↳ heat required to change phase, $\Delta Q = mL \rightarrow$ Latent heat, J/kg or cal/g

Latent heat of fusion

$$L_f = \frac{Q}{m}$$

Latent heat of Vaporisation

$$L_v = Q/m$$

(i) L_f for ice = 80 cal/g { By loosing 80 cal, 1g of water changes to ice at 0°c & Vice-Versa

(ii) L_v for steam = 540 cal/g

(iii) During phase change T is const.

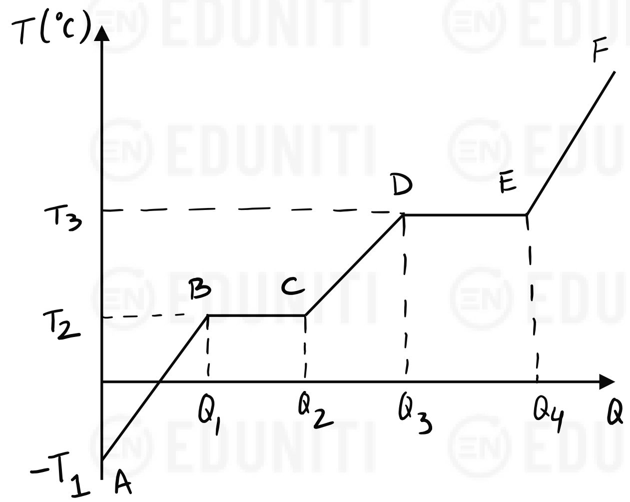
(i) AB → solid

(ii) T_2 → Melting pt.

(iii) CD → Liquid

(iv) T_3 → Boiling pt.

(v) EF → Vapour

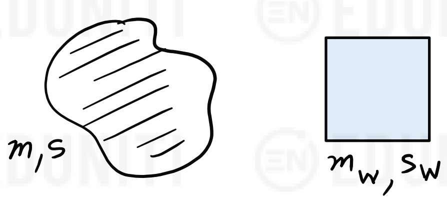


$Q_1 = mS_s(T_2 + T_1)$ $\quad Q_2 - Q_1 = mL_f$

$Q_3 - Q_2 = mS_L(T_3 - T_2)$ $\quad Q_4 - Q_3 = mL_v$

12. Water Equivalent

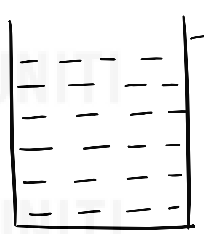
↳ Amount of water which requires the same amount of heat for same ΔT as that of object.



$$msΔT = m_w s_w ΔT \Rightarrow \boxed{m_w = \frac{ms}{s_w}}$$

→ W (water Equivalent)

Ex:



→ suppose water equivalent of container is 50g

⇒ Instead of Container, take 50g water

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Heat Transfer – Part 2

1. Modes of Heat Transfer
2. Thermal Conduction
3. Analogy with Ohm's Law
4. Interface & Junction Temperature
5. Temperature variation at Steady State
6. Equivalent Thermal Conductivity
7. Radial Heat Conduction
8. Freezing of Lake
9. Thermal Radiation
10. Stefan – Boltzmann Law
11. Newton's Law of Cooling
12. Variation of body temperature as per Newton's Law
13. Average form of Newton's Law of Cooling
14. Wien's Displacement Law

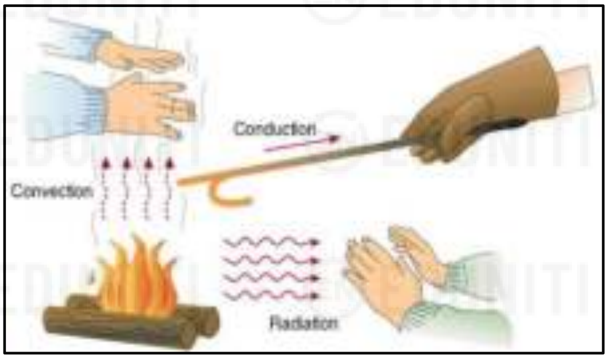
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Modes of heat transfer

Conduction
Need medium
No particle transfer
atoms vibrate

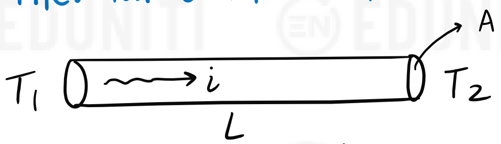
Convection
Need medium
Particle tranfers

Radiation
No medium
Energy transfer
through Em waves



Part 2 – Heat Transfer

2. Thermal Conduction



If $T_1 > T_2$, at steady state

Heat current, $i = \dfrac{dQ}{dt} = \dfrac{KA(T_1 - T_2)}{L}$ {J/s}

(i) $\dfrac{dQ}{dt}$ is constant

(ii) K is thermal conductivity {W/m-K}

3. Analogy with Ohm's law

$I = \dfrac{V_1 - V_2}{R}$

where $R = \dfrac{\rho L}{A}$

Similarily, $i = \dfrac{T_1 - T_2}{L/KA} = \dfrac{T_1 - T_2}{R_{th}}$

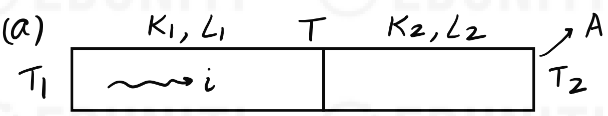
$\therefore R_{th} = \dfrac{L}{KA}$

Thermal Resistance

Note:
Resistance in series & parallel concept valid

4. Interface & Junction Temperature

(a)



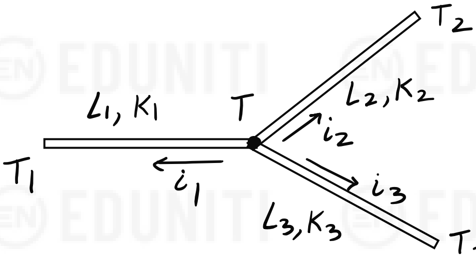
$\therefore$ i is same both rods

$\Rightarrow \dfrac{T_1 - T}{L_1/K_1A} = \dfrac{T - T_2}{L_2/K_2A}$

Solve for T.

$R_1 = \dfrac{L_1}{K_1A}$, $R_2 = \dfrac{L_2}{K_2A}$

(b)

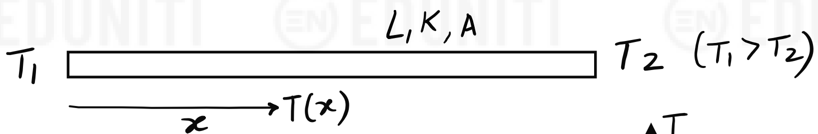


$i_1 + i_2 + i_3 = 0$

$\Rightarrow \dfrac{T - T_1}{R_1} + \dfrac{T - T_2}{R_2} + \dfrac{T - T_3}{R_3} = 0$

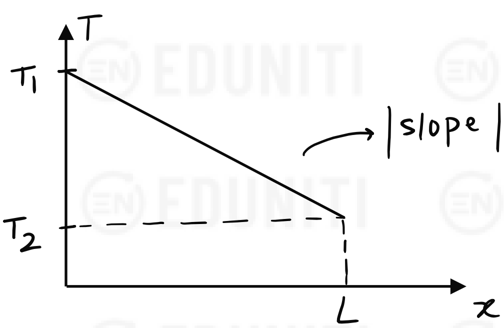
where, $R_1 = \dfrac{L_1}{K_1A_1}$, $R_2 = \dfrac{L_2}{K_2A_2}$, $R_3 = \dfrac{L_3}{K_3A_3}$

5. Temperature Variation at steady state



$\dfrac{T_1 - T(x)}{x/KA} = \dfrac{T_1 - T_2}{L/KA}$

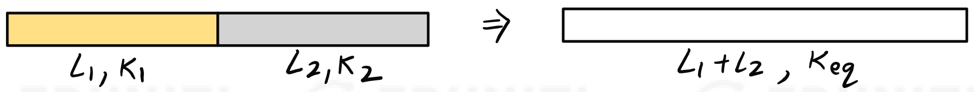
$\Rightarrow \boxed{T(x) = -\left(\dfrac{T_1 - T_2}{L}\right)x + T_1}$



$|\text{slope}| = \dfrac{T_1 - T_2}{L}$

6. Equivalent Thermal Conductivity (K_{eq})

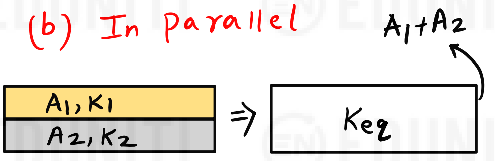
(a) In Series



$$R_1 + R_2 = R_{eq} \Rightarrow \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} = \frac{L_1 + L_2}{K_{eq} A}$$

$$\Rightarrow K_{eq} = \frac{K_1 K_2 (L_1 + L_2)}{L_1 K_2 + L_2 K_1}$$

(b) In parallel

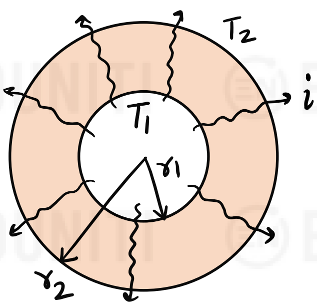


$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}$$

$$\Rightarrow \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L} = \frac{K_{eq}(A_1 + A_2)}{L} \quad \therefore K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

7. Radial Heat Conduction

(a)

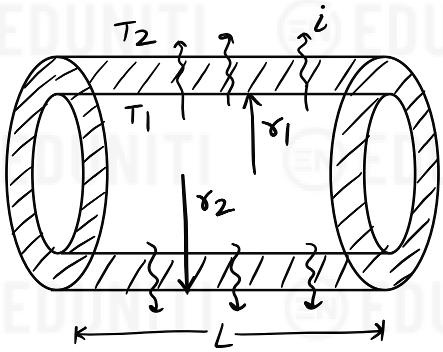


Spherical shell

$$i = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

(b)

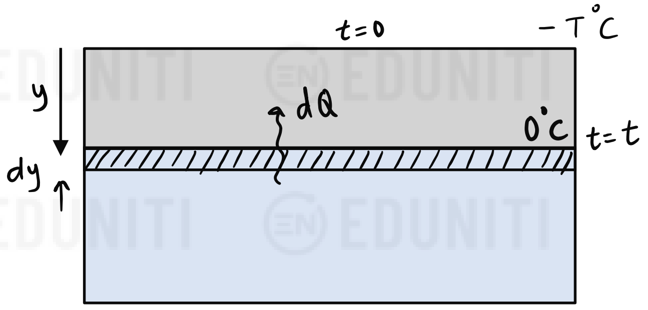


Cylindrical shell

$$i = \frac{T_1 - T_2}{R_{th}}$$

$$R_{th} = \frac{\ln(r_2/r_1)}{2\pi K L}$$

8. Freezing of Lake (time taken to freez depth y)



(i) at $t=0$ Lake starts to freez from top.

(ii) at time t 'y' depth freezes

(iii) In next dt time dy freezes by loosing dQ heat.

$$\frac{dQ}{dt} = \frac{KA}{y} \cdot T \Rightarrow dm \cdot L_f = \frac{KA}{y} \cdot T \cdot dt$$

$$\Rightarrow \rho A dy \cdot L_f = \frac{KA}{y} \cdot T \cdot dt \Rightarrow \frac{\rho L_f}{KT} \int_0^y y\,dy = \int_0^t dt \quad \therefore \boxed{t = \frac{\rho L_f}{2KT} \cdot y^2}$$

9. Thermal Radiation

Part 2 – Heat Transfer

Prevost Theory

Radiated (If $T > 0K$)

(i) $T > T_s$

$\Rightarrow$ Radiated > absorbed

& vice-versa

(ii) $T = T_s$

$\Rightarrow$ Radiated = absorbed

Kirchoff's Law

Good emitter are also good absorber

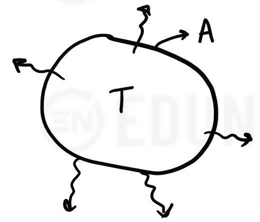
$(e = a < 1)$

e → emissivity, a → absorpitivity

For BLACK Body

$e = a = 1$

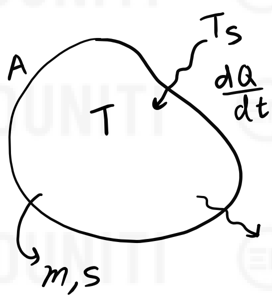
Stefan's Law



$\frac{dQ}{dt} = \sigma A T^4$

↳ rate of heat transfer for Black-Body

10. Stefan-Boltzmann Law



$\frac{dQ}{dt} = \sigma e A T_s^4$

$\frac{dQ}{dt} = \sigma e A T^4$

Net rate of heat Loss by body

$\frac{dQ}{dt} = \sigma e A (T^4 - T_s^4)$, $T > T_s$

σ → Stefan-Boltzmann Const.

Rate of Cooling

$\Delta Q = m s \Delta T \Rightarrow \frac{dQ}{dt} = -ms\frac{dT}{dt}$ $\left\{ \begin{array}{l} -ve \\ as \\ T\downarrow \end{array} \right.$

$$\Rightarrow \frac{dT}{dt} = -\frac{\sigma e A}{ms}(T^4 - T_s^4)$$

11. Newton's Law of Cooling

↳ Applicable if T is close to T_s $(T - T_s = \Delta T$ or $T = T_s + \Delta T)$

$$\frac{dT}{dt} = -\frac{\sigma e A}{ms}\left[(T_s + \Delta T)^4 - T_s^4\right] = -\frac{\sigma e A}{ms} \cdot T_s^4\left[\left(1 + \frac{\Delta T}{T_s}\right)^4 - 1\right]$$

$\because \Delta T$ is small

$(1+x)^n \simeq 1 + nx$

$$\Rightarrow \frac{dT}{dt} = -\frac{\sigma e A}{ms} \cdot T_s^4\left(1 + 4\frac{\Delta T}{T_s} - 1\right)$$

$$\Rightarrow \frac{dT}{dt} = -\frac{4\sigma e A T_s^3 \Delta T}{ms} \Rightarrow \frac{dT}{dt} = -\frac{4\sigma e A T_s^3}{ms}(T - T_s)$$

$$\Downarrow$$

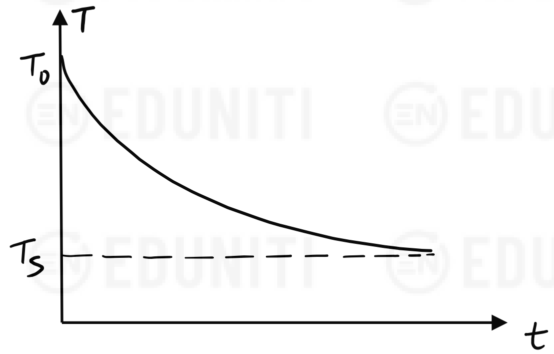
$$\frac{dT}{dt} \propto (T - T_s)$$

12. Variation of body temp^n as per Newton's Law

$$\frac{dT}{dt} = -\frac{4\sigma e A T_s^3}{ms}(T-T_s) \quad \text{or} \quad \frac{dT}{dt} = -c(T-T_s) \quad \left\{ c = \frac{4\sigma e A T_s^3}{ms} \right.$$

$$\Rightarrow \int_{T_0}^{T} \frac{dT}{T-T_s} = -c\int_0^t dt \quad \Rightarrow \boxed{T = T_s + (T_0 - T_s)e^{-ct}}$$

T_0 is bodies temp^n at $t=0$



13. Average form of Newton's Law of cooling

↳ If body cools from T_2 to T_1 in time t,

$$\frac{T_2 - T_1}{t} = c\left(\frac{T_2+T_1}{2} - T_s\right)$$

JEE 2021, July

Ex 7. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is ______°C.

Sol^n: $\frac{75-65}{5} = c\left(\frac{75+65}{2} - 25\right) \Rightarrow c = \frac{2}{45}$

$$\frac{65-T}{5} = \frac{2}{45}\left(\frac{65+T}{2} - 25\right) \Rightarrow \boxed{T = 57^\circ C} \text{ Ans.}$$

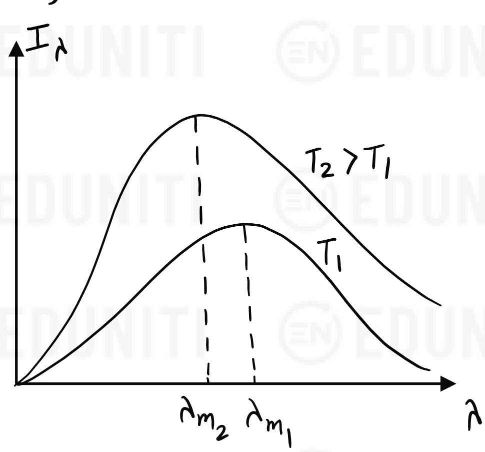
14. Wien's Displacement Law

↳ Energy distribution curve of black body radiation

(i) For a Given T of body, Various wavelength is emitted.

(ii) $\lambda_m \propto \frac{1}{T} \Rightarrow \lambda_m T = b$

↳ Wien's Const. 2.8×10^{-3} m-K



Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in KINETIC THEORY OF GASES– PART 1

1.Postulates of KTG
2.Boyle's Law
3.Charles Law
4.Dalton's Law of Partial Pressures
5.Different forms of Ideal Gas Law
6.Pressure exerted by a Gas
7.Maxwell's distribution of velocities
8.Degree of Freedom
9.Equipartition of energy and Internal energy
10.Mixing Gases
11.Mean Free Path
12.Mean Free Time

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Postulates of KTG :

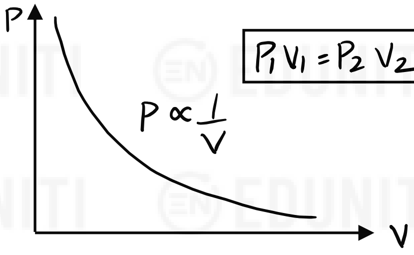
1. Molecule size negligible
2. Molecule is assumed to be hard sphere and all collision elastic
3. No interaction among molecules => Total Energy = K.E + P.E = K.E
4. Collision time is negligible
5. Effect of Gravity on molecule is neglected
6. Real gases obeys Ideal gas law PV=nRT at very high T and very low P

At STP, [P=1atm, T=273K] $PV = nRT$

$$\Rightarrow V = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5} = 2.24 \times 10^{-2}\,m^3 = \boxed{22.4\,L}$$

2. BOYLE'S LAW , T = const.

Boyle's law states that the pressure of a fixed amount of a gas varies inversely with the volume if the temperature is maintained constant.

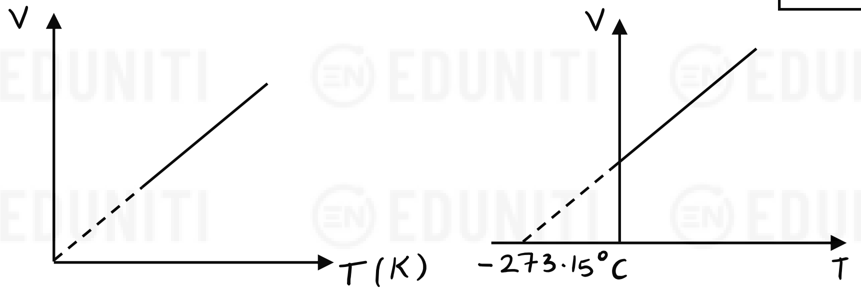


$$\boxed{P_1 V_1 = P_2 V_2}$$

3. CHARLES LAW, $P =$ const.

Charles law states that the pressure remaining constant, the volume of a fixed amount of a gas varies directly with its absolute temperature.

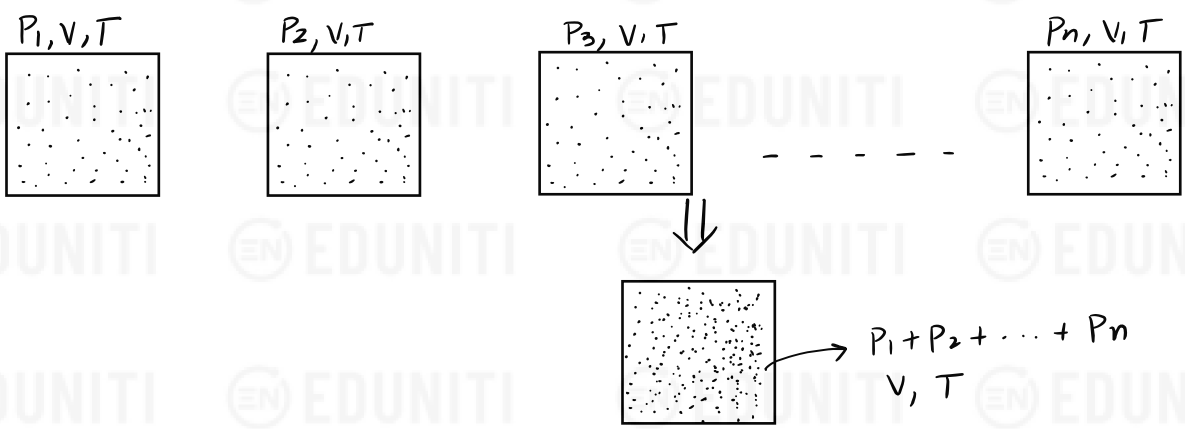
$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$



4. DALTON'S LAW OF PARTIAL PRESSURES

Pressure exerted by a mixture of non interacting gases is equal to the sum of their partial pressures. Hence, for a mixture of n gases, the total pressure of the gas is given by

$$\boxed{P = P_1 + P_2 + P_3 + \cdots + P_n}$$



5. DIFFERENT FORMS OF IDEAL GAS LAW

N : Total number of molecules of gas.

N_A : Avogadro No., 6.023×10^{23}

m : mass of gas

M : molecular mass

$$PV = nRT$$

$$PV = \frac{N}{N_A}RT$$

$$\Rightarrow \boxed{PV = NkT}$$

where, $\boxed{k = \frac{R}{N_A}}$ is Boltzmann's Constant.

$$k = 1.38 \times 10^{-23}\ J/K.$$

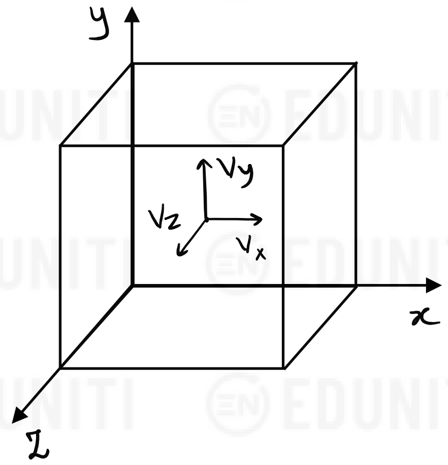
$$PV = \frac{m}{M}RT$$

$$\Rightarrow P = \frac{m}{V}\frac{RT}{M}$$

$$\Rightarrow \boxed{P = \frac{\rho RT}{M}}$$

6. PRESSURE EXERTED BY GAS:

Momentum transferred to a container wall per second per unit of its surface area is avg pressure, $P = \frac{1}{3}\rho v_{rms}^2$

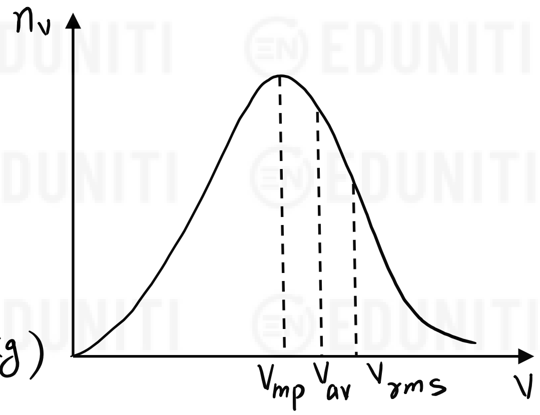


7. MAXWELL'S DISTRIBUTION OF VELOCITIES

A particle speed probability distribution shows how the speeds of molecules are distributed for an ideal gas.

n_v : no. of molecules per unit range of speeds.

$V_{mp} = \sqrt{\frac{2RT}{M}}$ $V_{av} = \sqrt{\frac{8RT}{\pi M}}$ $V_{rms} = \sqrt{\frac{3RT}{M}}$



NOTE:

(1.) T : Temp. in K
M : Molecular mass (kg)
V : speed in m/s

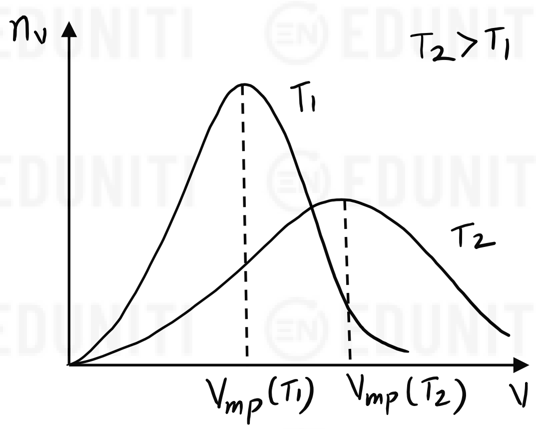
(2) $V_{mp} < V_{av} < V_{rms}$ for a given gas at T temp.

(3) Area under curve gives total no. of molecules

ON Increasing T:

(1.) V_{mp} increases.

(2.) More molecules have high speed.



8. DEGREE OF FREEDOM, f

Number of ways a molecule can Participate in contributing to the total mechanical energy of that molecule

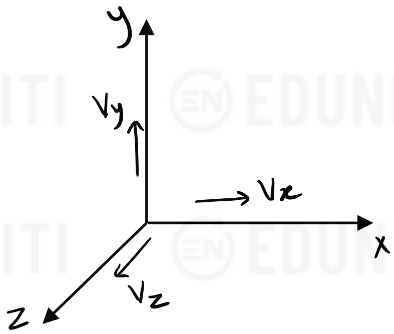
MODES

VIBRATIONAL

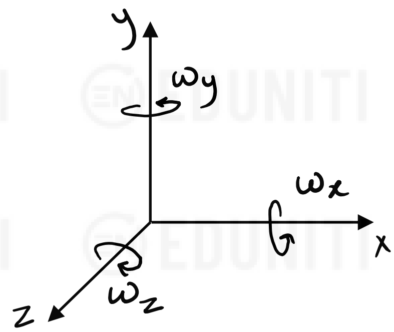
(1.) 2 DOF for each mode

(2.) Occurs at Very temp$^\circ$

TRANSLATIONAL

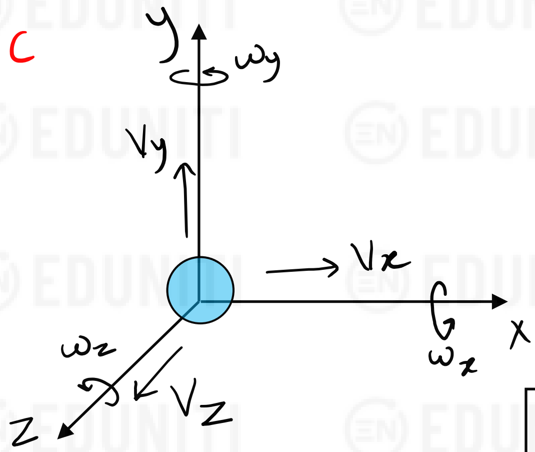


ROTATIONAL



(A) MONOATOMIC

(Eg: He, Ne, Ar)



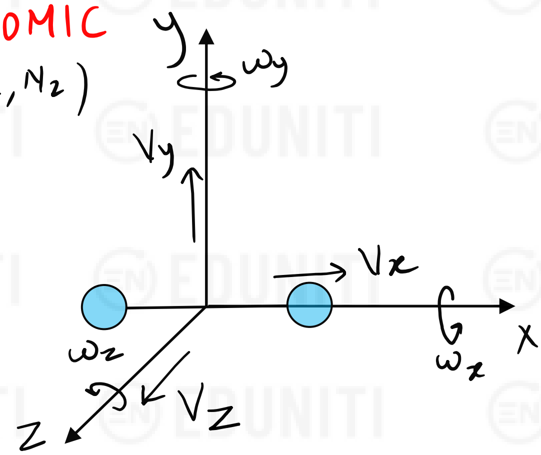
NOTE:

(1.) SINCE molecules size is assumed to be zero implies $\frac{1}{2}I\omega_y^2$, $\frac{1}{2}I\omega_x^2$, $\frac{1}{2}I\omega_z^2$ is zero as $I = 0$.

$\boxed{f = 3}$ Due to translational.

(B) DIATOMIC

(Eg: H_2, O_2, N_2)



NOTE:

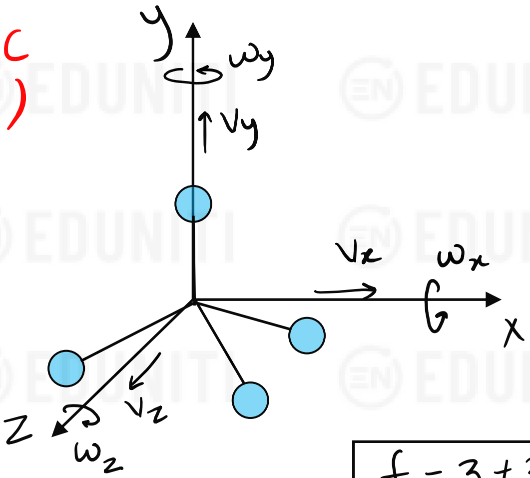
(1.) SINCE molecules are along x-axis, implies $\frac{1}{2}I\omega_x^2 = 0$

$\boxed{f = 3 + 2 = 5}$ At low temp$^\circ$

$\boxed{f = 3 + 2 + 2 = 7}$ At high temp$^\circ$

(C) POLYATOMIC (NON-LINEAR)

Eg: H_2O, CH_4



$$\boxed{f = 3 + 3 = 6}$$

NOTE:

(1.) LINEAR Polyatomic

Eg: CO_2, $f = 3 + 2 = 5$

(2.) Vibrational DOF will be mentioned in problems.

9. EQUIPARTION OF ENERGY AND INTERNAL ENERGY, U

According to this law, for any system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom with the average energy associated with each degree of freedom equal to $kT/2$ per molecule $RT/2$ or per mole.

Implies If 1 mole of gas has f DOF,

$$U = \frac{fRT}{2}$$

$\therefore$ For n mole of gas

$$\boxed{U = \frac{fnRT}{2}}$$ Total Kinetic energy.

10. MIXING GASES

(1.) Equivalent f for a gaseous mixture (Gases mixed at same temp°)

$$\boxed{f_{mix} = \frac{n_1 f_1 + n_2 f_2 + \cdots + n_N f_N}{n_1 + n_2 + \cdots + n_N}}$$

(2.) FINAL temp° of gaseous mixture (Gases at different temp° mixed at Const. volume in thermally insulated vessel, U remains Const.)

$$\boxed{T_f = \frac{f_1 n_1 T_1 + f_2 n_2 T_2 + \cdots + f_N n_N T_N}{f_1 n_1 + f_2 n_2 + \cdots + f_N n_N}}$$

11. MEAN FREE PATH, λ

The free path travelled by a molecule between two successive collisions

$$\boxed{\lambda = \frac{1}{\sqrt{2}\pi d^2 n}}$$

d : diameter of molecule

n : no. of molecules per unit volume, $\frac{N}{V}$

NOTE: using $PV = NkT$ or $\frac{P}{kT} = \frac{N}{V} = n$

$$\boxed{\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}}$$

Replacing n by P/kT

12. MEAN FREE TIME, τ

Time between two successive collisions

$$\boxed{\tau = \frac{\lambda}{V_{av}}}$$

NOTE: for τ calculation
Depending on problem we need to find dependence of τ with variables.

Eg: If T and P are changing

where, $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$ and $V_{av} = \sqrt{\frac{8RT}{M}}$

$\lambda \propto \frac{T}{P}$ and $V_{av} \propto \sqrt{T}$

$$\Rightarrow \tau \propto \frac{T}{P} \times \frac{1}{\sqrt{T}} \Rightarrow \boxed{\tau \propto \frac{\sqrt{T}}{P}}$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in THERMODYNAMICS – PART 2

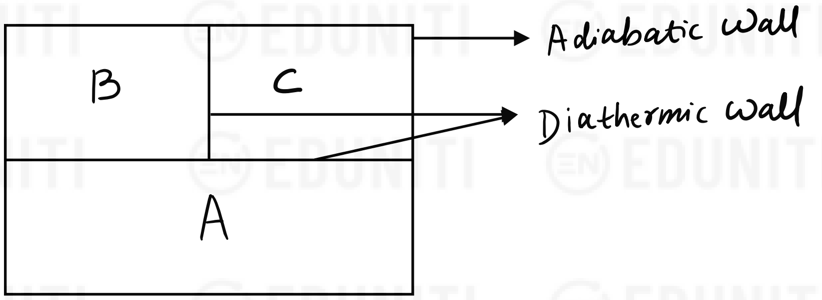
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. THERMAL EQUILIBRIUM

Two different temperature body in contact exchange heat until common temperature is reached. This is Called state of Thermal Equilibrium

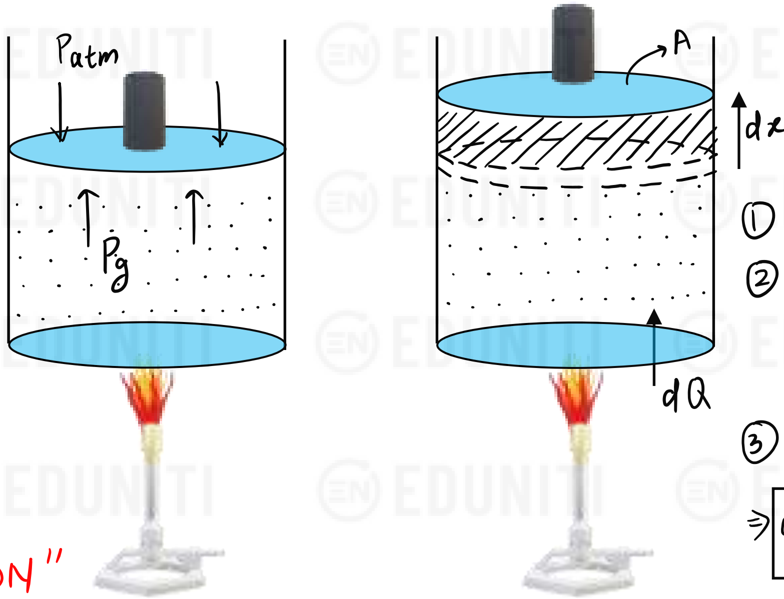
2. ZEROTH LAW OF THERMODYNAMICS

If two systems (B and C) are separately in thermal equilibrium with a third one (A), then they themselves are in thermal equilibrium with each other.



3. FIRST LAW OF THERMODYNAMICS

$$\boxed{dQ = dU + dW}$$



① dQ
② $dW = P_g A\,dx$
$= \boxed{P_g\,dV}$
③ $U = \frac{fRnT}{2}$
$\Rightarrow \boxed{dU = \frac{fRn\,dT}{2}}$

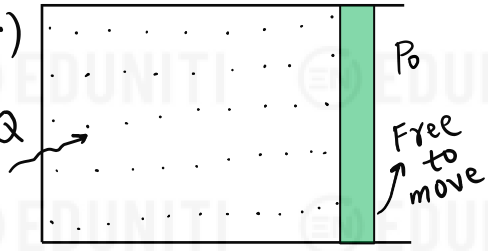
"SIGN CONVENTION"

(1) If heat is supplied, ΔQ is +VE
If heat is rejected, ΔQ is −VE

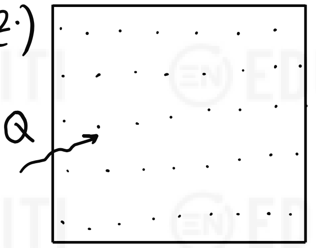
(2) If gas does work (volume of gas increases), ΔW is +VE
If work is done on the gas (volume of gas decreases), ΔW is −VE

(3) $T\uparrow \Rightarrow \Delta U$ is +VE
$T\downarrow \Rightarrow \Delta U$ is −VE

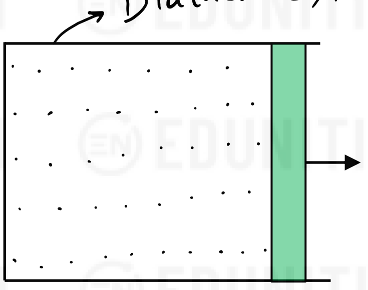
4. THERMODYNAMIC PROCESS

(1.)


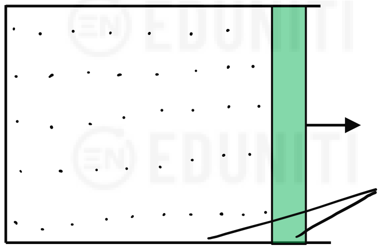
Isobaric Process $V \propto T$

(2.)


Isochoric Process $P \propto T$

(3.)


Isothermal Process $P \propto \frac{1}{V}$

(4.)


Adiabatic Process $PV^{\gamma} = C$

5. MOLAR HEAT CAPACITY, C

Molar heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of a substance by a unit degree.

$$\boxed{C = \frac{dQ}{n\,dT}} \Rightarrow \int dQ = \int_{T_1}^{T_2} nC\,dT$$

UNIT : $J\,mol^{-1}K^{-1}$

$$\Delta Q = nC(T_2 - T_1)$$

$$\Rightarrow \Delta Q = nC\Delta T$$

NOTE:

C depends on the way of heating / process.

Const. V → C_V

Const. P → C_P

C_P, C_V

(1.) $C_V = \dfrac{dQ}{n\,dT} = \dfrac{dU + \cancel{dW}^{\,0}}{n\,dT}$

$$= \frac{(fRn\,dT)/2}{n\,dT} \Rightarrow \boxed{C_V = \frac{fR}{2}}$$

Also,

$$U = \frac{fRnT}{2}$$

$$\Rightarrow U = nC_VT$$

or

$$\boxed{dU = nC_V\,dT}$$

(2.) $C_P = \dfrac{dQ}{n\,dT} = \dfrac{dU + dW}{n\,dT} = \dfrac{nC_V\,dT + P\,dV}{n\,dT}$

$$\Rightarrow C_P = C_V + \frac{P\,dV}{n\,dT} \quad \left(\begin{array}{l} PV = nRT \\ \Rightarrow P\,dV = nR\,dT \end{array}\right)$$

$$\Rightarrow C_P = C_V + R$$

$\boxed{C_P - C_V = R}$ MAYOR'S RELATION

$\boxed{C_P = \left(\frac{f+2}{2}\right)R}$

6. ADIABATIC CONSTANT, γ

$$\boxed{\gamma = \frac{C_P}{C_V}} \Rightarrow \boxed{\gamma = 1 + \frac{2}{f}} \Rightarrow \boxed{f = \frac{2}{\gamma - 1}}$$

GAS	f	C_P	C_V	γ
Monoatomic	3	$\frac{5R}{2}$	$\frac{3R}{2}$	$5/3$
Diatomic	5	$\frac{7R}{2}$	$\frac{5R}{2}$	$7/5$
NON-LINEAR Polyatomic	6	4R	3R	$4/3$

* Vibration mode only if mentioned

C_P, C_V in terms of γ and R

$$C_V = \frac{fR}{2} = \boxed{\frac{R}{\gamma - 1}}$$

$$C_P = \gamma C_V = \boxed{\frac{\gamma R}{\gamma - 1}}$$

7. C_p, C_v and γ FOR MIXTURE OF GASES

Gases at same temp° T,

n_1	C_{v_1}	C_{p_1}	f_1
n_2	C_{v_2}	C_{p_2}	f_2
n_3	C_{v_3}	C_{p_3}	f_3
⋮	⋮	⋮	⋮

$$C_{v_{mix}} = \boxed{\frac{n_1 C_{v_1} + n_2 C_{v_2} + \cdots}{n_1 + n_2 + \cdots}}$$

$$C_{p_{mix}} = \boxed{\frac{n_1 C_{p_1} + n_2 C_{p_2} + \cdots}{n_1 + n_2 + \cdots}}$$

$$f_{mix} = \boxed{\frac{n_1 f_1 + n_2 f_2 + \cdots}{n_1 + n_2 + \cdots}}$$

$$\gamma_{mix} = \boxed{\frac{C_{p\,mix}}{C_{v\,mix}}} \quad \text{or} \quad \boxed{1 + \frac{2}{f_{mix}}}$$

8. WORK DONE IN VARIOUS PROCESSES

$$\Delta W = \int_{V_1}^{V_2} P\,dV, \quad PV = nRT$$

ISOCHORIC

V = const.

$$\boxed{\Delta W = 0}$$

ISOBARIC

P = const.

$$\boxed{\Delta W = P(V_2 - V_1)}$$

OR

$$\boxed{\Delta W = nR(T_2 - T_1)}$$

ISOTHERMAL

T = const.

$$\boxed{\Delta W = nRT \ln\left(\frac{V_2}{V_1}\right)}$$

OR

$$\boxed{\Delta W = nRT \ln\left(\frac{P_1}{P_2}\right)}$$

ADIABATIC

$\Delta Q = 0$

$\Rightarrow \Delta W = -\Delta U$

$\Rightarrow \Delta W = -nC_v \Delta T$

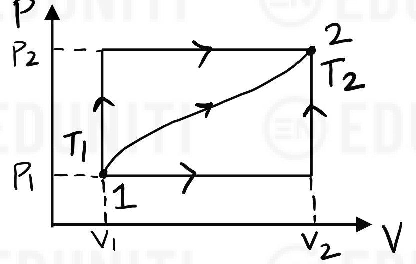
$$\boxed{\Delta W = \frac{nR(T_2 - T_1)}{1 - \gamma}}$$

OR

$$\boxed{\Delta W = \frac{P_2V_2 - P_1V_1}{1 - \gamma}}$$

9. INDICATOR DIAGRAM

When a gas is heated/cooled/compressed etc. thermodynamic parameters Pressure, Volume and Temperature changes. P, V and T combinedly defines the state of a Gas.



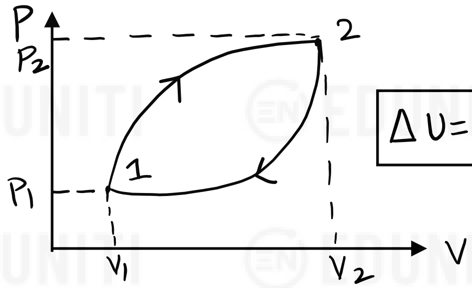
NOTE:

(1.) ΔU is state variable, only depends T_1 and T_2 and not path dependent.

(2.) ΔQ and ΔW both depends on path or how P and V are related.

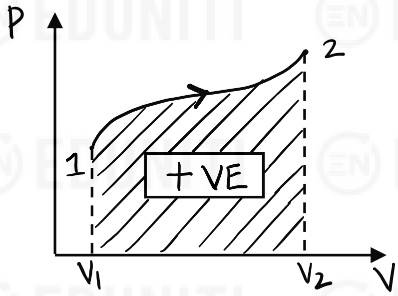
CYCLIC PROCESSES

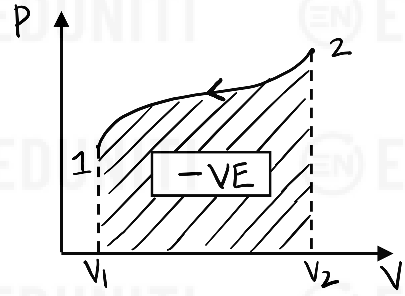
A process in which initial and final state are same (state means P, V and T are same)

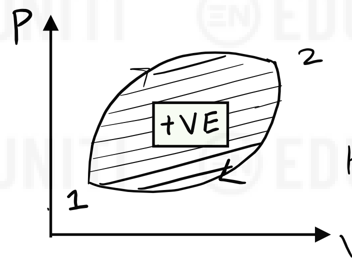


$\boxed{\Delta U = 0}$

10. ΔW USING P-V INDICATOR DIAGRAM



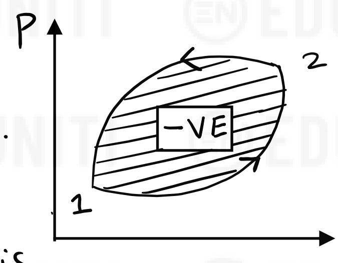




ΔU = 0

ΔQ = ΔW

Heat supplied



ΔQ = ΔW

Heat Rejected

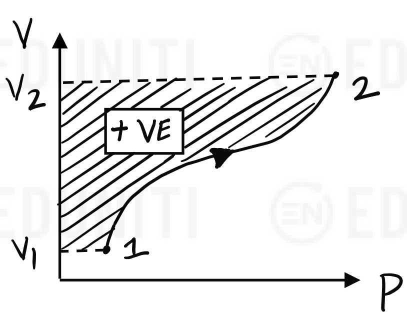
NOTE:

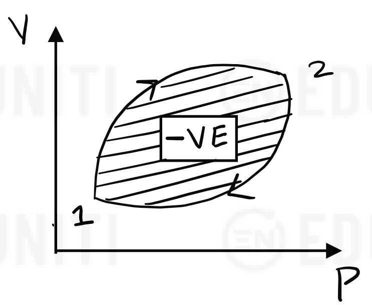
(1.) For Non-cyclic, area is between curve and V-axis.

$V\uparrow \Rightarrow \Delta W$ is +VE

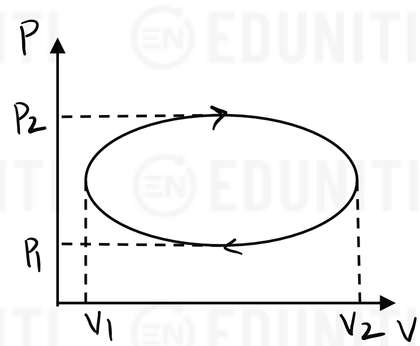
(2.) For cyclic, area is of the loop. If clockwise ⇒ ΔW is +VE. (P along y axis and V along x-axis)

... CONTINUED WITH Ex.





Ex:



→ Ellipse ($A = \pi ab$)

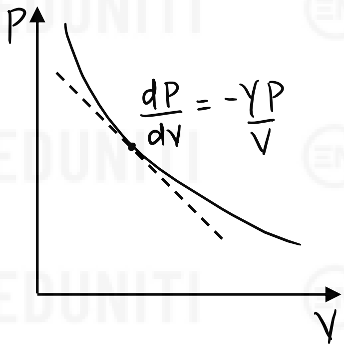
$$\Delta W = \pi\left(\frac{V_2 - V_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$$

11. ADIABATIC PROCESS, $\Delta Q = 0$

(1.) $\boxed{PV^{\gamma} = C}$

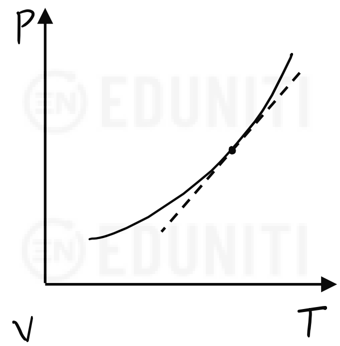
$$\frac{dP}{dV} \cdot V^{\gamma} + P\gamma V^{\gamma-1} = 0$$

$$\Rightarrow \boxed{\frac{dP}{dV} = -\gamma \frac{P}{V}}$$



(2.) $\boxed{T^{\gamma} P^{1-\gamma} = C}$

$$\frac{dP}{dT} = \frac{\gamma}{(\gamma-1)} \frac{P}{T}$$



(3.) $\boxed{TV^{\gamma-1} = C}$

$$\frac{dV}{dT} = -\frac{1}{\gamma-1} \frac{V}{T}$$

12. ISOTHERMAL V/S ADIABATIC : PV slope

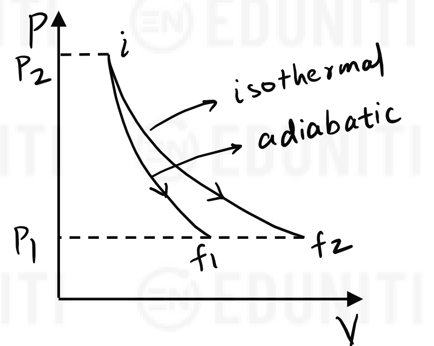
$\hookrightarrow PV = \text{const.}$

$$\Rightarrow \frac{dP}{dV} \cdot V + P = 0$$

$$\Rightarrow \boxed{\frac{dP}{dV} = -\frac{P}{V}}$$

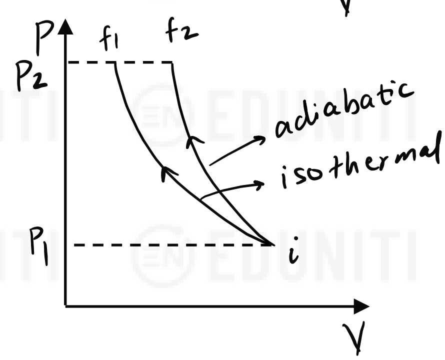
$\hookrightarrow PV^{\gamma} = \text{const.}$

$$\Rightarrow \boxed{\frac{dP}{dV} = -\frac{\gamma P}{V}}$$



Thus,

$$\left|\frac{dP}{dV}\right|_{ad} > \left|\frac{dP}{dV}\right|_{iso}$$



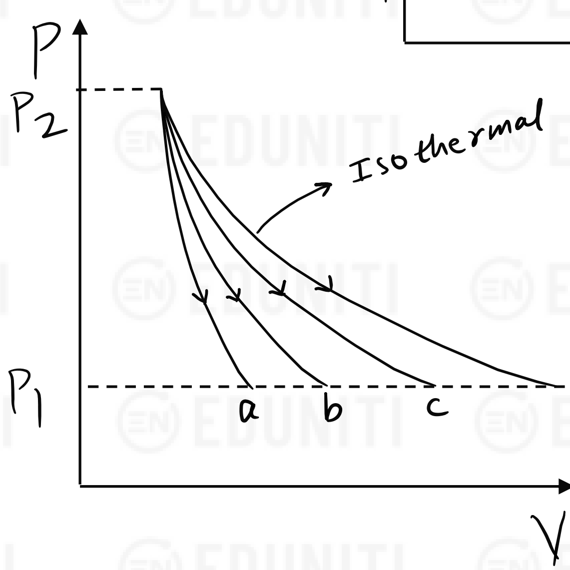
PV SLOPE AMONG GASES

$$\left|\frac{dP}{dV}\right| = \gamma \frac{P}{V} \quad \text{(adiabatic)}$$

a : monoatomic, $\gamma = \frac{5}{3} = 1.67$

b : diatomic, $\gamma = \frac{7}{5} = 1.4$

c : Polyatomic, $\gamma = \frac{4}{3} = 1.3$



13. REVERSIBLE POLYTROPIC PROCESS

Reversible Processes are Quasi-Static (very slow process such that system is always in thermodynamic Equilibrium) and Non-dissipative (no loss in energy due to friction, viscous force etc.)

$PV^x = \text{Const.}$ → Molar Heat Capacity

x : Polytropic Index

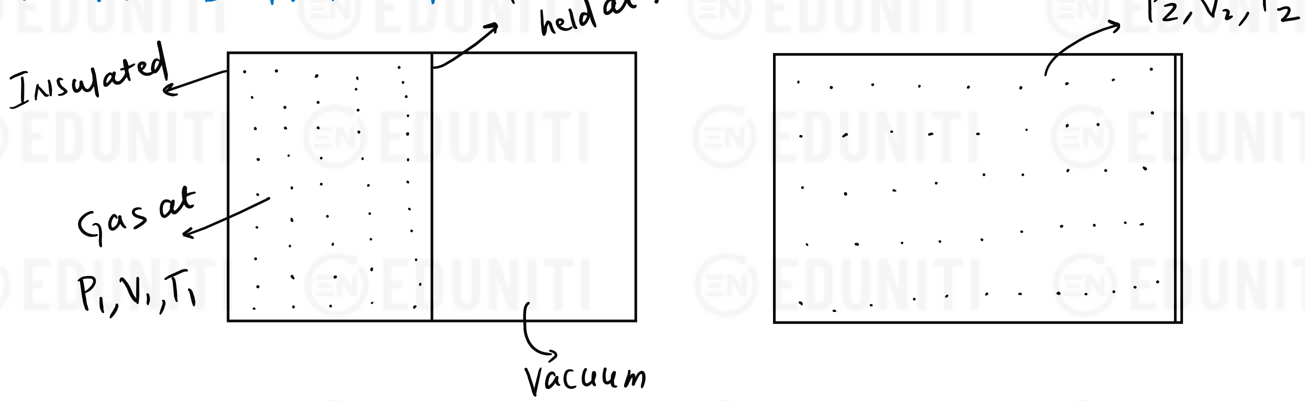
$$\boxed{C = C_V + \frac{R}{1-x}}$$

- $x \to \infty$ → $C = C_V$
- $x = 0$ → $C = C_V + R = C_P$
- $x = 1$ → $C \to \infty, T = \text{Const.}$
- $x = \gamma$ → $C = 0$

$$\boxed{\Delta Q = nC\Delta T}$$

$$\boxed{\Delta W = \frac{nR(T_2 - T_1)}{1-x}}$$

14. FREE EXPANSION



NOTE:

(1.) If no opposition ⇒ $\boxed{\Delta W = 0}$

(2) $\Delta Q = 0$ ⇒ $\Delta U = 0$ or $\boxed{T = \text{Const.}}$ ⇒ $T_1 = T_2$

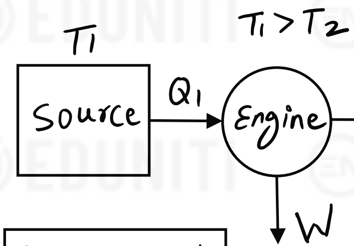
(3) $\boxed{P_1V_1 = P_2V_2}$ * Imp for problem solving.

15. SECOND LAW OF THERMODYNAMICS

Kelvin-Planck Statement → Impossible to convert 100% heat into Work (Heat Engines: Heat is supplied so that system does work)

Clausius Statement → Heat cannot flow from colder to hotter body without doing any external work. (Refrigerators: Work is done on the system to extract heat)

16. THERMAL EFFICIENCY OF A HEAT ENGINE



$\boxed{Q_1 - Q_2 = W}$

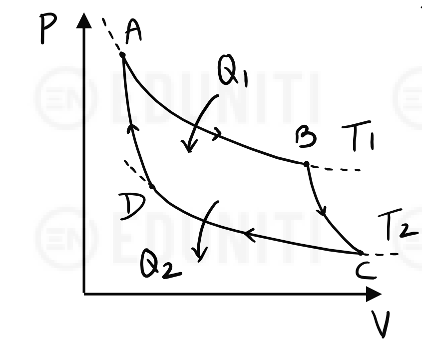
$\boxed{\eta = \frac{W}{Q_1}}$

or

$\boxed{\eta = 1 - \frac{Q_2}{Q_1}}$

* Q_1 and Q_2 are magnitudes.

Carnot Engine:
- Based on Carnot cycle
- Max. possible η (Ideal case)



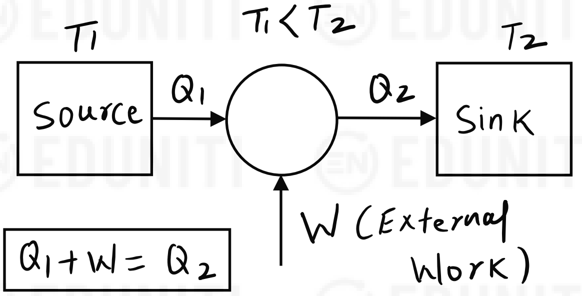
T = const. in A→B and C→D

$\Delta Q = 0$ in B→C and D→A

$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}}$

$\therefore \boxed{\eta = 1 - \frac{T_2}{T_1}}$

17. COEFFICIENT OF PERFORMANCE OF A REFRIGERATOR



$\boxed{Q_1 + W = Q_2}$

A best refrigeration cycle is one that removes the greatest amount of heat Q_1 from inside the refrigerator for least amount of external work W.

$\boxed{COP = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1}}$

If it follows Carnot Cycle i.e. $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$\Rightarrow \boxed{COP = \frac{T_1}{T_2 - T_1}}$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in String Waves – PART 1

1. Waves (*Transverse & Longitudinal*)
2. Wave Equation
3. Path & Phase Difference
4. Direction of Wave through equation
5. Particle velocity & acceleration
6. Direction of particle velocity
7. Wave speed on strings
8. Average Power & Intensity Transmitted

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Waves (Transverse & Longitudnal)

Energy is transmitted from one point to another

↳ also called Travelling waves

Transverse

Disturbance travels in x-dir^n but particle moves in dir^n ⊥ to x-axis

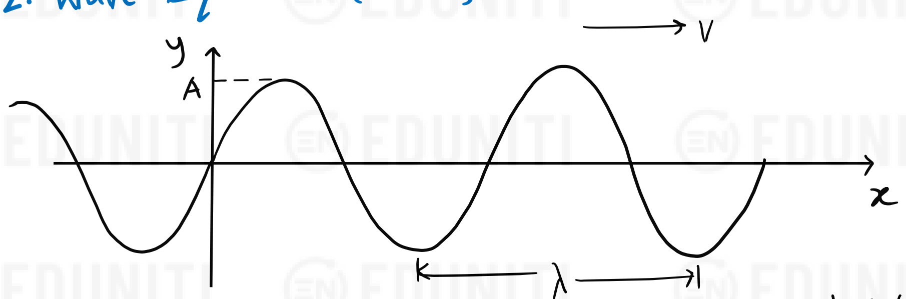
↳ string waves

Longitudnal

Particle moves along the same dir^n as disturbance

↳ sound waves

2. Wave Equation (string waves)



Wave speed

$V = \frac{\text{Coeff of } t}{\text{Coeff of } x} = \frac{\omega}{K}$

$= f\lambda$

$y = A\sin(\omega t - Kx + \phi)$

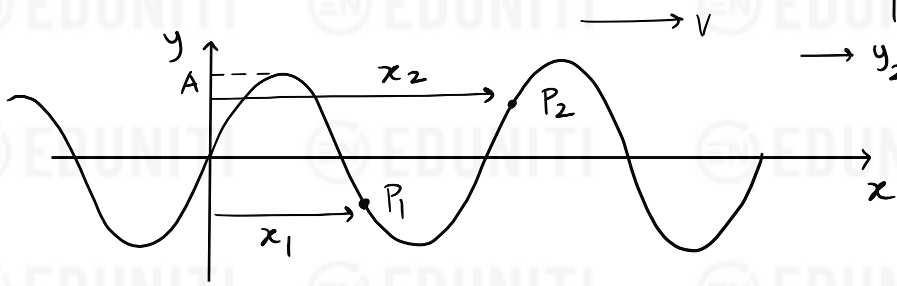
Angular wave no, $K = \frac{2\pi}{\lambda}$

Initial phase Const.

Angular freq, $\omega = 2\pi f = \frac{2\pi}{T}$

Amplitude

3. Path & Phase difference



$\rightarrow y_1 = A\sin(\omega t - Kx_1 + \phi)$

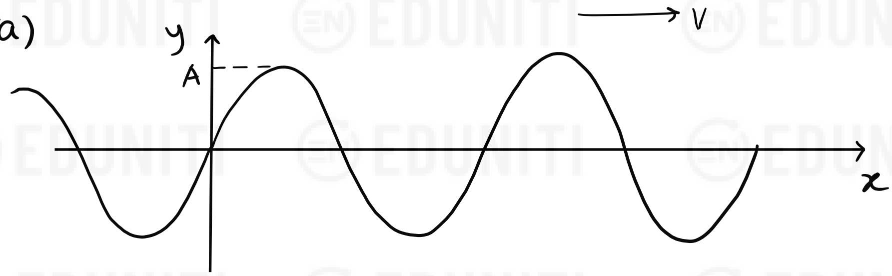
$\rightarrow y_2 = A\sin(\omega t - Kx_2 + \phi)$

Phase difference, $\Delta\phi = \phi_1 - \phi_2 = K(x_2 - x_1)$

$\Rightarrow \boxed{\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x}$ Δx is path difference

4. Direction of Wave Motion in Eq^n

(a)

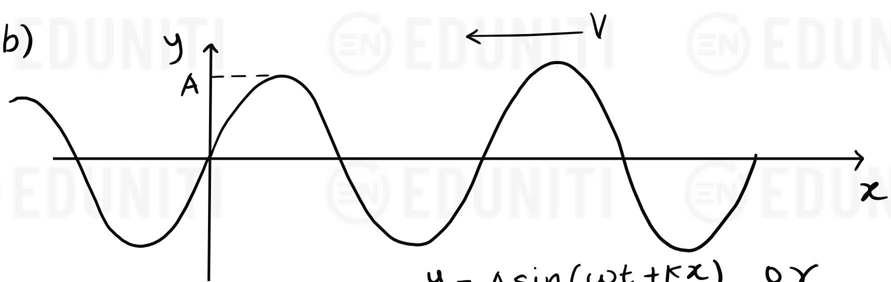


$y = A\sin(\omega t - Kx)$

or

$= A\sin(-\omega t + Kx)$

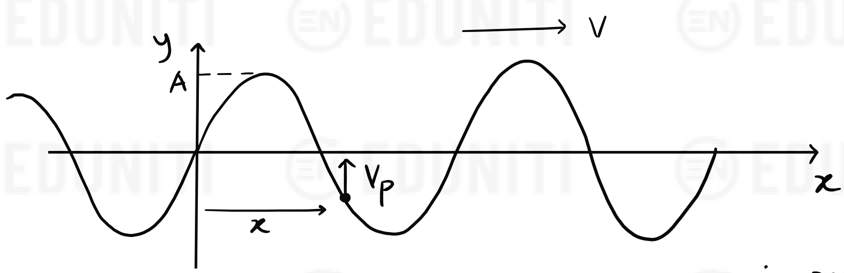
(b)



$y = A\sin(\omega t + Kx)$ or

$= A\sin(-\omega t - Kx)$

5. Particle Velocity & Acceleration



$y = A\sin(\omega t - Kx + \phi)$

$\downarrow$

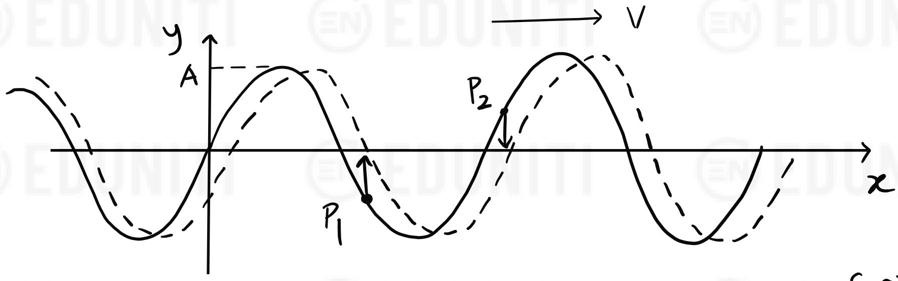
$V_p = \partial y / \partial t$

$\Rightarrow V_p = A\omega\cos(\omega t - Kx + \phi)$

or, $\boxed{V_p = \omega\sqrt{A^2 - y^2}}$ $\left\{ \begin{array}{l} y \text{ is pos}^n \text{ of particle at} \\ x \text{ distance at time } t \end{array} \right.$

and, $a_p = \frac{\partial V_p}{\partial t}$ $\Rightarrow \boxed{a_p = -\omega^2 y}$

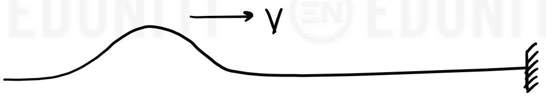
6. Particle Direction at time t



Method 1 : $\boxed{v_p = -V \times \text{slope}}$ $\left\{ \begin{array}{l} \text{at } P_1 \text{ slope is } -ve \\ \therefore v_p \text{ is } +ve \Rightarrow \uparrow \end{array} \right.$

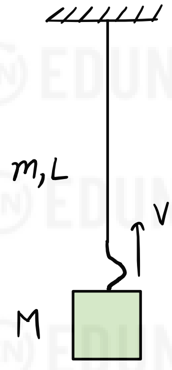
Method 2 : Draw wave after dt time
↳ Same result.

7. Wave speed on strings



$V = \sqrt{\dfrac{T}{\mu}}$ → T: Tension in string, μ: mass/Length

Ex5.



$M >> m$
Find time to reach top.

sol^n: $\because M >> m \Rightarrow T$ in string $\simeq Mg$

$$\therefore V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}} \Rightarrow t = \frac{L}{V} = \sqrt{\frac{mL}{Mg}}$$

8. Avg. Power & Intensity Transmitted

↳ $P_{av} = \frac{1}{2} \rho s v \omega^2 A^2$

or

$P_{av} = 2\pi^2 f^2 A^2 \rho v s$

ρ : density of string
s : Cross-sec^n area of string
v : wave speed, $\sqrt{T/\mu}$
ω : $2\pi f$
A : Amplitude

↳ $I = P_{av}/s = 2\pi^2 f^2 A^2 \rho v$

Space to add concepts learnt from PYQs if any

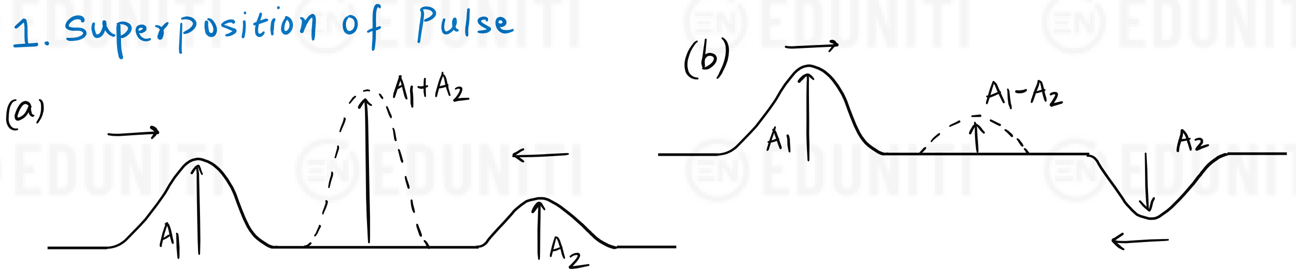
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

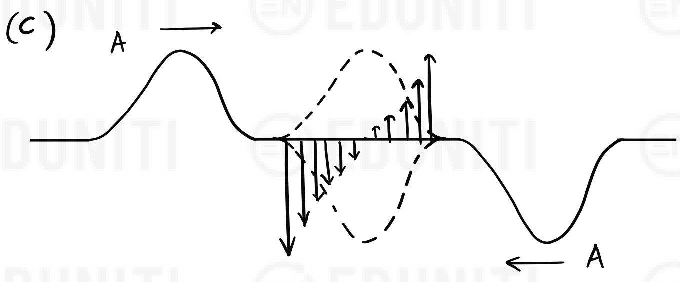
Topics to cover in String Waves – PART 2

1. Superposition of Pulse
2. Reflection from fixed & free ends
3. Reflection & Transmission between two strings
4. Standing or Stationary Waves
5. Key Points in Standing Waves
6. Equation of Standing Waves
7. Standing wave in Clamped string (*between 2 fixed ends*)
8. Standing wave in Clamped String (*1 fixed & 1 free end*)
9. Standing wave in Composite String
10. Sonometer

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Superposition of Pulse

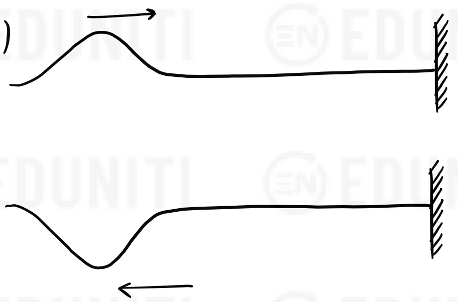




Although displacement is zero but all energy is in form of K.E.

2. Reflection from fixed & free ends

(a)

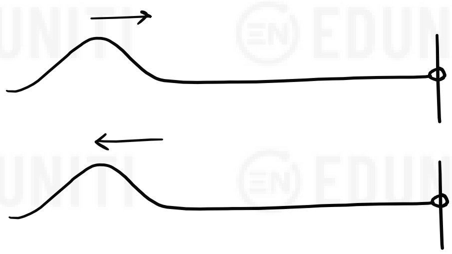


$y_i = A\sin(\omega t - Kx)$

Phase change of π

$y_r = A\sin(\omega t + Kx + \pi)$

(b)

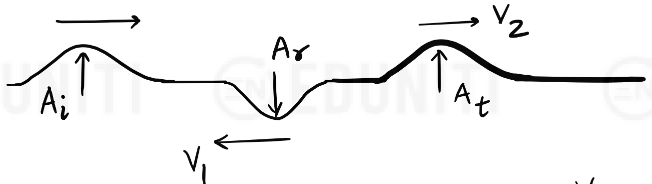


$y_i = A\sin(\omega t - Kx)$

$y_r = A\sin(\omega t + Kx)$

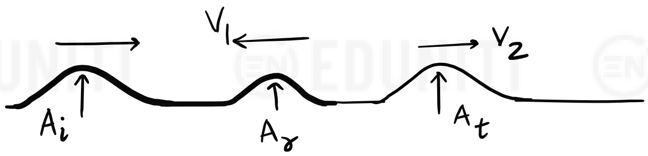
3. Reflection & transmission between two strings

(a)



$V = \sqrt{\frac{T}{\mu}} \Rightarrow V \propto \frac{1}{\sqrt{\mu}} \Rightarrow V_1 > V_2$

(b)



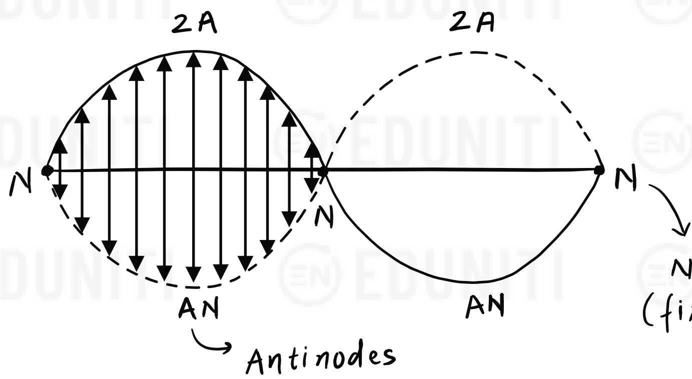
$V_1 < V_2$

$A_t = \frac{2V_2}{V_1 + V_2} A_i$, $A_r = \frac{V_2 - V_1}{V_2 + V_1} A_i$

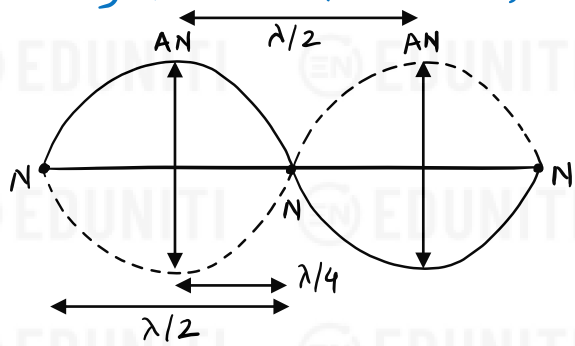
4. Standing waves or Stationary waves

↳ formed due to superposⁿ of two coherent waves travelling in opp direction

↳ both waves had 'A' Amp.



5. Key points in standing waves



(i) All points between two successive NODES are in same phase

(ii) All points across a NODE are in opp phase $(\Delta\phi = \pi)$

(iii) separation betn two consecutive NODES or ANTINODES is $\lambda/2$.

(iv) and betn consecutive NODES and ANTINODE is $\lambda/4$.

6. Equation of standing Waves

$\hookrightarrow$ $y = \underbrace{2A\sin(Kx+\phi_1)}_{\text{Amplitude of a point at distance } x}\sin(\omega t+\phi_2)$

$\phi_1 = 0$

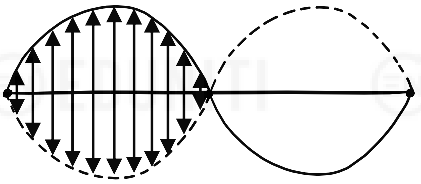
$\Rightarrow y = 2A\sin Kx \sin(\omega t+\phi_2)$

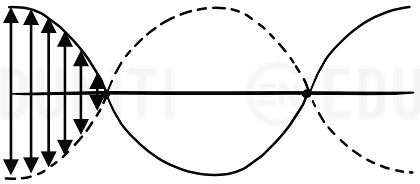
(at $x=0$, NODE)

$\phi_1 = \pi/2$

$y = 2A\cos Kx \sin(\omega t+\phi_2)$

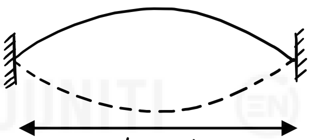
(at $x=0$, AN)





7. Standing Wave in clamped string (fixed at both ends)

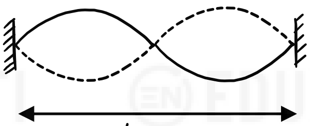
(a)



$f_1 = \frac{V}{\lambda} \Rightarrow f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$\hookrightarrow$ fundamental freq or 1^{st} Harmonic

(b)



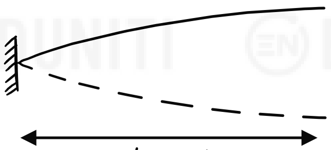
$f_2 = \frac{1}{L}\sqrt{\frac{T}{\mu}} = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$ → 2^{nd} Harmonic → 1^{st} overtone

$$\boxed{n^{th}\text{ Harmonic or } (n-1)^{th}\text{ overtone} \Rightarrow f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = nf_1}$$

NOTE: If $f_1 = 100\,Hz \Rightarrow$ string will be in resonance with tuning fork of 100, 200, 300 Hz...

8. Standing Wave in clamped string (fixed at one end)

(a)

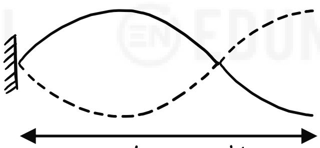


$L = \lambda/4$

$f_1 = \frac{v}{\lambda} = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$

↳ Fundamental freq or 1^{st} Harmonic

(b)



$L = 3\lambda/4$

$f_3 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$ → 3^{rd} Harmonic → 1^{st} overtone

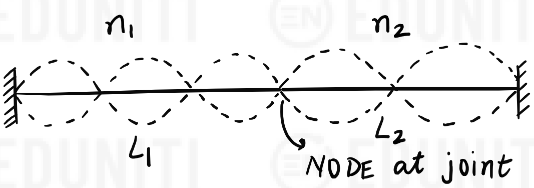
$$\boxed{f = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}} = (2n+1) f_1}$$

↳ $(2n+1)^{th}$ Harmonic

↳ n^{th} overtone

NOTE: If $f_1 = 100\text{ Hz} \Rightarrow$ String will be in resonance with tuning fork of 100, 300, 500 Hz...

9. Standing Wave in Composite string



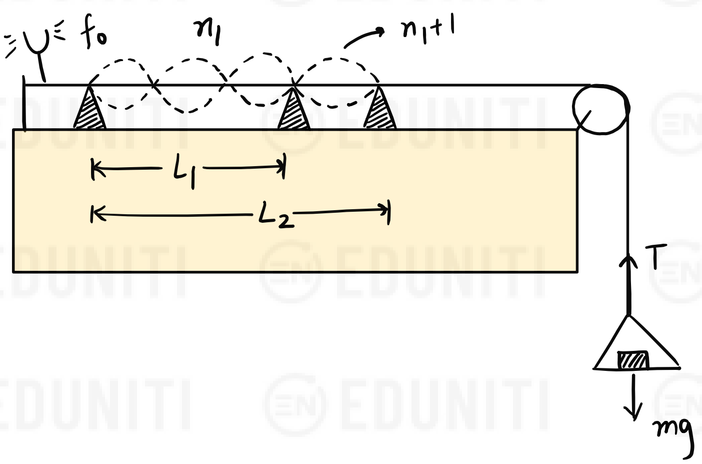
$\text{freq}_{,1} = \text{freq}_{,2}$

$\Rightarrow \frac{n_1}{2L_1}\sqrt{\frac{T}{\mu_1}} = \frac{n_2}{2L_2}\sqrt{\frac{T}{\mu_2}}$

$\Rightarrow \frac{n_1}{n_2} = \frac{L_1}{L_2} \times \sqrt{\frac{\mu_1}{\mu_2}}$

↳ will learn its application in Example ahead

10. Sonometer (to find wave speed in strings)



$\therefore L_2 - L_1 = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2(L_2 - L_1)$

$\therefore V = f_0\lambda = 2f_0(L_2 - L_1)$

↳ Simply saying, it is Standing waves betn 2 fixed ends.

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Sound Waves – PART 3

1. Displacement & Pressure waves
2. Speed of sound waves
3. Beat Frequency
4. Doppler's Effect
5. Different cases (how to write sign)
6. Effect of Wind Speed
7. Sound reflected from fixed wall/object
8. Source and observer not in same line

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Displacement and Pressure Wave

$\hookrightarrow s = s_0 \sin\omega\left(t - \frac{x}{v}\right)$

$\hookrightarrow P = P_0 \cos\omega\left(t - \frac{x}{v}\right)$, $P_0 = \frac{B\omega s_0}{v}$

Here P_0 is amplitude pressure

2. Speed of sound waves

$$V_{liquid} = \sqrt{\frac{B}{\rho}}, \quad V_{solid} = \sqrt{\frac{Y}{\rho}}, \quad V_{gas} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

B : Bulk Modulus

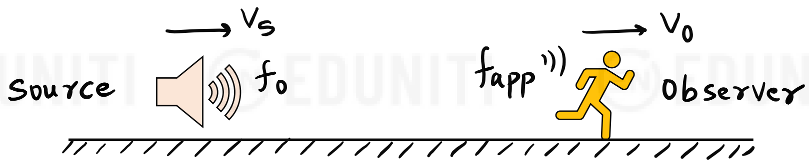
Y : Young's Modulus, γ : Adiabatic constant

3. Beat frequency : superposition of two waves of almost equal frequencies.

$$f_b = |f_1 - f_2|$$

4. DOPPLER'S EFFECT

When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency.



MOST GENERAL

$$f_{app} = f_o\left(\frac{V \pm V_W \pm V_o}{V \pm V_W \pm V_s}\right)$$

V_s : speed of source
V_o : speed of observer
f_o : Source sound frequency
f_{app} : Apparent frequency heard by observer
V : speed of sound in still air

5.(i) CASES (V_s and V_o in same direction)

NOTE:
If any speed tries to decrease separation between source and observer, sign is taken so as to $\uparrow f_{app}$ and vice-versa.

Source at rest: $f_{app} = f_o\left(\frac{V - V_o}{V}\right)$

Observer at rest: $f_{app} = f_o\left(\frac{V}{V - V_s}\right)$

Both moves as shown: $f_{app} = f_o\left(\frac{V - V_o}{V - V_s}\right)$

Source at rest: $f_{app} = f_o\left(\frac{V + V_o}{V}\right)$

Observer at rest: $f_{app} = f_o\left(\frac{V}{V + V_s}\right)$

Both moves as shown: $f_{app} = f_o\left(\frac{V + V_o}{V + V_s}\right)$

5(ii) CASES (V_s and V_o in opposite direction)

(a)

$$\Rightarrow f_{app} = f_o\left(\frac{V+V_o}{V-V_s}\right)$$

(b)

$$\Rightarrow f_{app} = f_o\left(\frac{V-V_o}{V+V_s}\right)$$

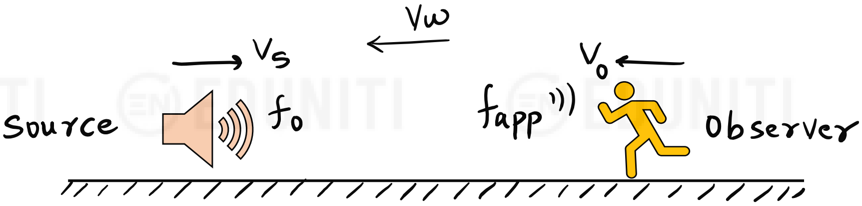
6. EFFECT OF WIND SPEED (V_w)

↳ If "V" is along $V_w \Rightarrow V+V_w$

↳ If "V" is opposite $V_w \Rightarrow V-V_w$

$$f_{app} = f_o\left(\frac{V-V_w+V_o}{V-V_w-V_s}\right)$$

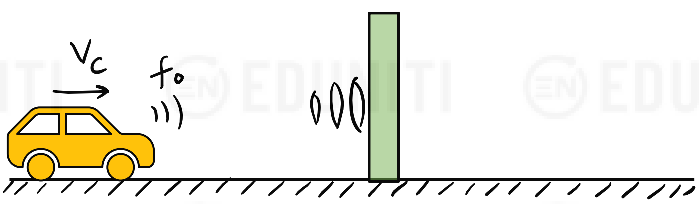
Ex:

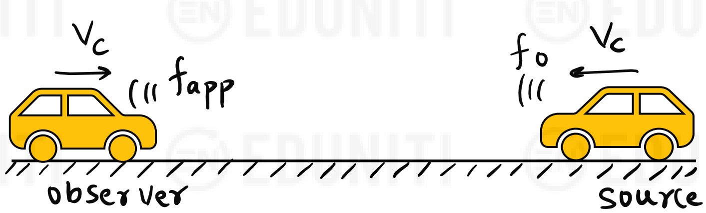


7. SOUND REFLECTED FROM "FIXED WALL"

Car honk freq = f_o

f_{app} heard by driver after reflection.



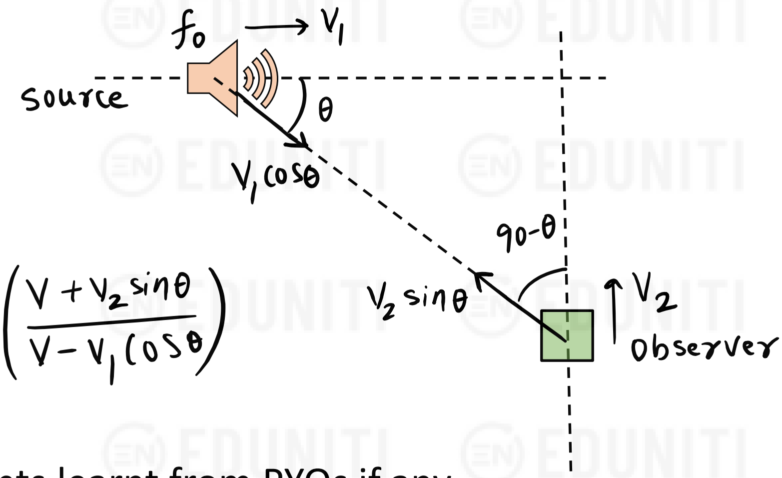


$$f_{app} = f_o\left(\frac{V+V_c}{V-V_c}\right)$$

Beat frequency, $f_b = f_{app} - f_o$

8. SOURCE AND OBSERVER NOT IN SAME LINE

Take components of velocities along line joining them.



$$f_{app} = f_0\left(\frac{V + V_2 \sin\theta}{V - V_1 \cos\theta}\right)$$

<u>Space to add concepts learnt from PYQs if any</u>

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

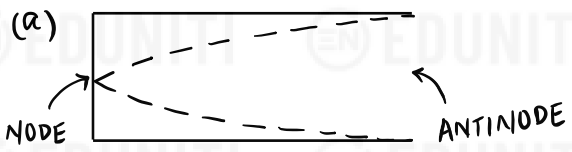
Topics to cover in Sound Waves – PART 4

1. Closed Organ Pipe
2. Open Organ Pipe
3. End Correction
4. Resonance Tube

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. CLOSED ORGAN PIPE (STATIONARY WAVES)

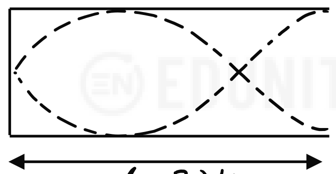
(a)



$l = \lambda/4$

$\therefore \lambda = 4l$, $\boxed{f_0 = \frac{v}{4l}}$ → fundamental frequency or 1^{st} Harmonic

(b)

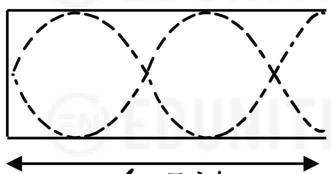


$l = 3\lambda/4$

$\Rightarrow \lambda = 4l/3$

$\therefore f_3 = \frac{3v}{4l}$ → 3^{rd} Harmonic or 1^{st} overtone

(c)



$l = 5\lambda/4$

$\Rightarrow \lambda = 4l/5$

$\therefore f_5 = \frac{5v}{4l}$ → 5^{th} Harmonic or 2^{nd} overtone

$v = \sqrt{\frac{\gamma RT}{M}}$ or $\sqrt{\frac{\gamma P}{\rho}}$

$\gamma = C_p/C_v$

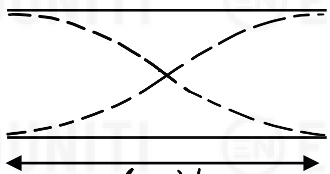
If frequency of tuning fork matches with odd Multiple of fundamental frequency, Resonance occurs

$f_n = (2n+1)f_0$

↳ n^{th} overtone

2. OPEN ORGAN PIPE (STATIONARY WAVES)

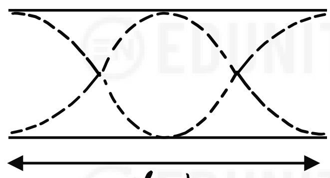
(a)



$l = \lambda/2$

$\therefore \lambda = 2l$, $\boxed{f_0 = \dfrac{V}{2l}}$ → fundamental frequency or 1^{st} Harmonic

(b)

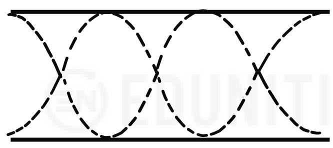


$l = \lambda$

$\Rightarrow \lambda = l$

$\therefore f_2 = \dfrac{2V}{2l}$ → 2nd Harmonic or 1^{st} Overtone

(c)



$l = 3\lambda/2$

$\Rightarrow \lambda = 2l/3$

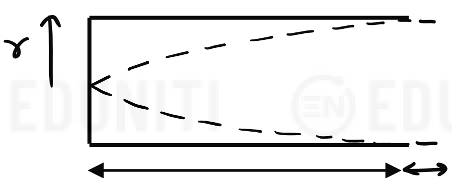
$\therefore f_3 = \dfrac{3V}{2l}$ → 3rd Harmonic or 2^{nd} Overtone

If frequency of tuning fork matches with integer Multiple of fundamental frequency, Resonance occurs

$f_n = (n+1) f_0$
↳ n^{th} Overtone

3. END CORRECTION (e)

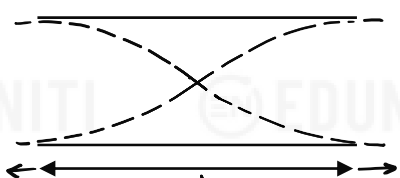
↳ At open side, ANTINODE is formed a little outside.



r ↑ L e

⇓

$f_0 = \dfrac{V}{4(L+e)}$



e L e

⇓

$f_0 = \dfrac{V}{2(L+2e)}$

$\boxed{e = 0.6r}$

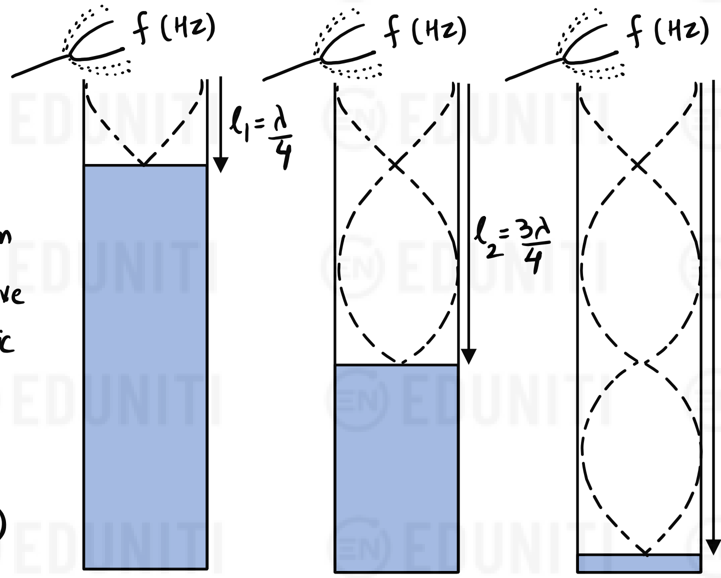
r = Pipe radius.

L = Pipe true length

4. RESONANCE TUBE

↳ Used to find Speed of sound in air.

NOTE: ONCE f is fixed, λ is also fixed.



(1.) Difference betn any two consecutive Resonance/harmonic is $\lambda/2$.

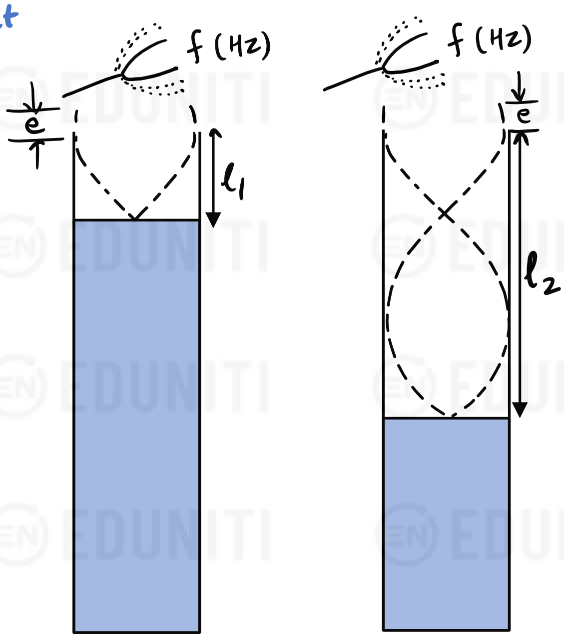
$l_2 - l_1 = \lambda/2$

$\Rightarrow \lambda = 2(l_2 - l_1)$

$\therefore V = \lambda f = 2f(l_2 - l_1)$

end correction effect

l_1 and l_2 are length measured on scale.



$(l_2+e)-(l_1+e)=\frac{\lambda}{2}$

$\Rightarrow l_2-l_1=\frac{\lambda}{2}$

$\therefore v=f\times 2(l_2-l_1)$

Hence end correction has no effect on final result of experiment.

Space to add concepts learnt from PYQs if any