Assignment Two

Q1. Write the following two MATLAB functions:

A. *Function [F] = FiniteDifference(x, f)*, a function to compute the Newton form of the interpolating polynomial for function values **f** through the nodes provided by x. The distinct points are given by:
$x(1) = x_0, x(2) = x_1, \ldots, x(n + 1) = x_n$
The returned vector F should save the coefficients of the Newton form of the interpolating polynomial in terms of finite differences:
$F(1) = f[x_0], F(2) = f[x_0, x_1], \ldots, F(n + 1) = f[x_0, x_1, x_2, \ldots, x_n]$.

B. *Function [P] = EvalFiniteDiff(xt, x, F)*, a function that evaluates the Newton form of the finite differences at a list of points. Here, x is the vector containing each point used to construct F, and *xt* is a list of points to evaluate the polynomial.

Q2. Consider the function

$$f(x) = \frac{1}{1 + 25x^2}$$

a) Let x_i be 11 equally spaced points on [−1, 1]:
$x_i = -1 + i/5$, *for* $i = 0, 1, 2, \ldots, 10$.

b) Use your code from the previous problem (**Q1**) to construct the finite differences with these points x_i.

c) Sample the polynomial at 500 uniformly chosen points, *z = linspace(−1, 1, 500)*, by calling *P = EvalFiniteDiff(z, x, F)*

d) Make a single MATLAB plot that contains: (z, P), $(z, f(z))$, and the data points $(x_i, f(x_i))$ for $i = 0, 1, 2, \ldots 10$. Make the $P(z)$ blue solid line, $f(z)$ a red dashed line, and use *x*'s, dots or stars for the data points.

e) Now, let x_i be 11 unequally spaced points on [−1, 1]:

$$x_i = \cos\left(\frac{\pi(i + 0.5)}{11}\right), \quad \text{for} \quad i = 0, 1, 2, \ldots, 10.$$

Repeat steps (a)–(d) for these unequally spaced data points.

Please submit your source code, and the plots of your results, adhere to the following regulations:

- Each student must complete the assignment **individually**.
- Only **MATLAB** programming languages can be used.
- Resources: in your course Moodle Page