How likely is the null hypothesis?

Andy J. Wills

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In our example, we estimated that the probability the null hypothesis is true, prior to data collection, was .95. So $P(H_0) = .95$. Our t-test gave $p = .049$, so $P(data|H_0) = .049$. To work out $P(H_0|data)$, we use Bayes' theorem. Bayes' is most typically written as:

$$P(H_0|data) = \frac{P(data|H_0)P(H_0)}{P(data)} \tag{1}$$

but can equivalently be expressed as:

$$P(H_0|data) = \frac{P(data|H_0)P(H_0)}{P(data|H_0)P(H_0) + P(data|\overline{H_0})P(\overline{H_0})} \tag{2}$$

where $P(\overline{H_0})$ means the probability the null hypothesis is false[1]. Equation 2 is more useful in this case, as $P(\overline{H_0}) = 1 - P(H_0)$ and $P(data|\overline{H_0}) = 1 - P(data|H_0)$. By substitution, we can see that:

$$P(null|data) = \frac{0.049 \times .95}{.049 \times .95 + .951 \times .05} = 0.49 \tag{3}$$

which leads to the claim that the probability of the null hypothesis after a significant t-test is, in this case, close to 50 : 50.

[1]Note $\overline{H_0}$ is a different thing to the probability that the experimental hypothesis is true, $P(H_1)$. Null-hypothesis significance testing does not give you that number