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Solving multi-level multi-objective linear programming problems through fuzzy goal programming approach

Ibrahim A. Baky *

Department of Basic Sciences, Benha Higher Institute of Technology, Benha University, El-Kalyoubia, Egypt

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ABSTRACT

In this paper, two new algorithms are presented to solve *multi-level multi-objective linear programming (ML-MOLP) problems* through the fuzzy goal programming (FGP) approach. The membership functions for the defined fuzzy goals of all objective functions at all levels are developed in the model formulation of the problem; so also are the membership functions for vectors of fuzzy goals of the decision variables, controlled by decision makers at the top levels. Then the fuzzy goal programming approach is used to achieve the highest degree of each of the membership goals by minimizing their deviational variables and thereby obtain the most satisfactory solution for all decision makers.

The first suggested algorithm groups the membership functions for the defined fuzzy goals of the objective functions at all levels and the decision variables for each level except the lower level of the multi-level problem. The second proposed algorithm lexicographically solves MOLP problems of the ML-MOLP problem by taking into consideration the decisions of the MOLP problems for the upper levels. An illustrative numerical example is given to demonstrate the algorithms.

1. Introduction

The standard mathematical programming problem involves finding an optimal solution for just one decision maker. Nevertheless, many planning problems contain an hierarchical decision structure, each with independent, and often conflicting objectives. These types of problems can be modeled using a multi-level mathematical programming (MLMP) approach. The basic concept of the MLMP technique is that the first-level decision maker (FLDM) sets his/her goal and/or decision, and then asks each subordinate level of the organization for their optima, that calculated in isolation. The lower level decision makers' decisions are then submitted and modified by the FLDM in consideration of the overall benefit for the organization. The process continues until a satisfactory solution is reached.

Most of the developments in MLP problems focus on bi-level linear programming as a class of MLP [1–4]. Bi-level nonlinear programming was studied in [5,6]. In [7], an interactive algorithm for bi-level multi-objective programming is presented and explained using the concept of satisfactoriness. The bi-level multi-objective with multiple interconnected decision makers is discussed in [8]. Three-level programming (TLP) is another class of MLP problems in which there are three independent decision-makers (DMs) [9,10]. Each DM attempts to optimize his objective function and is affected by the actions of the other DMs. Several three-level programming problems such as:

* Tel.: +20 13 323 75 22.
E-mail address: ibrahimbaky@yahoo.com

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1. the hybrid extreme-point search algorithm [3,11],
2. mixed-integer problem with complementary slackness [9],
3. the penalty function approach [6,9], and
4. the balance space approach [12–14]

are studied and introduced along with their solution methods.

A bibliography of the related references on bi-level and multi-level programming in both linear and non-linear cases, which is updated biannually, can be found in [15]. The use of the fuzzy set theory [16] for decision problems with several conflicting objectives was first introduced by Zimmermann [17] Thereafter, various versions of fuzzy programming (FP) have been investigated and widely circulated in literature [9,11,18–21].

In a hierarchical decision making context, it has been realized that each DM should have a motivation to cooperate with the other, and a minimum level of satisfaction of the DM at a lower-level must be considered for the overall benefit of the organization. The use of the concept of membership function of fuzzy set theory to multi-level programming problems for satisfactory decisions was first introduced by Lai [22] in 1996. Thereafter, Lai's satisfactory solution concept was extended by Shih et al. [23] and a supervised search procedure with the use of max–min operator of Bellman and Zadeh [24] was proposed. Abo-Sinna [5,10] extended the fuzzy approach for multi-level programming problems of Shih et al. [23] for solving bi-level and three-level non-linear multi-objective programming problems. The basic concept of these fuzzy programming (FP) approaches is the same as implies that each lower level decision maker optimizes his/her objective function, taking a goal or preference of the first level decision makers into consideration. In the decision process, the membership functions of the fuzzy goals for the decision variables of all the decision makers are taken into consideration and an FP problem is solved with a constraint on an overall satisfactory degree of any upper levels. If the proposed solution is not satisfactory to any upper levels, the solution search is continued by redefining the elicited membership functions until a satisfactory solution is reached [19,25].

The main difficulty arises with the FP approach of Shih et al. is that there is possibility of rejecting the solution again and again by the FLDM and re-evaluation of the problem is repeatedly needed to reach the satisfactory decision, where the objectives of the DMs are over conflicting. Even inconsistency between the fuzzy goals of the objectives and the decision variables may arise. This makes the solution process a lengthy one [19,25]. The fuzzy goal programming (FGP) technique introduced by Mohamed [18] – for proper distribution of decision powers to the DMs to arrive at a satisfying decision for the overall benefit of the organization – was developed to overcome the above undesirable situation. The FGP of Mohamed [18] was extended to solve multiobjective linear fractional programming problems in [20], bi-level programming problems in [19], bi-level quadratic programming problems in [25]. In [26], his FGP is further extended to multi-level programming problems with a single objective function at each level.

In this article, the FGP approach introduced by Mohamed [18] is extend to solve multi-level multi-objective linear programming (ML-MOLP) problems. Two FGP procedures are presented in this article to ML-MOLP problems. To formulate any of these two proposed FGP models of the TL-MOLP problem, the fuzzy goals of the objectives are determined by finding individual optimal solutions. They are then characterized by the associated membership functions. These functions are transformed into fuzzy flexible membership goals by means of introducing over and under deviational variables and assigning highest membership value (unity) as aspiration level to each of them. To elicit the membership functions of the decision vectors controlled by any level DM, the optimal solution of the corresponding MOLP problem is separately determined. A relaxation of the decisions are considered to avoid decision deadlock.

The first proposed FGP procedure makes an extension of the work of Pal et al. [19] and Pramanik and Roy [26]. Pal et al. [19] deals with bi-level linear single objective programming problems; and Pramanik and Roy [26] propose an FGP procedure to multi-level programming problems with a single linear objective at each level. The final fuzzy model of Pramanik and Roy groups the membership functions for the defined fuzzy goals of the objective functions and the decision variables at all levels which are evaluated separately for each level except the lower level of the multi-level problem.

The second proposed procedure may be seen as lexicographic methods for solving multiobjective programming problems. Firstly, it formulates the FGP model of the first level problem to obtain a satisfactory solution to the FLDM problem. A relaxation of the FLDM decisions is considered to avoid a decision deadlock. These decisions of the FLDM are modeled by membership functions of fuzzy set theory and passed to the second level DM (SLDM) as additional constraints. Then, the SLDM formulates its FGP model that takes into consideration the membership goals of the objectives and decision variables of the FLDM. Thereafter, the attained solution is sent to the third-level DM (TLDM) who seeks the solution in a similar manner. The process continues until the lower level. This procedure may be considered as extension of the fuzzy mathematical programming algorithm of Shih et al. concept [23] that modified by Sinha in [21,28] following the FGP approach of Mohamed [18].

2. Problem formulation

Consider a p-level programming problem of minimization-type multi-objective functions at each level. Let DM$_i$ denote the decision maker at the ith level that has control over the decision variable $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in_i}) \in R^{n_i}, \quad i = 1, 2, \ldots, p$, where $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, x_p) \in R^n$ and $n = n_1 + n_2 + \cdots + \boldsymbol{n_p}$ and furthermore assume that

$$F_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p) \equiv F_i(\boldsymbol{x}) : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_p} \to R^{m_i}, \quad i = 1, 2, \ldots, p \tag{1}$$

are the vector of objective functions to the DM_i, $i = 1,2,\ldots,p$. Mathematically, the ML-MOLP problem of minimization type may be formulated as follows [13,18,10,20,4,27,28]:

[1st Level]

$$\underset{x_1}{\mathrm{Min}}\, F_1(x) = \underset{x_1}{\mathrm{Min}}(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \ldots, f_{1m_1}(\boldsymbol{x}))$$

where $\boldsymbol{x}_2, \boldsymbol{x}_3, \ldots, \boldsymbol{x}_p$ solves

[2nd Level]

$$\underset{x_2}{\mathrm{Min}}\, F_2(x) = \underset{x_2}{\mathrm{Min}}(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \ldots, f_{2m_2}(\boldsymbol{x}))$$

$$\vdots$$

where $\boldsymbol{x}_p$ solves

[pth Level]

$$\underset{x_p}{\mathrm{Min}}\, F_p(\boldsymbol{x}) = \underset{x_p}{\mathrm{Min}}(f_{p1}(\boldsymbol{x}), f_{p2}(\boldsymbol{x}), \ldots, f_{pm_p}(\boldsymbol{x})) \tag{2}$$

subject to

$$\boldsymbol{x} \in \boldsymbol{G} = \left\{ \boldsymbol{x} \in R^n | A_1 x_1 + A_2 x_2 + \cdots + A_p x_p \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \boldsymbol{b}, \boldsymbol{x} \geqslant 0, \boldsymbol{b} \in R^m \right\} \neq \phi \tag{3}$$

where

$$\begin{aligned} f_{ij}(\boldsymbol{x}) &= \boldsymbol{c}_1^{ij}\boldsymbol{x}_1 + \boldsymbol{c}_2^{ij}\boldsymbol{x}_2 + \cdots + \boldsymbol{c}_p^{ij}\boldsymbol{x}_p,\ i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i \\ &= c_{11}^{ij}x_{11} + c_{12}^{ij}x_{12} + \cdots + c_{1n_1}^{ij}x_{1n_1} + c_{21}^{ij}x_{21} + c_{22}^{ij}x_{22} + \cdots + c_{2n_2}^{ij}x_{2n_2} + \cdots + c_{p1}^{ij}x_{p1} + c_{p2}^{ij}x_{p2} + \cdots + c_{pn_p}^{ij}x_{pn_p} \end{aligned} \tag{4}$$

and where $\boldsymbol{G}$ is the multi-level convex constraints feasible choice set, m_i, $i = 1,2,\ldots,p$, are the number of DM_i's objective functions, m is the number of the constraints, $\boldsymbol{c}_k^{ij} = (c_{k1}^{ij}, c_{k2}^{ij}, \ldots, c_{kn_k}^{ij}),\ k = 1,2,\ldots,p,\ c_{kn_k}^{ij}$ are constants, and A_i are coefficients matrices of size $m \times n_i$, $i = 1,2,\ldots,p$.

3. Fuzzy goal programming formulation

In ML-MOLP problems, if an imprecise aspiration level is assigned to each of the objectives in each level of the ML-MOLP, then these fuzzy objectives are termed as fuzzy goals. They are characterized by their associated membership functions by defining the tolerance limits for achievement of their aspired levels.

3.1. Construction of membership functions

Since all the DMs are interested in minimizing their own objective functions over the same feasible region defined by the system of constraints (3), the optimal solutions of both of them calculated in isolation can be taken as the aspiration levels of their associated fuzzy goals.

Let $\boldsymbol{x}^{ij} = (\boldsymbol{x}_1^{ij}, \boldsymbol{x}_2^{ij}, \ldots, \boldsymbol{x}_p^{ij}); f_{ij}^{\min},\ i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i$ be the optimal solutions of DMs objective functions, respectively, when calculated in isolation. Let $g_{ij} \geqslant f_{ij}^{\min}$ be the aspiration level assigned to the ijth objective $f_{ij}(\boldsymbol{x})$ (the subscript ij means that $j = 1,2,\ldots,m_p$ when $i = p$ for DM_p problem). Also, let $\boldsymbol{x}^{i^*} = (\boldsymbol{x}_1^{i^*}, \boldsymbol{x}_2^{i^*}, \ldots, \boldsymbol{x}_p^{i^*}),\ i = 1,2,\ldots,p-1$, the optimal solution of the pth-level MOLP problems. Then, the fuzzy goals of the decision makers' objective functions at each level and the vector of fuzzy goals of the decision variables controlled by upper $p-1$ level decision makers appear as:

$$f_{ij}(\boldsymbol{x}) \lesssim g_{ij}, \quad i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i, \quad \text{and} \quad \boldsymbol{x}_i \underset{\sim}{=} \boldsymbol{x}_i^{i*}, \quad i = 1,2,\ldots,p-1,$$

where "$\lesssim$" and "$\underset{\sim}{=}$" indicates the fuzziness of the aspiration levels, and is to be understood as "essentially less than" and "essentially equal to", respectively, [27,17].

It may be noted that the solutions $\boldsymbol{x}^{ij} = (\boldsymbol{x}_1^{ij}, \boldsymbol{x}_2^{ij}, \ldots, \boldsymbol{x}_p^{ij}); i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i$ are usually different because the objectives of all the DMs are conflicting in nature. Therefore, it can be assumed reasonably that the values $f_{\ell m}(\boldsymbol{x}_1^{\ell m}, \boldsymbol{x}_2^{\ell m}, \ldots, \boldsymbol{x}_p^{\ell m}) \geqslant f_{ij}^{\min} \forall \ell = 1,2,\ldots,p, \quad m = 1,2\ldots,m_i$, and $ij \neq \ell m$ and all values greater than $f_{\ell m}^u = max[f_{ij}(\boldsymbol{x}_1^{\ell m}, \boldsymbol{x}_2^{\ell m}, \ldots, \boldsymbol{x}_p^{\ell m}), i = 1,2,\ldots,p, j = 1,2\ldots,m_i$, and $ij \neq \ell m]$ are absolutely unacceptable to the objective function $f_{\ell m}(\boldsymbol{x}) \equiv f_{\ell m}(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p)$. As such, $f_{\ell m}(\boldsymbol{x})$ can be considered as the upper tolerance limit $u_{\ell m}$ of the fuzzy goal to the objective functions. Then, membership functions $\mu_{f_{ij}}(f_{ij}(\boldsymbol{x}))$ for the ijth fuzzy goal can be formulated as (Fig. 1):

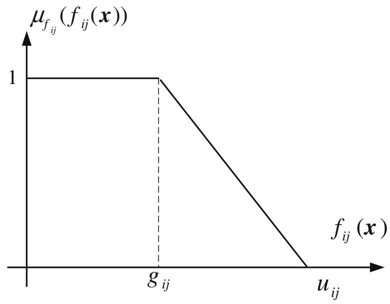

Fig. 1. Membership function of minimization-type objective functions.

$$\mu_{f_{ij}}(f_{ij}(\boldsymbol{x})) = \begin{cases} 1 & \text{if } f_{ij}(\boldsymbol{x}) \leqslant g_{ij} \\ \frac{u_{ij}-f_{ij}((\boldsymbol{x}))}{u_{ij}-g_{ij}} & \text{if } g_{ij} \leqslant f_{ij}(\boldsymbol{x}) \leqslant u_{ij}\,, \; i=1,2,\ldots,p, \; j=1,2,\ldots,m_i \\ 0 & \text{if } f_{ij}(\boldsymbol{x}) \geqslant u_{ij} \end{cases} \tag{5}$$

To build the membership functions for the fuzzy goals of the decision variables controlled by DM_i, the optimal solutions of the ith-level MOLP problems, $\boldsymbol{x}^{i^*} = (\boldsymbol{x}_1^{i^*}, \boldsymbol{x}_2^{i^*}, \ldots, \boldsymbol{x}_p^{i^*}), \; i=1,2,\ldots,p-1$, should be determined first following any MOLP approach (see Appendix A).

Let $t_k^{i_L}$ and $t_k^{i_R}, \; i=1,2,\ldots,p-1, \; k=1,2,\ldots,n_i$ be the maximum negative and positive tolerance values on the decision vectors considered by the ith-level DM. The tolerances $t_k^{i_L}$ and $t_k^{i_R}$ are not necessarily same.

The linear membership functions (Fig. 2) for each of the n_i components of the decision vector $\boldsymbol{x}_i^{i^*} = (x_{i1}^*, x_{i2}^*, \ldots, x_{in_i}^*)$ controlled by the upper $p-1$ levels decision makers can be formulated as:

$$\mu_{x_{ik}}(x_{ik}) = \begin{cases} \frac{x_{ik}-(x_{ik}^*-t_k^{i_L})}{t_k^{i_L}} & \text{if } x_{ik}^* - t_k^{i_L} \leqslant x_{ik} \leqslant x_{ik}^* \\ \frac{(x_{ik}^*+t_k^{i_R})-x_{ik}}{t_k^{i_R}} & \text{if } x_{ik}^* \leqslant x_{ik} \leqslant x_{ik}^* + t_k^{i_R} \\ 0 & \text{otherwise} \end{cases}; \quad i=1,2,\ldots,p-1, \; k=1,2,\ldots,n_i. \tag{6}$$

It may be noted that, the decision maker may desire to shift the range of x_{ik}. Following Pramanik and Roy [26] and Sinha [28], this shift can be achieved.

Now, in a fuzzy decision environment, the fuzzy goals consist of the decision maker's objective functions at each level and the vector of fuzzy goals of the decision variables controlled by upper $p-1$ level decision makers. Their achievement to their aspired levels to the extent possible is actually represented by the possible achievement of their respective membership values to the highest degree. Regarding this aspect of fuzzy programming problems, a goal programming approach seems to be most appropriate for the solution of the upper pth-level multi-objective linear programming problems and the multi-level multi-objective linear programming problem [19].

3.2. Fuzzy goal programming approach

In fuzzy programming approaches, the highest degree of membership function is one. So, as in Mohamed [18], for the defined membership functions in (5) and (6), the flexible membership goals with the aspired level 1 can be presented as:

$$\mu_{f_{ij}}(f_{ij}(\boldsymbol{x})) + d_{ij}^- - d_{ij}^+ = 1, \quad i=1,2,\ldots,p, \; j=1,2,\ldots,m_i, \tag{7}$$

$$\mu_{x_{ik}}(x_{ik}) + d_{ik}^- - d_{ik}^+ = 1, \quad i=1,2,\ldots,p-1, \; k=1,2,\ldots,n_i \tag{8}$$

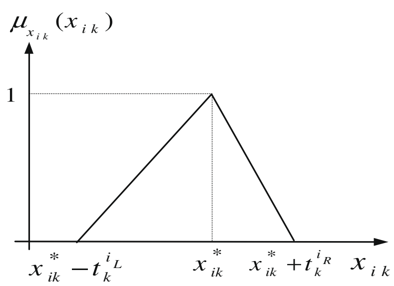

Fig. 2. Membership functions of decision vectors x_{ik}.

or equivalently as:

$$\frac{u_{ij} - \left(\boldsymbol{c}_1^{ij}\boldsymbol{x}_1 + \boldsymbol{c}_2^{ij}\boldsymbol{x}_2 + \cdots + \boldsymbol{c}_p^{ij}\boldsymbol{x}_p\right)}{u_{ij} - g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \quad i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i, \tag{9}$$

$$\frac{x_{ik} - \left(x_{ik}^* - t_k^{i_L}\right)}{t_k^{i_L}} + d_{ik}^{L-} - d_{ik}^{L+} = 1, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i, \tag{10}$$

$$\frac{\left(x_{ik}^* + t_k^{i_R}\right) - x_{ik}}{t_k^{i_R}} + d_{ik}^{R-} - d_{ik}^{R+} = 1, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i, \tag{11}$$

where $d_{ik}^- = \left(d_{ik}^{L-}, d_{ik}^{R-}\right),\ d_{ik}^+ = \left(d_{ik}^{L+}, d_{ik}^{R+}\right)$, and $d_{ij}^-, d_{ik}^{L-}, d_{ik}^{R-}, d_{ij}^+, d_{ik}^{L+}, d_{ik}^{R+} \geqslant 0$ with $d_{ij}^- \times d_{ij}^+ = 0,\ d_{ik}^{L-} \times d_{ik}^{L+} = 0$, and $d_{ik}^{R-} \times d_{ik}^{R+} = 0,\ i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i$, represent the under- and over-deviations, respectively, from the aspired levels.

In conventional goal programming (GP), the under- and/or over-deviational variables are included in the achievement function for minimizing them and that depends upon the type of the objective functions to be optimized. In the proposed procedures, the over-deviational variables for the fuzzy goals of objective functions, $d_{ij}^+,\ i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i$, and the over-deviational and the under-deviational variables for the fuzzy goals of the decision variables, $d_{ik}^{L-}, d_{ik}^{L+},\ d_{ik}^{R-}$, and $d_{ik}^{R+},\ i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i$ are required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that any under-deviation from a fuzzy goal indicates the full achievement of the membership value [20]. In addition, $d_{ij}^+ = 0$ (when a membership goal is fully achieved), and $d_{ij}^+ = 1$ (are when its achievement is zero) are found in the solution [20].

4. FGP algorithms to TL-MOLP

The FGP approach to multiobjective programming problems presented by Mohamed [18] is extended here to formulate two FGP algorithms to multi-level multi-objective linear programming problems.

4.1. The first FGP algorithm to TL-MOLP

The first FGP procedure proposed in this article, as mentioned in the introduction, groups the membership functions for the defined fuzzy goals of the objective functions at all levels; it also groups the membership functions of the fuzzy goals of the decision variables of the $p-1$ upper levels problems, which are evaluated separately. Therefore, considering the goal achievement problem at the same priority level, the equivalent proposed fuzzy multi-level multi-objective linear goal programming model of the problem can be presented under the framework of *minsum* GP as follows:

$$\begin{aligned}
\min Z = {} & \sum_{j=1}^{m_1} w_{1j}^+ d_{1j}^+ + \sum_{j=1}^{m_2} w_{2j}^+ d_{2j}^+ + \cdots + \sum_{j=1}^{m_p} w_{pj}^+ d_{pj}^+ \\
& + \sum_{k=1}^{n_1} [w_{1k}^L (d_{1k}^{L-} + d_{1k}^{L+}) + w_{1k}^R (d_{1k}^{R-} + d_{1k}^{R+})] \\
& + \sum_{k=1}^{n_2} [w_{2k}^L (d_{2k}^{L-} + d_{2k}^{L+}) + w_{2k}^R (d_{2k}^{R-} + d_{2k}^{R+})] \\
& \vdots \\
& + \sum_{k=1}^{n_{p-1}} [w_{p-1k}^L (d_{p-1k}^{L-} + d_{p-1k}^{L+}) + w_{p-1k}^R (d_{p-1k}^{R-} + d_{p-1k}^{R+})]
\end{aligned}$$

subject to

$$\begin{aligned}
& \mu_{f_{1j}}(f_{1j}(\boldsymbol{x})) + d_{1j}^- - d_{2j}^+ = 1, \quad j = 1,2,\ldots,m_1 \\
& \mu_{f_{2j}}(f_{2j}(\boldsymbol{x})) + d_{2j}^- - d_{2j}^+ = 1, \quad j = 1,2,\ldots,m_2 \\
& \vdots \\
& \mu_{f_{pj}}(f_j(\boldsymbol{x})) + d_{pj}^- - d_{pj}^+ = 1, \quad j = 1,2,\ldots,m_p \\
& \mu_{x_{1k}}(x_{1k}) + d_{1k}^- - d_{1k}^+ = 1, \quad k = 1,2,\ldots,n_1 \\
& \mu_{x_{2k}}(x_{2k}) + d_{2k}^- - d_{2k}^+ = 1, \quad k = 1,2,\ldots,n_2
\end{aligned}$$

$$\begin{aligned}
&\vdots\\
&\mu_{x_{p-1k}}(x_{p-1k}) + d_{p-1k}^{-} - d_{p-1k}^{+} = 1, \quad k = 1,2,\ldots,n_{p-1}\\
&A_1x_1 + A_2x_2 + \cdots + A_px_p \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \boldsymbol{b},\ \boldsymbol{x} \geqslant 0\\
&d_{ij}^{-}, d_{ij}^{+} \geqslant 0 \text{ and } d_{ij}^{-} \times d_{ij}^{+} = 0,\ i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i\\
&d_{ik}^{-}, d_{ik}^{+} \geqslant 0 \text{ and } d_{ik}^{-} \times d_{ik}^{+} = 0,\ i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i
\end{aligned} \tag{12}$$

The above problem can be rewritten as:

$$\begin{aligned}
\min Z = &\sum_{j=1}^{m_1} w_{1j}^{+} d_{1j}^{+} + \sum_{j=1}^{m_2} w_{2j}^{+} d_{2j}^{+} + \cdots + \sum_{j=1}^{m_p} w_{pj}^{+} d_{pj}^{+}\\
&+ \sum_{k=1}^{n_1} [w_{1k}^{L}(d_{1k}^{L-} + d_{1k}^{L+}) + w_{1k}^{R}(d_{1k}^{R-} + d_{1k}^{R+})]\\
&+ \sum_{k=1}^{n_2} [w_{2k}^{L}(d_{2k}^{L-} + d_{2k}^{L+}) + w_{2k}^{R}(d_{2k}^{R-} + d_{2k}^{R+})]\\
&\vdots\\
&+ \sum_{k=1}^{n_{p-1}} [w_{p-1k}^{L}(d_{p-1k}^{L-} + d_{p-1k}^{L+}) + w_{p-1k}^{R}(d_{p-1k}^{R-} + d_{p-1k}^{R+})]
\end{aligned}$$

subject to

$$\begin{aligned}
&\frac{u_{ij} - \left(\boldsymbol{c}_1^{ij}\boldsymbol{x}_1 + \boldsymbol{c}_2^{ij}\boldsymbol{x}_2 + \cdots + \boldsymbol{c}_p^{ij}\boldsymbol{x}_p\right)}{u_{ij} - g_{ij}} + d_{ij}^{-} - d_{ij}^{+} = 1, \quad i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i\\
&\frac{x_{ik} - \left(x_{ik}^{*} - t_k^{i_L}\right)}{t_k^{i_L}} + d_{ik}^{L-} - d_{ik}^{L+} = 1, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i\\
&\frac{\left(x_{ik}^{*} + t_k^{i_R}\right) - x_{ik}}{t_k^{i_R}} + d_{ik}^{R-} - d_{ik}^{R+} = 1, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i\\
&A_1x_1 + A_2x_2 + \ldots + A_px_p \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \boldsymbol{b},\ \boldsymbol{x} \geqslant 0\\
&d_{ij}^{-}, d_{ij}^{+} \geqslant 0 \text{ and } d_{ij}^{-} \times d_{ij}^{+} = 0, \quad i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i\\
&d_{ik}^{L-}, d_{ik}^{L+} \geqslant 0 \text{ and } d_{ik}^{L-} \times d_{ik}^{L+} = 0, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i\\
&d_{ik}^{R-}, d_{ik}^{R+} \geqslant 0 \text{ and } d_{ik}^{R-} \times d_{ik}^{R+} = 0, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i
\end{aligned} \tag{13}$$

where Z represents the fuzzy achievement function consisting of the weighted over-deviational variables $d_{ij}^{+},\ i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i$ of the fuzzy goals g_{ij} and the under-deviational and the over-deviational variables $d_{ik}^{R-}, d_{ik}^{R+}, d_{ik}^{L-}$, and $d_{ik}^{L+},\ i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i$ for the fuzzy goals of all the decision variables for the upper $p-1$ levels. The numerical weights $w_{ij}^{+},\ w_{ik}^{R}$, and w_{ik}^{L} represent the relative importance of achieving the aspired levels of the respective fuzzy goals subject to the constraints set in the decision situation.

To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested by Mohamed [18] can be used to assign the values to $w_{ij}^{+},\ w_{ik}^{R}$, and w_{ik}^{L}. In the present formulation, these values are determined as:

$$w_{ij}^{+} = \frac{1}{u_{ij} - g_{ij}}, \quad i = 1,2,\ldots,p,\ j = 1,2,\ldots,m_i \tag{14}$$

$$w_{ik}^{L} = \frac{1}{t_k^{i_L}} \quad \text{and} \quad w_{ik}^{R} = \frac{1}{t_k^{i_R}}, \quad i = 1,2,\ldots,p-1,\ k = 1,2,\ldots,n_i \tag{15}$$

The FGP model (13) provides the most satisfactory decision for the DMs at all levels by achieving the aspired levels of the membership goals to the extent possible in the decision environment. The solution procedure is straightforward and illustrated via the illustrative example in Section 5.

Following the above discussion, the first proposed FGP algorithm for solving ML-MOLP problems is given as follows:

FGP Alg I:

Step 1. Calculate the individual minimum and maximum values of all the objective functions for all levels under the given constraints.

Step 2. Set the goals and the upper tolerance limits – u_{ij}, g_{ij}, $i = 1,2,\ldots,p$, $j = 1,2,\ldots,m_i$ – for all the objective functions in all levels.
Step 3. Evaluate the weights $w_{ij}^{+} = \frac{1}{u_{ij}-g_{ij}}$, $i = 1,2,\ldots,p-1$, $j = 1,2,\ldots,m_i$
Step 4. Set $\ell = 1$
Step 5. Elicit the membership functions $\mu_{f_{\ell j}}(f_{\ell j}(\boldsymbol{x}))$, $j = 1,2,\ldots,m_\ell$
Step 6. Formulate the Model (A1) for the ℓth-level MOLP problem.
Step 7. Solve the Model (A1) to get $\boldsymbol{x}^{\ell^*} = (\boldsymbol{x}_1^{\ell^*}, \boldsymbol{x}_2^{\ell^*}, \ldots, \boldsymbol{x}_p^{\ell^*})$.
Step 8. Set the maximum negative and positive tolerance values on the decision vector $\boldsymbol{x}_\ell^{\ell^*} = (x_{\ell 1}^*, x_{\ell 2}^*, \ldots, x_{\ell n_\ell}^*)$, $t_k^{\ell L}$ and $t_k^{\ell R}$, $k = 1,2,\ldots,n_\ell$.
Step 9. Evaluate the weights $w_{\ell k}^{L} = \frac{1}{t_k^{\ell L}}$ and $w_{\ell k}^{R} = \frac{1}{t_k^{\ell R}}$, $k = 1,2,\ldots,n_\ell$.
Step 10. Elicit the membership functions $\mu_{x_{\ell k}}(x_{\ell k})$, $k = 1,2,\ldots,n_\ell$ for the decision vector $\boldsymbol{x}_\ell^{\ell^*} = (x_{\ell 1}^*, x_{\ell 2}^*, \ldots, x_{\ell n_\ell}^*)$ in Eq. (6).
Step 11. If $\ell > p-1$, then go to Step 12; otherwise go to Step 5.
Step 12. Elicit the membership functions $\mu_{f_{pj}}(f_{pj}(\boldsymbol{x}))$, $j = 1,2,\ldots,m_p$ for the objective functions in the pth-level.
Step 13. Evaluate the weights $w_{pj}^{+} = \frac{1}{u_{pj}-g_{pj}}$, $j = 1,2,\ldots,m_p$
Step 14. Formulate the Model (13) for the ML-MOLP problem
Step 15. Solve the Model (13) to get the satisfactory solution of the ML-MOLP problem

4.2. The second FGP algorithm to TL-MOLP

In Alg I, the final model contains the membership functions for the fuzzy goals of the decision variables controlled by $p-1$ upper levels, which separately solved for the ith-level MOLP problem, $i = 1,2,\ldots,p-1$. The second proposed algorithm lexicographically solves p MOLP problems that take into consideration the decisions of the upper levels. As mentioned in the introduction, after the initialization steps – step 1 to step 3 in Alg I – the solution procedure starts with the MOLP problem of the DM_1 obtaining the satisfactory solution. A relaxation of the DM_1 decisions is considered to avoid decision deadlock. This decisions of the DM_1 are modeled by membership functions of fuzzy set theory and passed to the DM_2 as additional constrains. Then, DM_2 take into consideration the membership goals of the objectives and decision variables of the DM_1. Thereafter, the attained solution is sent to the DM_3 who seeks the solution in a similar manner. The process is repeated until the lower level is reached. Following this discussion, we are now in a position to introduce the second proposed FGP algorithm for solving ML-MOLP problems:

FGP Alg II:

Step 1. Calculate the individual minimum and maximum values of all the objective functions for all levels under the given constraints.
Step 2. Set the goals and the upper tolerance limits – u_{ij}, g_{ij}, $i = 1,2,\ldots,p$, $j = 1,2,\ldots,m_i$ – for all the objective functions in all levels.
Step 3. Evaluate the weights $w_{ij}^{+} = \frac{1}{u_{ij}-g_{ij}}$, $i = 1,2,\ldots,p-1$, $j = 1,2,\ldots,m_i$
Step 4. Set $\ell = 1$
Step 5. Elicit the membership functions $\mu_{f_{\ell j}}(f_{\ell j}(x))$, $j = 1,2,\ldots,m_\ell$
Step 6. Formulate the Model (A1) for the ℓth-level MOLP problem
Step 7. Solve the Model (A1) to get $\boldsymbol{x}^{\ell^*} = (\boldsymbol{x}_1^{\ell^*}, \boldsymbol{x}_2^{\ell^*}, \ldots, \boldsymbol{x}_p^{\ell^*})$.
Step 8. Set the maximum negative and positive tolerance values on the decision vector $\boldsymbol{x}_\ell^{\ell^*} = (x_{\ell 1}^*, x_{\ell 2}^*, \ldots, x_{\ell n_\ell}^*)$, $t_k^{\ell L}$ and $t_k^{\ell R}$, $k = 1,2,\ldots,n_\ell$.
Step 9. Evaluate the weights $w_{\ell k}^{L} = \frac{1}{t_k^{\ell L}}$ and $w_{\ell k}^{R} = \frac{1}{t_k^{\ell R}}$,
Step 10. Elicit the membership functions $\mu_{x_{\ell k}}(x_{\ell k})$, $k = 1,2,\ldots,n_\ell$ for decision vector $\boldsymbol{x}_\ell^{\ell^*} = (x_{\ell 1}^*, x_{\ell 2}^*, \ldots, x_{\ell n_\ell}^*)$.
Step 11. Formulate the Model (13) for the ML-MOLP problem with $p = \ell$.
Step 12. Solve the Model (13) to get the solution $\boldsymbol{x}^{\ell^*} = (\boldsymbol{x}_1^{\ell^*}, \boldsymbol{x}_2^{\ell^*}, \ldots, \boldsymbol{x}_p^{\ell^*})$
Step 13. $\ell = \ell + 1$
Step 14. if $\ell > p$, then stop with a satisfactory solution $\boldsymbol{x}^{\ell^*} = (\boldsymbol{x}_1^{\ell^*}, \boldsymbol{x}_2^{\ell^*}, \ldots, \boldsymbol{x}_p^{\ell^*})$ to the ML-MOLP problem; otherwise go to Step 8.

According to the solution preference, Alg II can be used to direct the solution of the ML-MOLP problem to the decisions of DM_1. And then direct the solution to the decisions of DM_2 preserving the solution nearby the decisions of DM_1. Thereafter the process go on until the last level of the ML-MOLP preserving the solution nearby the decisions of the upper levels.

5. Numerical example

To demonstrate the proposed FGP procedures, consider the following there level multi-objective linear programming problem:

[1st Level]

$$\underset{x_1}{\mathrm{Min}}\,(f_{11} = x_1 - x_2 - 4x_3, f_{12} = -x_1 + 3x_2 - 4x_3)$$

where x_2 and x_3 solve

[2nd Level]

$$\underset{x_2}{\mathrm{Min}}\,(f_{21} = 2x_1 - x_2 + 2x_3, f_{22} = 2x_1 + x_2 - 3x_3, f_{23} = 3x_1 - x_2 + x_3)$$

where x_3 solves

[3rd Level]

$$\underset{x_3}{\mathrm{Min}}\,(f_{31} = 7x_1 + 3x_2 - 4x_3, f_{32} = x_1 + x_3)$$

subject to

$$\begin{aligned}
&x_1 + x_2 + x_3 \leqslant 3, \qquad x_1 + x_2 - x_3 \leqslant 1,\\
&x_1 + x_2 + x_3 \geqslant 1, \qquad -x_1 + x_2 + x_3 \leqslant 1,\\
&x_3 \leqslant 0.5, \text{and} \qquad x_1, x_2, x_3 \geqslant 0.
\end{aligned}$$

Table 1 summarizes minimum and maximum individual optimal solutions, of all objectives functions for the three levels of the ML-MOLP problem, subjected to given constraints, G. To demonstrate the proposed algorithms, the aspiration levels and upper tolerance limits to the objective functions can be taken as the minimum and maximum individual optimal solutions.

The first Algorithm, Alg I, can be explained through the solution procedure of the second algorithm, Alg II. Then, following Alg II, the proposed FGP procedure to the multi-level multi-objective linear programming problem proceeds as:

Then, following Alg II, the proposed FGP procedure to solve this problem proceeds as:

First level DM FGP model:

$$\begin{aligned}
&\min Z_1 = 0.286d_{11}^+ + 0.154d_{12}^+\\
&\text{subject to}\\
&\quad -0.286x_1 + 0.286x_2 + 1.143x_3 + d_{11}^- - d_{11}^+ = 0.714,\\
&\quad 0.154x_1 - 0.154x_2 + 0.62x_3 + d_{12}^- - d_{12}^+ = 0.54,\\
&\quad x_1 + x_2 + x_3 \leqslant 3, \quad x_1 + x_2 - x_3 \leqslant 1,\\
&\quad x_1 + x_2 + x_3 \geqslant 1, \quad -x_1 + x_2 + x_3 \leqslant 1,\\
&\quad x_3 \leqslant 0.5, \quad x_1, x_2, x_3 \geqslant 0,\\
&\quad d_{ij}^-, d_{ij}^+ \geqslant 0 \text{ and } d_{ij}^- \times d_{ij}^+ = 0,\ i = 1\ j = 1, 2.
\end{aligned}$$

Using the LP-ILP linear and integer programming software program, version 1 for windows, the optimal solution of this problem is $\boldsymbol{x}^{1^*} = (\boldsymbol{x}_1^{1^*}, \boldsymbol{x}_2^{1^*}, \boldsymbol{x}_3^{1^*}) = 0.5, 0, 0.5)$. Let the first level DM decide $\boldsymbol{x}_1^{1^*} = 0.5$ with the negative and positive tolerances $t_1^{1_L} = t_1^{1_R} = 0.5$ with weights $w_{11}^L = w_{11}^R = \frac{1}{0.5} = 2$.

Second level DM FGP model:

$$\min Z_2 = 0.286d_{11}^+ + 0.154d_{12}^+ + 0.2d_{21}^+ + 0.33d_{22}^+ + 0.167d_{23}^+ + 2\left[d_{11}^{L-} + d_{11}^{L+} + d_{11}^{R-} + d_{11}^{R+}\right]$$

subject to

$$\begin{aligned}
&-0.286x_1 + 0.286x_2 + 1.143x_3 + d_{11}^- - d_{11}^+ = 0.714,\\
&0.154x_1 - 0.154x_2 + 0.62x_3 + d_{12}^- - d_{12}^+ = 0.54,\\
&-0.4x_1 + 0.2x_2 - 0.4x_3 + d_{21}^- - d_{21}^+ = 0.2,\\
&-0.667x_1 - 0.33x_2 + x_3 + d_{22}^- - d_{22}^+ = 0.33,\\
&-0.5x_1 + 0.167x_2 - 0.167x_3 + d_{23}^- - d_{23}^+ = 0.17,\\
&2x_1 + d_{11}^{L-} - d_{11}^{L+} = 1, \quad 2x_1 + d_{11}^{R-} - d_{11}^{R+} = 1,\\
&x_1 + x_2 + x_3 \leqslant 3, \quad x_1 + x_2 - x_3 \leqslant 1,\\
&x_1 + x_2 + x_3 \geqslant 1, \quad -x_1 + x_2 + x_3 \leqslant 1,\\
&x_3 \leqslant 0.5, \quad x_1, x_2, x_3 \geqslant 0,\\
&d_{ij}^-, d_{ij}^+ \geqslant 0 \text{ and } d_{ij}^- \times d_{ij}^+ = 0,\ i = 1, 2,\ j = 1, 2,\\
&d_{11}^{L-}, d_{11}^{L+}, d_{11}^{R-}, d_{11}^{R+} \geqslant 0, \quad d_{11}^{L-} \times d_{11}^{L+} = 0, d_{11}^{R-} \times d_{11}^{R+} = 0.
\end{aligned}$$

Table 1
Minimum and maximum individual optimal solutions.

	f_{11}	f_{12}	f_{21}	f_{22}	f_{23}	f_{31}	f_{32}
$\min_G f_{ij}$	−2.5	−3.5	−1	−1	−1	−0.5	0
$\max_G f_{ij}$	1	3	4	2	5	8.5	2

The optimal solutions of this linear programming problem are $\boldsymbol{x}^{2^*} = (\boldsymbol{x}_1^{2^*}, \boldsymbol{x}_2^{2^*}, \boldsymbol{x}_3^{2^*})$=(0.5, 0, 0.5), (0.5, 0.998, 0.5), and (0.5, 0.5, 0) (alternative optima). Let the second level DM decide $x_1^{2^*} = 0.998$ with the negative and positive tolerances $t_1^{2_L} = 0.75$ and $t_1^{2_R} = 0.25$ with weights $w_{21}^L = \frac{1}{0.75} = 1.333$ and $w_{21}^R = \frac{1}{0.25} = 4$ respectively.

Third Level DM – and the final FGP model to the three-level MOLP problem – FGP model:

$$
\begin{aligned}
\min Z_3 = {} & 0.286d_{11}^+ + 0.154d_{12}^+ + 0.2d_{21}^+ + 0.33d_{22}^+ + 0.167d_{23}^+ + 0.111d_{31}^+ + 0.5d_{32}^+ \\
& + 2\left[d_{11}^{L-} + d_{11}^{L+} + d_{11}^{R-} + d_{11}^{R+}\right] + 1.33(d_{21}^{L-} + d_{21}^{L+}) + 4(d_{21}^{R-} + d_{21}^{R+}) \\
& \text{subject to} \\
& -0.286x_1 + 0.286x_2 + 1.143x_3 + d_{11}^- - d_{11}^+ = 0.714, \\
& 0.154x_1 - 0.154x_2 + 0.62x_3 + d_{12}^- - d_{12}^+ = 0.54, \\
& -0.4x_1 + 0.2x_2 - 0.4x_3 + d_{21}^- - d_{21}^+ = 0.2, \\
& -0.667x_1 - 0.33x_2 + x_3 + d_{22}^- - d_{22}^+ = 0.33, \\
& -0.5x_1 + 0.167x_2 - 0.167x_3 + d_{23}^- - d_{23}^+ = 0.17, \\
& -0.78x_1 - 0.33x_2 + 0.44x_3 + d_{31}^- - d_{31}^+ = 0.06, \\
& -0.5x_1 - 0.5x_3 + d_{32}^- - d_{32}^+ = 0, \\
& 2x_1 + d_{11}^{L-} - d_{11}^{L+} = 1, \quad 2x_1 + d_{11}^{R-} - d_{11}^{R+} = 1, \\
& 1.33x_2 + d_{21}^{L-} + d_{21}^{L+} = 1.33, \quad 4x_2 + d_{21}^{R-} + d_{21}^{R+} = 3.99, \\
& 2x_1 + d_{11}^{L-} - d_{11}^{L+} = 1, \quad 2x_1 + d_{11}^{R-} - d_{11}^{R+} = 1, \\
& x_1 + x_2 + x_3 \leqslant 3, \quad x_1 + x_2 - x_3 \leqslant 1, \\
& x_1 + x_2 + x_3 \geqslant 1, \quad -x_1 + x_2 + x_3 \leqslant 1, \\
& x_3 \leqslant 0.5, \quad x_1, x_2, x_3 \geqslant 0, \\
& d_{ij}^-, d_{ij}^+ \geqslant 0 \text{ and } d_{ij}^- \times d_{ij}^+ = 0,\ i = 1,2,3,\ j = 1,2, \\
& d_{11}^{L-}, d_{11}^{L+}, d_{11}^{R-}, d_{11}^{R+} \geqslant 0, \quad d_{11}^{L-} \times d_{11}^{L+} = 0, \quad d_{11}^{R-} \times d_{11}^{R+} = 0, \\
& d_{21}^{L-}, d_{21}^{L+}, d_{21}^{R-}, d_{21}^{R+} \geqslant 0, \quad d_{21}^{R-} \times d_{21}^{R+} = 0, \quad d_{21}^{L-} \times d_{21}^{L+} = 0.
\end{aligned}
$$

The satisfactory solution of the ML-MOLP problem is $\boldsymbol{x}^{3^*} = (\boldsymbol{x}_1^{3^*}, \boldsymbol{x}_2^{3^*}, \boldsymbol{x}_3^{3^*}) = (0.5, 0.998, 0.5)$ with objective function values $f_{11} = -2.498$, $f_{12} = 0.494$, $f_{21} = 1.002$, $f_{22} = 0.498$, $f_{23} = 1.002$, $f_{31} = 4.493$, and $f_{32} = 1$ and with membership function values $\mu_{11} = 0.999$, $\mu_{21} = 0.39$, $\mu_{21} = 0.6$, $\mu_{22} = 0.5$, $\mu_{23} = 0.67$, $\mu_{31} = 0.45$, and $\mu_{32} = 0.5$ respectively.

Note: The solution of the problem following the fuzzy approach of Abo-Sinna [5,10] that extends the fuzzy approach of Shih et al. [23] is $\boldsymbol{x}^{3^*} = (\boldsymbol{x}_1^{3^*}, \boldsymbol{x}_2^{3^*}, \boldsymbol{x}_3^{3^*}) = (0.5, 0.9975, 0.5)$ with objective function values $f_{11} = -2.498$, $f_{12} = 0.494$, $f_{21} = 1.002$, $f_{22} = 0.498$, $f_{23} = 1.002$, $f_{31} = 4.493$, and $f_{32} = 1$ and with membership function values $\mu_{11} = 0.999$, $\mu_{21} = 0.39$, $\mu_{21} = 0.6$, $\mu_{22} = 0.5$, $\mu_{23} = 0.67$, $\mu_{31} = 0.45$, and $\mu_{32} = 0.5$ respectively.

A comparison given in Table 2 between the proposed FGP approach and that given in [5,10] by Abo-Sinna clearly shows that the two solutions are close to one another. But as said previously, the proposed FGP approach treats the main difficulties arise with the FP approaches of Shih et al. [23] and Abo-Sinna [5,10].

Table 2
Comparison of optimal solutions and satisfactory solutions of the illustrative example based on the proposed FGP and the fuzzy approach of Abo-Sinna [5,10].

The proposed FGP approach		The fuzzy approach		The optimal solutions
$f_{11} = -2.498$	$\mu_{11} = 0.999$	$f_{11} = -2.21$	$\mu_{11} = 0.92$	$f_{11} = -2.5$
$f_{12} = 0.494$	$\mu_{21} = 0.39$	$f_{12} = -0.569$	$\mu_{21} = 0.55$	$f_{12} = -3.5$
$f_{21} = 1.002$	$\mu_{21} = 0.6$	$f_{21} = 1.88$	$\mu_{21} = 0.56$	$f_{21} = -1$
$f_{22} = 0.498$	$\mu_{22} = 0.5$	$f_{22} = -0.09$	$\mu_{22} = 0.7$	$f_{22} = -1$
$f_{23} = 1.002$	$\mu_{23} = 0.67$	$f_{23} = 1.09$	$\mu_{23} = 0.65$	$f_{23} = -1$
$f_{31} = 4.493$	$\mu_{31} = 0.45$	$f_{31} = 2.62$	$\mu_{31} = 0.65$	$f_{31} = -0.5$
$f_{32} = 1$	$\mu_{32} = 0.5$	$f_{32} = 0.899$	$\mu_{32} = 0.55$	$f_{32} = 0$
$\boldsymbol{x}^* = (0.5, 0.998, 0.5)$		$\boldsymbol{x}^* = (0.339, 0.61, 0.5)$		

6. Summary and conclusion

This paper presents two fuzzy goal programming procedures for solving multi-level multi-objective linear programming problems. A fuzzy goal programming model is developed to minimize the group regret of degree of satisfactions of all the decision makers, and to achieve the highest degree (unity) of each of the defined membership function goals to the extent possible by minimizing their deviational variables and thereby obtain the most satisfactory solution for all the decision makers. The main advantage of the proposed fuzzy goal programming algorithm is that the possibility of rejecting the solution again and again by the upper decision makers and re-evaluation of the problem repeatedly by redefining the elicited membership functions needed to reach the satisfactory decision does not arise.

The first proposed algorithm groups the membership functions for the defined fuzzy goals of the objective functions at all levels as well as the membership functions of the fuzzy goals of the decision variables for each level except the lower level of the multi-level problem. The second proposed algorithm, lexicographically solve MOLP problems of the ML-MOLP problem take into consideration, the decisions of the MOLP problems for the upper levels. An illustrative numerical example is given to demonstrate the proposed algorithms.

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Appendix A. The FGP model to MOLP problems

The FGP model of Mohamed [18], for solving single-level MOLP problem, is presented here to facilitate the achievement of $\mathrm{x}^{\ell^*} = (\mathrm{x}_1^{\ell^*}, \mathrm{x}_2^{\ell^*}, \ldots, \mathrm{x}_p^{\ell^*}), \ell = 1, 2, \ldots, \mathrm{p} - 1$. Following the same symbols of this paper, the FGP model formulation of any ℓth-level MOLP problem is:

$$
\begin{aligned}
&\min Z = \sum_{j=1}^{m_\ell} w_{\ell j}^{+} d_{\ell j}^{+} \\
&\text{subject to} \\
&\quad \boldsymbol{c}_1^{\ell j}\boldsymbol{x}_1 + \boldsymbol{c}_2^{\ell j}\boldsymbol{x}_2 + \cdots + \boldsymbol{c}_p^{\ell j}\boldsymbol{x}_p + d_{\ell j}^{-} - d_{\ell j}^{+} = 1, \quad j = 1, 2, \ldots, m_\ell \\
&\quad A_1x_1 + A_2x_2 + \cdots + A_px_p \begin{pmatrix} \leqslant \\ = \\ \geqslant \end{pmatrix} \boldsymbol{b},\ \boldsymbol{x} \geqslant 0 \\
&\quad d_{\ell j}^{-} \times d_{\ell j}^{+} = 0,\ \text{and}\ d_{\ell j}^{-}, d_{\ell j}^{+} \geqslant 0,\ j = 1, 2, \ldots, m_\ell
\end{aligned}
\tag{A1}
$$

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