Details of Instance Generation

We first recall the uni-parametric linear complementarity problem (upLCP). Let $\mathscr{A} = \{\mu\theta + \upsilon : \mu, \upsilon \in \mathbb{R}\}$, the set of affine functions of a scalar parameter θ. Then upLCP is as follows:

> Given $M(\theta) \in \mathscr{A}^{h\times h}$ and $q(\theta) \in \mathscr{A}^h$, for each $\theta \in \Theta$ find vectors $w(\theta)$ and $z(\theta)$ that satisfy the system
>
> $$\begin{aligned} w - M(\theta)z &= q(\theta) \\ w^\top z &= 0 \\ w, z &\geq 0 \end{aligned} \tag{1}$$
>
> or show that no such vectors exist.

Additionally, as the article for which the provided instances were generated relies on two assumptions, we recall those as well.

Assumption 1 *The matrix $M(\theta)$ is sufficient for all $\theta \in \Theta$.*

Assumption 2 *System* (1) *is feasible for all $\theta \in \Theta$.*

Now, the instances provided here are derived from two classes of problems. Instances of the first class are obtained from biobjective quadratic programs (QPs) having convex objectives that are scalarized using the weighted-sum approach and then reformulated as upLCPs. We refer to these instances as `boQP` instances. Instances of the second class are obtained using some of the techniques outlined by Illés and Morapitiye [1]. We refer to these instances as `sufLCP` instances. Exact details as to how each set of instances is generated are contained in the appropriately named sections below. We use the notations $\mathsf{Tri}(\cdot)$ and $\mathsf{Uni}(\cdot)$, together with appropriate parameters, to denote values taken from the triangular and uniform distributions, respectively. When vectors and matrices are generated with all elements drawn from the same distribution, we denote this by adding an appropriate subscript to the aforementioned notations. Additionally, all randomly generated data described in the following sections is rounded to the nearest integer.

`boQP` Instances

Prior to reformulation as upLCP, instances of type `boQP` have the following form:

$$\begin{aligned} \min_x \quad & \frac{1}{2}x^\top Q(\theta)x + c(\theta)^\top x \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \\ & \theta \in \Theta \end{aligned} \tag{2}$$

To generate such an instance, we begin by selecting the dimension h. We then select $n \sim \mathsf{Tri}(0, \frac{h}{2}, h)$ and define $Q(\theta) \in \mathscr{A}^{n\times n}$, $c(\theta) \in \mathscr{A}^{n}$, $A \in \mathbb{R}^{n\times(h-n)}$ and $b \in \mathbb{R}^{(h-n)}$ as follows.

- $Q(\theta) = (1-\theta)Q_1Q_1^\top + \theta Q_2Q_2^\top$, where $Q_1, Q_2 \sim \mathsf{Uni}_{n\times n}(-2, 2)$.
- $c(\theta) = (1-\theta)c_1 + \theta c_2$, where $c_1, c_2 \sim \mathsf{Uni}_{n\times 1}(-2, 2)$.
- $A \sim \mathsf{Uni}_{n\times(h-n)}(-2.5, 2.5)$.
- $b = Ap$, where $p \sim \mathsf{Uni}_{(h-n)\times 1}(0, 2.5)$.

The associated upLCP is obtained by setting $M(\theta) = \begin{bmatrix} 0 & -A \\ A^\top & Q(\theta) \end{bmatrix}$ and $q(\theta) = \begin{bmatrix} b \\ c(\theta) \end{bmatrix}$. By letting $\Theta = [0, 1]$, we know that Assumptions 1 and 2 are satisfied for all $\theta \in \Theta$. Satisfaction of Assumption 1 comes from the fact that, by construction, $M(\theta)$ is postive semi-definite (PSD) for all $\theta \in \Theta$ and it is well known that all PSD matrices are sufficient. Satisfaction of Assumption 2 comes from the fact that, by construction, Problem (2) containing $Q(\theta)$, $c(\theta)$, A, and b as derived above is feasible for all $\theta \in \mathbb{R}$ and has a convex objective function for all $\theta \in \Theta$.

There are five `boQP` instances for each $h \in \{50, 75, 100, 125\}$.

0.1 `sufLCP` Instances

Instances of type `sufLCP` have the form of System (1). For each instance, we begin by selecting the dimension h. We then generate the instance's defining data in the following two phases: (i) we generate data that we use to construct $M(\theta)$, (ii) then we generate data that we use to construct $q(\theta)$.

Matrix $M(\theta)$ relies on $n_1, n_2, n_3 \in \mathbb{R}$, $d^1, d^2 \in \mathbb{R}^{n_3}$, and $H \in \mathbb{R}^{n_1\times n_1}$. We first generate $n_1 \sim \mathsf{Tri}(0, \frac{h}{2}, h)$, $n_2 \sim \mathsf{Uni}(0, h-n_1)$, and $n_3 = h - n_1 - n_2$, subject to the restrictions that $n_1, n_2 > 0$ and $n_3 \le \frac{h}{2}$. We then generate $H \sim \mathsf{Uni}_{n_1\times n_1}(-2, 2)$ and $d^1 \sim \mathsf{Uni}_{n_3\times 1}(1, 5)$. Next, we randomly select $I \subset \{1, \ldots, n_3\}$ so that $|I| = \min\left\{n_3, \left\lfloor \frac{h}{5} \right\rfloor\right\}$ and define d^2 so that $d_i^2 \sim \begin{cases} \mathsf{Uni}(-d_i^1, d_i^1) & \text{if } i \in I \\ 0 & \text{otherwise} \end{cases}$. Finally, we set $M(\theta) = \begin{bmatrix} HH^\top & E & F \\ -E^\top & 0 & 0 \\ -F^\top & 0 & D(\theta) \end{bmatrix}$ where: (i) $E \in \mathbb{R}^{n_1\times n_2}$ has entries $e_{ij} = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{otherwise} \end{cases}$; (ii) $F \in \mathbb{R}^{n_1\times n_3}$ has entries $f_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$; and (iii) $D(\theta) = diag(d^1 + \theta d^2)$.

The vector $q(\theta)$ relies on $a^1, a^2, q^1, q^2 \in \mathbb{R}^h$. We first generate $a^1 \sim \mathsf{Uni}_{h\times 1}(0, 5)$ and then generate a^2 so that $a_i^2 \sim \begin{cases} \mathsf{Uni}(0, 5) & \text{if } a_i^1 = 0 \\ 0 & \text{otherwise} \end{cases}$. We then set $q^1 = -M(0)a^1 + a^2$, define $J = \{j : q_i^1 > 0\}$, and randomly select $L \subseteq J$ so that $|L| = \min\left\{|J|, \left\lfloor \frac{h}{5} \right\rfloor\right\}$. Finally, we define q^2 so that $q_l^2 \sim \begin{cases} \mathsf{Uni}(-q_l^1, q_l^1) & \text{if } l \in L \\ 0 & \text{otherwise} \end{cases}$ and set $q(\theta) = q^1 + \theta q^2$.

For all instances of type `sufLCP` we again set $\Theta = [0, 1]$. We note that with $M(\theta)$ defined as above, by Lemmas 1, 6, and 7 of [1], Assumption 1 is satisfied for all $\theta \in \Theta$. Additionally, although the method given above for generating $q(\theta)$ guarantees that each instance of upLCP is feasible for $\theta = 0$, it does not guarantee that Assumption 2 will be met for all $\theta \in \Theta$. We note, however, that during testing our implementation successfully partitioned the entirety of Θ for each instance.

There are five `sufLCP` instances for each $h \in \{50, 75, 100, 125, 150, 175\}$.

References

[1] Tibor Illés and Sunil Morapitiye. Generating sufficient matrices. In *Short papers of the 8th VOCAL optimization conference: advanced algorithms, Published by Pázmány Péter Catholic University, Budapest*, page 56, 2018.