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Finding a Less Risky Alternative to the SP500

Motivation

We want to find a portfolio of stocks which can deliver our clients a lower risk than the SP500 index but still have as much return as the SP500.

Tools

We will use the programming language R to conduct a search for such a portfolio of ten stocks, finding their log returns, their risk-adjusted returns, kurtosis and minimizing the variance of the portfolio to reach appropriate weights to reduce portfolio volatility.

Conduct Preliminary Study to Find 10 Stocks for our Portfolio

We begin by grabbing 25 stocks from Yahoo that have good chart technicals since the end of the Great Recession of 2007-2009. By that, we mean stock charts where the 50-week moving average stays above the 200-week moving average for most of the life of the chart. Moreover, a stock that bounces off its 200-week moving average or never reaches such moving average is also a good indicator of long-term upward strength. The list of the 25 stocks is listed in the Appendix.

We next calculate the mean of the annual log returns of the 25 stocks to determine the average return of each stock over the last 13 years (2009-2022).

We are also interested in the risk-adjusted return, where the excess return is adjusted for volatility; the more volatility the less "adjusted" return the stock has. The balancing act between return and risk is necessary since we want to find a portfolio of stocks with as much return but with lower risk than the SP500 index. We use 2.28% for the risk-free 10-year treasury rate, since that number is the average over the last 13 years. (Our data source was: https://www.macrotrends.net/2016/10-year-treasury-bond-rate-yield-chart.)

And lastly, we find excess kurtosis which, if positive, indicates a fatter tail log return distribution than the normal distribution. All the stocks sampled had a fatter tail. But the sample size of 13 is fairly small to make a good inference on which stock log returns have fatter tails than others.

We select the top ten stocks for better curation and easier asset management that have the highest risk-adjusted ratios (Sharpe ratios), which can be viewed in the Appendix.

Portfolio Return and Volatility

If we choose equal weighting for my selected ten stocks, such that each stock is 1/10 of our portfolio, we can find both the average log return and the standard deviation of the returns.

We can clearly see the portfolio has less volatility than any one of the portfolio's components:

DG	COST	MA	AAPL	PGR	DHR	V	HD	BRK.B	TMO	My Portfolio
0.1417521	0.1461525	0.212711	0.2436664	0.1308821	0.169148	0.1902117	0.1986749	0.1213073	0.2139314	0.1199106

The average log return per year for our chosen portfolio is 17.64%, which according the table in the Appendix, is about the median return in the individual stock names. But our portfolio volatility is the least value of any individual stock name at 0.1199.

If we compare our portfolio return and risk to the SP500, we find that our chosen portfolio has 1.678 times higher average annual log return and about equal risk. We will use SPY, which is a popular, liquid exchange-traded fund that closely tracks the SP500 index but trades like an individual stock.

Given 2009-2022 data	SPY	Equal-Weighted Portfolio
Average Annual Log Return	10.51%	17.64%
Volatility of Returns	0.1205	0.1199

Optimize the Portfolio

Suppose we want to find a portfolio to satisfy our risk aversion. We want a portfolio that closely follows the returns of the SP500 but which has less risk than the SP500. The famous mantra is that we "can't have our cake and eat it too". That is, we can't expect on average the same return as the SP500 but assume lesser risk.

But using Modern Portfolio Theory, we can plot the envelope of possible portfolios by changing the weights of the components. And finding the minimal variance in the course of changing the each of the component's weight from 1/10 to something different, we can achieve a less risky portfolio. The weights of course can be positive, where we are long the stock component, or negative, where we are short the stock component. And each of the ten weights will differ from one another.

Construct the Envelope of Possible Portfolios

We want to minimize the covariance of the assets of the portfolio, thusly:

$$\begin{aligned} \min_{\boldsymbol{w}} \operatorname{var}(r_p) \quad &= \min_{\boldsymbol{w}} \boldsymbol{w}\Sigma_p \boldsymbol{w}' \\ \text{subject to} \quad & \mathrm{E}(\boldsymbol{r})\boldsymbol{w}' = \mu = \mathrm{E}(r_p), \\ & \boldsymbol{w}\mathbf{1}_{col} = 1, \end{aligned}$$

We need only two construct two portfolios, with different weights of the ten stocks, to create the convex curve of possible portfolios. Indeed, every other portfolio can be created from just two portfolios, w and v, using convex combinations: a***w**+(1-a) ***v**, where w and v are just vectors of different weights and a is a constant needed to represent a different portfolio on the envelope curve.

To construct portfolios w and v, which must lie on the envelope, follow the below three steps, where capital sigma is the covariance-variance matrix of the ten assets and E(**r**) is the expected return vector.

1. Choose an arbitrary constant c. Common choice is 0.
2. Compute $\boldsymbol{z} = \Sigma_p^{-1}(\mathrm{E}(\boldsymbol{r}) - c)'$. Note that Σ_p is a $n \times n$ matrix, $(\mathrm{E}(\boldsymbol{r}) - c)'$ is a $n \times 1$ column vector.
3. The envelope portfolio is $w_i = \frac{z_i}{\sum z_i}$, $i = 1, ..., n$.

Once portfolios w and v are constructed, we will create the envelope of portfolios. The envelope is the set of portfolios that have lowest variance given the same expected returns. Specifically, we want to create the efficient frontier, where the maximum or highest portfolio return is selected given that we have the same variance in two or more possible portfolios.

Mathematically, we can find the efficient frontier in this way:

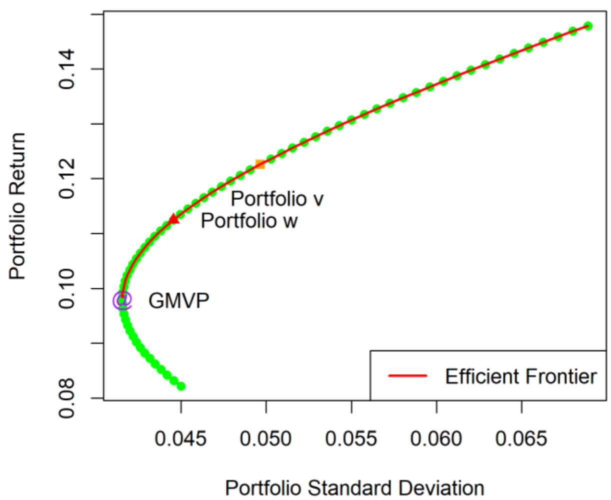
$$
\begin{aligned}
\max_{\boldsymbol{w}} \mathrm{E}(r_p) \quad & = \max_{\boldsymbol{w}} \mathrm{E}(\boldsymbol{r})\boldsymbol{w}' \\
\text{subject to} \quad & \mathrm{var}(r_p) = \boldsymbol{w}\Sigma_p\boldsymbol{w}' = \sigma^2, \\
& \boldsymbol{w}\mathbf{1}_{col} = 1
\end{aligned}
$$

This efficient frontier is denoted by the red line in our envelope chart below, which plots portfolio returns against portfolio volatility. All portfolios to the right of the frontier are not efficient, and all points to the left are not mathematically possible (infeasible) given our current ten stocks.

Lastly, we want to find the minimum portfolio variance, such that our portfolio volatility is minimized and equals less than that of the SP500 while maintaining similar annual returns.

We obtain the minimum via the Global Minimum Variance Portfolio calculation, minimizing the covariance among the assets by changing the ten weights (possibly positive, negative or both):

$$
\begin{aligned}
\min_{\boldsymbol{w}} \mathrm{var}(r_p) \quad & = \min_{\boldsymbol{w}} \boldsymbol{w}\Sigma_p\boldsymbol{w}' \\
\text{subject to} \quad & \boldsymbol{w}\mathbf{1}_{col} = 1,
\end{aligned}
$$



Conclusion

The mean annual portfolio return for the GMVP is 9.79%, while SPY is similar at 10.51%. Meanwhile, we dramatically reduced the volatility of the portfolio to 0.0415, from its equal-weighted volatility at 0.1199 and the SPY volatility at 0.1205.

The weights of our preferred portfolio, the Global Minimum Variance Portfolio, are:

	DG	COST	MA	AAPL	PGR	DHR	V	HD	BRK.B	TMO
Weights for GMVP	-0.03852478	0.819763	-0.272239	-0.1740391	0.417465	0.1192749	0.4518381	-0.309318	0.404144	-0.4183642

To understand the details of the optimization problems above, where we find the minimum variance of the portfolio returns by changing the weights, please see the R code implementation.

Future iterations of the above will include finding a <u>different</u> set of ten stocks such that its portfolio variance remains at about 0.04-0.05 but the annual portfolio return is about 20-25%. Moreover, we will include restrictions on short sales since management of short sale positions is often difficult.

Appendix

25 asset candidates

	13-Year Average of Log Yearly Returns	Sharpe Ratio	Excess Kurtosis (normal=0, thinner tails<0, fatter tails>0
DG	0.16224901	0.9837529	1.651316
COST	0.15990376	0.9380870	4.147376
MA	0.21608947	0.9086954	1.752642
AAPL	0.24272054	0.9025475	2.048086
PGR	0.14057866	0.8998838	3.665846
DHR	0.17132209	0.8780601	3.219662
V	0.18171979	0.8354892	2.338287
HD	0.18058636	0.7941937	3.987459
BRK.B	0.11903675	0.7933304	2.309343
TMO	0.18973155	0.7803040	2.514321
MSFT	0.18156665	0.7776887	2.101295
INTU	0.19343388	0.7347012	5.156169
AMZN	0.24039468	0.6951731	2.360763
ADBE	0.18944323	0.6843947	2.568051
AMGN	0.11671735	0.6825285	1.867764
GOOGL	0.16901674	0.6566662	1.964546
NKE	0.14873014	0.6184353	3.851511
ABT	0.12101899	0.5767249	2.382868
NVDA	0.30167705	0.5674507	1.957427
LMT	0.11816463	0.4836396	2.212729
ICE	0.10981956	0.4507895	4.935271
AMD	0.21926244	0.3186895	2.115006
INTC	0.07073781	0.2852095	1.797545
WMT	0.06665616	0.2612907	3.799193
TMUS	0.09666489	0.1908156	3.191926