claim If φ_i is strictly increasing then (ii) is satisfied by our allocation rule

If $\pi_i(\theta_i, \theta_{-i}) > 0$, then $\pi_i(\hat{\theta}_i, \theta_{-i}) = 1$. if $\hat{\theta}_i > \theta_i$.

$\Rightarrow \pi_i(\hat{\theta}_i, \theta_{-i}) \geq \pi_i(\theta_i, \theta_{-i})$

If $\pi_i(\theta_i, \theta_{-i}) = 0$ then

$\Rightarrow \pi_i(\hat{\theta}_i, \theta_{-i}) \geq \pi_i(\theta_i, \theta_{-i}) \quad \forall \hat{\theta}_i \geq \theta_i$

$\alpha_i(\theta_i) = \underset{\theta_{-i}}{E}[\pi_i(\theta_i, \theta_{-i})]$ is non-decreasing

Revenue

$E[\max(\varphi_1(\theta_1)\ \varphi_2(\theta_2) \cdots \varphi_N(\theta_N),\ 0]$

$r_i(\theta_i, \theta_{-i}) = \min\{z_i : \varphi_i(z_i) \geq 0 \text{ and } \varphi_i(z_i) \geq \max_{j \neq i} \varphi_j(\theta_j)\}$

r_i is the smallest valuation of the bidder i such that she is guranted to win the item.

$$\pi_i(\theta_i, \theta_{-i}) = \begin{cases} 1 & \text{if } \theta_i > r_i(\theta_{-i}) \\ 0 & \text{if } \theta_i < r_i(\theta_{-i}) \end{cases}$$

$$q_i(\theta_i, \theta_{-i}) = \theta_i \pi_i(\theta_i, \theta_{-i}) - \int_0^{\theta_i} \pi_i(\theta_i, \theta_{-i})\, d\theta$$

$$= \theta_i I_{\theta_i > r_i} - \int_0^{\theta_i} I_{\theta > r_i}\, d\theta$$

If $\theta_i > r_i(\theta_i)$ then.

$q_i(\theta_i, \theta_{-i}) = \theta_i - (\theta_i - r_i) = r_i$

If $\theta_i < r_i$

$q_i(\theta_i, \theta_{-i}) = 0$

Revenue - optimal Mechanism.

① the bidder with the largest virtual valuation $\varphi_i(\theta_i)$ wins

If $\max_j \varphi_j(\theta_j) \geq 0$

If $\max_j \varphi_j(\theta_j) < 0$ the Seller does not sell the item.

② the winner (If there is one) pays a amount equal to the smallest value that would still win

---

Example If f_i are all identical

$$\varphi(\theta_i) = \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)}$$

$r_i(\theta_i) = \min\{z_i : \varphi(z_i) \geq 0,\ \varphi(z_i) \geq \max_{j \neq i} \varphi(\theta_j)\}$ 由单调得到

$= \min\{z_i,\ z_i \geq \varphi^{-1}(0),\ z_i \geq \max_{j \neq i} \theta_j\}$

This is like a second price auction with a reserve price of $\varphi^{-1}(0)$.

A: $\varphi(\theta_i) < 0$

B: $\varphi(\theta_1) > 0$, $\varphi(\theta_i) < 0$ $i \neq 1$