Probabilistic Structural Response Analysis of an SDOF System Subjected to Stochastic Ground Motion Using Monte Carlo Simulation

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1 Introduction

The dynamic loads of structural systems due to earthquake excitation have highly uncertain values owing to the randomness of the earthquake ground motion. Unlike other loads, earthquake loads cannot be determined with certainty owing to the complexity of the geophysical processes involved. Thus, modern earthquake engineering is increasingly turning to probabilistic modeling approaches for structural response estimation under uncertain dynamic loads. In deterministic analysis, a single representative record of ground motion is normally taken. However, this approach cannot capture the uncertainties that exist in real earthquakes. Even earthquakes of similar intensity and distance can produce vastly different structural responses. Thus, uncertainty quantification (UQ) tools have become essential in structural dynamics and seismic reliability analysis.

The objective of this research project is to develop a computational model that captures the stochastic process of earthquake excitation and propagates the uncertainty through a structural dynamic system. Specifically, a single-degree-of-freedom (SDOF) system is analyzed under simulated stochastic ground motion using a spectral representation method with a Kanai-Tajimi power spectral density (PSD) model. Monte Carlo simulation is then used to provide probabilistic estimates of the structural response. This research project will demonstrate how uncertainties in earthquake excitation influence structural response and how statistical analysis can be used to quantify uncertainties in structural response. This is in accordance with modern performance-based engineering practice that requires probabilistic estimates of structural demand rather than a single deterministic value.

1.1 Problem Definition

The physical problem is the dynamic behavior of a linear elastic SDOF structural system under earthquake ground acceleration. The equation of motion for a base-excited system is given by:

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = -m\,a_g(t), \tag{1}$$

where $a_g(t)$ denotes the ground acceleration acting at the base of the structure. Dividing by the mass m leads to

$$\ddot{u}(t) + \frac{c}{m}\dot{u}(t) + \frac{k}{m}u(t) = -a_g(t). \tag{2}$$

To express the equation in standard dynamic form, the natural circular frequency of the system is introduced as

$$\omega_j = \sqrt{\frac{k}{m}}, \tag{3}$$

which represents the intrinsic vibration rate determined by the stiffness-to-mass ratio. The corresponding natural period is related through

$$T = \frac{2\pi}{\omega_j}, \qquad \text{or equivalently} \qquad \omega_j = \frac{2\pi}{T}. \tag{4}$$

Dissipation of vibrational energy is characterized by the damping ratio

$$\zeta = \frac{c}{2m\omega_j}, \tag{5}$$

which measures the level of energy loss relative to critical damping. For civil engineering structures, ζ typically lies between 0.03 and 0.10, and a representative value of $\zeta = 0.05$ is commonly adopted in seismic analysis.

Substituting these definitions into the equation of motion yields the normalized form

$$\boxed{\ddot{u}(t) + 2\zeta\omega_j\,\dot{u}(t) + \omega_j^2\,u(t) = -a_g(t),} \tag{6}$$

demonstrating the dependence of the structural response on the natural frequency, damping properties, and the applied ground acceleration. The structural response is highly dependent on the properties of the applied ground motion. Because earthquake acceleration is random, it is treated as a stochastic process.

The parameter of interest in this research is the response spectrum, which is the spectral acceleration $S_a(T)$, which is the pseudo acceleration of an SDOF system with a natural period of T. Response spectra are commonly used in structural design because they provide a summary of the maximum structural response for various structural periods.

1.2 Structure of the Computational Model

The computational framework consists of three sequential modules.

1.2.1 Ground Motion Generator

Earthquake acceleration is generated as a stochastic process using a spectral representation technique. The stationary part of the ground motion is characterized by the Kanai–Tajimi power spectral density (PSD) function:

$$S_{a_g}(\omega) = S_0\,\frac{\omega_g^4 + (2\zeta_g\omega_g\omega)^2}{\left(\omega_g^2 - \omega^2\right)^2 + (2\zeta_g\omega_g\omega)^2}, \tag{7}$$

which describes the site filtering effects, with ω_g denoting the dominant ground frequency, ζ_g denoting soil damping, and S_0 being a parameter controlling the level of intensity.

The stochastic acceleration is synthesized by summing up harmonic waves with random phases:

$$a_s(t) = \sum_{k=1}^{N} \sqrt{2S(\omega_k)\Delta\omega}; \cos! \left(\omega_k t + \phi_k\right), \tag{8}$$

where the phases are randomly sampled as $\phi_k \sim \mathrm{Unif}(0, 2\pi)$ to provide statistical randomness between realizations. To simulate a realistic earthquake duration, a deterministic envelope function is multiplied with the stationary signal. This leads to a nonstationary ground motion that resembles recorded seismic events.

1.2.2 Structural Dynamic Solver

For each simulated ground motion, the equation of motion for the structure is solved numerically with the Newmark-β method. The Newmark-β method is popular in the numerical solution of the equation of motion in structural dynamics because it is unconditionally stable if proper values of the parameters are chosen. The solver provides time histories of displacement, velocity, and acceleration. The absolute acceleration is obtained by adding the ground acceleration to the relative acceleration. The maximum value of the absolute acceleration is determined for each period of the oscillators

1.2.3 Response Spectrum Calculation

The above process is repeated for a series of natural periods T. For each natural period, the maximum absolute acceleration is stored, and a response spectrum curve for the ground motion realization is obtained.

Therefore, the computational model relates the ground motion to the structural response:

$$Y = M(X), \tag{9}$$

where X is the stochastic input signal and Y is the structural response.

1.3 Modeling and Propagation of Uncertainties

The stochastic ground motion simulator introduces uncertainty into the model. Although the soil properties and structural characteristics are deterministic, randomness is introduced through the random phase angles employed in the spectral representation. The phases introduce different earthquake time histories for the same spectral parameters.

This defines the input random vector:

$$X = a_g(t; \Phi), \tag{10}$$

where Φ denotes the collection of random phase variables.

The Monte Carlo simulation is employed to propagate the uncertainty through the structural model. The steps involved are:

1. A random ground motion is generated.
2. The structural response is solved.
3. The response spectrum is calculated.
4. The process is repeated many times.

Each simulation generates a different response spectrum due to the different ground motion input. After N simulations, the statistical properties are estimated.

Mean response:

$$\mu(T) = \frac{1}{N}\sum_{i=1}^{N} S_a^{(i)}(T). \tag{11}$$

Standard deviation:

$$\sigma(T) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(S_a^{(i)}(T) - \mu(T)\right)^2}. \tag{12}$$

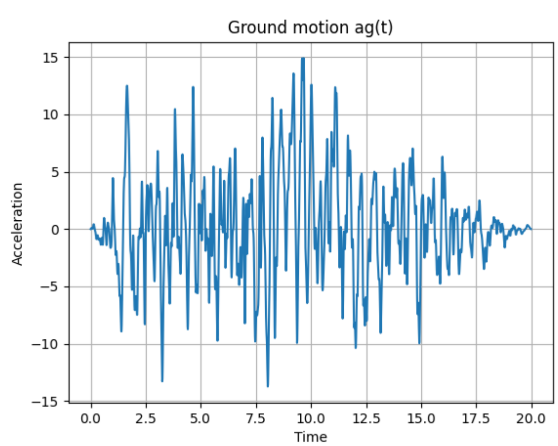
These statistics quantify the expected demand and its variability.

1.4 Results of Stochastic Simulations

Four main results were generated from the simulation.

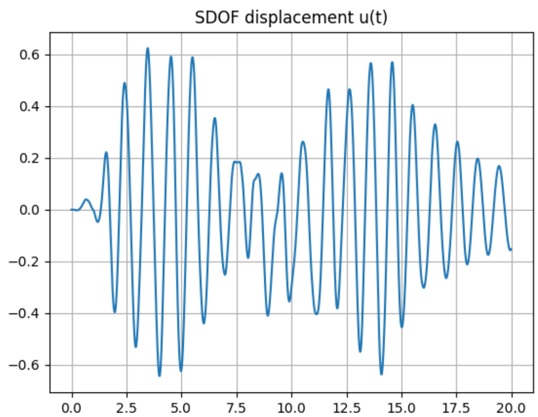
1.4.1 Ground Motion Realization

The acceleration time history has irregular oscillations with varying amplitude. This is an indication that the stochastic generator is producing realistic earthquake motions and not simple harmonic waves. The amplitude modulation is an indication of nonstationarity, which is a characteristic of real earthquake motions.



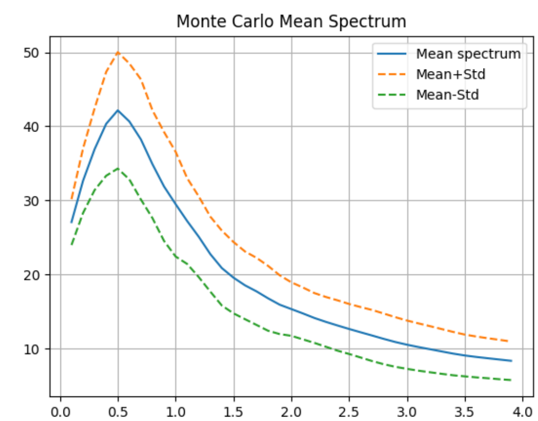
1.4.2 Structural Response

The displacement time history shows that the structure is responding to the ground motion in a dynamic manner, with the oscillations depending on both the properties of the structure and the characteristics of the input. The peaks occur when the input frequency content matches the natural frequency of the oscillator.



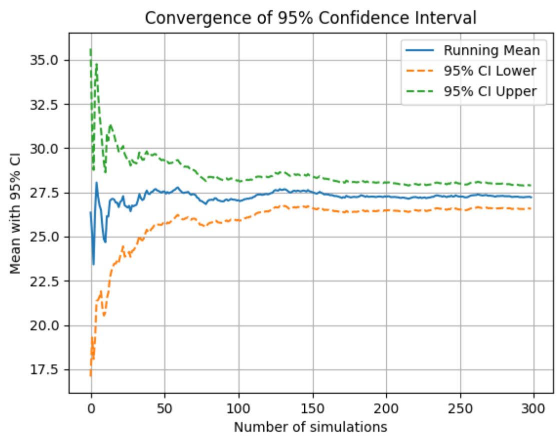
1.4.3 Mean Response Spectrum

The Monte Carlo mean response spectrum shows the classical form as expected from structural dynamics theory. The spectral acceleration has its maximum at shorter periods, peaks around the dominant frequency, and reduces for longer periods. This is an indication that the computational model is physically consistent. The band corresponding to mean $\pm$ standard deviation represents the uncertainty of structural demand. The wider bands show higher variability due to differences in ground motion realization.



1.4.4 Convergence Behavior

The outcome of the Monte Carlo simulations reveals that the structural response cannot be modeled properly by a single deterministic analysis.Each simulated earthquake motion provides a slightly different result, and it is only after a large number of simulations that a stable estimate of the expected result emerges. As the number of simulations increases, the running mean approaches a constant, and the spread of the results diminishes. This diminishment of spread follows the well-known $1/\sqrt{N}$ rule, which indicates that additional simulations provide diminishing returns. The graph of the confidence interval confirms that the estimate of the response becomes better with an increasing number of samples. In summary, the above discussion makes it clear that the uncertainty of the seismic excitation needs to be represented in a probabilistic way, and that Monte Carlo simulation is a valuable tool for estimating the expected structural response as well as the associated uncertainty.



1.5 Discussion

The result indicates that the structural response cannot be represented appropriately using a single deterministic earthquake record. Even when the soil parameters are maintained constant, the differences in the ground motion phase characteristics result in differences in the response spectra. This highlights the importance of probabilistic analysis in seismic design. The result also indicates that the uncertainty propagation analysis provides not only the expected response value but also a reliability measure and risk. The engineers can make use of this probabilistic information to perform the safety margin analysis, design robustness, and performance-based design. Moreover, the computational model is modular and can be easily extended. Other uncertainties, such as structural stiffness, damping, or mass, can be considered by using random parameter distributions