Dynamic Airline Ticket Pricing: A Stochastic Programming Approach for Online Travel Agencies.

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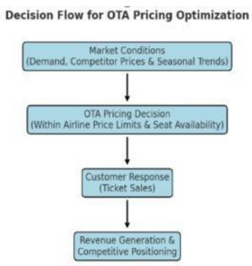
Introduction

In the highly competitive travel industry, online travel agencies (OTAs) must set optimal prices for flight tickets to maximize revenue while remaining competitive. Poorly set prices can lead to lost revenue opportunities (if set too low) or decreased sales (if set too high). This report presents both deterministic and stochastic pricing optimization models for an **Online Travel Agency (OTA),** responsible for setting ticket prices for flights to Paris, London, and New York while competing with another travel agency. The goal is to maximize expected revenue while accounting for demand fluctuations and competitor price uncertainty.

Uncertainty Factors: Demand fluctuates by season (LowDemand, Medium Holiday, HighDemand PeakSeason), and competitor price changes, driven by demand or market actions, influence the OTA's pricing.

Data Source & Assumptions

- The dataset uses synthetic data generated in Python with logical assumptions for realism, incorporating knowledge from actual airline pricing strategies found online. The price sensitivity factor (b) was estimated based on typical consumer behaviour in airline markets.
- Both models assume demand changes linearly with price, independent destination markets that don't affect each other and in the stochastic approach, the OTA can identify current market conditions to implement appropriate pricing strategies.



Model Formulation

A) **Deterministic Model:** *(The deterministic model can be found in Appendix A)*

The deterministic model finds the best single price that maximizes revenue for tickets across three destinations (Paris, London, and New York) using weighted averages of all possible demand scenarios. The model considers how customers respond to prices, what competitors charge, and airline pricing and capacity limits. This approach is simpler but less flexible and cannot adapt to specific market conditions as they emerge.

B) **Stochastic Model**: *(The stochastic model can be found in Appendix B)*

The stochastic model creates a dynamic pricing strategy that changes based on market conditions. Instead of one fixed price, it determines different optimal prices for each destination across five distinct scenarios (LowDemand, Medium, High Demand, Holiday and PeakSeason). By assigning probabilities to these different scenarios, the model maximizes expected revenue across all possible outcomes. This approach captures the value of adaptability in uncertain markets, allowing the OTA to charge premium rates when demand is high and offer more competitive prices when customers are more price-sensitive while respecting the same constraints as the deterministic model.

Results and Analysis

The analysis compares deterministic and stochastic pricing approaches for airline tickets across three destinations and five demand scenarios.

Stochastic Model Performance:

The stochastic model delivers €604,950 in expected revenue, while the deterministic model optimistically forecasts €623,740 in revenue, however its actual performance across scenarios **(EEV)** is only €586,283 *(Figure1).* This "optimism gap" demonstrates the risk of ignoring uncertainty in pricing decisions.

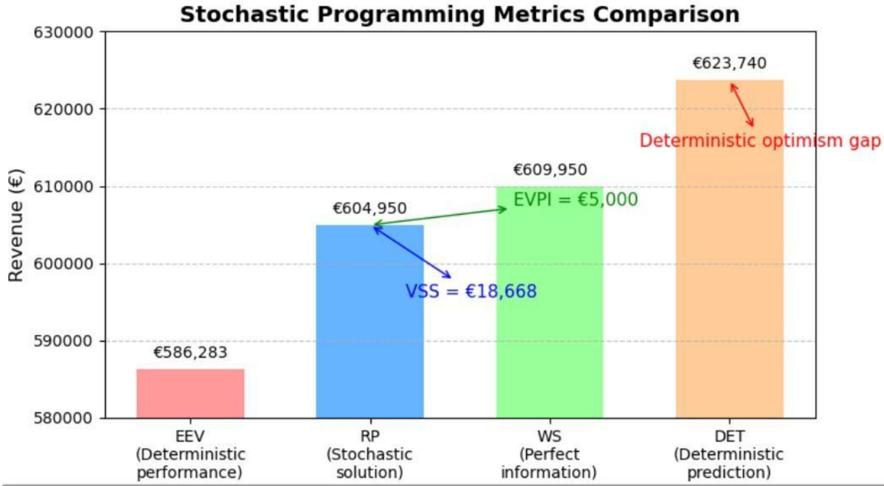
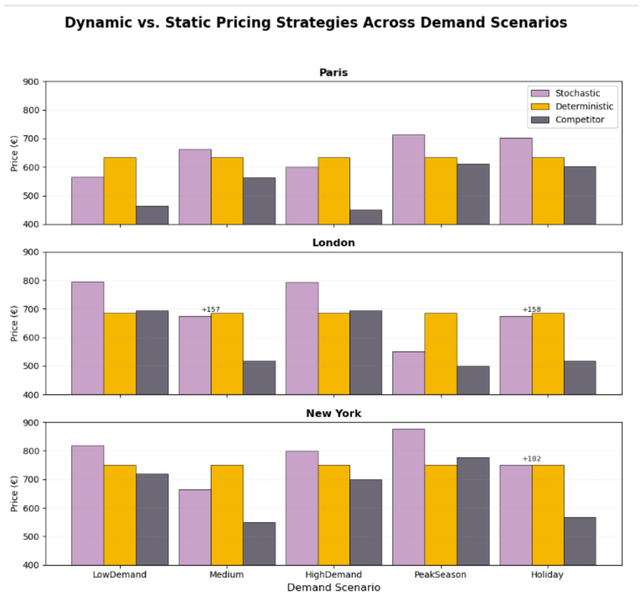


Figure 1: Comparison of stochastic programming metrics showing the value of the stochastic approach (VSS) and the maximum value of perfect information (EVPI). Note the gap between deterministic prediction and actual performance.

The **Value of Stochastic Solution (VSS)** of €18,668 represents a 3.2% revenue improvement with no additional operational costs. With the **Expected Value of Perfect Information (EVPI)** at only €5,000, it implies that the stochastic model already captures most of the potential value available from perfect forecasting.

Dynamic Pricing Strategy: The stochastic model's key advantage is its ability to implement dynamic pricing that adapts to different demand scenarios *(Figure 2).* This creates three strategic benefits:

1. **Price Differentiation:** For New York, prices range from €664.78 (Medium demand) to €875.87 (Peak Season) – a €210 difference that captures additional revenue by matching prices to demand sensitivity.



2. **Competitive Positioning:** For London during Low Demand, pricing €100 above competitors maximizes profit, while during Peak Season, pricing closer to competitors maintains market share when demand is naturally high.
3. **Market Responsiveness:** Paris prices vary by 26% across scenarios, demonstrating flexibility that static pricing cannot achieve.

Sensitivity Analysis: Our sensitivity analysis (*Figure 3)* shows that the value of stochastic pricing increases dramatically with higher price elasticity. As elasticity increases from 0.10 to 0.20, the VSS rises from €12,450 to €31,250 – a 151% increase.

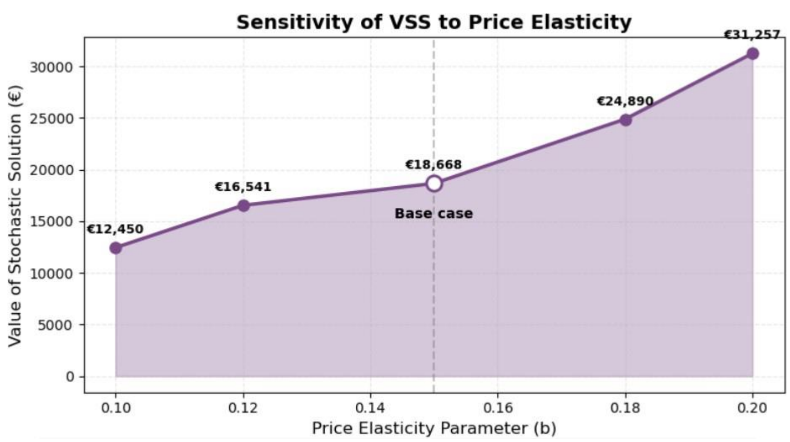


Figure 3: Sensitivity of VSS to Price Elasticity.

This analysis offers practical guidance for implementation: stochastic pricing provides smaller benefits for luxury routes where customers are less price-sensitive (b≈0.10), reasonable gains for average markets (b≈0.15), and becomes crucial for highly competitive routes (b≥0.18) where price sensitivity can yield over €24,890 in extra revenue.

The results demonstrate that dynamic pricing through stochastic optimization significantly outperforms traditional fixed-pricing approaches, particularly in markets with fluctuating demand patterns and higher price sensitivity.

Limitations

1. **Non-reactive Competition**: While the model considers different competitor prices across scenarios, it doesn't account for competitors' direct reactions to our pricing decisions.
2. **Uniform Price Elasticity**: The model applies the same elasticity across all destinations and scenarios, ignoring route-specific sensitivities.
3. **Single-stage Approach**: Our model uses a single-stage approach rather than a two-stage stochastic formulation, assuming we can identify the demand scenario when setting prices instead of modelling the sequential nature of pricing decisions as booking patterns become clear.

Future Improvements:Implement a two-stage stochastic model with real-time updates: A model that sets initial prices early in the booking window, then adjusts them as actual demand patterns become clearer, using live data to update forecasts and optimize revenue. **Integrate AI – powered competitor response prediction:** Deploy machine learning tools that not only forecast demand scenarios but also predict competitor price reactions, enabling more strategic pricing and capturing additional value beyond the €5,000, EVPI identified.

Appendices

Appendix A – Deterministic Model

Sets: *Destinations*

$i \in \{Paris, London, New\ York\}$

Parameters

- $D_{base,i}$ $(Expected\ base\ demand\ for\ city\ i)$
- b $(Price\ sensitivity/Elasticity\ factor)$
- $E[C_i]$ $(Expected\ competitor\ price\ for\ city\ i)$
- $P_{min,i}$, $P_{max,i}$ $(Min\ and\ max\ ticket\ prices\ for\ city\ i)$
- $MaxCapacity_i$, $(Maximum\ number\ of\ seats\ on\ plane\ to\ city\ i)$

Decision Variables

$P_i(Ticket\ price\ set\ by\ the\ OTA\ \ for\ city\ i)$

Model

Objective Function

This objective function maximizes the total expected revenue across all destinations by multiplying each destination's optimal price by its expected demand.

$$max \sum_{\{i\ \in\{Paris, London\ New\ York\}} P_i \times D_i$$

where:

P_i = Ticket price set by the OTA for city i

D_i = Demand for city i, given by:

$$D_i = D_{base,i} - b \times (P_i - E[C_i])$$

where: $D_{base,i}$ = Expected base demand for city i,b = Price sensitivity factor and $E[C_i]$ = Expected competitor price for city i bringing it to:

$$max \sum_i P_i \times (D_{base,i} - b \times (P_i - E[C_i]))$$

Constraints

1. **Price Bound Constraints:** $P_{min,i} \leq P_i \leq P_{max,i}, \ \forall i.$

   This constraint means that the ticket price $P_i$must lie within the airline's minimum and maximum allowed range:

2. **Non-Negative Demand Constraint:** $D_i \geq 0, \ \forall i$

   Demand must be non-negative**,** meaning price increases should not result in a negative number of tickets sold:

3. **Price Elasticity Constraint:** $P_i \leq E[C_i] + 100, \ \forall i$

   To ensure competitive pricing, the OTA should not price too high above competitors: OTA prices can't be more than €100 above what competitors are charging.

Appendix B – Stochastic Model

Sets: -*Destinations and scenarios*

$i \in \{Paris, London, New\ York\}$

$s \in Scenarios = \{LowDemand, Medium, HighDemand, PeakSeason, Holiday\}$

Parameters

- $D_{base,i,s}$, $(Base\ demand\ for\ city\ i\ under\ scenario\ s)$
- b $(Price\ sensitivity\ factor\ )$
- $C_{\{i,s\}}$ $(Competitor\ price\ for\ city\ i\ under\ scenario\ s\ )$
- $P_{min,i}$, $P_{max,i}$ $(Min\ and\ max\ ticket\ prices\ for\ city\ i)$
- $\{\boldsymbol{\pi}\}_s$ $(Probability\ of\ scenario\ s$
- $MaxCapacity_i$, $(Maximum\ ticket\ sales\ allowed\ for\ city\ i)$

Scenario Probabilities (π_s):

LowDemand	Medium	HighDemand	PeakSeason	Holiday
0.15	0.30	0.25	0.15	0.15

Decision Variables

- $P_{i,s}$ $(Ticket\ price\ for\ city\ i\ under\ scenario\ s\ )$
- $D_{i,s}$ $(Demand\ for\ city\ i\ under\ scenario\ s\ )$

Model:

Objective Function

The objective function of this stochastic model maximizes the expected total revenue across all destinations and demand scenarios. It calculates this by:

1. For each scenario-destination pair, multiplying the optimal price by the corresponding demand
2. Weighting each scenario's revenue by its probability of occurrence
3. Summing these probability-weighted revenues across all scenarios and destinations

$$max \sum_{s \in Scenarios} \sum_{i \in \{Paris, London, New\ York\}} \boldsymbol{\pi}_{\boldsymbol{s}} P_{i,s} D_{i,s}$$

where:

$P_{i,s}$ = Ticket price for city i in scenario s.

$D_{i,s}$ = Demand for city i, in scenario s, given by: $\boldsymbol{D}_{\{i,s\}} = \boldsymbol{D}_{\{\{base\},i,s\}} - \boldsymbol{b} \times (\boldsymbol{P}_{\{i,s\}} - \boldsymbol{C}_{\{i,s\}})$

where $\boldsymbol{C}_{\{i,s\}}$ *= (Competitor's ticket price for city i in scenario s) and* $\boldsymbol{\pi}_{s}$ *= (Probability of scenario s occurring.)*

The demand function shows how customers respond to price differences - they buy more when our prices are lower than competitors and less when our prices are higher. The b parameter controls how strongly customers react to these price differences.

Constraints

Price Bound Constraints: $P_{min,i} \leq P_{i,s} \leq P_{max,i}, \ \forall i, s.$

Ticket prices must remain within airline-imposed limits:

Non-Negative Demand Constraint:

$$D_{\{i,s\}} \geq 0, \qquad \forall i, s$$

Demand must be **non-negative**, ensuring that no negative ticket sales occur:

Capacity Constraint:

The number of tickets sold cannot exceed the maximum available seat capacity per route:

$$D_{\{i,s\}} \leq \{MaxCapacity\}_i, \qquad \forall i, s$$

Price Elasticity Constraint:

To prevent excessive pricing beyond competitors' range, OTA prices can't be more than €100 above what competitors are charging.

$$\boldsymbol{P_{\{i,s\}} \leq C_{\{i,s\}} + 100, \qquad \forall i, s}$$