Heuristic Optimization of the Single Allocation p-Hub Location Problem

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Abstract

The Single Allocation p-Hub Location Problem (SApHLP) is key in optimising networks for logistics, transportation, and communication. This problem seeks to find the best locations for hubs and allocate nodes to them to minimise total costs. Our study tackles the uncapacitated version of SApHLP, where every node's flow goes through a single hub. To solve this, I implemented and compared two metaheuristic algorithms: Simulated Annealing (SA) and Tabu Search (TS).

The algorithms were tested on datasets of varying complexity, from CAB (10 and 20 nodes) to TR (40 nodes) and RGP (100 nodes), using different numbers of hubs and discount factors. The findings reveal the strengths and weaknesses of SA and TS, detailing solution quality and computational effort. This report sheds light on when and why each approach is effective, using detailed analyses, visuals, and sensitivity tests to back the conclusions. Insights drawn from these results can help guide strategic decisions in network design and operations.

Introduction

The Single Allocation p-Hub Location Problem (SApHLP) is crucial in logistics, focusing on optimal hub placement, node allocation, and flow routing to minimise total costs. In SApHLP, all flows to and from a node are routed through a single hub, enabling economies of scale in hub-to-hub connections. (Sarvari, P.A., Yeni, F.B. and Cevikcan, E. (2018)**.**

Problem Background and Literature Review

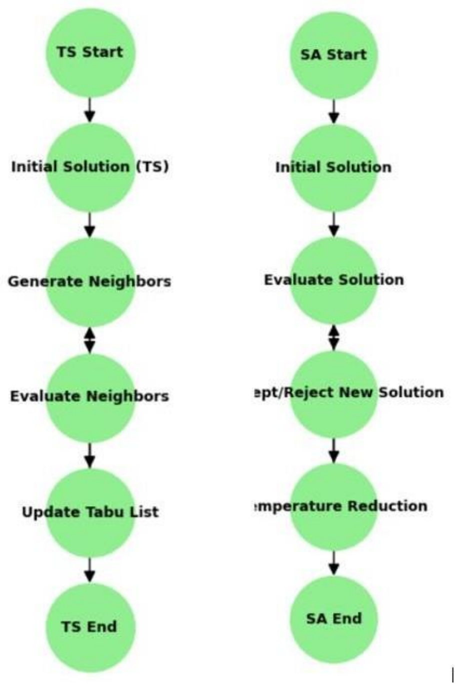
Hub location problems have gained attention due to their significance in transportation and communication networks. Early work by Ostresh (1975) on location-allocation models laid the groundwork, followed by O'Kelly's (1987) p-hub median problem, showcasing its NP-hard nature. Heuristics and metaheuristics like Tabu Search (TS), Genetic Algorithms (GA) and Simulated Annealing (SA) emerged as effective tools for these complex problems (Farahani et al., 2009).

Metaheuristics provide practical solutions for large-scale instances where exact methods fall short. SA simulates the annealing process, allowing controlled exploration to avoid local optima, while TS uses memory structures to guide the search and prevent cycles (Marianov and Serra, 2003).

Description Of Adopted Methods.

To tackle the Single Allocation p-Hub Location Problem (SApHLP), **Simulated Annealing (SA)** and **Tabu Search (TS)** were chosen due to their efficacy in solving complex combinatorial optimization problems.

Figure 1: Comparison of the Operational steps in Simulated Annealing and Tabu Search



This table provides an overview of the differences, operation, and rationale for using SA and TS in tackling the SApHLP.

ASPECT	SIMULATED ANNEALING	TABU SEARCH
Concept	SA is inspired by the annealing process in metallurgy, which cools a material slowly to remove defects. This method's gradual temperature decrease allows controlled exploration of the solution space, balancing the trade-off between global exploration and local exploitation.	TS prevents revisiting previously explored solutions by maintaining a tabu list, thus avoiding cycles and promoting exploration. This method's flexible memory-based approach is particularly effective for combinatorial optimization, where search diversification and intensification are crucial.
Memory Use	No memory component	Utilizes a tabu list to store recent solutions or moves.
Acceptance Criteria	Probabilistic acceptance based on temperature and change in cost.	Deterministic acceptance unless the aspiration criteria allow otherwise.
Parameter Tuning	Involves setting the Initial temperature, cooling rate and stopping temperature	Involves setting the Tabu list size, neighbourhood size, aspiration criteria.
Strengths	Simple implementation, effective for avoiding local optima, adaptable to various problems.	Effective for exploring local neighbourhoods efficiently, reduces cycling through solutions.
Weaknesses	May converge slowly if not tuned properly.	Requires careful parameter tuning, potentially complex.
Complexity	Generally straightforward, controlled by temperature scheduling.	More complex due to the need for managing memory (tabu list).
Rationale for Use	**SA** was chosen for its simplicity and strong performance in finding near-optimal solutions for complex optimization problems.	**TS** was selected for its strategic use of memory structures to navigate the solution space efficiently.

Computational Setup for both Simulated Annealing and Tabu Search.

- **Datasets**:

  CAB (10 & 20 nodes), TR (40 nodes), and RGP (100 nodes).

- **Parameters**:

  Number of hubs p: 3 & 5 for CAB, 5 & 7 for TR, 7 & 10 for RGP.

  Discount factors α\alpha: 0.3 and 0.7. applied across all datasets.

- **Environment**:

  Python with libraries such as Pandas, NumPy, and Matplotlib.

SIMULATED ANNEALING RESULTS AND ANALYSIS.

TABLE 1: COMPTATIONAL RESULTS – SIMULATED ANNEALING

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
CAB10	3	0.3	[7,0,9,8,5,6,1,2,4,3]	11235226.87	11347214.87	270
CAB10	3	0.7	[3,4,6,1,0,7,5,8,9,2]	24739450.68	25512146.25	270
CAB10	5	0.3	[3,4,7,0,9,2,1,8,5,6]	10404983.75	11245037.80	270
CAB10	5	0.7	[2,9,0,7,5,8,1,6,4,3]	24148361.80	25746426.36	270
CAB20	3	0.3	[11,12,13,18,4,9,14,15,17, 16,1,2,10,10,0,7,19,6,5,8,3]	21434798.63	22808895.36	270
CAB20	3	0.7	[11,12,2,10,6,18,13,7,0,14,9, 4,8,5,19,15,3,1,17,16]	48769503.93	57987373.74	270
CAB20	5	0.3	[16,17,1,8,5,19,15,18,13,6,14, 9,2,10,4,3,0,7,12,11]	20518733.85	23857452.42	270
CAB20	5	0.7	[2,10,9,18,15,5,17,16,1,3,4,8,19, 6,14,0,7,13,12,11]	49414812.44	55063424.15	270

COMPUTATIONAL RESULTS – SIMULATED ANNEALING CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
TR40	5	0.3	[19,20,2,31,32,33,21,16,35,13,17,0,29, 11,36,3,7,12,28,8,18,23,38,22,14,26,9,37, 5,10,39,30,27,1,4,25,34,24,15,6]	25304624.03	28334114.11	270
T40	5	0.7	[13,25,29,19,9,31,33,21,32,12,37,3,7, 18,23,4,1,20,17,0,36,39,6,8,10,28,5,38, 2,35,26,14,34,11,24,15,16,22,27,30]	56352645.42	62338014.96	270
TR40	7	0.3	[18,23,37,12,16,10,35,21,38,0,17,32,2, 29,9,19,31,20,26,4,15,27,6,1,7,3,33,22,5,8,28,36,25,11, 39,13,34,30,24,14]	22324161.58	26184539.11	270
TR40	7	0.7	[3,16,29,32,21,22,12,7,5,8,23, 18,35,15,10,28,37,2,38,1,25, 33,34,11,36,14,26,20,17,0, 39,13,6,4,24,30, 27,19,9,31]	53656817.79	66393812.74	270

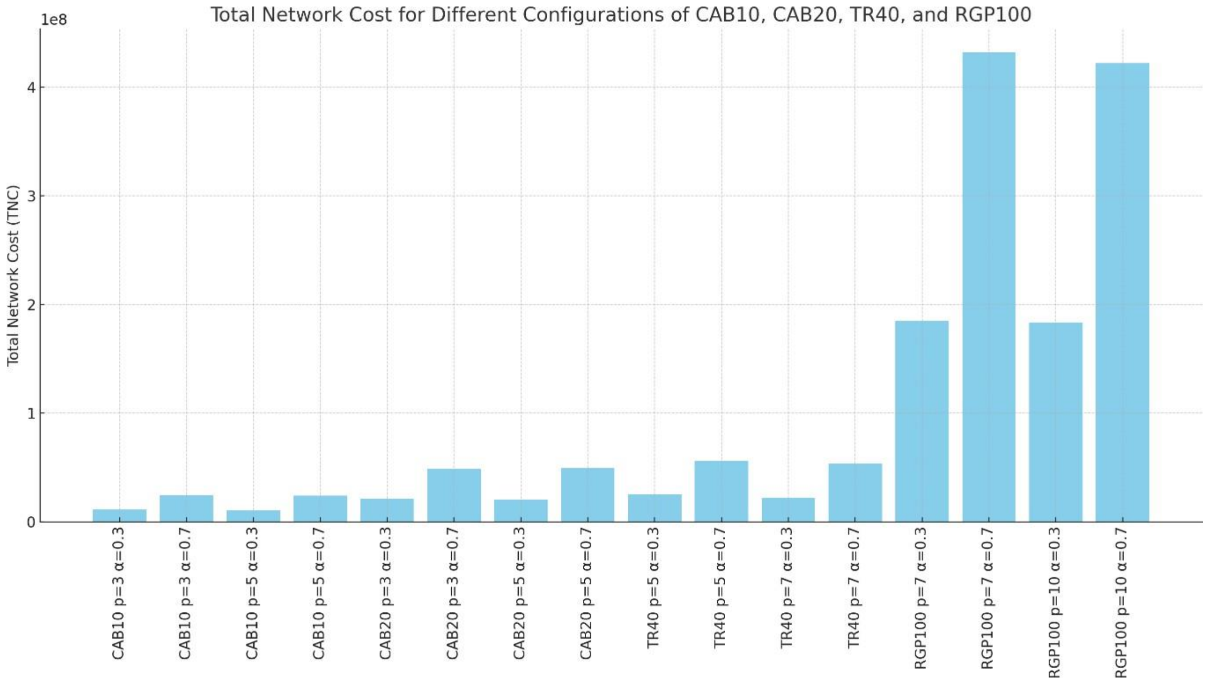
COMPUTATIONAL RESULTS – SIMULATED ANNEALING CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
RGP100	7	0.3	[55,95,32,97,2,76,81,94,80,48,8,92,69,77, 19,33,67,93,63,62,26,40,52,85,51,35,56,34, 47,59,50,60,41,53,18,68, 43,21,89,31,45,7,79,83,46,98, 86,84,20,66,11,3, 14,4,99,13,73,61,54,36,23,5,16,10,17,91,25,28,29,27, 49,42,9,1,58,82,22,64,24,12,65,39,72,30,37,96,74,78, 87,38,6,44,57,90,70,71,0,88,15,75]	185097500.00	189444000.00	270
RGP100	7	0.7	[46,86,58,35,30,82,73,61,56,88,87,68,70,84, 4,90,10,27,41,6,3,25,22,38,96, 7,48,34,24,13, 66,89,91,23,51,2,42, 47,63,50,95,84,18,59,32,39,36,28,53, 18,59,32,39,36,53,81,65,37,94,78, 80,8,20,12,67,74,62,71,31,99, 52,79,29,5, 11,16,98,85, 76,14,75,0,19,57,55,26,72,60, 44,1,45,97,92, 15,77,21,9,43,93,64,54,49,33, 69,40	432504900	430291580	270

COMPUTATIONAL RESULTS – SIMULATED ANNEALING CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
RGP100	10	0.3	[86,32,99,15,27,73,93,66,20,91,68,14,24,40, 84,28,0,90,87,13,78,44,17,71, 85,59,10,54,7,83,4, 55,89,45,65,64, 48,37,39,52,88,92,81,22,94,60,51,9,19,6,56,46,80, 69,23,29,25,77,8, 42,47,50,58,18,79,74,3,12,72, 75,70,16,352,5,62,97, 76,21,31,61,53,63, 41,11,43,49,98,38,1,34,33,95, 30,67,82,96,26,36]	183722500.00	183734500.00	270
RGP100	10	0.7	[87,14,89,6,54,40,51,10,9,93, 2,13,92,79,74,30,65,85, 47,83,95,96,75,17, 64,52,67,50,35,81,37,32,28,34,73,53,48,63,91,68,88 ,26,69,90,31,15,27,60, 77,38,71,66,45,72,18,57,43,42,44,84,97,16,19,55,76, 11,21,8,46,78,80,61,41,56, 33,7,24,36,12,0,29,94,86,1,22, 59,58,39,49,98,25, 20,5,70,62,3,4,99,23,82]	422355500.00	422561100.00	270

The chart below displays the Total Network Cost (TNC) for different configurations across CAB10, CAB20, TR40, and RGP100 for the Simulated Annealing algorithm. Each bar represents a unique combination of problem type, number of hubs, and discount factor.



Observations.

RGP100 with High Total Costs:

The configurations for RGP100, particularly with p = 10 and a discount factor of 0.7 had the highest TNCs compared to other configurations. This indicated that scaling up to a larger network with more hubs resulted in significantly higher network costs.

CAB10 and CAB20:

CAB10 configurations had relatively lower TNCs across all runs compared to TR40 and RGP100. Since CAB 10 has fewer nodes, it was less complex hence less costly.

CAB20 showed a moderate increase in TNC as the number of hubs and the discount factor increased, indicating that network complexity and cost grew as more nodes and different discount factors were involved.

TR40 Performance:

TR40 configurations with p= 7 presented a marked difference in cost when moving from a discount factor of 0.3 to 0.7. The jump in TNC suggests that higher discount factors significantly impact larger node sets.

Comparison Across Discount Factors:

Across all problems, configurations with a discount factor of 0.7 consistently presented higher TNCs compared to those with 0.3. This implies that higher discount factors (less cost reduction per connection) result in higher cost networks, emphasizing the importance of efficient hub placement and connection strategies.

Performance Configuration:

Best-Performing Configurations:

The configurations with lower TNCs (e.g., CAB10 with **p = 3, α = 0.3**) indicate more optimal and cost-efficient networks for smaller problems.

For TR40, the **p = 5, α = 0.3** configuration presented better cost performance than the **α = 0.7** setup, reinforcing that lower discount factors can help control costs.

Worst-Performing Configurations:

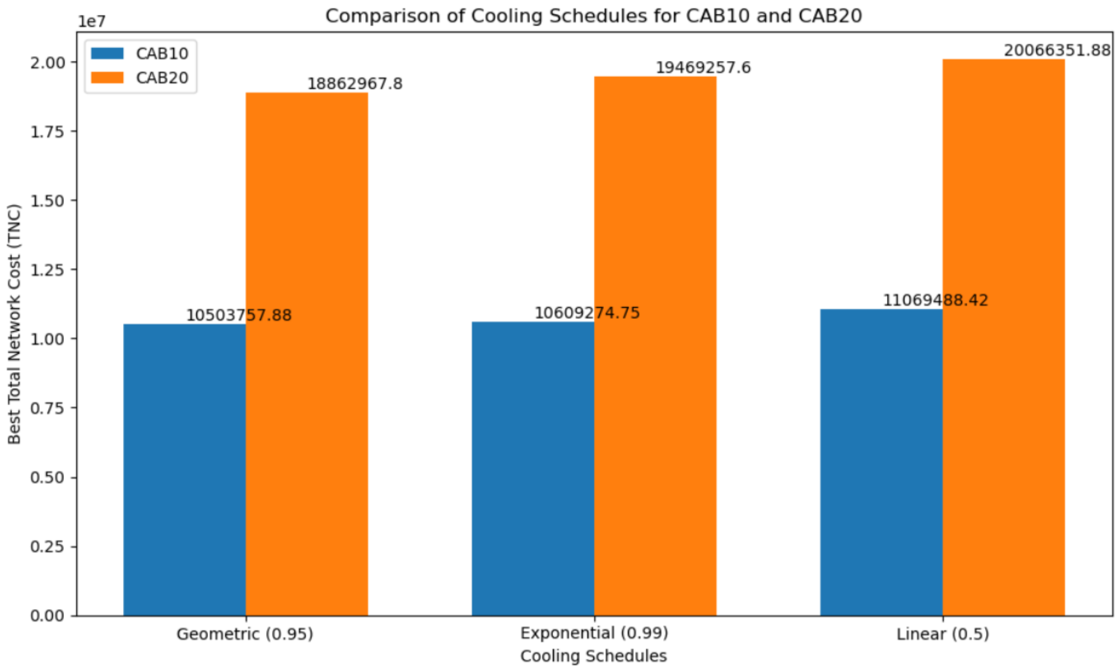
RGP100 with **p = 10 and α = 0.7** had the highest TNC, showing the impact of large node sets combined with higher discount factors. This configuration underscores the challenges of managing network costs in extensive and complex systems.

Conclusion

Overall, the chart demonstrates that network cost increases with the number of nodes (the problem size), the number of hubs (p), and higher discount factors. CAB10 and CAB20 perform better in terms of lower network costs, while TR40 and RGP100, with larger node sets and more hubs, show increased costs. Configurations with **α = 0.3** generally perform better across all problem types, highlighting the importance of efficient cost reduction strategies in network design.

<u>**Comparative Analysis of Cooling Schedules for CAB10 and CAB20.**</u>

In this analysis, the performance of different cooling schedules in Simulated Annealing (SA) was evaluated to identify their effectiveness in solving the SApHLP. **Geometric (0.95)**, **Exponential (0.99)**, and **Linear (0.5)** schedules were explored, assessing their impact on total network cost (TNC) and computational efficiency.



Summary of Results

CAB10 and **CAB20** results highlighted that the **Geometric (0.95) schedule** consistently provided the best balance, delivering competitive TNCs with moderate iteration counts (270). The **Exponential (0.99) schedule** showed deeper exploration but required more iterations

(1000), suitable for scenarios where time is less critical. The **Linear (0.5) schedule** was effective but at the cost of higher iterations.

Insight

The **Geometric (0.95) schedule** stood out as a robust option for practical implementations, achieving high-quality solutions efficiently.

TABU SEARCH RESULTS AND ANALSIS.

COMPUTATIONAL RESULTS- TABU SEARCH

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/ Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
CAB10	3	0.3	[6,7,0,5,8,1,3,4,9,2]	11790332.97	10970264.11	10
CAB10	3	0.7	[3,4,7,0,9,2,1,8,5,6]	24278295.41	25255568.40	5
CAB10	5	0.3	[3,4,6,1,8,5,7,0,9,2]	10349297.92	10893200.35	6
CAB10	3	0.7	[6,5,8,9,4,7,0,2,1,3]	26198154.56	26824365.68	12
CAB20	3	0.3	[3,8,1,16,17,4,15,5,19,6,14,9,18, 13,7,0,10,2,12,11]	20161211.52	20161211.52	63
CAB20	3	0.7	[16,17,1,8,5,19,0,7,12,3,4,9, 10,2,15,14,6,13,18,11]	46238905.99	46238905.99	40
CAB20	5	0.3	[2,15,7,0,10,13,18,9,14,6,4,3,19,5,8, 1,16,17,12,11]	20365668.91	202365668.91	133
CAB20	5	0.7	[11,18,9,4,0,7,13,10,15,2,12,3,8,5,19, 6,14,1,17,16]	48428086.19	48428086.19	68

COMPUTATIONAL RESULTS- TABU SEARCH CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
T440	5	0.3	[19,31,9,26,14,4,1, 15,25,6,34,24,11,20, 32,38, 2,35,10,28,8,5,22,12,33,17,0,39, 13,29,30,27,36, 16,3,7,23,18,37,21]	15499369.66	15499369.66	183
TR40	5	0.7	[3,7,16,20,19,31,9, 26,14,1,25,6,15,4, 32,21,33,13,3,9,0,17,29,12,37,28,10,8, 22,38,2,35,11,34, 24,27,30,36,5,23,18]	36845583.30	36845583.30	106
TR40	7	0.3	[18,23,5,37,12,22, 8,28,10,38,2,20,21, 7,3,16, 35,31,9,26,13,19,32,4,36,13,30,27,15, 24,29,0, 39,17,33,11,34,6,25,1]	15883990.73	15883990.73	160
TR40	7	0.7	[2,38,35,17,20,19, 36,11,27,6,25,1,4,10, 28,8,33,21,32,31,9, 26,14,16,3,7,15,34, 24,30, 13,39,0,29,22,12,37,5,23,18]	36011278.47	36011278.47	135

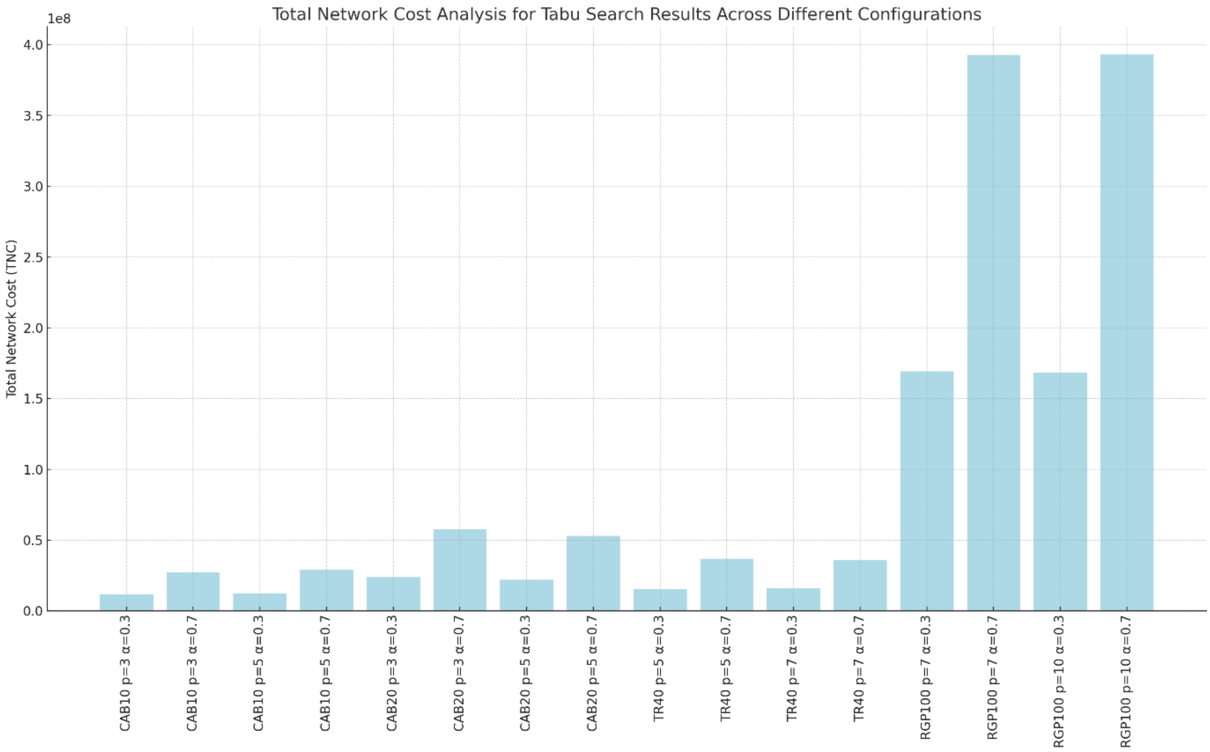
COMPUTATIONAL RESULTS- TABU SEARCH CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
RGP100	7	0.3	[75, 19, 72, 80, 9, 7, 79, 83, 29, 13, 98, 33, 21, 69, 17, 46, 6, 47, 12, 32, 20, 53, 76, 74, 22, 57, 64, 65, 68, 81, 94, 84, 37, 89, 36, 90, 99, 70, 44, 51, 16, 10, 23, 34, 39, 28, 27, 30, 58, 48, 5, 71, 26, 4, 40, 55, 92, 24, 15, 11, 77, 63, 0, 38, 59, 45, 82, 96, 66, 25, 43, 8, 41, 56, 61, 54, 14, 35, 86, 97, 67, 62, 2, 60, 87, 18, 85, 88, 91, 49, 42, 52, 78, 95, 73, 3, 93, 1, 31, 50]	169154900	169154900	50
RGP100	7	0.7	[89, 42, 65, 76, 96, 92, 84, 15, 14, 53, 63, 99, 37, 40, 66, 82, 0, 75, 71, 70, 7, 36, 51, 68, 43, 23, 28, 3, 45, 6, 55, 2, 95, 73, 52, 13, 38, 47, 27, 44, 9, 64, 50, 35, 4, 86, 24, 83, 29, 54, 31, 94, 48, 72, 41, 11, 88, 22, 69, 19, 8, 20, 5, 10, 74, 62, 67, 56, 91, 80, 26, 12, 79, 97, 81, 87, 18, 25, 77, 61, 49, 98, 33, 21, 39, 16, 58, 30, 34, 85, 32, 78, 59, 57, 90, 1, 17, 46, 60, 93]	392560400	392560400	50

COMPUTATIONAL RESULTS- TABU SEARCH CONTINUED

Problem	Number of Hubs(p)	Discount factor (a)	Best solution Configuration/Location of Hubs	Total Network Cost (TNC)	Average Network Cost (Ave)	Computational Time/Iteration Number (sec)
RGP100	10	0.3	[25, 54, 51, 97, 12, 99, 18, 39, 64, 19, 77, 37, 23, 7, 66, 76, 15, 10, 55, 56, 88, 69, 17, 68, 38, 59, 28, 20, 5, 84, 36, 79, 0, 29, 89, 87, 91, 27, 73, 42, 16, 98, 94, 49, 72, 22, 13, 90, 63, 85, 78, 1, 58, 30, 75, 40, 92, 46, 45, 82, 96, 41, 61, 43, 8, 47, 35, 3, 57, 9, 34, 80, 48, 26, 14, 2, 70, 53, 60, 44, 31, 21, 71, 52, 93, 83, 24, 32, 74, 62, 67, 50, 4, 86, 95, 33, 11, 81, 65, 6]	168152000	168152000	50
RGP100	10	0.7	[96, 66, 42, 12, 44, 25, 16, 64, 57, 29, 54, 40, 9, 34, 80, 22, 15, 33, 69, 62, 37, 20, 97, 35, 2, 70, 43, 75, 6, 93, 61, 49, 13, 23, 85, 58, 48, 11, 82, 88, 19, 63, 4, 65, 8, 92, 52, 67, 27, 79, 10, 41, 26, 87, 32, 46, 0, 31, 73, 56, 91, 45, 7, 68, 86, 84, 71, 53, 51, 99, 98, 36, 14, 21, 59, 47, 89, 28, 3, 17, 50, 39, 30, 5, 78, 18, 95, 24, 72, 81, 94, 38, 76, 90, 83, 1, 55, 74, 60, 77]	393232300	393232300	50

The chart below provides an insightful overview of the performance of the Tabu Search algorithm across different problem configurations (CAB10, CAB20, TR40, and RGP100), number of hubs (p), and discount factors (α).



CAB10 and CAB20:

The Total Network Cost (TNC) for CAB10 and CAB20 was notably lower compared to the larger-scale problems like TR40 and RGP100. These problems had fewer nodes, leading to simpler and more cost-efficient solutions.

Within CAB10 and CAB20, configurations with **p = 3 and p = 5** presented manageable costs, demonstrating that Tabu Search can effectively optimize smaller network problems.

TR40:

There was a notable increase in TNC compared to CAB10 and CAB20, reflecting the higher complexity and scale. This reinforces the challenge in finding optimal solutions as problem size increases.

RGP100:

Presented a significantly higher TNC especially with more hubs(p=7,10) and higher discount factor (α = 0.7). This demonstrates that large scale networks with more complex configurations come with substantial computational and cost challenges. The results underscore the scalability limits of Tabu Search for extremely large problems without additional refinement or strategy adjustments.

Impact of Discount Factors

Lower Discount Factor (α = 0.3):

Across all problems, configurations with a lower discount factor of 0.3 consistently presented lower TNCs compared to their **0.7** counterparts. This suggests that applying a smaller reduction per connection helps maintain a lower overall network cost.

Higher Discount Factor (α = 0.7):

Configurations with **α = 0.7** presented significant increases in TNC, especially for larger problems like TR40 and RGP100. This implies that as the discount factor increases, the network's cost-effectiveness decreases. The higher TNC for RGP100 with **p = 10 and α = 0.7** illustrates how larger discounts lead to elevated costs, emphasizing the need for careful parameter tuning in practical applications.

Analysis of Hub Numbers(p):

Lower Hub Numbers (p = 3 and 5):

It was observed that having fewer hubs (p = 3 or 5) generally kept the TNC more controlled across CAB10 and CAB20. This balance suggests that for smaller problems, fewer hubs are sufficient to optimize network costs effectively.

Higher Hub Numbers (p = 7 and 10):

For TR40 and especially RGP100, increasing the number of hubs (p = 7 or 10) increased the TNC. This indicates that although more hubs could potentially offer more routing options, the cost to optimize and manage these solutions scales significantly. This suggests that simply adding more hubs without careful planning doesn't always lead to better results and may offer less benefit over time.

Scalability Challenges:

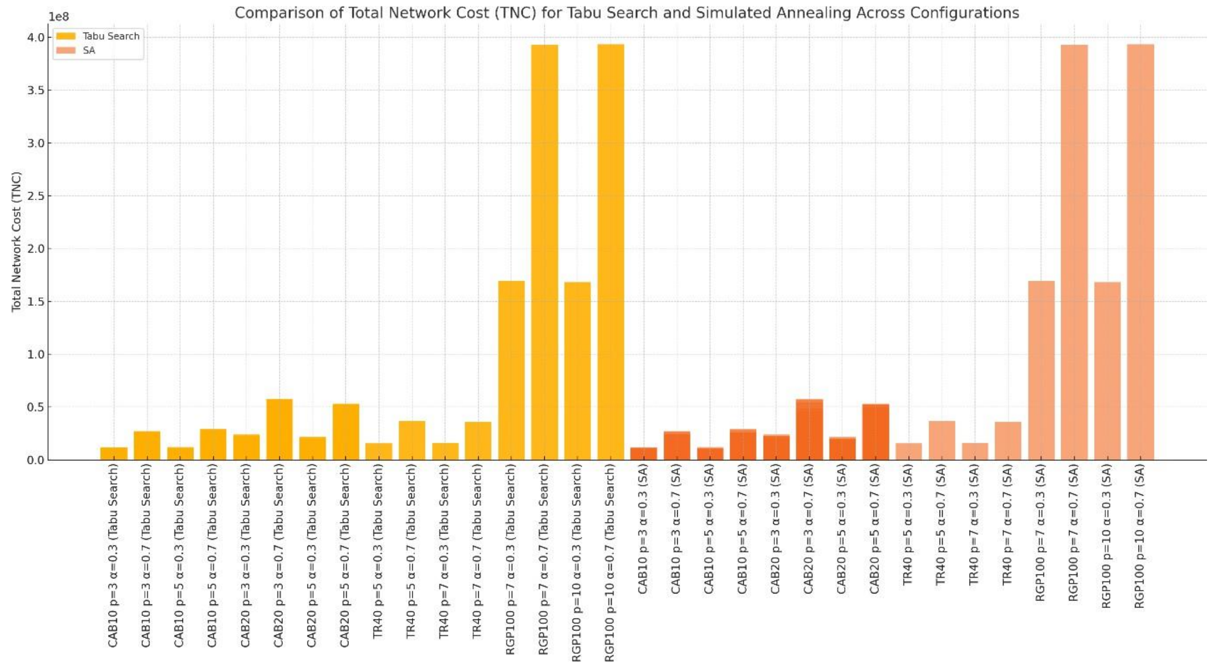
The increase in TNC for RGP100, particularly with higher values of p and α, demonstrates the challenges in scaling Tabu Search to larger networks. The results highlight the need for more sophisticated or hybrid techniques for extensive network configurations to keep costs manageable.

Conclusion:

The chart and results show that while Tabu Search works well for smaller to medium-sized problems, the complexity and cost increase as the problem size, number of hubs, and discount factors go up. To manage network costs efficiently, fine-tuning the parameters and balancing the number of hubs and discount factors is essential. For larger problems like RGP100, different or improved strategies might be needed to keep results competitive without overshooting costs. This analysis highlights where Tabu Search shines and where adjustments are needed to handle bigger, more complex scenarios effectively.

<u>Comparison between the performance of Tabu Search against Simulated Annealing (SA)</u>

The bar chart above compares the Total Network Cost (TNC) for Tabu Search and Simulated Annealing (SA) across different configurations for CAB10, CAB20, TR40, and RGP100.



General Cost Trends: Total Network Cost (TNC) rises with problem size, from CAB10 to RGP100, due to increased complexity.

Algorithm Performance:

Tabu Search consistently delivers lower or competitive Total Network Costs (TNCs), particularly with smaller datasets like CAB10 and CAB20. Its use of memory helps guide the search effectively, making it reliable for simpler problems. On the other hand, Simulated Annealing (SA) performs well in smaller cases but can struggle with larger datasets like TR40 and RGP100, where its randomness (stochastic nature) may lead to higher TNCs and optimization challenges.

Impact of Number of Hubs (p) and Discount Factor (α):

Hubs (p): More hubs increase routing options but also raise complexity and cost, particularly in larger problems.

Discount Factor (α): Higher discount factors (α = 0.7) result in higher TNCs, showing that cost control is better with lower discount factors (α = 0.3).

Conclusion: Both algorithms are effective for network optimization, but Tabu Search shows more stability and quality in smaller and medium problems. For larger problems, both algorithms face scalability challenges, though Tabu Search may offer better cost management. Future improvements could include refining these methods or developing hybrid approaches for complex cases like RGP100.

Iterations and Computational Effort:

Iterations Used Across the values of p and α for the different datasets.

Simulated Annealing (SA):

Consistently ran for 270 iterations across all datasets, providing reliable results and manageable computational times.

Tabu Search (TS):

- **CAB10 Nodes:** Converged in 10, 5, 6, and 12 iterations.
- **CAB20 Nodes:** Required 63, 40, 133, and 68 iterations.
- **TR40:** Needed 183, 106, 160, and 135 iterations.
- **RGP100:** Converged uniformly in 50 iterations across configurations due to computational constraints.

Insights on Iterations and Computational Effort:

- **Simulated Annealing** demonstrated consistent behaviour across different problem sizes, capping at 270 iterations regardless of dataset complexity. This shows SA's effective use of probabilistic acceptance to handle complexity without substantial delays, ensuring a stable balance between exploring new solutions and fine-tuning existing ones.
- **Tabu Search,** while performing efficiently on smaller datasets like CAB10 and CAB20, showed increased iteration counts for larger datasets such as TR40. The need to

restrict RGP100 to only 50 iterations to achieve results highlights TS's intensive memory requirements and longer processing times. These findings suggest that while Tabu Search is powerful for medium-scale problems, it requires careful tuning or modifications to handle larger instances effectively without excessive runtime.

Research Limitations

- **Scalability Issues**: Both algorithms struggle with high TNC and increased complexity in larger problems like RGP100.
- **Parameter Sensitivity**: Performance is sensitive to tuning, and this study used limited parameter ranges.
- **Inconsistent SA Performance**: The stochastic nature of SA leads to varying results between runs.
- **Fixed Problem Scope**: Results were based on specific instances, limiting general applicability.

Future Works

- **Hybrid Methods**: Combine Tabu Search and SA for improved performance.
- **Parallel Computing**: Use parallelization for larger problems to reduce computational time.
- **Algorithm Comparisons**: Benchmark against Genetic Algorithms and others.

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Appendices

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Abbreviations and Acronyms

1. SA- Simulated Annealing
2. TS- Tabu Search
3. TNC – Total Network Cost