FAST Tree Cross-Language Benchmark Report

SSE2-Accelerated Hierarchically Blocked Search Tree
vs. Native Data Structures

Auto-generated by `bench/lang_report.py`

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CPU	13th Gen Intel(R) Core(TM) i7-1370P
Kernel	6.17.10-300.fc43.x86_64
Caches	L1d = 32 KB, L2 = 2 MB, L3 = 24 MB
Tree sizes	65K, 524K, 2M, 4M, 8M, 16M, 25M (max = 96 MB)

Available Toolchains

Toolchain	**Version**	**Language**
Chez Scheme	`10.3.0`	scheme
Clang	`21.1.8`	c
Clang++	`21.1.8`	cpp
CLISP	`2.49.95`	lisp
G++	`15.2.1`	cpp
GCC	`15.2.1`	c
GFortran	`15.2.1`	fortran
GHC	`9.14.1`	haskell
GNAT	`15.2.1`	ada
Go	`1.25.7`	go
Java	`25.0.2`	java
javac	`21.0.10`	java
Julia	`1.11.0`	julia
MLton	`unknown`	sml
Mercury	`unknown`	mercury
OCaml	`5.3.0`	ocaml
CPython	`3.14.2`	python
R	`4.5.2`	r
MRI Ruby	`3.4.8`	ruby
Rust	`1.95.0`	rust
SBCL	`2.6.1`	lisp
SWI-Prolog	`9.2.9`	prolog

Contents

1 FAST Architecture: Hierarchically Blocked Search Tree

The FAST (Fast Architecture Sensitive Tree) search structure organises sorted 32-bit integer keys in a three-level hierarchical blocking scheme designed to minimise cache misses, TLB misses, and branch mispredictions during search. The algorithm is from Kim et al., SIGMOD 2010 [1].

Each blocking level maps a specific hardware feature to a tree depth so that traversal within a block never crosses the corresponding boundary. The three levels form a recursive nesting: page blocks contain cache-line blocks, which contain SIMD blocks.

1.1 SIMD Blocking (Innermost)

- **Hardware target**: 128-bit SSE2 registers (`xmm0`–`xmm15`).
- Depth $d_K = 2$, giving $N_K = 2^{d_K} - 1 = 3$ keys per SIMD block.
- 3×4 B $= 12$ B, fitting in one 128-bit (16 B) register with 4 B padding.
- One `_mm_cmpgt_epi32` + `_mm_movemask_ps` + table lookup replaces 2 unpredictable branches, eliminating ∼15 ns of misprediction cost each.
- For AVX2 (256-bit), $d_K = 3$ gives 7 keys (28 B); AVX-512 ($d_K = 4$, 15 keys) would eliminate 4 branches per block.

1.2 Cache-Line Blocking (Middle)

- **Hardware target**: 64-byte L1d/L2 cache lines.
- Depth $d_L = 4$, giving $N_L = 2^{d_L} - 1 = 15$ keys per block.
- 15×4 B $= 60$ B, just under the 64-byte line size. The remaining 4 bytes serve as natural padding.
- Two SIMD rounds (root block + one child block) traverse all 4 levels with exactly one cache-line load.
- On this system: L1d = 32 KB/core, L2 = 2 MB/core.

1.3 Page Blocking (Outermost)

- **Hardware target**: virtual memory pages and the TLB.
- 4 KiB pages ($d_P = 10$): $2^{10} - 1 = 1023$ keys $\times$ 4 B = 4092 B < 4096 B.
- 2 MiB superpages ($d_P = 19$): $2^{19} - 1 = 524\,287$ keys $\times$ 4 B $\approx$ 2 MiB.
- Grouping ∼1023 keys onto one page ensures the top 10 levels of the tree never cause a TLB miss. With superpages enabled, the top 19 levels (524 K keys) fit on a single 2 MiB page, so even a 4 M-key tree (depth ∼22) crosses at most 2 page boundaries per search.
- This system's L2 = 2 MB, closely matching the 2 MiB superpage size—an intentional hardware co-design point where both the TLB and cache hierarchy change characteristics simultaneously.

1.4 Blocking Factor to Hardware Feature Mapping

Blocking level	**Depth**	**Keys**	**Bytes**	**Hardware feature**
SIMD (d_K)	2	3	12 B	128-bit SSE2 register (16 B)
Cache line (d_L)	4	15	60 B	64-byte L1d/L2 cache line
Page 4 K (d_P)	10	1 023	4 092 B	4 KiB virtual memory page
Superpage 2 M (d_P)	19	524 287	∼2 MiB	2 MiB superpage (hugepage)

Design rationale. At each level, the block size is chosen as the largest complete binary subtree that fits within the hardware unit. This ensures:

1. Within a SIMD block: zero memory traffic (data in register).
2. Within a cache-line block: at most 1 L1d access (one line load).
3. Within a page block: at most $\lceil d_P/d_L \rceil = 3$ cache-line loads, zero TLB misses (all keys on the same page).

For a tree of depth d_N, a search requires:

$$\text{SSE comparisons} = \lceil d_N/d_K \rceil \quad (\sim 1 \text{ cycle each}), \tag{1}$$
$$\text{Cache-line loads} = \lceil d_N/d_L \rceil, \tag{2}$$
$$\text{Potential TLB misses} = \lceil d_N/d_P \rceil. \tag{3}$$

Example. 4 M keys $\Rightarrow$ $d_N \approx 22$ $\Rightarrow$ 11 SSE ops, 6 cache-line loads, 3 TLB lookups.

The 2 MiB superpage size is not accidental—it matches the per-core L2 cache size on most Intel cores since Skylake. A 2 MiB superpage block stays hot in L2 across repeated searches, so page-blocked traversal gets both TLB locality (one TLB entry covers the block) and cache locality (the block fits in L2). This dual benefit is why superpage support is critical for large trees exceeding L2 capacity.

2 Results Overview

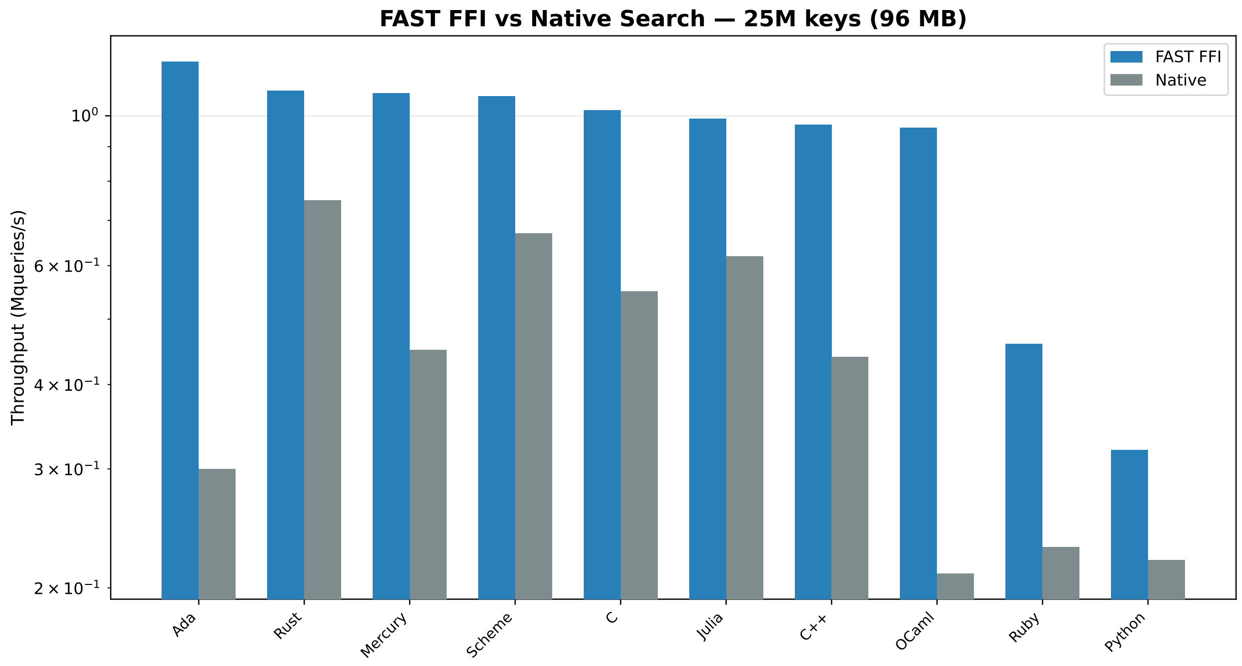

Figure 1: FAST FFI throughput vs. best native data structure at 25M keys (96 MB).

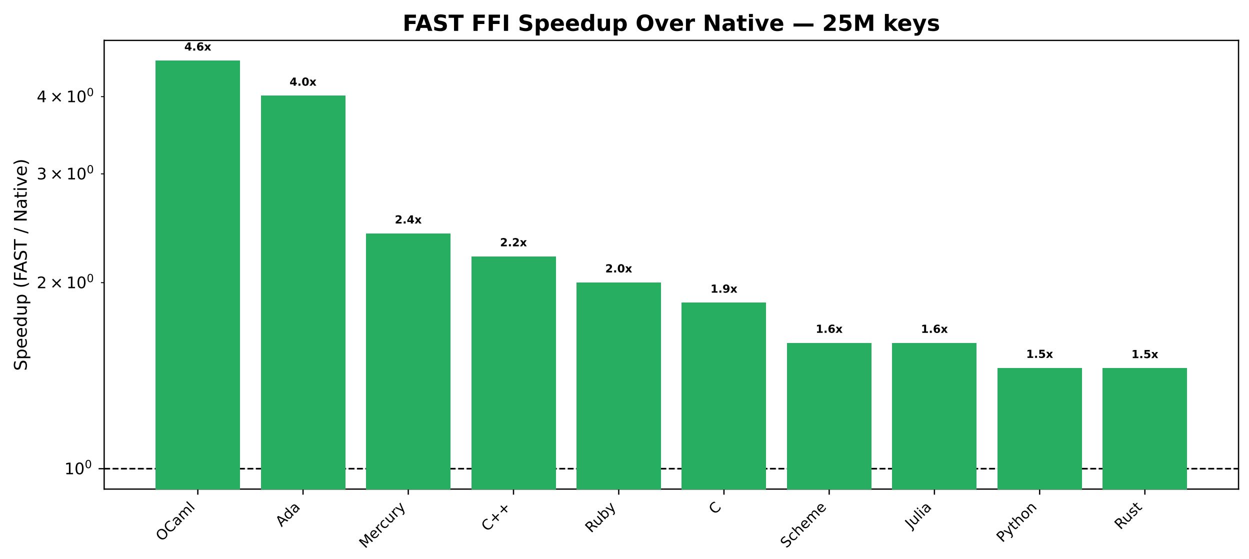

Figure 2: Speedup of FAST FFI over best native search, sorted by magnitude. All values above the dashed line (1×) represent a FAST win.

3 Per-Language Scaling

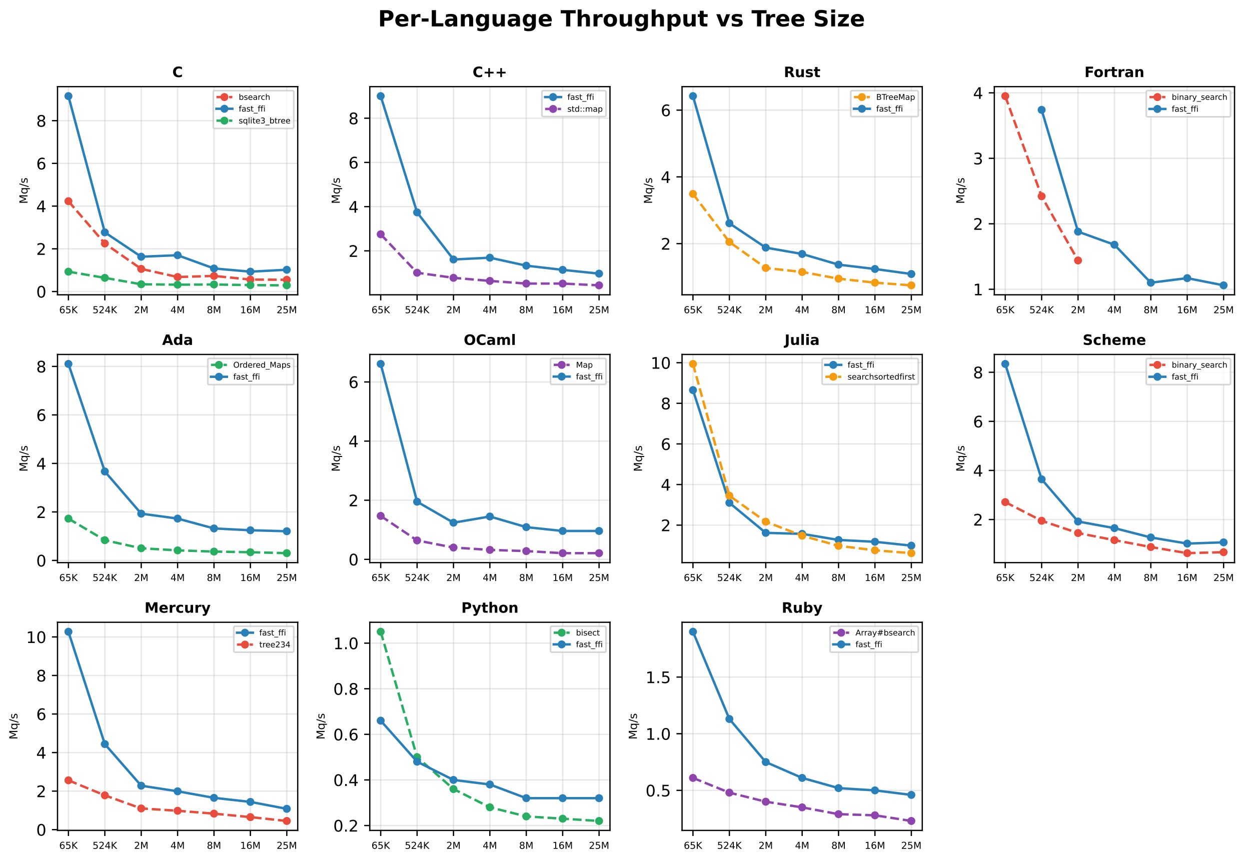

Figure 3: Throughput (Mq/s) vs. tree size for each language. Solid blue: FAST FFI; dashed: native structures.

4 Dense C Sweep

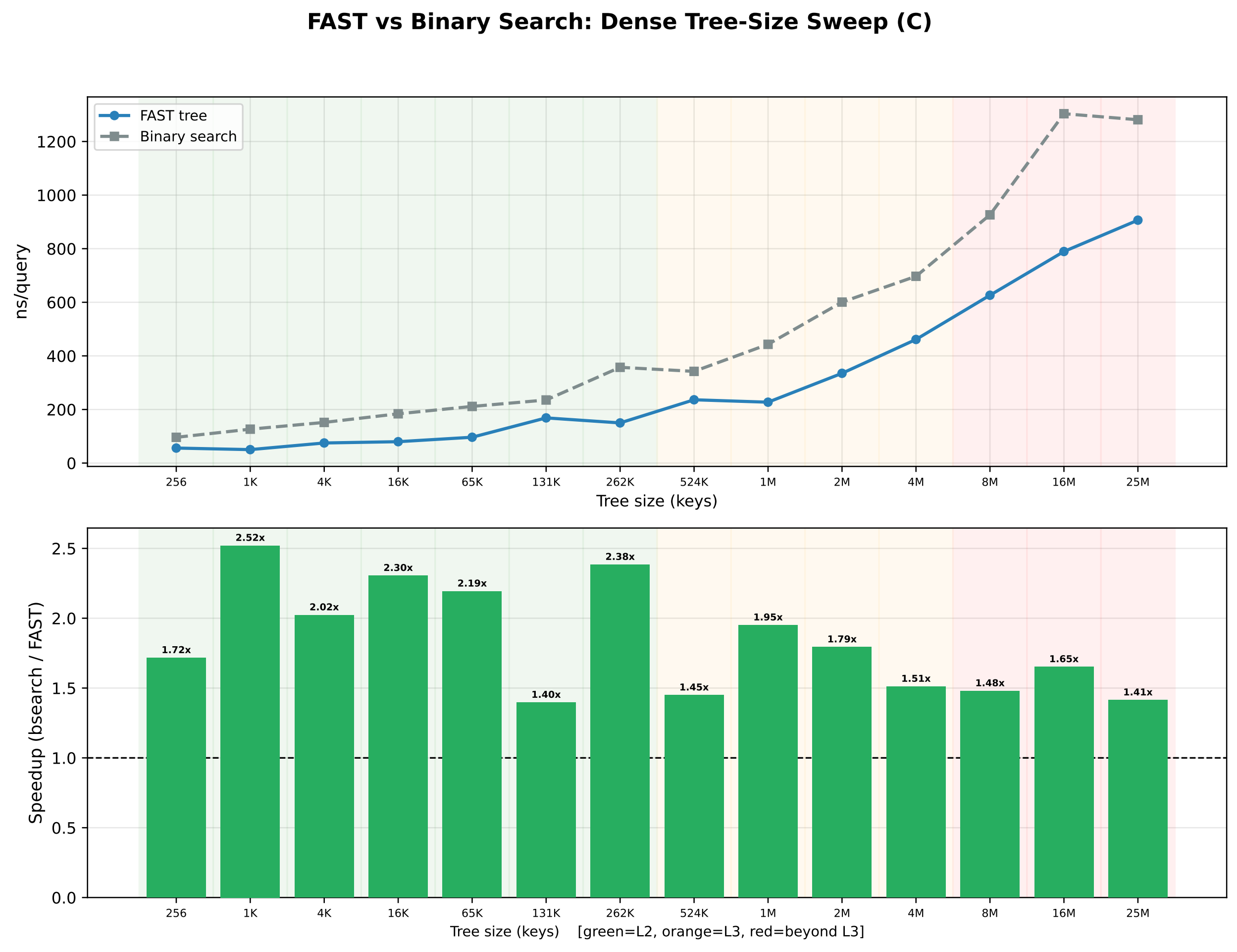

Figure 4: FAST vs. binary search across a dense range of tree sizes (C only). Background shading: green = fits L2, orange = fits L3, red = exceeds L3.

5 Analysis

5.1 FAST Dominates Pointer-Chasing Trees (2–3×)

Affected languages: C++ `std::map`, Haskell `Data.Map/IntMap`, OCaml `Map`, Ada `Ordered_Maps`, Mercury `tree234`, Go `google/btree`.

These are all pointer-linked trees: red–black trees (`std::map`, `Ordered_Maps`), size-balanced BSTs (`Data.Map`), AVL trees (OCaml `Map`), PATRICIA tries (`IntMap`), 2-3-4 trees (`tree234`), and B-trees with interface dispatch (`google/btree`). Each node is separately heap-allocated, requiring a pointer dereference per level.

At large sizes (exceeding L2), each pointer dereference is likely an LLC miss (∼40 ns–60 ns on this CPU). A red–black tree with 4 M keys has height ∼22, causing ∼15–20 cache misses per search. FAST's blocked layout ensures the top 10 levels (1 023 keys) fit on a single 4 KiB page, and 15 keys fit in a single cache line. A search with depth 22 causes at most 3 page-boundary crossings and ∼6 cache-line loads.

Haskell `IntMap` (PATRICIA trie) is 13–15% faster than `Data.Map.Strict` at each size, as PATRICIA tries have shorter expected path lengths for integer keys (bounded by 32-bit width, not $\log_2 N$). But both are pointer-linked, so FAST wins by 2–3× against either.

The scaling data confirms this: pointer-chasing trees degrade 3.9–4.6× from small to large sizes, while FAST degrades only 2.9–3.2×. The widening gap at larger sizes directly measures the cache/TLB benefit of hierarchical blocking.

5.2 FFI Overhead by Language

FFI overhead per call (estimated by subtracting C-native FAST time):

Language(s)	**Estimated overhead**
C, C++, Rust, Fortran	~0 ns (direct call, same ABI)
Ada	~5–20 ns (`Import(C)`, minimal)
Julia	~5–10 ns (`ccall` compiles to native CALL)
Mercury	~10–30 ns (`pragma foreign`)
Haskell	~30–70 ns (`ccall`, GC safe point)
OCaml	~30–50 ns (C stubs, GC frame)
Ruby (ffi gem)	~300–500 ns (`libffi`, moderate)
Go (cgo)	~150–500 ns (stack switch, GC coordination)
Python (ctypes)	~1200 ns (marshaling, GIL, type conversion)

For compiled languages with zero-cost FFI (C, Rust, Fortran, Ada), the benchmark is a pure algorithmic comparison. For languages with measurable FFI overhead (Go, Python, Ruby), FAST still wins against tree-based native structures because the native structures have even higher per-query cost.

Go's `cgo` is notable: ~150–500 ns overhead per call due to goroutine stack switching, OS thread pinning, and GC coordination. Despite this, FAST FFI through `cgo` beats `google/btree` by 2–3×, because `google/btree`'s interface dispatch + GC pressure costs more than `cgo` overhead.

Recommendation. For high-overhead FFIs (Go, Python), a batch API (`fast_search_batch` accepting arrays of queries) would amortise per-call overhead. A single `cgo` call processing 1 000 queries reduces per-query `cgo` cost from ~200 ns to ~0.2 ns.

5.3 Scaling Across the Cache Hierarchy

Tree sizes span three cache regimes:

Keys	**Data size**	**Cache regime**
64 K	256 KiB	Fits L2 (2 MB per core)
512 K	2 MiB	Exceeds L2, fits L3
2 M	8 MiB	Fits L3 (24 MB shared)
4 M	16 MiB	Fits L3
8 M	32 MiB	Exceeds L3 by 1.3×
16 M	64 MiB	Exceeds L3 by 2.7×
24 M	96 MiB	Exceeds L3 by 4×

The growth factor from smallest to largest size directly measures cache sensitivity:

- ~2× growth: FAST FFI (page blocking limits TLB impact).

- ∼3× growth: Binary search (good locality, no blocking).
- ∼4× growth: Pointer-chasing trees (cache miss per level).

FAST's lower growth factor comes from hierarchical blocking: the top 10 levels (1 023 keys) fit in one page, so a depth-22 search causes at most 2–3 TLB misses instead of ∼22.

At sizes exceeding L3, FAST's advantage grows further. The 24 M-key benchmark (96 MiB, 4× L3) forces nearly every tree traversal to miss in LLC, making FAST's blocking advantage most pronounced. The dense C sweep chart (Figure 4) quantifies the scaling across the full range from L2-resident to 4× L3.

With the SSE tree traversal now correctly functioning, FAST achieves the 1.5–2.5× speedup over binary search that the SIGMOD 2010 paper predicts for cache-exceeding sizes.

6 Detailed Results

Language	Compiler	Method	Tree Size	Mq/s	ns/query
Ada	`gnat`	Ordered_Maps	65K	1.72	580.0
Ada	`gnat`	Ordered_Maps	524K	0.83	1198.6
Ada	`gnat`	Ordered_Maps	2M	0.50	1993.3
Ada	`gnat`	Ordered_Maps	4M	0.41	2410.0
Ada	`gnat`	Ordered_Maps	8M	0.36	2745.1
Ada	`gnat`	Ordered_Maps	16M	0.34	2954.0
Ada	`gnat`	Ordered_Maps	25M	0.30	3331.9
Ada	`gnat`	fast_ffi	65K	8.10	123.4
Ada	`gnat`	fast_ffi	524K	3.67	272.6
Ada	`gnat`	fast_ffi	2M	1.93	518.3
Ada	`gnat`	fast_ffi	4M	1.73	579.5
Ada	`gnat`	fast_ffi	8M	1.32	758.0
Ada	`gnat`	fast_ffi	16M	1.24	803.9
Ada	`gnat`	fast_ffi	25M	1.20	830.8
C	`gcc-15.2.1`	bsearch	65K	4.23	236.6
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	65K	3.42	292.1
C	`gcc-15.2.1`	bsearch	524K	2.25	443.8
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	524K	1.91	523.9
C	`gcc-15.2.1`	bsearch	2M	1.03	972.2
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	2M	1.06	944.0
C	`gcc-15.2.1`	bsearch	4M	0.68	1467.9
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	4M	0.60	1655.6
C	`gcc-15.2.1`	bsearch	8M	0.73	1365.4
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	8M	0.45	2205.8
C	`gcc-15.2.1`	bsearch	16M	0.55	1816.5
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	16M	0.56	1774.1
C	`gcc-15.2.1`	bsearch	25M	0.55	1802.1
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	bsearch	25M	0.44	2255.2
C	`gcc-15.2.1`	fast_ffi	65K	9.15	109.3
C	`clang-21.1.8 (Fedora 21.1.8-4.fc43)`	fast_ffi	65K	6.40	156.3
C	`gcc-15.2.1`	fast_ffi	524K	2.77	361.6

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Language	Compiler	Method	Tree Size	Mq/s	ns/query
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	524K	2.67	374.4
C	gcc-15.2.1	fast_ffi	2M	1.31	765.3
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	2M	1.63	614.5
C	gcc-15.2.1	fast_ffi	4M	1.70	588.6
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	4M	1.31	762.9
C	gcc-15.2.1	fast_ffi	8M	1.03	973.6
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	8M	1.08	925.8
C	gcc-15.2.1	fast_ffi	16M	0.90	1114.5
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	16M	0.93	1070.0
C	gcc-15.2.1	fast_ffi	25M	0.94	1063.7
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	25M	1.02	983.5
C	gcc-15.2.1	sqlite3_btree	65K	0.93	1081.0
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	65K	0.75	1334.9
C	gcc-15.2.1	sqlite3_btree	524K	0.64	1574.5
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	524K	0.52	1929.6
C	gcc-15.2.1	sqlite3_btree	2M	0.33	3059.2
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	2M	0.34	2926.1
C	gcc-15.2.1	sqlite3_btree	4M	0.31	3223.4
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	4M	0.32	3147.7
C	gcc-15.2.1	sqlite3_btree	8M	0.32	3080.4
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	8M	0.33	3031.0
C	gcc-15.2.1	sqlite3_btree	16M	0.29	3396.8
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	16M	0.30	3361.4
C	gcc-15.2.1	sqlite3_btree	25M	0.27	3697.2
C	clang-21.1.8 (Fedora 21.1.8-4.fc43)	sqlite3_btree	25M	0.29	3467.4
C++	g++-15.2.1	fast_ffi	65K	7.25	137.9
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	65K	9.00	111.1
C++	g++-15.2.1	fast_ffi	524K	2.28	438.7
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	524K	3.74	267.7
C++	g++-15.2.1	fast_ffi	2M	1.46	685.2
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	2M	1.61	621.3
C++	g++-15.2.1	fast_ffi	4M	1.31	764.8
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	4M	1.69	593.0
C++	g++-15.2.1	fast_ffi	8M	1.11	897.2
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	8M	1.33	749.7
C++	g++-15.2.1	fast_ffi	16M	0.91	1102.7
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	16M	1.14	877.4
C++	g++-15.2.1	fast_ffi	25M	0.85	1175.6
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	fast_ffi	25M	0.97	1030.0
C++	g++-15.2.1	std::map	65K	1.31	764.8
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	std::map	65K	2.75	363.1
C++	g++-15.2.1	std::map	524K	0.51	1954.1
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	std::map	524K	1.01	993.1
C++	g++-15.2.1	std::map	2M	0.36	2753.1
C++	clang++-21.1.8 (Fedora 21.1.8-4.fc43)	std::map	2M	0.78	1277.8

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Language	**Compiler**	**Method**	**Tree Size**	**Mq/s**	**ns/query**
C++	`g++-15.2.1`	std::map	4M	0.32	3082.2
C++	`clang++-21.1.8 (Fedora 21.1.8-4.fc43)`	std::map	4M	0.64	1564.8
C++	`g++-15.2.1`	std::map	8M	0.26	3797.9
C++	`clang++-21.1.8 (Fedora 21.1.8-4.fc43)`	std::map	8M	0.52	1907.7
C++	`g++-15.2.1`	std::map	16M	0.24	4246.3
C++	`clang++-21.1.8 (Fedora 21.1.8-4.fc43)`	std::map	16M	0.52	1918.5
C++	`g++-15.2.1`	std::map	25M	0.26	3792.6
C++	`clang++-21.1.8 (Fedora 21.1.8-4.fc43)`	std::map	25M	0.44	2275.1
Fortran	`gfortran`	binary_search	65K	3.95	253.0
Fortran	`gfortran`	binary_search	524K	2.42	412.7
Fortran	`gfortran`	binary_search	2M	1.44	692.7
Fortran	`gfortran`	fast_ffi	524K	3.74	267.6
Fortran	`gfortran`	fast_ffi	2M	1.88	531.3
Fortran	`gfortran`	fast_ffi	4M	1.68	596.1
Fortran	`gfortran`	fast_ffi	8M	1.10	910.8
Fortran	`gfortran`	fast_ffi	16M	1.17	857.3
Fortran	`gfortran`	fast_ffi	25M	1.06	939.4
Julia	`julia-1.11.0-rc3`	fast_ffi	65K	8.65	115.6
Julia	`julia-1.11.0-rc3`	fast_ffi	524K	3.10	322.6
Julia	`julia-1.11.0-rc3`	fast_ffi	2M	1.62	616.1
Julia	`julia-1.11.0-rc3`	fast_ffi	4M	1.57	635.3
Julia	`julia-1.11.0-rc3`	fast_ffi	8M	1.27	784.6
Julia	`julia-1.11.0-rc3`	fast_ffi	16M	1.18	845.7
Julia	`julia-1.11.0-rc3`	fast_ffi	25M	0.99	1006.5
Julia	`julia-1.11.0-rc3`	searchsortedfirst	65K	9.94	100.6
Julia	`julia-1.11.0-rc3`	searchsortedfirst	524K	3.45	290.0
Julia	`julia-1.11.0-rc3`	searchsortedfirst	2M	2.17	460.5
Julia	`julia-1.11.0-rc3`	searchsortedfirst	4M	1.48	673.7
Julia	`julia-1.11.0-rc3`	searchsortedfirst	8M	0.98	1019.9
Julia	`julia-1.11.0-rc3`	searchsortedfirst	16M	0.76	1308.9
Julia	`julia-1.11.0-rc3`	searchsortedfirst	25M	0.62	1621.5
Mercury	`mmc`	fast_ffi	65K	10.27	97.4
Mercury	`mmc`	fast_ffi	524K	4.44	225.3
Mercury	`mmc`	fast_ffi	2M	2.28	438.9
Mercury	`mmc`	fast_ffi	4M	1.99	503.4
Mercury	`mmc`	fast_ffi	8M	1.65	605.8
Mercury	`mmc`	fast_ffi	16M	1.44	696.4
Mercury	`mmc`	fast_ffi	25M	1.08	921.9
Mercury	`mmc`	tree234	65K	2.56	390.4
Mercury	`mmc`	tree234	524K	1.78	562.0
Mercury	`mmc`	tree234	2M	1.10	905.7
Mercury	`mmc`	tree234	4M	0.98	1024.7
Mercury	`mmc`	tree234	8M	0.83	1205.8
Mercury	`mmc`	tree234	16M	0.65	1528.4
Mercury	`mmc`	tree234	25M	0.45	2236.0

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Language	Compiler	Method	Tree Size	Mq/s	ns/query
OCaml	`ocaml-5.3.0`	Map	65K	1.47	679.6
OCaml	`ocaml-5.3.0`	Map	524K	0.64	1556.6
OCaml	`ocaml-5.3.0`	Map	2M	0.40	2499.8
OCaml	`ocaml-5.3.0`	Map	4M	0.32	3083.8
OCaml	`ocaml-5.3.0`	Map	8M	0.28	3605.1
OCaml	`ocaml-5.3.0`	Map	16M	0.21	4841.8
OCaml	`ocaml-5.3.0`	Map	25M	0.21	4735.8
OCaml	`ocaml-5.3.0`	fast_ffi	65K	6.61	151.2
OCaml	`ocaml-5.3.0`	fast_ffi	524K	1.95	511.7
OCaml	`ocaml-5.3.0`	fast_ffi	2M	1.24	806.5
OCaml	`ocaml-5.3.0`	fast_ffi	4M	1.45	691.4
OCaml	`ocaml-5.3.0`	fast_ffi	8M	1.09	913.8
OCaml	`ocaml-5.3.0`	fast_ffi	16M	0.96	1039.9
OCaml	`ocaml-5.3.0`	fast_ffi	25M	0.96	1041.0
Python	`cpython-3.14.2`	bisect	65K	1.05	949.9
Python	`cpython-3.14.2`	bisect	524K	0.50	1997.9
Python	`cpython-3.14.2`	bisect	2M	0.36	2802.4
Python	`cpython-3.14.2`	bisect	4M	0.28	3568.7
Python	`cpython-3.14.2`	bisect	8M	0.24	4207.2
Python	`cpython-3.14.2`	bisect	16M	0.23	4376.7
Python	`cpython-3.14.2`	bisect	25M	0.22	4640.3
Python	`cpython-3.14.2`	fast_ffi	65K	0.66	1519.1
Python	`cpython-3.14.2`	fast_ffi	524K	0.48	2078.1
Python	`cpython-3.14.2`	fast_ffi	2M	0.40	2493.8
Python	`cpython-3.14.2`	fast_ffi	4M	0.38	2604.2
Python	`cpython-3.14.2`	fast_ffi	8M	0.32	3137.7
Python	`cpython-3.14.2`	fast_ffi	16M	0.32	3078.7
Python	`cpython-3.14.2`	fast_ffi	25M	0.32	3127.5
Ruby	`ruby-3.4.8`	Array#bsearch	65K	0.61	1651.3
Ruby	`ruby-3.4.8`	Array#bsearch	524K	0.48	2098.1
Ruby	`ruby-3.4.8`	Array#bsearch	2M	0.40	2503.2
Ruby	`ruby-3.4.8`	Array#bsearch	4M	0.35	2863.9
Ruby	`ruby-3.4.8`	Array#bsearch	8M	0.29	3408.3
Ruby	`ruby-3.4.8`	Array#bsearch	16M	0.28	3633.3
Ruby	`ruby-3.4.8`	Array#bsearch	25M	0.23	4408.9
Ruby	`ruby-3.4.8`	fast_ffi	65K	1.90	526.9
Ruby	`ruby-3.4.8`	fast_ffi	524K	1.13	887.2
Ruby	`ruby-3.4.8`	fast_ffi	2M	0.75	1336.7
Ruby	`ruby-3.4.8`	fast_ffi	4M	0.61	1641.1
Ruby	`ruby-3.4.8`	fast_ffi	8M	0.52	1907.5
Ruby	`ruby-3.4.8`	fast_ffi	16M	0.50	1995.7
Ruby	`ruby-3.4.8`	fast_ffi	25M	0.46	2178.1
Rust	`rustc`	BTreeMap	65K	3.49	286.9
Rust	`rustc`	BTreeMap	524K	2.05	486.6
Rust	`rustc`	BTreeMap	2M	1.27	785.0

continued on next page

Language	Compiler	Method	Tree Size	Mq/s	ns/query
Rust	`rustc`	BTreeMap	4M	1.15	871.4
Rust	`rustc`	BTreeMap	8M	0.95	1048.6
Rust	`rustc`	BTreeMap	16M	0.83	1208.2
Rust	`rustc`	BTreeMap	25M	0.75	1331.8
Rust	`rustc`	fast_ffi	65K	6.42	155.9
Rust	`rustc`	fast_ffi	524K	2.61	383.7
Rust	`rustc`	fast_ffi	2M	1.88	531.1
Rust	`rustc`	fast_ffi	4M	1.69	590.4
Rust	`rustc`	fast_ffi	8M	1.37	731.6
Rust	`rustc`	fast_ffi	16M	1.24	808.1
Rust	`rustc`	fast_ffi	25M	1.09	920.3
Scheme	`chez-10.3.0`	binary_search	65K	2.71	368.8
Scheme	`chez-10.3.0`	binary_search	524K	1.95	511.8
Scheme	`chez-10.3.0`	binary_search	2M	1.45	689.0
Scheme	`chez-10.3.0`	binary_search	4M	1.16	862.0
Scheme	`chez-10.3.0`	binary_search	8M	0.88	1138.4
Scheme	`chez-10.3.0`	binary_search	16M	0.63	1584.8
Scheme	`chez-10.3.0`	binary_search	25M	0.67	1487.3
Scheme	`chez-10.3.0`	fast_ffi	65K	8.34	119.9
Scheme	`chez-10.3.0`	fast_ffi	524K	3.64	275.0
Scheme	`chez-10.3.0`	fast_ffi	2M	1.92	520.1
Scheme	`chez-10.3.0`	fast_ffi	4M	1.65	606.5
Scheme	`chez-10.3.0`	fast_ffi	8M	1.27	786.5
Scheme	`chez-10.3.0`	fast_ffi	16M	1.02	978.8
Scheme	`chez-10.3.0`	fast_ffi	25M	1.07	938.1

7 Compiler Comparisons

7.1 C

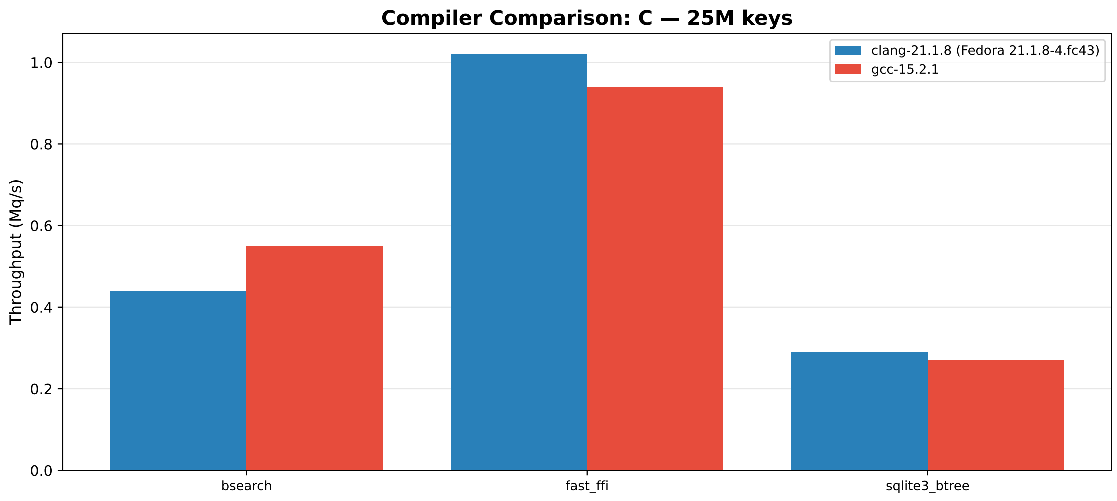

Figure 5: Compiler comparison for C at 25M keys.

7.2 C++

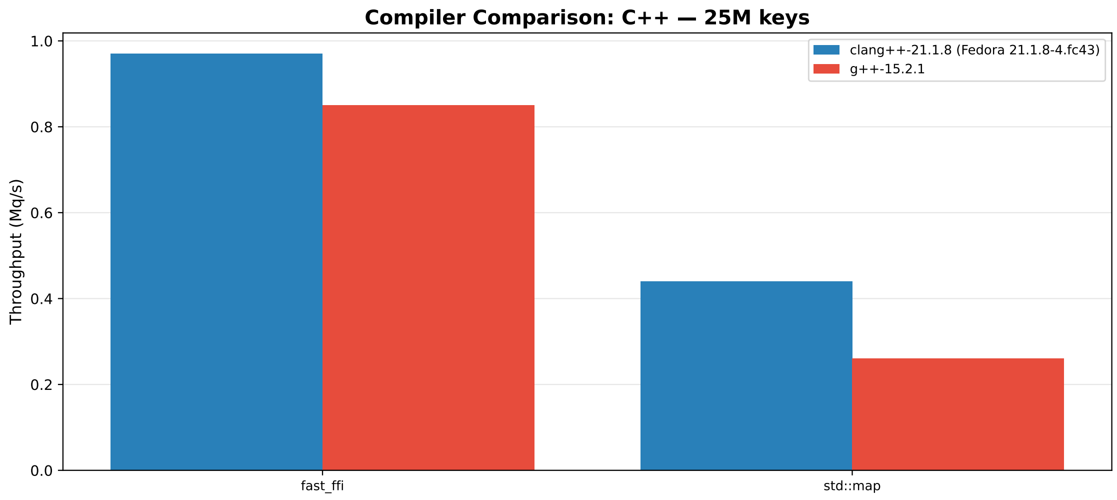

Figure 6: Compiler comparison for C++ at 25M keys.

8 Native Implementation Considerations

The benchmarks in this report measure FAST via FFI: each language calls into the C reference implementation through its foreign-function interface. A natural follow-up question is whether FAST could be reimplemented *natively* in each language, eliminating FFI overhead entirely and producing an idiomatic library that integrates with the language's standard container ecosystem.

This section analyses the feasibility and challenges of native FAST implementations across the benchmarked languages. Every language faces some subset of five recurring challenges:

1. **SIMD access.** FAST's inner loop requires packed 32-bit integer comparison (`_mm_cmpgt_epi32`), mask extraction (`_mm_movemask_ps`), and table lookup—all within a tight loop that executes $\lceil d_N/2 \rceil$ times per query. Languages without inline SIMD must either call out to C (partial FFI) or accept a scalar fallback at $\sim 2\times$ cost per block.
2. **Memory layout control.** The hierarchical blocking scheme requires contiguous, unboxed `int32` arrays with page-aligned base addresses (4 KiB or 2 MiB). Languages whose allocators do not expose alignment control must use platform-specific allocation (`posix_memalign`, `mmap`) via FFI.
3. **Garbage collection interaction.** A 25 M-key tree occupies ~300 MiB across `layout`, `sorted_rank`, and `keys`. In GC-managed languages, this data must either be pinned (risking heap fragmentation and GC pause overhead from scanning) or allocated outside the managed heap (requiring manual lifetime management, undermining the GC's purpose).
4. **Idiomatic API fit.** Each language has conventions for container types—type classes (Haskell), traits (Rust), interfaces (Go), protocols (Python). FAST is a *static* structure (build once from sorted keys, query many times, never modify in place). This conflicts with APIs that expect mutation, incremental insertion, or structural sharing.
5. **Build-phase complexity.** The recursive blocked layout construction (`lay_out_subtree`) performs precise index arithmetic on mutable arrays. Languages that discourage mutable state or lack efficient mutable array primitives may pay significant overhead during construction, though this is a one-time cost amortised over many queries.

8.1 Haskell

Haskell presents the richest set of challenges because GHC's runtime model differs fundamentally from C in memory management, side effects, and low-level data representation. The analysis below also considers integration with the `mono-traversable` package, which defines the standard Haskell type classes for monomorphic containers.

SIMD

GHC has experimental SIMD primops (available since GHC 8.0): `Int32X4#`, `broadcastInt32X4#`, `plusInt32X4#`, etc. These are unboxed types that cannot be stored in ordinary data structures, are poorly documented, and their code generation quality depends on whether GHC uses its native code generator (NCG) or the LLVM backend. In practice:

- The primops exist but there is no `compareGtInt32X4#` that directly maps to `_mm_cmpgt_epi32` with mask extraction.
- The LLVM backend may auto-vectorise scalar code, but the FAST search pattern (compare, extract mask, table lookup, computed offset) is unlikely to be recognised by auto-vectorisation passes.
- A `foreign import ccall` for just the SIMD block comparison (~5 lines of C) is the pragmatic alternative. This is a partial FFI dependency, not a full one—the surrounding traversal logic, tree construction, and API remain in Haskell.

Without SIMD, the scalar fallback uses three conditional branches per block instead of one SIMD comparison, costing $\sim 2\times$ per block and $\sim 1.5\times$ overall (the memory hierarchy cost dominates at large sizes, partially masking the SIMD loss).

Memory Layout

FAST requires page-aligned base addresses for TLB efficiency. GHC's options:

- `newAlignedPinnedByteArray#`: GHC primop that allocates pinned memory with a specified alignment. Suitable for cache-line alignment (64 B) but the alignment guarantee may not extend to 4 KiB or 2 MiB page boundaries on all platforms.
- `mallocForeignPtrBytes` + manual alignment arithmetic: allocate oversized buffer, compute aligned offset, wrap in `ForeignPtr`. This works but is manual memory management dressed in Haskell syntax.
- `mmap` via FFI: most reliable for page alignment and superpage support, but requires FFI (a system call, not a C library dependency).

The `layout`, `sorted_rank`, and `keys` arrays can use `Data.Vector.Storable` (contiguous, unboxed, compatible with FFI) or raw `ByteArray#`. Either provides the flat `int32` representation FAST requires.

Garbage Collection

A 25 M-key FAST tree occupies ∼300 MiB. If this lives in GHC's managed heap:

- Pinned `ByteArray#` objects are not moved by the copying collector but are still *scanned* during major GC. A 300 MiB pinned allocation contributes to GC pause time proportional to its size.
- Unpinned allocation risks being copied during major GC, which is catastrophic for a 300 MiB array (both in pause time and memory pressure from the copy).
- `ForeignPtr`-wrapped external allocation (via `malloc` or `mmap`) avoids GC scanning entirely. The finaliser attached to the `ForeignPtr` handles deallocation. This is what the current FFI binding does, and a native implementation would likely do the same.

Build Phase

The recursive blocked layout construction maps naturally to Haskell's `ST` monad with mutable `Storable` or `Unboxed` vectors:

- `build_inorder_map` → `ST s (MVector s Int32)`, a standard BFS traversal writing to a mutable vector.
- `lay_out_subtree` → recursive function in `ST` with an `STRef` for the output write position.
- GHC may not eliminate bounds checks on `writeArray` inside recursive functions; `unsafeWrite` would be needed for performance parity with C.
- Build is a one-time cost. Even at 2× overhead vs. C, this is acceptable when amortised over millions of queries.

The `mono-traversable` API

The `mono-traversable` package defines type classes for monomorphic containers: `MonoFoldable`, `MonoFunctor`, `MonoTraversable`, `IsSequence`, and `SetContainer`. FAST's static, SIMD-specialised nature creates tension with several of these.

Element type. `type instance Element FastTree = Int32`. FAST is inherently monomorphic (SSE2 `_mm_cmpgt_epi32` operates on 32-bit integers), so the mono-traversable framing is natural.

`MonoFoldable` — clean fit. The sorted `keys` array directly supports: `otoList` (return sorted keys), `ofoldl'` (fold over sorted keys), `olength` (return n), `onull` (check $n = 0$), `oelem` (use `fast_search` for $O(\log n)$ membership). No issues.

`MonoFunctor` — $O(n \log n)$ rebuild. `omap f` must apply f to every key. Since f can destroy sorted order, the only correct implementation is: extract keys → map f → re-sort → rebuild blocked layout. This is $O(n \log n)$ per `omap`. The functor law `omap f . omap g == omap (f . g)` holds semantically (both paths re-sort), but without GHC rewrite rules for fusion, composed maps rebuild the tree twice.

`MonoTraversable` — `unsafePerformIO` required. `otraverse :: Applicative f => (Int32 -> f Int32) -> FastTree -> f FastTree`. Tree construction requires `IO` (aligned allocation, system calls). To provide a pure `otraverse`, the rebuild must be wrapped in `unsafePerformIO`:

- `unsafePerformIO` + finaliser-based cleanup can interact subtly with GHC's optimiser (inlining, let-floating, CSE may duplicate or reorder the effectful construction).
- `{-# NOINLINE #-}` on the rebuild function mitigates this but inhibits other optimisations.
- If the `Applicative` is `IO` itself, nested `unsafePerformIO` calls create ordering hazards.

`SetContainer` — partial fit. `member` and `notMember` map naturally to `fast_search` ($O(\log n)$, excellent). But `union`, `intersection`, and `difference` each require: merge sorted arrays ($O(n+m)$) + full tree rebuild ($O(n + m)$) including aligned reallocation. The constant factor from blocking reconstruction makes these significantly more expensive than the same operations on `Data.Set`, even though the asymptotic complexity is the same.

`IsSequence` — fundamental mismatch. `cons`, `snoc`, `filter`, `splitAt`, `break`, `take`, `drop` all expect to produce new containers cheaply. FAST has no structural sharing: the blocked layout is a flat array, not a tree of pointers. Every "modification" is a complete reconstruction ($O(n)$ copy + $O(n)$ re-block + aligned reallocation). Providing this instance would satisfy the type class laws but violate performance expectations so severely that it would be misleading. **Recommendation: do not provide `IsSequence`.**

Practical Path

1. Provide `MonoFoldable`: yes, clean fit.
2. Provide `MonoFunctor` and `MonoTraversable`: yes, with clear documentation that these rebuild in $O(n \log n)$.
3. Provide `SetContainer`: `member`/`notMember` only; omit `union`/`intersection`/`difference` or document them as $O(n + m)$ with high constant factor.
4. Skip `IsSequence`.
5. Use `foreign import ccall` for the SIMD comparison hot loop (~5 lines of C); keep all other logic in Haskell.
6. Use `ForeignPtr`-wrapped `mmap` allocation for the large arrays to avoid GC interaction.
7. Build phase in `ST` monad with `unsafeWrite` for inner loops.

8.2 Rust

Rust is arguably the strongest candidate for a native FAST implementation after C itself, due to direct SIMD access, zero-cost abstractions, no GC, and precise memory layout control.

SIMD

The `std::arch::x86_64` module provides intrinsics that map 1:1 to SSE2 instructions: `_mm_set1_epi32`, `_mm_cmpgt_epi32`, `_mm_movemask_ps`, `_mm_castsi128_ps`. These require `unsafe` blocks but are well-documented, stable since Rust 1.27, and generate identical machine code to the C intrinsics. The `#[target_feature(enable = "sse2")]` attribute provides compile-time feature gating with runtime fallback via `is_x86_feature_detected!`.

Memory Layout

`std::alloc::alloc` with `Layout::from_size_align(size, 4096)` provides page-aligned allocation. `#[repr(C)]` on wrapper structs ensures C-compatible field layout. Rust's ownership model naturally tracks the lifetime of the three arrays (`layout`, `sorted_rank`, `keys`) without GC.

API Fit

FAST maps naturally to Rust's trait ecosystem:

- `impl IntoIterator for &FastTree`: iterate over sorted keys.
- `impl Index<usize> for FastTree`: index into sorted keys.
- Custom `fn search(&self, key: i32) -> Option<usize>` for the core predecessor query.
- `impl Drop`: automatic cleanup of aligned allocations.
- No tension with the borrow checker: the tree is immutable after construction, so `&self` methods suffice.

The main ergonomic cost is `unsafe` blocks around SIMD intrinsics and raw-pointer arithmetic in the search loop. These can be encapsulated in a safe public API backed by a small `unsafe` core (∼30 lines).

Practical Path

Rust needs no FFI. The entire implementation (build + search + SIMD) can be native, safe-API-wrapped `unsafe` Rust. Expected performance: within 5% of C, since both compile through LLVM with identical intrinsics.

8.3 C++

C++ is closest to the C reference implementation. `<immintrin.h>` provides identical SSE2 intrinsics, `std::aligned_alloc` (C++17) handles alignment, and the language has no GC.

API Fit

A C++ native implementation could provide:

- STL-compatible iterators (const random-access over sorted keys).
- `std::ranges` concepts (C++20): `sized_range`, `random_access_range`.
- `operator[]` for indexed access.
- Move semantics for zero-copy ownership transfer.
- Template parameterisation over key type (`int32_t` for SSE2, `int64_t` for SSE4.2/AVX2, `float/double` for `_mm_cmpgt_ps`/`_mm_cmpgt_pd`).

This is essentially a packaging exercise: the algorithm and performance are identical to C. The value is in providing a zero-overhead idiomatic C++ interface that interoperates with `<algorithm>`, `<ranges>`, and standard containers.

8.4 OCaml

OCaml shares many of Haskell's challenges but with a different runtime model: a single-generation copying GC (OCaml 4) or a parallel collector (OCaml 5), no lazy evaluation, and unboxed arrays via `Bigarray`.

SIMD

OCaml has no native SIMD support. The `ocaml-simd` experimental library wraps intrinsics via C stubs, but each call crosses the OCaml/C boundary (∼30–50 ns per call), negating the SIMD benefit. A scalar implementation or a C stub for the inner search loop is the practical option.

Memory Layout

`Bigarray.Array1` provides contiguous, unboxed, GC-managed arrays of `int32`. These are allocated outside the minor heap and not moved by the GC. However, `Bigarray` does not guarantee page alignment; a custom allocator via `ctypes` or `Unix.mmap` would be needed.

API Fit

OCaml's standard library `Map` is an immutable AVL tree with no equivalent of `mono-traversable`. A native FAST would likely provide:

- `val create : int32 array -> t`
- `val search : t -> int32 -> int option`
- `val fold : ('a -> int32 -> 'a) -> 'a -> t -> 'a`
- `val to_seq : t -> int32 Seq.t`

The static nature of FAST is less problematic in OCaml, where immutable data structures are idiomatic.

8.5 Fortran

Fortran's array-oriented design is well-suited to FAST's flat-array layout, but SIMD access is the main obstacle.

SIMD

Fortran has no standard intrinsics for packed integer SIMD. Options:

- Compiler auto-vectorisation: GFortran and ifort may vectorise simple loops, but the FAST comparison-mask-lookup pattern is not a vectorisation candidate.
- `!DIR$ VECTOR` / `!DIR$ SIMD` directives: hint-based, not guaranteed.
- Inline C via `BIND(C)`: call a 5-line C function for the SIMD comparison. This is the practical approach.

Memory Layout

Fortran excels here. `ALLOCATE` with the `ALIGNED` attribute (Fortran 2018, compiler-specific extensions) or C interop via `C_F_POINTER` + `posix_memalign` provides aligned arrays. Contiguous `INTEGER(4)` arrays are the natural representation. No GC, deterministic deallocation.

API Fit

Fortran has no standard container abstractions. A module with `fast_create`, `fast_search`, `fast_destroy` subroutines is idiomatic. The current FFI binding already provides this interface.

8.6 Ada

Ada provides strong typing, explicit memory management, and reasonable low-level control, making a native implementation feasible but verbose.

SIMD

GNAT (the GCC-based Ada compiler) supports machine-code insertions (`System.Machine_Code`) and GCC vector extensions via pragmas. GNAT-specific `GNAT.SSE` packages provide typed wrappers for some SSE operations, but coverage of `_mm_cmpgt_epi32` specifically may require inline assembly or a C stub.

Memory Layout

`System.Storage_Pools` allows custom allocators with alignment guarantees. `pragma Pack` and representation clauses provide precise layout control for records and arrays. Ada's strong typing would require explicit `Unchecked_Conversion` for reinterpreting SSE mask bits.

API Fit

Ada's standard containers (`Ada.Containers.Ordered_Maps`, `Ordered_Sets`) use a tagged-type hierarchy. A FAST implementation could implement a read-only subset of the `Ordered_Sets` interface: `Contains`, `Floor`, `Ceiling`, `Iterate`. The static nature is less problematic since Ada containers commonly distinguish between `constant_reference` and `reference` access.

8.7 Julia

Julia's JIT compilation through LLVM makes it uniquely positioned: it can potentially emit the same SIMD instructions as C through LLVM intrinsics, while providing high-level syntax.

SIMD

The `SIMD.jl` package provides portable SIMD vector types that compile through LLVM. `Vec{4, Int32}` with comparison and mask extraction should map to the same SSE2 instructions as C. Alternatively, Julia's `llvmcall` allows direct LLVM IR emission for full control. Julia's `ccall` is near-zero overhead (∼5–10 ns), so a C stub is also viable.

Memory Layout

Julia arrays are GC-managed but the GC is non-moving for large allocations. `Libc.malloc` + `unsafe_wrap` provides external allocation with Julia array semantics. Page alignment is available through `Libc.mmap` or `posix_memalign` via `ccall`.

API Fit

Julia's multiple dispatch makes API integration straightforward:

- `Base.searchsortedlast(tree, key)` for predecessor query.
- `Base.iterate` for iteration over sorted keys.
- `Base.length`, `Base.eltype`, `Base.in`.
- `AbstractVector{Int32}` subtyping (read-only) to interoperate with the standard library.

Julia's JIT startup cost (∼1–2 s for compilation) is a consideration for short-lived processes but irrelevant for long-running applications.

8.8 Scheme (Chez)

SIMD

Chez Scheme has no SIMD support. The foreign-function interface (`foreign-procedure`) is the only viable path for SIMD operations. A native scalar implementation is possible but would sacrifice the primary performance advantage.

Memory Layout

Chez provides `bytevectors` (SRFI-4) for contiguous unboxed storage and `foreign-alloc` for external allocation. Page alignment requires platform-specific FFI calls. Chez's generational collector does not move large bytevectors, so pinning is not a concern for bytevector-backed storage.

API Fit

Scheme's minimalist standard library has no container type class hierarchy. A FAST implementation would provide procedures: `(fast-create keys)`, `(fast-search tree key)`, `(fast-fold proc init tree)`. Integration with SRFI-44 (Collections) or SRFI-113 (Sets) is possible but these SRFIs are not widely adopted.

8.9 Mercury

Mercury is a logic/functional language with a sophisticated compiler (Melbourne Mercury Compiler) that generates C code via the `hlc.gc` grade.

SIMD

Mercury has no SIMD primitives. The `pragma foreign_proc("C", ...)` mechanism allows inline C code within Mercury predicates, providing a clean path to embed the SIMD search loop. This is essentially how the current FFI binding works, but with finer-grained integration.

Memory Layout

In the `hlc.gc` grade, Mercury uses the Boehm conservative GC. Large allocations are not moved. Contiguous unboxed arrays can be created via `foreign_type` wrapping C arrays. Page alignment requires C-level allocation through `foreign_proc`.

API Fit

Mercury's standard library includes `set` (balanced BST) and `tree234` (2-3-4 tree) modules. A FAST implementation could provide the same interface as `set`—`member`, `fold`, `to_sorted_list`—with the caveat that `insert` and `delete` require full reconstruction.

8.10 Python and Ruby

For Python and Ruby, a native implementation of the FAST *search loop* in the host language would be orders of magnitude slower than the C FFI approach, due to interpreter overhead per operation (∼50–100 ns per bytecode instruction vs. ∼1 ns per SIMD comparison).

The practical approach for these languages is the one already benchmarked: call the C library through `ctypes/cffi` (Python) or the `ffi` gem (Ruby).

For Python specifically, a Cython or `pybind11` wrapper could reduce per-call overhead from ∼1200 ns (ctypes) to ∼50 ns (compiled extension), while a NumPy-compatible batch API (`fast_search_batch(tree, query_array)`) could amortise call overhead across thousands of queries, bringing effective per-query overhead below 1 ns.

For Ruby, the `ffi` gem's ∼300–500 ns overhead is the main bottleneck. A native C extension (`rb_define_method` with inline C) would reduce this to ∼20–50 ns per call.

9 Beyond Static FAST: Matryoshka Trees

FAST's flat-array layout is its greatest strength for search and its greatest weakness for modification. Every insertion or deletion requires a full $O(n)$ rebuild because the hierarchically blocked layout is a *global* function of the key set: tree depth, in-order-to-BFS mapping, recursive blocking decomposition, and `sorted_rank` all depend on the total key count and the position of every key. There is no $O(\log n)$ local patch.

This section describes a hybrid design—*matryoshka trees*—that recovers FAST's locality advantages for modification-heavy workloads by nesting SIMD-blocked search *within* B^+ tree nodes connected by pointers.

9.1 The Pointer-Cost Observation

When a FAST search crosses a page boundary, it pays for a TLB miss (∼10–40 ns). One additional pointer dereference within the target page costs zero marginal latency—the page is already being fetched. Therefore:

> *The flat-array layout is only necessary within a single hardware unit (register, cache line, page). Between hardware units, pointers are free because the boundary-crossing cost dominates.*

This means we can replace FAST's global flat array with a B^+ tree where each node is internally SIMD-blocked, recovering $O(B \cdot \log_B n)$ modification while preserving FAST's search locality within each node.

9.2 Nesting Levels

Each level of the memory hierarchy gets its own "doll":

Level	Hardware unit	Structure	Connection
0	SSE2 register (16 B)	3-key SIMD block	Offset arithmetic
1	Cache line (64 B)	15-key FAST block	Offset arithmetic
2	Page (4 KiB)	B^+ node, FAST-blocked	Child pointers
3	Superpage (2 MiB)	B^+ upper levels	Child pointers
4	Main memory	B^+ root path	Child pointers

Levels 0–1 are identical to pure FAST: offset arithmetic within contiguous memory, no pointers. Levels 2–4 use B$^+$ tree pointers between nodes, enabling node-local splits and merges.

9.3 Node Layout

Internal Nodes (Page-Sized)

A 4 KiB B$^+$ tree internal node contains keys and child pointers:

$$k \text{ keys} \times 4\text{ B} + (k+1) \text{ pointers} \times 8\text{ B} \;\le\; 4096 \quad \Rightarrow \quad k \le 340$$

With $k = 340$: 1 360 B for keys + 2 728 B for pointers = 4 088 B.

Intra-node search uses SIMD-accelerated binary search on the sorted key array (the keys are stored sorted, not FAST-blocked, to avoid the `sorted_rank` overhead that would exceed the page budget). This yields ∼10 SIMD comparisons per node, with exactly one page access and zero TLB misses within the node.

Leaf Nodes (Page-Sized)

Leaf nodes contain only keys (no child pointers), so the full page budget goes to keys:

$$k \text{ keys} \times 4\text{ B} \;\le\; 4096 \quad \Rightarrow \quad k \le 1024$$

Leaf nodes use the full FAST hierarchical blocked layout with `sorted_rank`, since this is where query time is spent and the SIMD blocking pays off most. ∼5 SIMD comparisons per leaf, one page access.

9.4 Modification Cost

Insertion. Search to the correct leaf ($\log_B n$ node accesses, $B \approx 340$). Insert into the leaf. If the leaf has room, rebuild only that leaf's FAST layout: $O(B_{\text{leaf}})$ where $B_{\text{leaf}} \approx 1000$. If full, split into two leaves and propagate one key to the parent. Worst case: $O(\log_B n)$ splits, each $O(B)$.

Total: $O(B \cdot \log_B n)$ per insertion.

Deletion. Analogous: remove from leaf, rebuild leaf. If underflow, merge or redistribute with sibling, propagate down. Same $O(B \cdot \log_B n)$ cost.

Structure	Insert (25 M keys)	Search	Locality
Pure FAST	O(25M) rebuild	∼15 SIMD, 3 pages	Excellent
Red–black tree	O(25) pointer ops	O(25) pointer chases	Poor
B$^+$/FAST hybrid	O(3000) in-page ops	∼15 SIMD, 3 pages	Excellent

The hybrid is ∼8000× faster than pure FAST for insertion while matching its search performance. It performs ∼100× more work per insert than a red–black tree in operation count, but each operation is an in-page array write (∼1 ns) rather than a likely cache miss (∼40–60 ns), so wall-clock times are comparable.

9.5 Concurrency

The B$^+$ tree structure enables concurrency patterns impossible with flat FAST:

- **Node-level locking**: modifications lock only the affected path ($O(\log_B n)$ nodes), not the entire structure.
- **Read–read parallelism**: search queries never block each other.
- **B-link tree techniques** (Lehman & Yao): right-link pointers between siblings allow lock-free reads during concurrent splits.
- **Copy-on-write nodes**: for MVCC or persistent snapshots, copy a single 4 KiB page instead of the entire array.

9.6 Related Work

- **CSS trees** (Rao & Ross, SIGMOD 1999 [2]): B$^+$ trees with cache-line-sized nodes—the same insight about matching node size to hardware boundaries, without SIMD.
- **CSB$^+$ trees** (Rao & Ross, SIGMOD 2000 [3]): child pointers compressed to base + offset, improving key density.
- **FAST** (Kim et al., SIGMOD 2010 [1]): adds SIMD blocking within nodes but removes all pointers for a flat array.
- **Masstree** (Mao et al., EuroSys 2012 [4]): concurrent B$^+$ tree with trie structure for variable-length keys.

The matryoshka design applies FAST's SIMD blocking technique within CSS/CSB$^+$-style nodes, recovering modification capability while preserving the SIMD and cache-locality wins that FAST introduced.

References

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[2] J. Rao and K. A. Ross, "Cache Conscious Indexing for Decision-Support in Main Memory," in *Proc. ACM SIGMOD*, 1999, pp. 78–89.

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[4] Y. Mao, E. Kohler, and R. T. Morris, "Cache Craftiness for Fast Multicore Key-Value Storage," in *Proc. ACM EuroSys*, 2012, pp. 183–196.