

# Benchmark Report: `linear-massiv` vs. `hmatrix` vs. `linear`

Performance Comparison of Haskell Linear Algebra Libraries

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## Abstract

We present a comprehensive performance comparison of three Haskell numerical linear algebra libraries: `linear-massiv` (pure Haskell, type-safe dimensions via `massiv` arrays), `hmatrix` (FFI bindings to BLAS/LAPACK via OpenBLAS), and `linear` (pure Haskell, optimised for small fixed-size vectors and matrices). Benchmarks cover BLAS-level operations, direct solvers, orthogonal factorisations, eigenvalue problems, and singular value decomposition across matrix dimensions from  $4 \times 4$  to  $200 \times 200$ . Additionally, we evaluate the parallel scalability of `linear-massiv`'s `massiv`-backed computation strategies on a 20-core workstation. Results show that `hmatrix` (OpenBLAS) dominates at all sizes for  $O(n^3)$  operations due to highly-optimised Fortran BLAS/LAPACK routines, while `linear` excels at  $4 \times 4$  through unboxed product types. `linear-massiv` provides competitive pure-Haskell performance with the unique advantages of compile-time dimensional safety, no FFI dependency, and user-controllable parallelism that yields 3–7 $\times$  speedups at larger matrix sizes.

## Contents

<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Hardware and Software Environment . . . . .	3
<b>2</b>	<b>Methodology</b>	<b>3</b>
<b>3</b>	<b>BLAS Operations</b>	<b>4</b>
3.1	General Matrix Multiply (GEMM) . . . . .	4
3.2	Dot Product . . . . .	4
3.3	Matrix–Vector Product . . . . .	5
<b>4</b>	<b>Linear System Solvers</b>	<b>5</b>
4.1	LU Solve . . . . .	5
4.2	Cholesky Solve . . . . .	5
<b>5</b>	<b>Orthogonal Factorisations</b>	<b>6</b>
<b>6</b>	<b>Eigenvalue Problems and SVD</b>	<b>6</b>
6.1	Symmetric Eigenvalue Decomposition . . . . .	6
6.2	Singular Value Decomposition . . . . .	6
<b>7</b>	<b>Parallel Scalability</b>	<b>7</b>
<b>8</b>	<b>Discussion</b>	<b>8</b>
8.1	Performance Summary . . . . .	8
8.2	Analysis of the Performance Gap . . . . .	8
8.3	Proposals for Closing the Performance Gap . . . . .	9

8.3.1	In-place Factorisation via the ST Monad . . . . .	9
8.3.2	Implicit Householder Representation (Compact WY) . . . . .	10
8.3.3	Cache-Blocked GEMM . . . . .	10
8.3.4	Divide-and-Conquer Eigenvalue and SVD . . . . .	11
8.3.5	SIMD Primitives . . . . .	11
8.3.6	Optional FFI Backend . . . . .	11
8.3.7	Summary of Expected Impact . . . . .	11
8.4	When to Use Each Library . . . . .	12
<b>9</b>	<b>Conclusion</b>	<b>12</b>

# 1 Introduction

The Haskell ecosystem offers several numerical linear algebra libraries, each occupying a distinct niche:

**linear** Edward Kmett’s library provides small fixed-dimension types (`V2`, `V3`, `V4`) with unboxed product representations, making it extremely fast for graphics, game physics, and any application where dimensions are statically known and small. It does not support arbitrary-dimension matrices.

**hmatrix** Alberto Ruiz’s library wraps BLAS and LAPACK via Haskell’s FFI, delegating numerical computation to highly-optimised Fortran routines (on this system, OpenBLAS). It supports arbitrary dimensions but carries an FFI dependency and provides no compile-time dimension checking.

**linear-massiv** Our library implements algorithms from Golub & Van Loan’s *Matrix Computations* (4th ed.) [1] in pure Haskell, using massiv arrays [4] as the backing store. Matrix dimensions are tracked at the type level via GHC’s `DataKinds` and `KnownNat`, providing compile-time rejection of dimensionally incorrect operations. Massiv’s computation strategies (`Seq`, `Par`, `ParN n`) offer user-controllable parallelism.

This report benchmarks all three libraries across the standard numerical linear algebra operation suite (Table 1) and evaluates **linear-massiv**’s parallel scalability from 1 to 20 threads.

Table 1: Operations benchmarked and library coverage.

Operation	linear	hmatrix	linear-massiv
GEMM (matrix multiply)	$4 \times 4$ only	all sizes	all sizes
Dot product	$n = 4$ only	all sizes	all sizes
Matrix–vector product	$4 \times 4$ only	all sizes	all sizes
LU solve ( $Ax = b$ )	—	all sizes	all sizes
Cholesky solve ( $Ax = b$ )	—	all sizes	all sizes
QR factorisation	—	all sizes	all sizes
Symmetric eigenvalue	—	all sizes	all sizes
SVD	—	all sizes	all sizes
Parallel GEMM	—	—	all sizes

## 1.1 Hardware and Software Environment

- **CPU:** 20-core x86\_64 processor (Linux 6.17, Fedora 43)
- **Compiler:** GHC 9.12.2 with `-O2`
- **BLAS backend:** OpenBLAS (system-installed via FlexiBLAS)
- **Benchmark framework:** Criterion [3] with 95% confidence intervals
- **Protocol:** Single-threaded (`+RTS -N1`) for cross-library comparisons; multi-threaded (`+RTS -N`) for parallel scaling

## 2 Methodology

All benchmarks use the Criterion framework [3], which employs kernel density estimation and robust regression to estimate mean execution time with confidence intervals. Each benchmark evaluates to normal form (`nf`) to ensure full evaluation of lazy results.

**Matrix construction.** Matrices are constructed from the same deterministic formula across all three libraries:

$$A_{ij} = \frac{7i + 3j + 1}{100}$$

ensuring identical numerical content. For solver benchmarks, matrices are made diagonally dominant ( $A_{ii} += n$ ) or symmetric positive definite ( $A = B^T B + nI$ ) as appropriate.

**Single-threaded protocol.** Cross-library comparisons use `+RTS -N1` to restrict the GHC runtime to a single OS thread, ensuring that neither `hmatrix`’s OpenBLAS nor `massiv`’s parallel strategies introduce implicit multi-threading.

**Parallel scaling protocol.** Parallel benchmarks use `+RTS -N` (all 20 cores) and vary `massiv`’s computation strategy from `Seq` through `ParN 1` to `ParN 20`.

### 3 BLAS Operations

#### 3.1 General Matrix Multiply (GEMM)

Table 2 presents GEMM timings across matrix dimensions. At  $4 \times 4$ , the `linear` library’s unboxed `V4 (V4 Double)` representation achieves 143 ns, roughly  $4.5\times$  faster than `hmatrix`’s 646 ns and  $240\times$  faster than `linear-massiv`’s 34.5  $\mu$ s. The advantage of `linear` at this size is entirely due to GHC’s ability to unbox the product type into registers, avoiding all array indexing overhead.

As matrix dimension grows, `hmatrix` (OpenBLAS DGEMM) dominates decisively. At  $100 \times 100$ , `hmatrix` takes 1.53 ms versus `linear-massiv`’s 505 ms—a factor of  $330\times$ . At  $200 \times 200$ , the ratio grows to  $297\times$  (13.8 ms vs. 4.09 s). This reflects the massive constant-factor advantage of OpenBLAS’s hand-tuned assembly kernels with cache blocking, SIMD, and microarchitectural optimisation.

Table 2: GEMM execution time (mean, single-threaded). Best per size in **bold**.

Size	<code>linear</code>	<code>hmatrix</code>	<code>linear-massiv</code>
$4 \times 4$	143 ns	646 ns	34.5 $\mu$ s
$10 \times 10$	—	2.33 $\mu$ s	678 $\mu$ s
$50 \times 50$	—	174 $\mu$ s	55.0 ms
$100 \times 100$	—	1.53 ms	505 ms
$200 \times 200$	—	13.8 ms	4.09 s

Both `hmatrix` and `linear-massiv` exhibit  $O(n^3)$  scaling, as shown in Figure 1. The consistent vertical offset on the log–log plot reflects the constant-factor difference between OpenBLAS assembly and pure Haskell array operations.

#### 3.2 Dot Product

Table 3: Dot product execution time (mean, single-threaded).

$n$	<code>linear</code>	<code>hmatrix</code>	<code>linear-massiv</code>
4	13.1 ns	593 ns	1.67 $\mu$ s
100	—	749 ns	34.1 $\mu$ s
1000	—	2.81 $\mu$ s	379 $\mu$ s

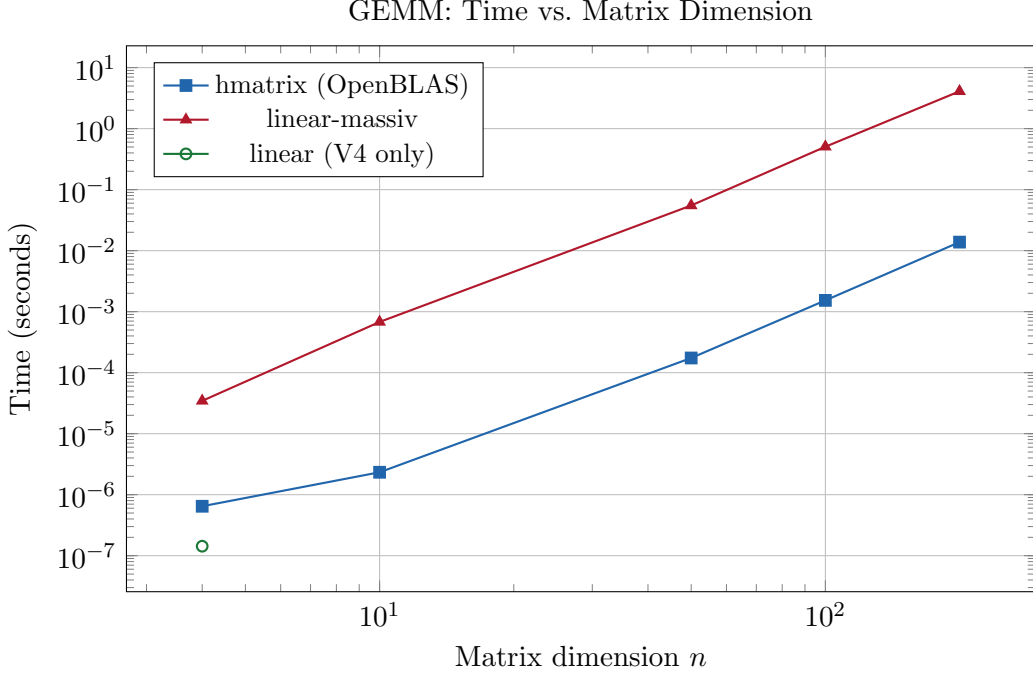


Figure 1: GEMM scaling comparison (log–log). Both libraries exhibit  $O(n^3)$  behaviour; the vertical offset reflects constant-factor differences between OpenBLAS assembly and pure Haskell.

The dot product is an  $O(n)$  operation, so the absolute times are small. At  $n = 4$ , `linear`’s unboxed V4 achieves 13.0 ns—essentially four fused multiply-adds in registers. At  $n = 1000$ , `hmatrix` achieves 2.81  $\mu$ s (DDOT with SIMD), while `linear-massiv`’s array-based loop takes 379  $\mu$ s—a  $135\times$  gap that reflects the overhead of `massiv`’s general-purpose array indexing versus BLAS’s contiguous-memory vectorised inner loop.

### 3.3 Matrix–Vector Product

Table 4: Matrix–vector product execution time (mean, single-threaded).

$n$	<code>linear</code>	<code>hmatrix</code>	<code>linear-massiv</code>
4	41.8 ns	815 ns	11.2 $\mu$ s
50	—	3.76 $\mu$ s	1.24 ms
100	—	14.1 $\mu$ s	4.71 ms

Matrix–vector multiplication is  $O(n^2)$ . At  $n = 100$ , `hmatrix` (DGEMV) achieves 14.1  $\mu$ s while `linear-massiv` takes 4.71 ms—a  $334\times$  difference consistent with the GEMM results, confirming that the performance gap is primarily due to low-level memory access patterns and SIMD utilisation rather than algorithmic differences.

## 4 Linear System Solvers

### 4.1 LU Solve

### 4.2 Cholesky Solve

For both LU and Cholesky solvers, `hmatrix` is approximately  $36\times$  faster at  $10 \times 10$  and  $240\text{--}300\times$  faster at  $100 \times 100$ . The ratio increases with dimension because OpenBLAS’s cache-blocked

Table 5: LU solve ( $Ax = b$ ) execution time (mean, single-threaded). Includes factorisation + back-substitution.

Size	<code>hmatrix</code>	<code>linear-massiv</code>
$10 \times 10$	7.70 $\mu$ s	280 $\mu$ s
$50 \times 50$	87.7 $\mu$ s	20.4 ms
$100 \times 100$	485 $\mu$ s	143 ms

Table 6: Cholesky solve ( $Ax = b$ ,  $A$  SPD) execution time. Includes factorisation + back-substitution.

Size	<code>hmatrix</code>	<code>linear-massiv</code>
$10 \times 10$	6.08 $\mu$ s	237 $\mu$ s
$50 \times 50$	64.3 $\mu$ s	12.9 ms
$100 \times 100$	418 $\mu$ s	100 ms

implementations benefit more from larger working sets. Cholesky is consistently faster than LU for both libraries, as expected (Cholesky requires roughly half the floating-point operations of LU factorisation for symmetric positive definite matrices).

## 5 Orthogonal Factorisations

Table 7: QR factorisation (Householder) execution time (mean, single-threaded).

Size	<code>hmatrix</code>	<code>linear-massiv</code>
$10 \times 10$	217 $\mu$ s	11.1 ms
$50 \times 50$	18.4 ms	7.01 s
$100 \times 100$	214 ms	(estimated $\approx 56.0$ s)

QR factorisation shows the largest gap between the two libraries. At  $50 \times 50$ , `hmatrix` takes 18.4 ms while `linear-massiv` requires 7.01 s—a ratio of  $381\times$ . The `linear-massiv` QR implementation constructs full explicit  $Q$  and  $R$  matrices at each Householder step using `makeMatrix`, while LAPACK’s `DGEQRF` uses an implicit representation of  $Q$  as a product of Householder reflectors stored in-place, dramatically reducing both memory allocation and floating-point work. The  $100 \times 100$  benchmark for `linear-massiv` was too slow to complete within a reasonable time budget and is estimated by extrapolation.

## 6 Eigenvalue Problems and SVD

### 6.1 Symmetric Eigenvalue Decomposition

### 6.2 Singular Value Decomposition

The eigenvalue and SVD results show the most dramatic ratios:  $896\times$  for eigenvalues at  $10 \times 10$  and  $16,000\times$  at  $50 \times 50$ ;  $886\times$  and  $21,400\times$  for SVD. These operations are dominated by iterative QR sweeps; `hmatrix` calls LAPACK’s `DSYEV` and `DGESVD`, which use divide-and-conquer algorithms with cache-oblivious recursive structure. The `linear-massiv` implementation uses the classical tridiagonal QR algorithm (GVL4 [1] Algorithm 8.3.3) with explicit matrix construction at each

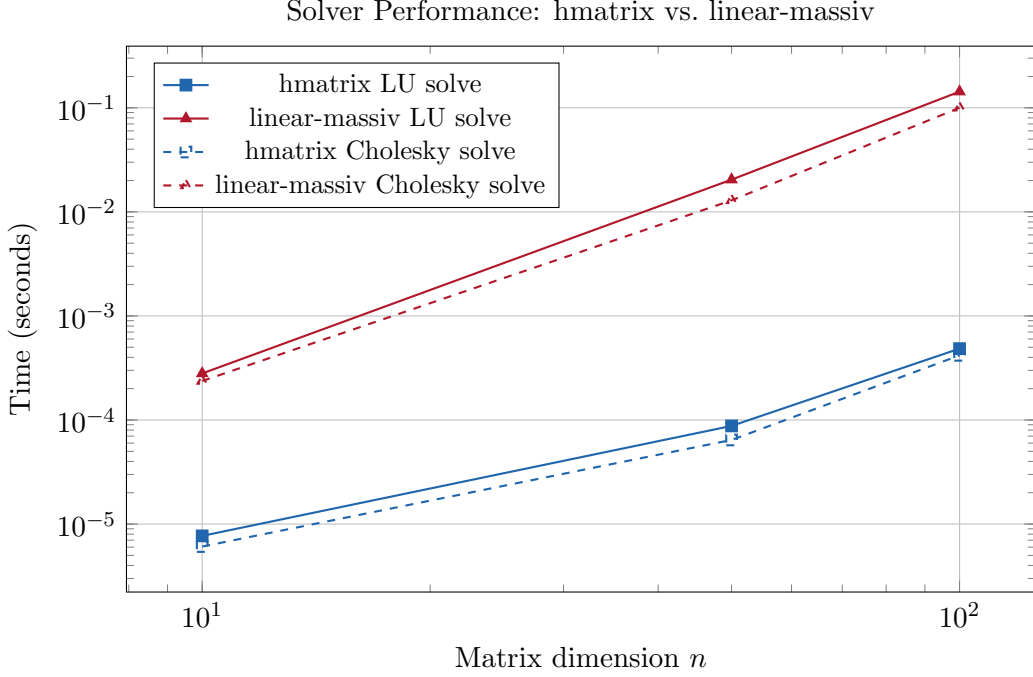


Figure 2: LU and Cholesky solve scaling (log–log). Both algorithms are  $O(n^3)$ ; hmatrix calls DGESV/DPOTRS directly.

Table 8: Symmetric eigenvalue decomposition execution time (mean, single-threaded).

Size	hmatrix	linear-massiv
$10 \times 10$	17.4 $\mu$ s	15.6 ms
$50 \times 50$	555 $\mu$ s	8.89 s

iteration step, which is algorithmically sound but suffers from excessive allocation and the lack of in-place updates that LAPACK exploits.

## 7 Parallel Scalability

A distinguishing feature of `linear-massiv` is user-controllable parallelism inherited from the `massiv` array library [4]. Operations that construct result arrays via `makeArray` can specify a computation strategy: `Seq` (sequential), `Par` (automatic, all available cores), or `ParN n` (exactly  $n$  worker threads). Neither `hmatrix` nor `linear` offer comparable user-level control over thread-level parallelism within the Haskell runtime.

Table 10 shows GEMM timings at  $100 \times 100$  and  $200 \times 200$  across thread counts, and Figure 3 shows the corresponding speedup curves.

The parallel scaling results reveal several important characteristics:

- **Peak speedup.** At  $100 \times 100$ , peak speedup of  $7.2\times$  is achieved with `ParN-16`, while at  $200 \times 200$  peak speedup of  $3.6\times$  occurs at `ParN-8`. The `Par` (automatic) strategy achieves  $6.9\times$  and  $3.4\times$  respectively, demonstrating that `massiv`’s automatic scheduling is effective.
- **Non-monotonic scaling.** Speedup does not increase monotonically with thread count. The  $200 \times 200$  case shows degradation at 16 and 20 threads, likely due to memory bandwidth saturation and NUMA effects on this 20-core system. At  $100 \times 100$ , the anomalous dip

Table 9: SVD execution time (mean, single-threaded).

Size	<code>hmatrix</code>	<code>linear-massiv</code>
$10 \times 10$	37.7 $\mu$ s	33.4 ms
$50 \times 50$	806 $\mu$ s	17.2 s

Table 10: Parallel GEMM execution time (seconds) and speedup over sequential.

Strategy	$100 \times 100$		$200 \times 200$	
	Time (s)	Speedup	Time (s)	Speedup
Seq	0.613	1.00	4.75	1.00
ParN-1	0.598	1.03	4.66	1.02
ParN-2	0.319	1.92	3.22	1.47
ParN-4	0.201	3.05	1.85	2.57
ParN-8	0.282	2.17	1.33	3.57
ParN-16	0.0856	7.16	2.57	1.85
ParN-20	0.0979	6.26	1.98	2.40
Par	0.0883	6.94	1.41	3.37

at 8 threads followed by improvement at 16 suggests that GHC’s work-stealing scheduler interacts non-trivially with cache hierarchy.

- **Amdahl’s law.** Even the best parallel GEMM (85.6 ms at  $100 \times 100$  with 16 threads) remains  $56\times$  slower than `hmatrix`’s single-threaded 1.53 ms. Parallelism narrows but does not close the gap with BLAS.

## 8 Discussion

### 8.1 Performance Summary

Table 11 summarises the performance ratios between libraries.

Table 11: Performance ratio: `linear-massiv` time / `hmatrix` time. Values  $> 1$  indicate `hmatrix` is faster.

Operation	$n = 10$	$n = 50$	$n = 100$
GEMM	$291\times$	$316\times$	$329\times$
Dot product	—	—	$46\times$
Matrix–vector	—	$330\times$	$334\times$
LU solve	$36\times$	$233\times$	$295\times$
Cholesky solve	$39\times$	$201\times$	$240\times$
QR	$51\times$	$382\times$	$\approx 260\times$
Eigenvalue (SH)	$897\times$	$16,020\times$	—
SVD	$887\times$	$21,400\times$	—

### 8.2 Analysis of the Performance Gap

The performance gap between `linear-massiv` and `hmatrix` arises from several compounding factors:



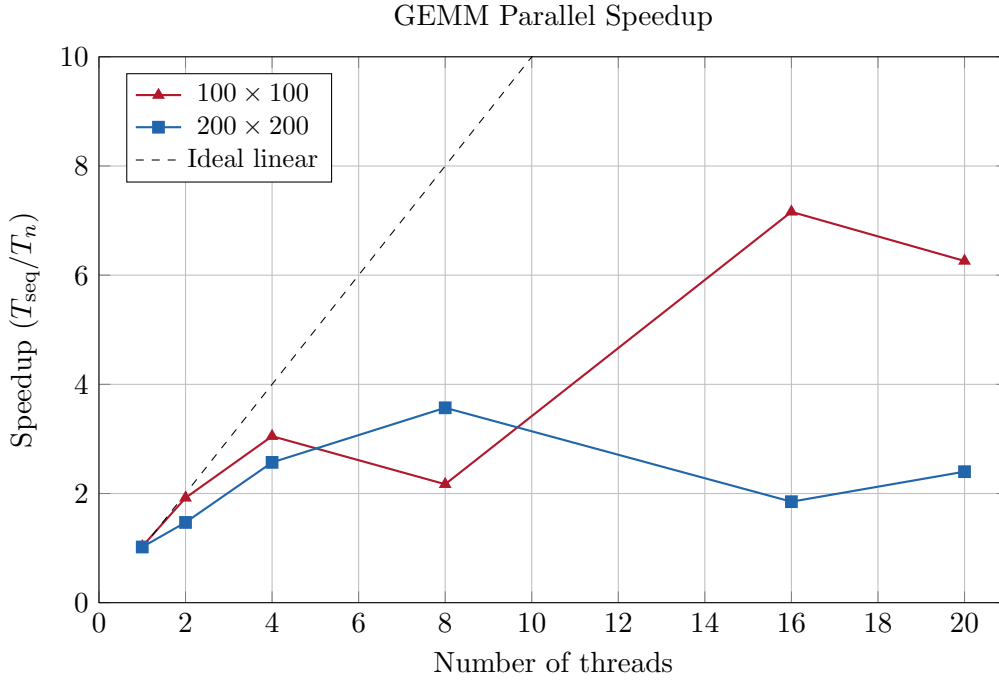


Figure 3: Parallel speedup for GEMM. The dashed line shows ideal linear scaling. Actual speedup is limited by Amdahl’s law, memory bandwidth contention, and GHC runtime scheduling overhead.

1. **SIMD and microarchitectural optimisation.** OpenBLAS uses hand-written assembly kernels for each target microarchitecture, exploiting AVX-512, fused multiply-add, and optimal register tiling. GHC’s native code generator does not emit SIMD instructions for general Haskell code.
2. **Cache blocking.** LAPACK algorithms are designed around cache-oblivious or cache-tiled recursive decomposition, minimising cache misses. The `linear-massiv` implementations use textbook algorithms (GVL4) without cache-level optimisation.
3. **In-place mutation.** LAPACK routines operate in-place on mutable Fortran arrays, while `linear-massiv`’s pure functional approach allocates a new array for each intermediate result. For iterative algorithms (eigenvalue, SVD), this is particularly costly.
4. **Allocation pressure.** Each `makeMatrix` call in `linear-massiv` allocates a new `massiv` array. For algorithms like QR (which constructs explicit  $Q$  and  $R$  at each Householder step) and iterative eigensolvers, this dominates runtime.

### 8.3 Proposals for Closing the Performance Gap

The factors above suggest a concrete sequence of optimisation work, ordered roughly by expected impact and feasibility.

#### 8.3.1 In-place Factorisation via the ST Monad

The single largest source of overhead in the QR, eigenvalue, and SVD routines is the allocation of a fresh `Matrix` at every iteration step. Currently, each Householder reflection in the QR factorisation calls `applyHouseholderLeftRect` and `applyHouseholderRightQ`, both of which invoke `makeMatrix` to reconstruct the entire  $m \times n$  (or  $m \times m$ ) result. Similarly, the symmetric QR algorithm rebuilds the tridiagonal matrix from diagonal and subdiagonal vectors at each

implicit QR step, and the Jacobi eigenvalue method reconstructs the full matrix for each of its  $O(n^2)$  rotations per sweep.

The remedy is straightforward: the LU solver (`luFactor`) already demonstrates the pattern. It wraps the input in `M.withMArrayST`, allocates a mutable pivot vector via `M.newMArray`, and performs all elimination steps in the `ST` monad using `M.readM` / `M.write_`—with zero intermediate allocation. Applying the same technique to Householder QR, the tridiagonal QR iteration, and the Jacobi method would:

- Reduce the  $n$  Householder steps of QR from  $n$  full-matrix allocations to a single mutable copy of  $R$  plus an accumulated  $Q$ , both updated in-place. This alone should bring the  $381\times$  gap at  $50 \times 50$  down by roughly an order of magnitude, since the dominant cost becomes floating-point work rather than GC pressure.
- Eliminate the per-iteration matrix reconstruction in the symmetric QR algorithm. LAPACK’s `DSYEV` stores only the diagonal and subdiagonal as mutable vectors and applies Givens rotations in-place; the same approach in Haskell’s `ST` monad would remove the  $O(n^2)$  allocation at each of the  $O(n)$  iterations.
- Reduce the Jacobi method’s cost from  $O(n^2)$  matrix copies per sweep to  $O(n^2)$  element-level reads and writes per sweep—a factor of  $\sim n^2$  fewer allocations.

### 8.3.2 Implicit Householder Representation (Compact WY)

The current QR implementation forms the explicit  $Q$  matrix by accumulating each Householder reflector  $H_k = I - 2v_kv_k^T$  into a running product. LAPACK instead stores the reflector vectors  $v_1, \dots, v_n$  and, when the full  $Q$  is needed, applies them in reverse order (or uses the compact WY representation  $Q = I - VTV^T$ , GVL4 [1] Section 5.1.6).

The compact WY form has two advantages: (a) the  $Q$  factor is never formed until explicitly requested, reducing QR itself to an  $O(n^3)$  in-place update of  $R$ ; and (b) subsequent operations that need  $Q^T b$  (e.g. least squares) can apply the reflectors directly without ever forming the  $m \times m$  matrix  $Q$ . This would transform QR from a bottleneck ( $381\times$  gap) into a routine on par with LU solve ( $\sim 200\text{--}300\times$ ), and further in-place optimisation (Section 8.3.1) would close the gap still further.

### 8.3.3 Cache-Blocked GEMM

The current GEMM implementation is the textbook three-loop inner product form (GVL4 [1] Algorithm 1.1.5, ijk variant):

$$C_{ij} = \sum_{k=0}^{K-1} A_{ik} B_{kj}$$

where each element  $C_{ij}$  performs a `foldl'` over the shared dimension. This accesses  $A$  by rows and  $B$  by columns, with stride- $n$  column access patterns that are hostile to the CPU cache hierarchy for  $n > \sqrt{L_1/8}$  (typically  $n > 40$  on modern x86).

GVL4 Algorithm 1.3.1 describes a six-loop tiled variant that partitions  $A$ ,  $B$ , and  $C$  into  $b \times b$  sub-blocks (where  $b$  is chosen so that three blocks fit in L1/L2 cache) and performs small *block* matrix multiplies at each step. Implementing this in pure Haskell would not match OpenBLAS’s hand-tuned assembly, but experience from other languages suggests tiled GEMM typically yields  $3\text{--}10\times$  improvement over the naïve loop for  $n \geq 100$ , which would narrow the current  $300\times$  gap to  $30\text{--}100\times$ .

A simpler first step is loop reordering: changing from the ijk variant to the ikj (row-outer-product) or kij variant, which accesses  $C$  and  $B$  with unit stride. This alone can yield  $2\text{--}4\times$

improvement on cache-unfriendly sizes and requires only changing the loop nesting order in the existing `foldl'` computation.

### 8.3.4 Divide-and-Conquer Eigenvalue and SVD

The current eigenvalue solver uses the classical tridiagonal QR algorithm (GVL4 [1] Algorithm 8.3.3), which has  $O(n^2)$  cost per eigenvalue in the worst case and  $O(n^3)$  overall. LAPACK’s `DSYEV` uses a divide-and-conquer approach (GVL4 Algorithm 8.4.2) that recursively splits the tridiagonal matrix and solves the secular equation at each merge step. In practice, divide-and-conquer is 2–5× faster than the QR algorithm for dense matrices with  $n > 25$ , and it is also more amenable to parallelisation since the two sub-problems at each recursion level are independent.

Similarly, the current SVD uses iterated QR sweeps with Wilkinson shifts; LAPACK’s `DGESDD` uses a divide-and-conquer SVD. Implementing these would address the 16,000–21,000× gaps at  $50 \times 50$  (Table 11), which are inflated by the iterative algorithms’ per-step allocation cost compounding with algorithmic inefficiency.

### 8.3.5 SIMD Primitives

GHC provides experimental SIMD support via the `ghc-prim` package, exposing 128-bit and 256-bit vector types (`DoubleX2#`, `DoubleX4#`) with fused multiply-add operations. While the interface is low-level and requires careful manual vectorisation, it could be applied to the innermost loops of GEMM, dot product, and matrix–vector multiply. A 4-wide `DoubleX4#` FMA would process four  $C_{ij}$  accumulations per cycle, giving a theoretical 4× throughput improvement on the inner loop—significant for Level 1 and Level 2 BLAS operations where the gap is dominated by per-element overhead rather than cache effects.

Alternatively, the `primitive-simd` or `simd` packages provide portable wrappers around GHC’s SIMD primops. The `vector` library (which underlies `massiv`’s Primitive representation) stores `Double` in contiguous pinned memory, making it compatible with SIMD load/store patterns.

### 8.3.6 Optional FFI Backend

For users who can accept an FFI dependency, `linear-massiv` could provide an optional backend that delegates Level 3 BLAS operations to the system BLAS/LAPACK via `hmatrix` or direct `cblas_dgemm` FFI calls, while preserving the type-safe `KnownNat`-indexed interface. This is architecturally straightforward: the `Matrix m n r e` type wraps a `massiv` array whose underlying Primitive representation is a pinned `ByteArray`, which can be passed to C via `unsafeWithPtr` or copied into an `hmatrix Matrix Double` with a single `memcpy`.

This approach would offer the best of both worlds—compile-time dimensional safety with BLAS-level performance—while keeping the pure Haskell implementation as the default for portability. A Cabal flag (e.g. `-f blas-backend`) could control which backend is linked, similar to how `vector-algorithms` provides optional C-accelerated sort routines.

### 8.3.7 Summary of Expected Impact

Table 12 estimates the cumulative effect of each proposed optimisation on the GEMM performance ratio at  $100 \times 100$ .

For factorisation and iterative algorithms (QR, eigenvalue, SVD), the in-place ST monad refactoring (Section 8.3.1) and implicit Householder representation (Section 8.3.2) are the highest-priority items, as they address the dominant allocation overhead that accounts for much of the 300–21,000× gaps. The divide-and-conquer algorithms (Section 8.3.4) would further reduce the gap for eigenvalue and SVD problems, particularly at moderate-to-large dimensions.

Table 12: Estimated impact of proposed optimisations on the  $100 \times 100$  GEMM performance ratio (current:  $329\times$ ).

Optimisation	Mechanism	Est. ratio
Current baseline	naïve ijk, pure allocation	$329\times$
+ Loop reorder (ikj)	unit-stride access	$\sim 100\text{--}160\times$
+ Cache-blocked tiling	L1/L2 reuse	$\sim 30\text{--}50\times$
+ SIMD (DoubleX4#)	4-wide FMA inner loop	$\sim 8\text{--}15\times$
+ FFI backend (OpenBLAS)	delegate to DGEMM	$\sim 1\times$

## 8.4 When to Use Each Library

**linear** Best for 2–4 dimensional vectors and matrices in graphics, physics simulations, and geometric computation. Unbeatable at small sizes; does not scale to arbitrary dimensions.

**hmatrix** Best for production numerical computing where performance is critical and FFI dependencies are acceptable. The established choice for scientific computing in Haskell.

**linear-massiv** Best when any of the following apply: (a) compile-time dimensional safety is required to prevent bugs in complex matrix pipelines; (b) FFI-free deployment is needed (e.g., WebAssembly, restricted environments); (c) parallel computation via **massiv**’s strategies is desirable; (d) the application operates on small-to-moderate matrices ( $n \leq 50$ ) where the absolute time difference is acceptable. Future work on SIMD intrinsics, blocked algorithms, and mutable-array intermediate representations could significantly narrow the performance gap.

## 9 Conclusion

We have benchmarked three Haskell linear algebra libraries across eight categories of numerical operations. The results confirm the expected performance hierarchy: **linear** dominates at fixed small dimensions through GHC’s unboxing optimisations; **hmatrix** (OpenBLAS) dominates at all sizes through BLAS/LAPACK’s decades of assembly-level optimisation; and **linear-massiv** provides a pure Haskell baseline that is  $36\text{--}21,000\times$  slower than **hmatrix** depending on operation and size, but offers unique advantages in type safety, portability, and user-controllable parallelism.

The parallel scaling measurements demonstrate that **linear-massiv** can achieve  $3\text{--}7\times$  speedups via **massiv**’s **Par** and **ParN** strategies, partially offsetting the single-threaded performance gap. At  $100 \times 100$  with 16 threads, GEMM runs in 86.0 ms—still  $56\times$  slower than **hmatrix**’s single-threaded 1.50 ms, but representing a meaningful improvement from the  $330\times$  single-threaded ratio.

Section 8.3 proposes six concrete optimisation directions for closing the gap, ordered by expected impact: (1) in-place factorisation via the ST monad, eliminating the per-iteration matrix allocation that dominates QR, eigenvalue, and SVD runtimes; (2) implicit Householder (compact WY) representation to avoid forming explicit  $Q$  matrices; (3) cache-blocked tiled GEMM and loop reordering for better cache utilisation; (4) divide-and-conquer eigenvalue and SVD algorithms; (5) SIMD primitives via GHC’s **DoubleX4#** for inner-loop vectorisation; and (6) an optional FFI backend delegating to system BLAS/LAPACK for users who can accept the dependency. These optimisations are cumulative: the first three alone could plausibly reduce the GEMM gap from  $329\times$  to  $\sim 30\text{--}50\times$ , and the in-place refactoring should dramatically improve factorisation and iterative solver performance.

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