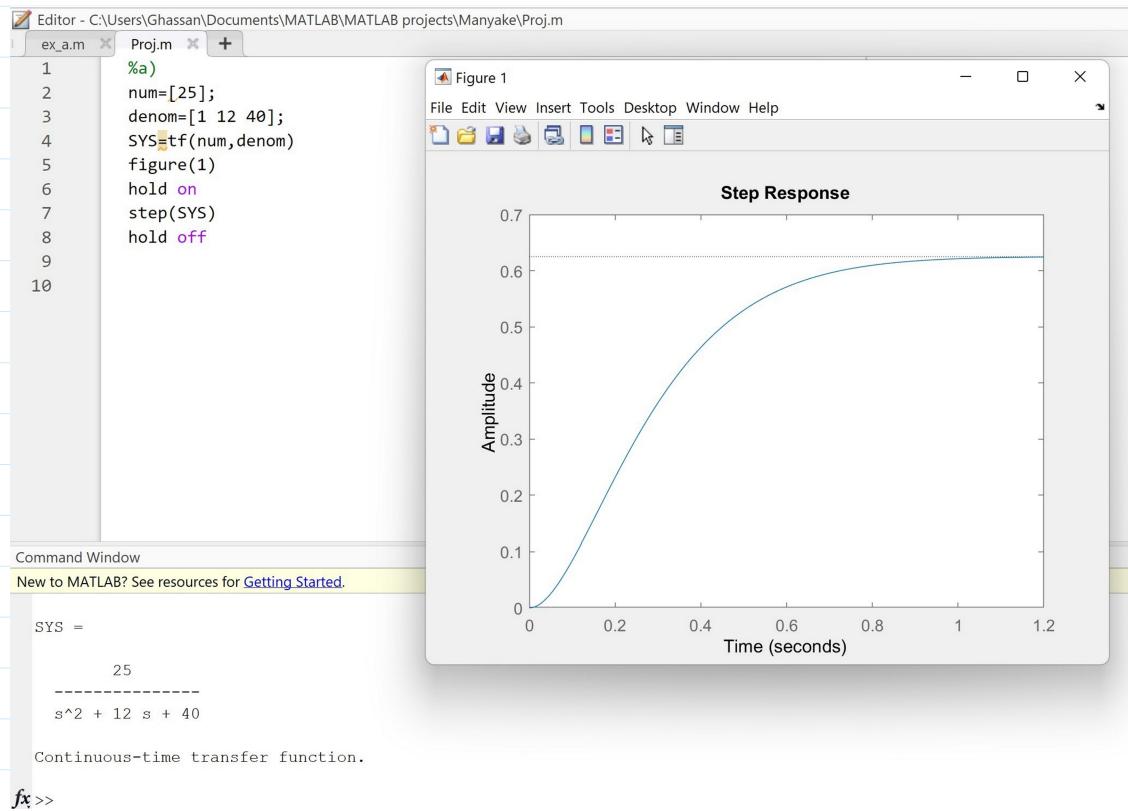


Problem 1:

a) The plot of the output representing the temperature at the back stabilizes at an "amplitude of 0.6" which is 18 degrees celsius. The temperature at the back is always gonna be less than the one at the front because the air traveling to the back is gonna loose energy along the way and become colder.



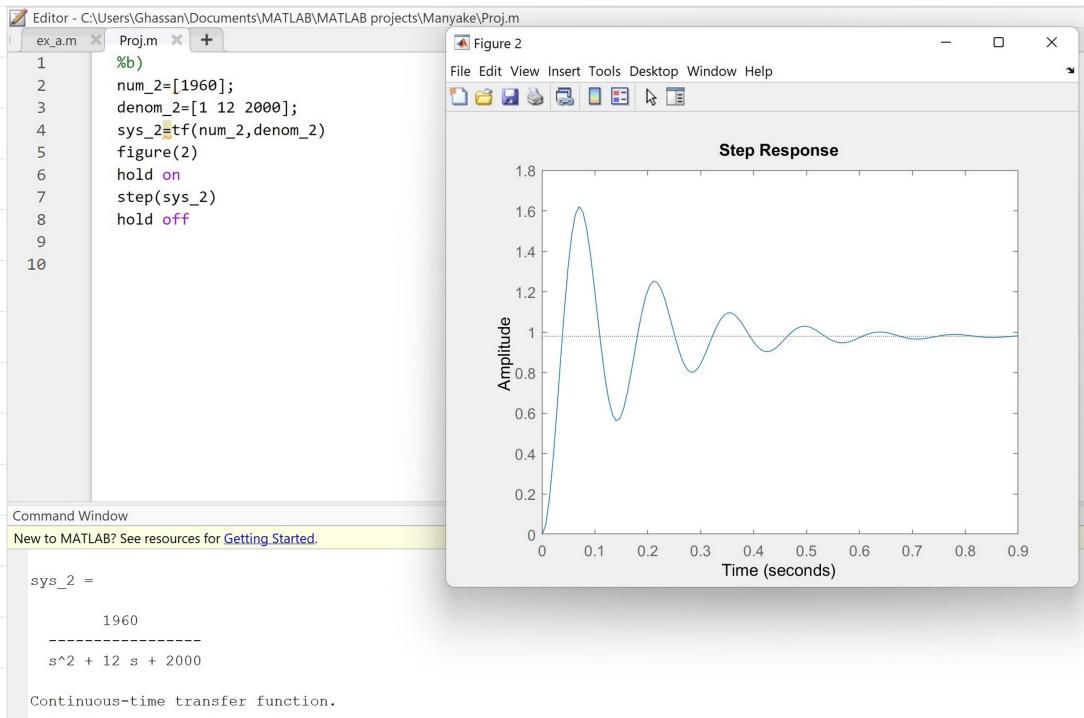
b)

$$b-1) \frac{E(s)}{R(s)} = \frac{1}{1+G(s)P(s)} = \frac{s^2 + 12s + 40}{s^2 + 12s + 40 + 2sK} \rightarrow E(s) = \frac{s^2 + 12s + 40}{s^2 + 12s + (40 + 2sK)}$$

$$(im e(t)) = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{s^2 + 12s + 40}{s^2 + 12s + 40 + 2sK} = \frac{40}{40 + 2sK} = 0,02$$

$$40 = 0,8 + 0,8K \rightarrow K = 78,4$$

$$b-2) H(s) = \frac{1960}{s^2 + 12s + 2000}$$



b_3) The presence of K in the denominator affects the widthness of the oscillations at the beginning so the bigger the K, the less wide the oscillations are and so the faster the output stabilizes. But the presence of K in the numerator is gonna affect the peak of the oscillations, the higher the K, the higher the temperature jumps and so we would be reaching undesirable temperatures at the beginning before reaching stability

$$c-1) H(s) = \frac{G(s)P(s)}{1+G(s)P(s)} = \frac{25K(1+0.05s)}{s^2 + 12s + 40 + 25K(1+0.05s)} = \frac{25 + 1.25s}{s^2 + 13.25s + 65}$$

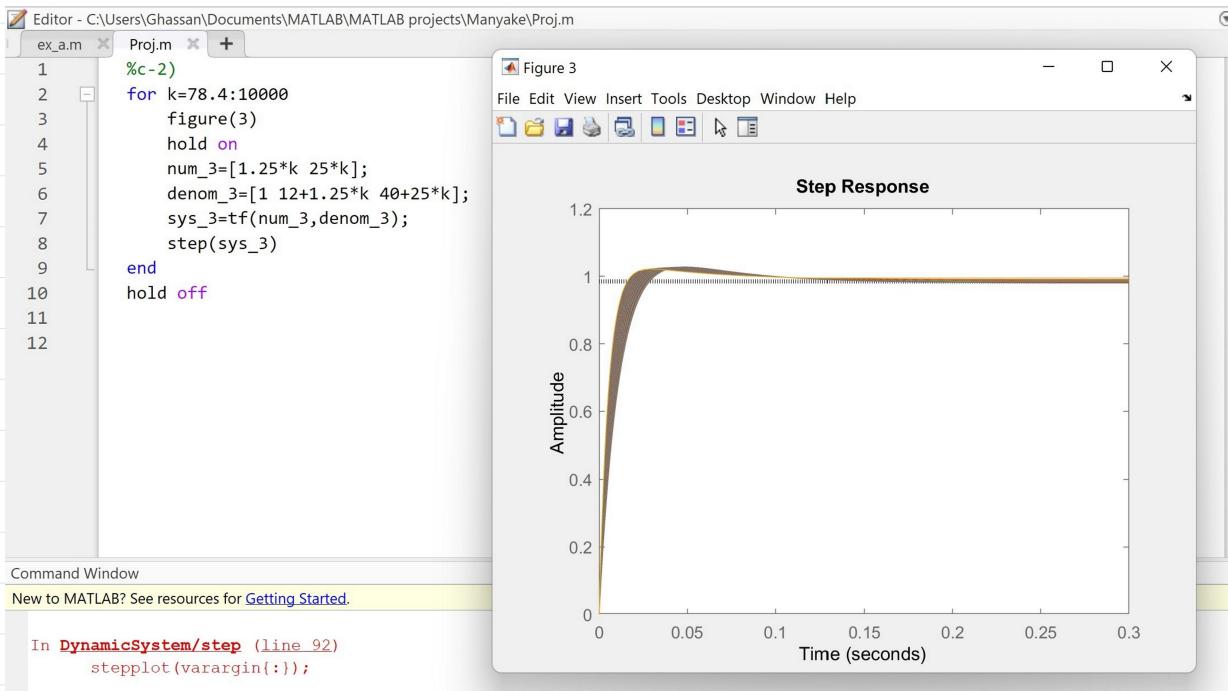
the zero is $s = -20$

$$c-2) E(s) = \frac{1}{R(s)} = \frac{1}{1+G(s)P(s)} = \frac{s^2 + 12s + 40}{s^2 + 12s + 40 + 25K(1+0.05s)}$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = \frac{40}{40+25K} \leq 0.02 \rightarrow K \geq 78.4$$

Using MATLAB we try $K = 78.4 \rightarrow y(t) < 1.5$
 the bigger the K, the smaller the peak of $y(t)$
 So any $K \geq 78.4$ will satisfy the 2 conditions

Using a for loop we vary K from 78.4 to 10000, we notice that none of the plot surpasses 1.5



$$d) \frac{E(s)}{R(s)} = \frac{s^2 + 12s + 40}{s^3 + (12 + 1.25K)s^2 + (25K + 40 + 0.25K)s} = \frac{s(s^2 + 12s + 40)}{s^3 + (12 + 1.25K)s^2 + (25K + 40)s + 25K}$$

$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s) = 0 \quad \forall K \in \mathbb{R}$
 We shall need to satisfy stability

R.H. Tables

$$s^3 \quad 1 \quad 25K+40$$

$$C_r = \frac{(12 + 1.25K)(25K + 40) - 25K}{12 + 1.25K}$$

$$s^2 \quad 12 + 1.25K \quad 25K$$

$$s \quad C_r \quad 0$$

$$s^0 \quad 25K$$

$$= \frac{300K + 480 + 31.25K^2 + 50K - 25K}{12 + 1.25K}$$

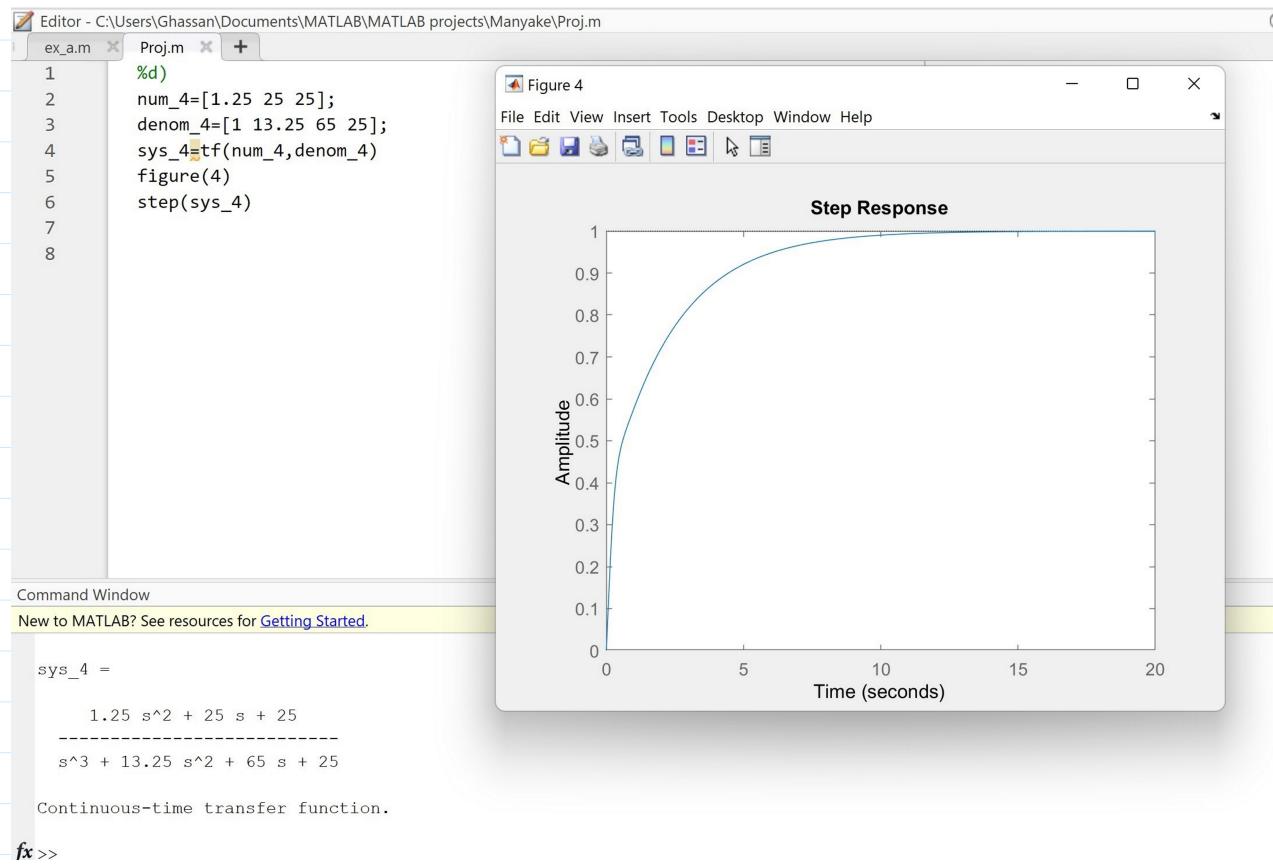
$$C_r = \frac{31.25K^2 + 325K + 480}{12 + 1.25K} > 0$$

$$12 + 1.25K > 0, \quad 25K > 0$$

$$K > -9.6 \quad K > 0 \quad K < -8.617 \text{ or } K > -1.78$$

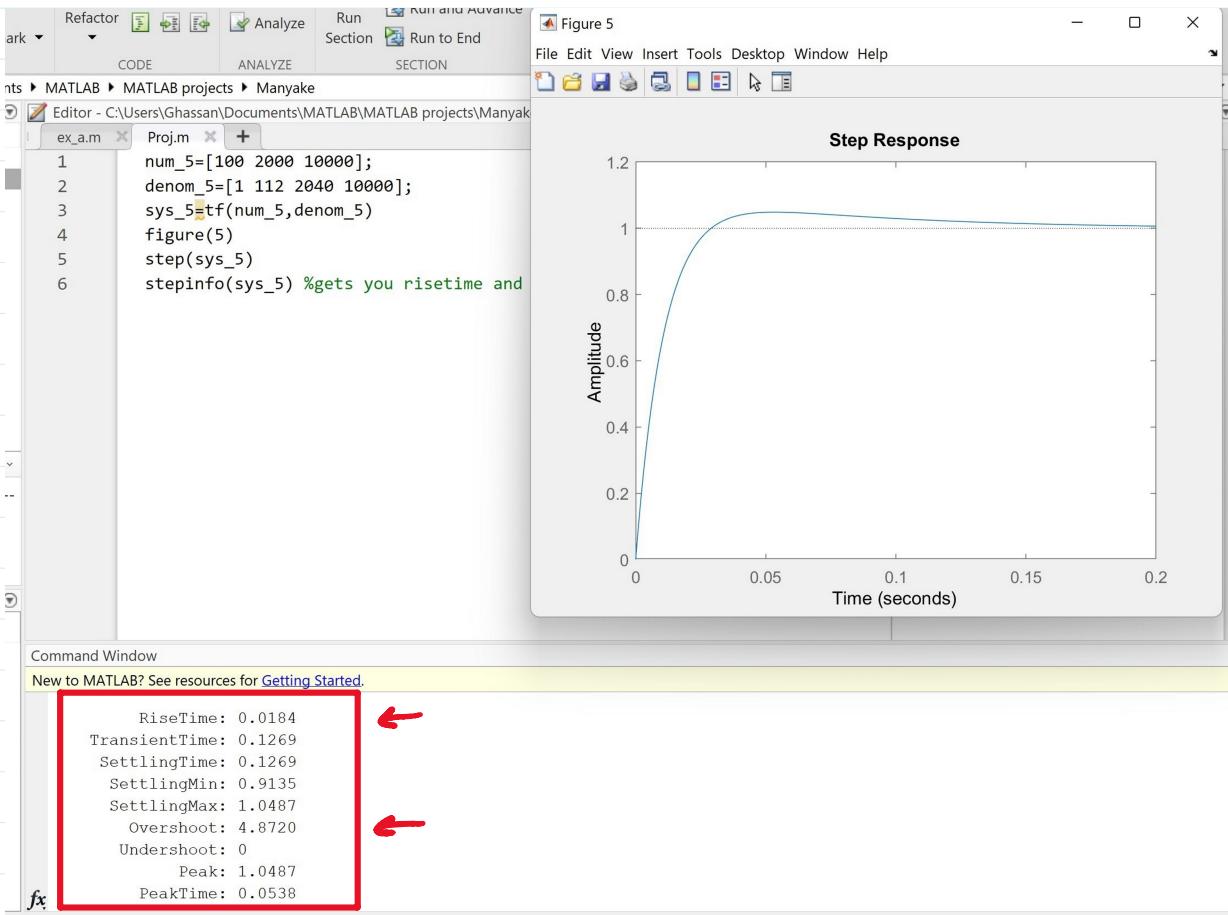
$S_0 | K > 0$

$$\frac{Y(s)}{R(s)} = \frac{G(s)P(s)}{1 + G(s)P(s)} = \frac{25(1 + 0.05s + 1/s)}{s^3 + 12s^2 + 40 + 25(1 + 0.05s + 1/s)} = \frac{25s^2 + 1.25s^3 + 25}{s^3 + (13.25)s^2 + 65s + 25}$$



e)

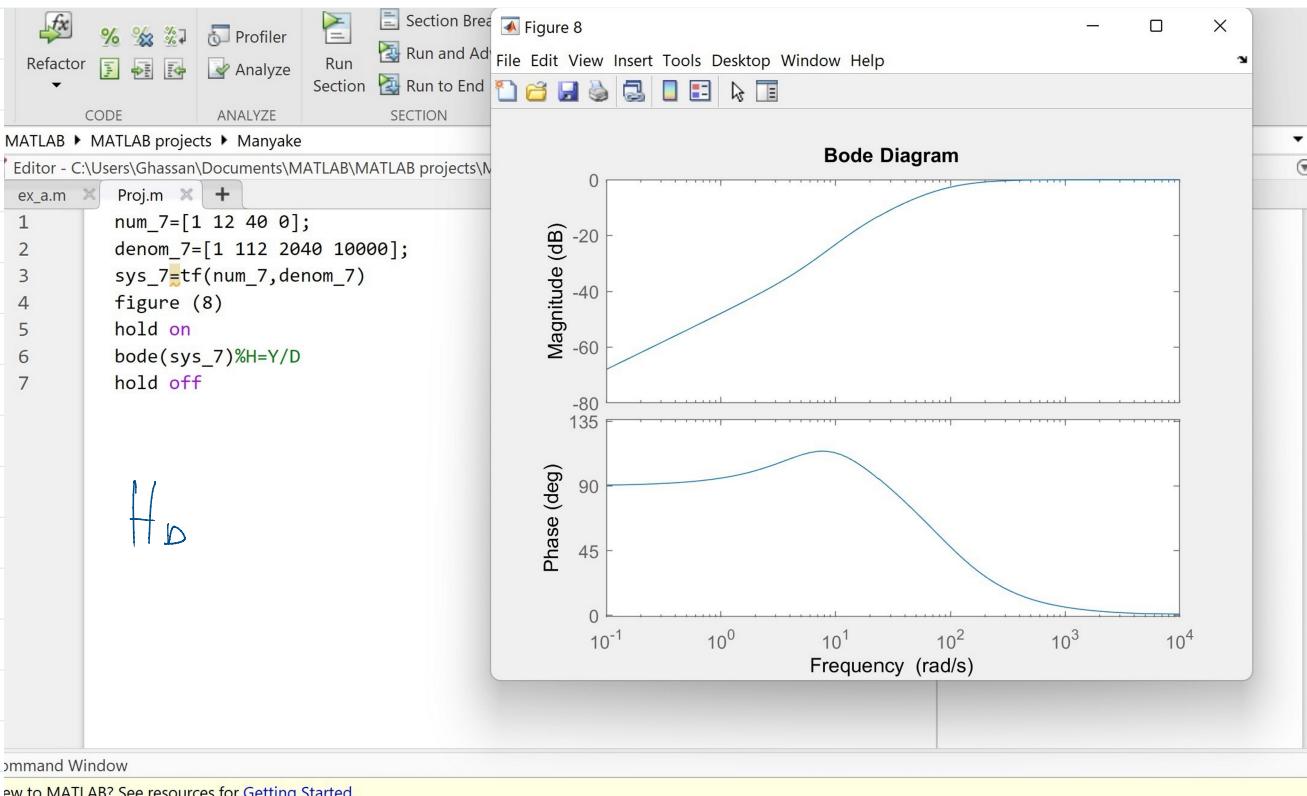
$$e-1) \frac{Y(s)}{R(s)} = \frac{2000s + 100s^2 + 10000}{s^3 + 112s^2 + 2040s + 10000}$$



$$e-2) \quad G(s)R(s) = \frac{80(1+0.05s + 5/s)s^2}{s^2 + 12s + 40} = \frac{100s^2 + 2000s + 10000}{(s^2 + 12s + 40)s}$$

$$H_o(s) = \frac{1}{1+G(s)R(s)} = \frac{(s^2 + 12s + 40)s}{s^3 + 112s^2 + 2040s + 10000}$$

$$H_N(s) = -\frac{G(s)R(s)}{1+G(s)R(s)}$$



Command Window

New to MATLAB? See resources for [Getting Started](#).

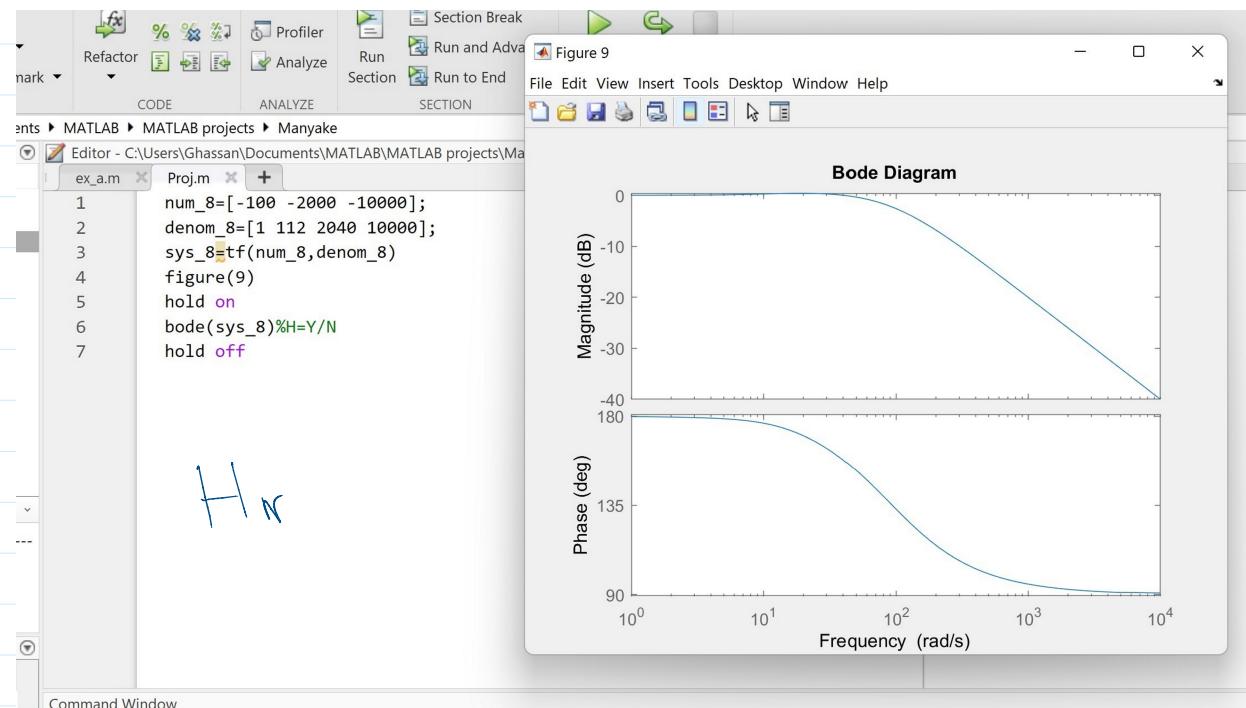
```

sys_7 =

```

$$\frac{s^3 + 12 s^2 + 40 s}{s^3 + 112 s^2 + 2040 s + 10000}$$

Continuous-time transfer function.

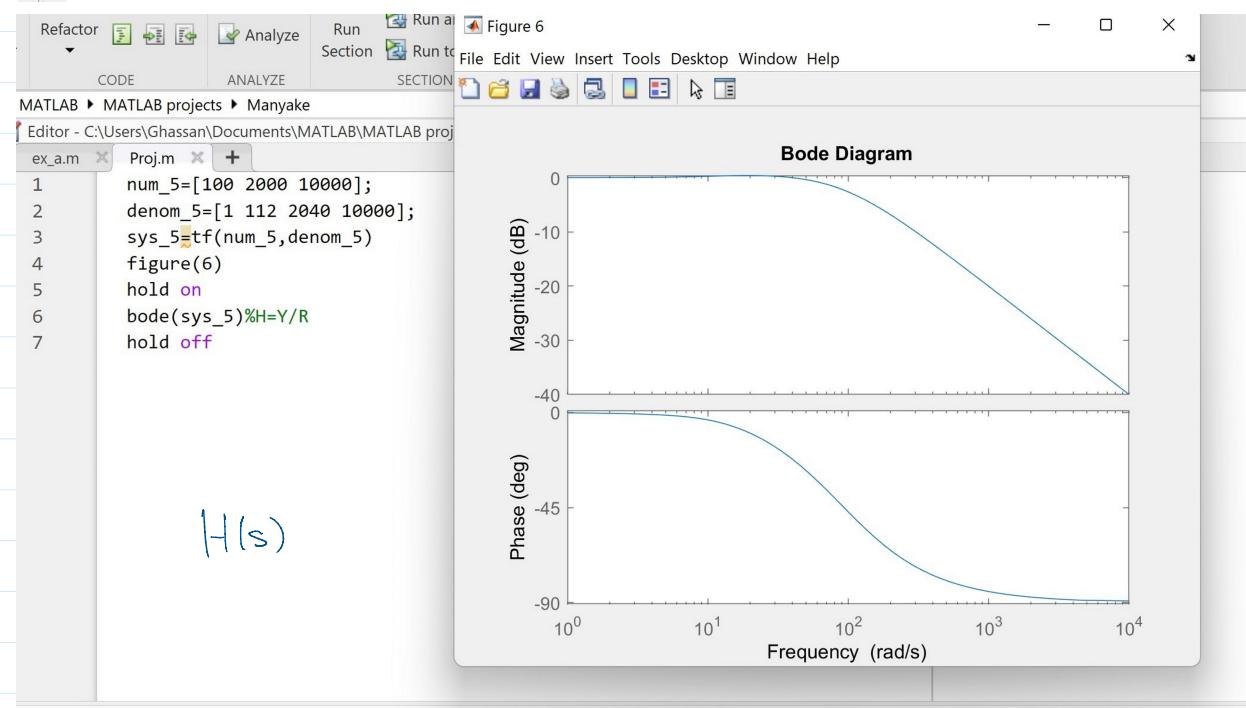


Command Window

New to MATLAB? See resources for [Getting Started](#).

```
sys_8 =
-100 s^2 - 2000 s - 10000
-----
s^3 + 112 s^2 + 2040 s + 10000

Continuous-time transfer function.
```

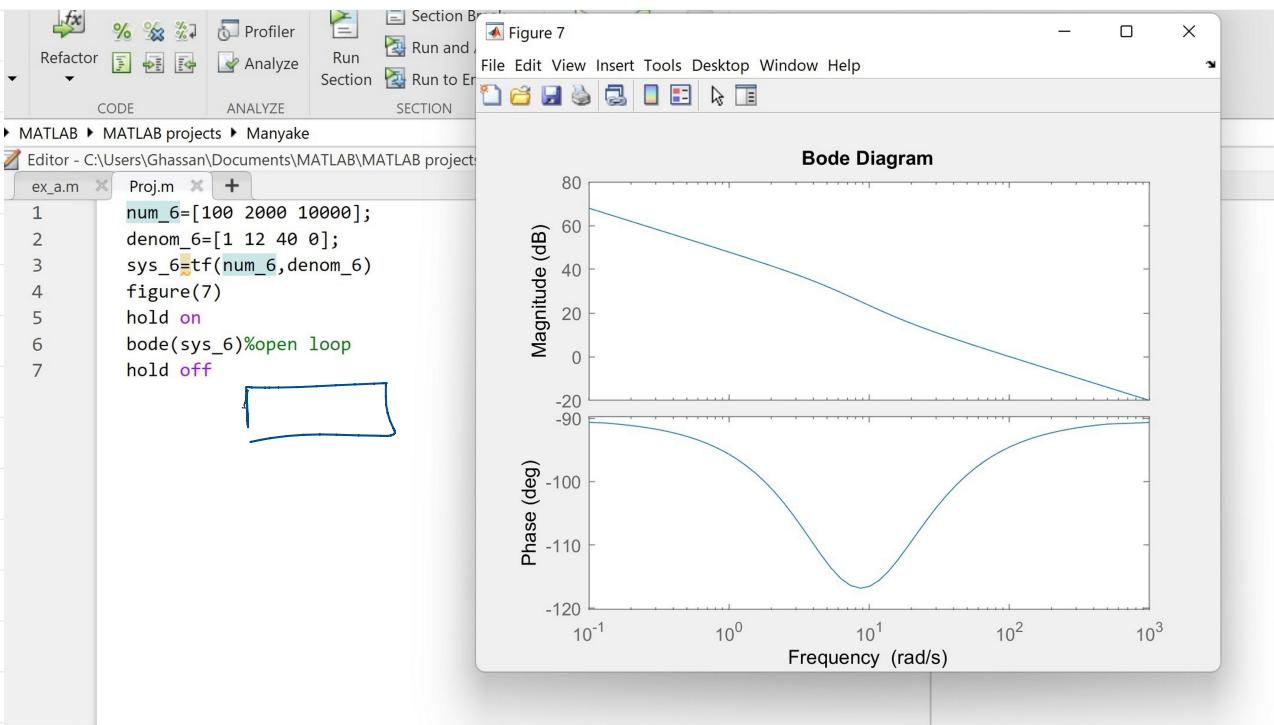


Command Window

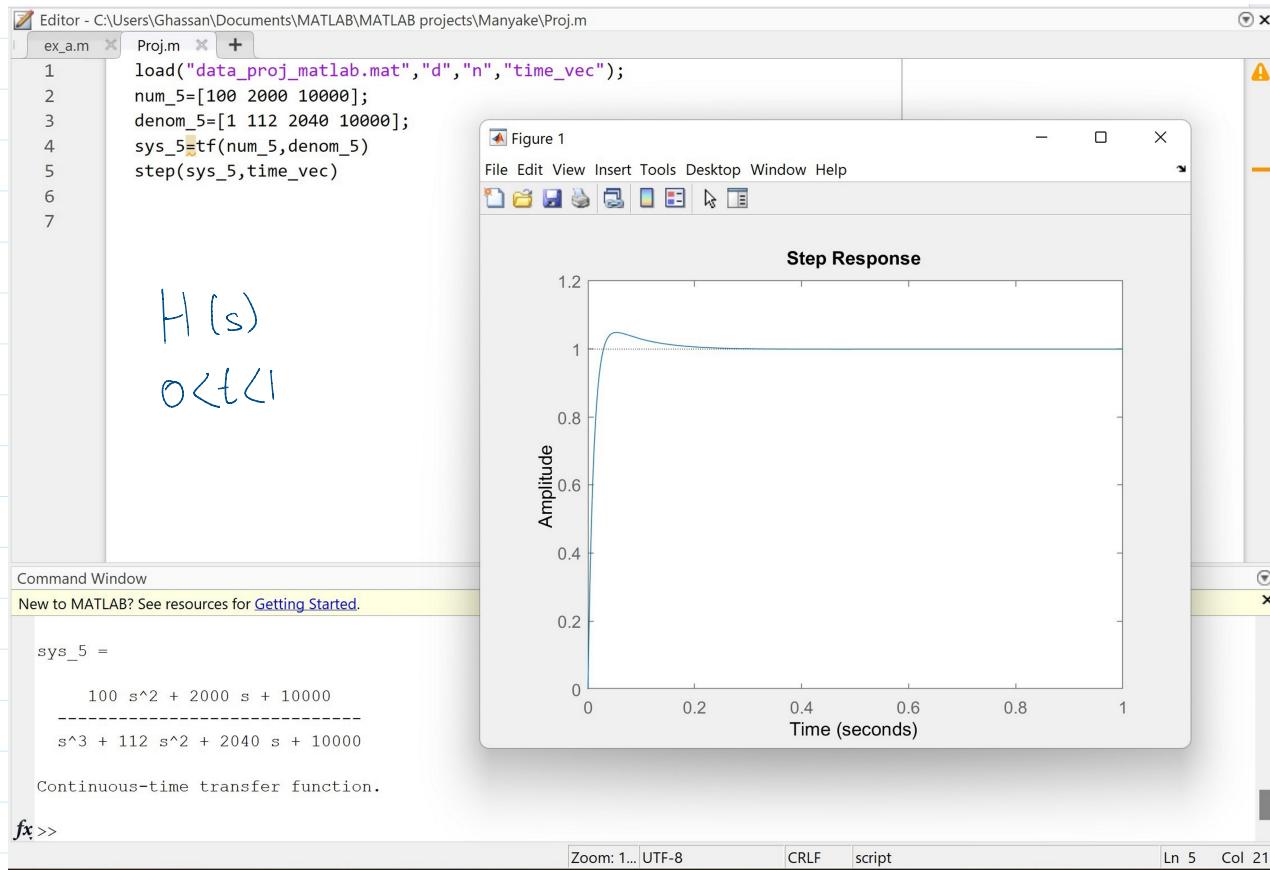
New to MATLAB? See resources for [Getting Started](#).

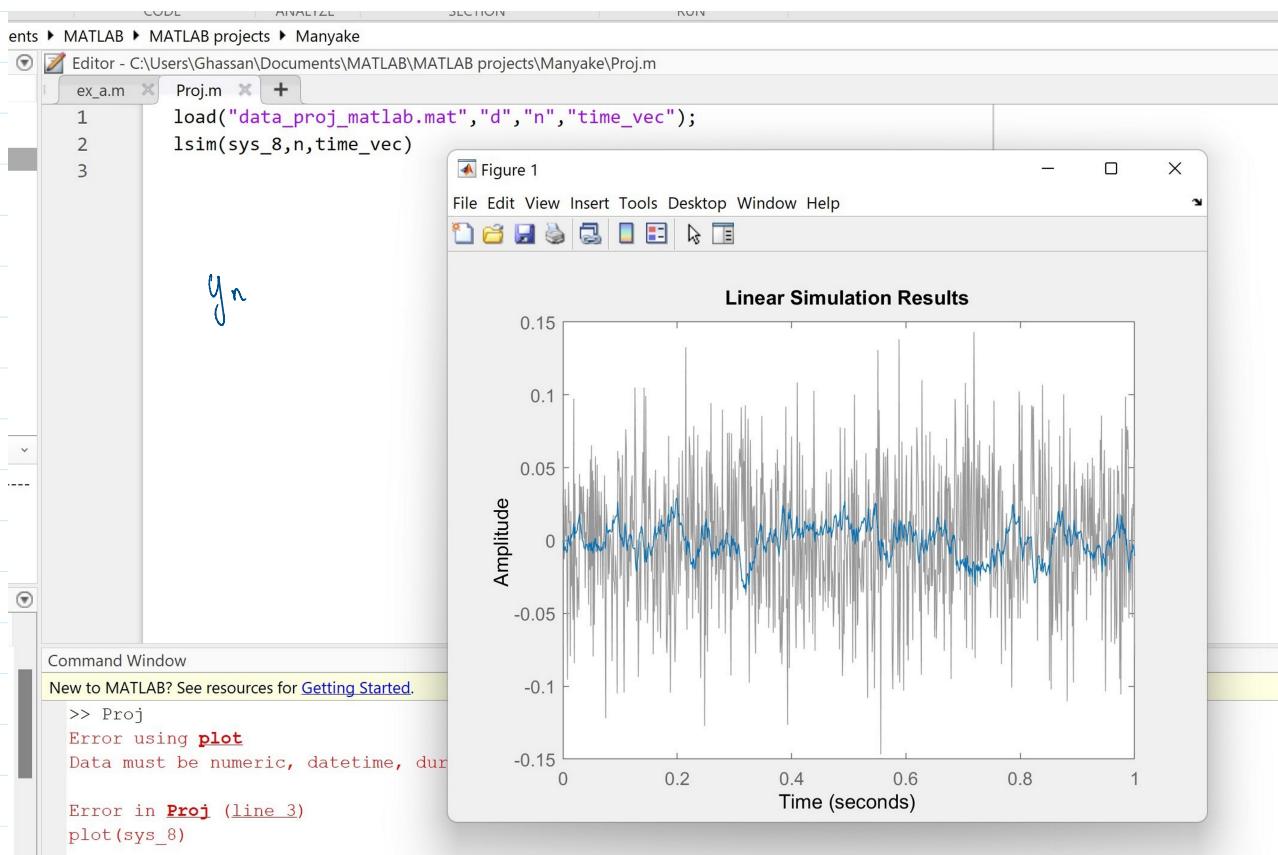
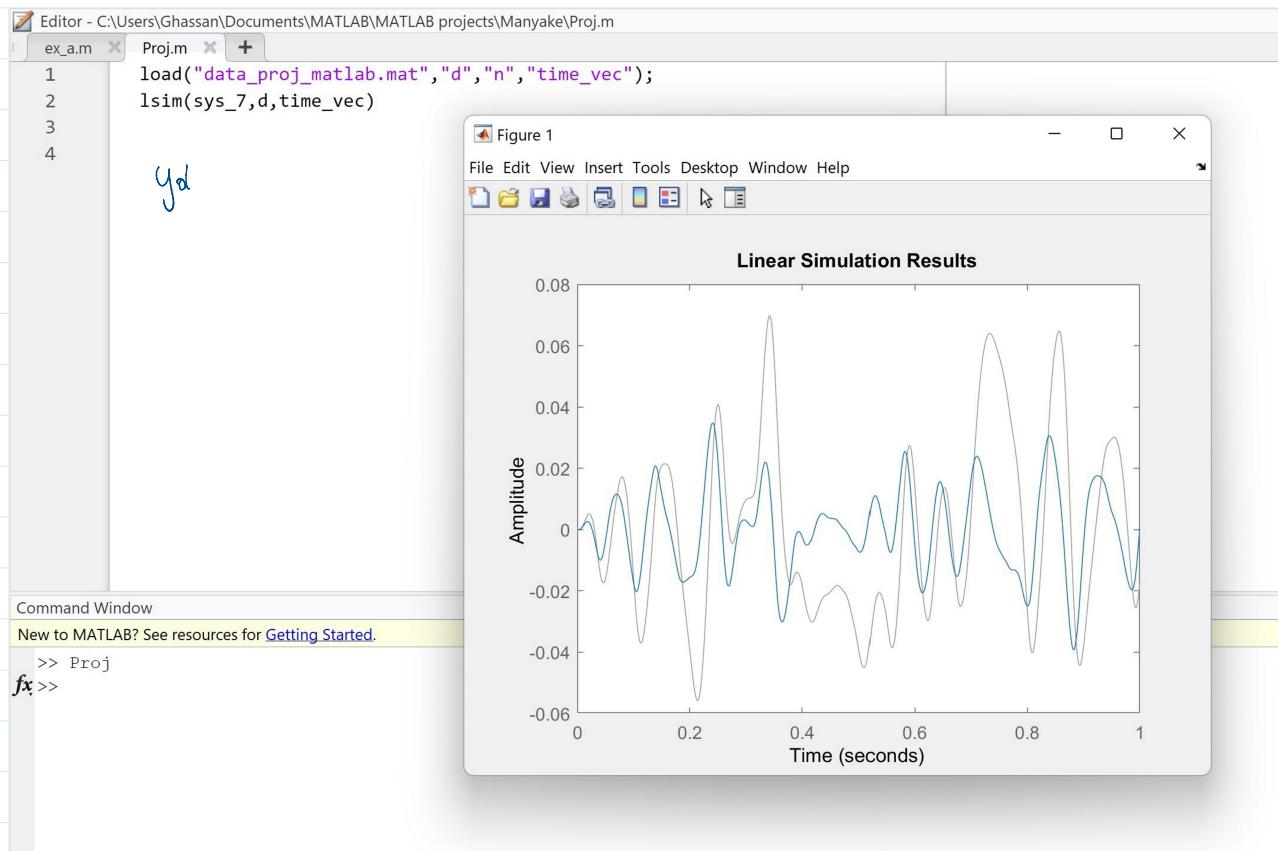
```
sys_5 =
100 s^2 + 2000 s + 10000
-----
s^3 + 112 s^2 + 2040 s + 10000

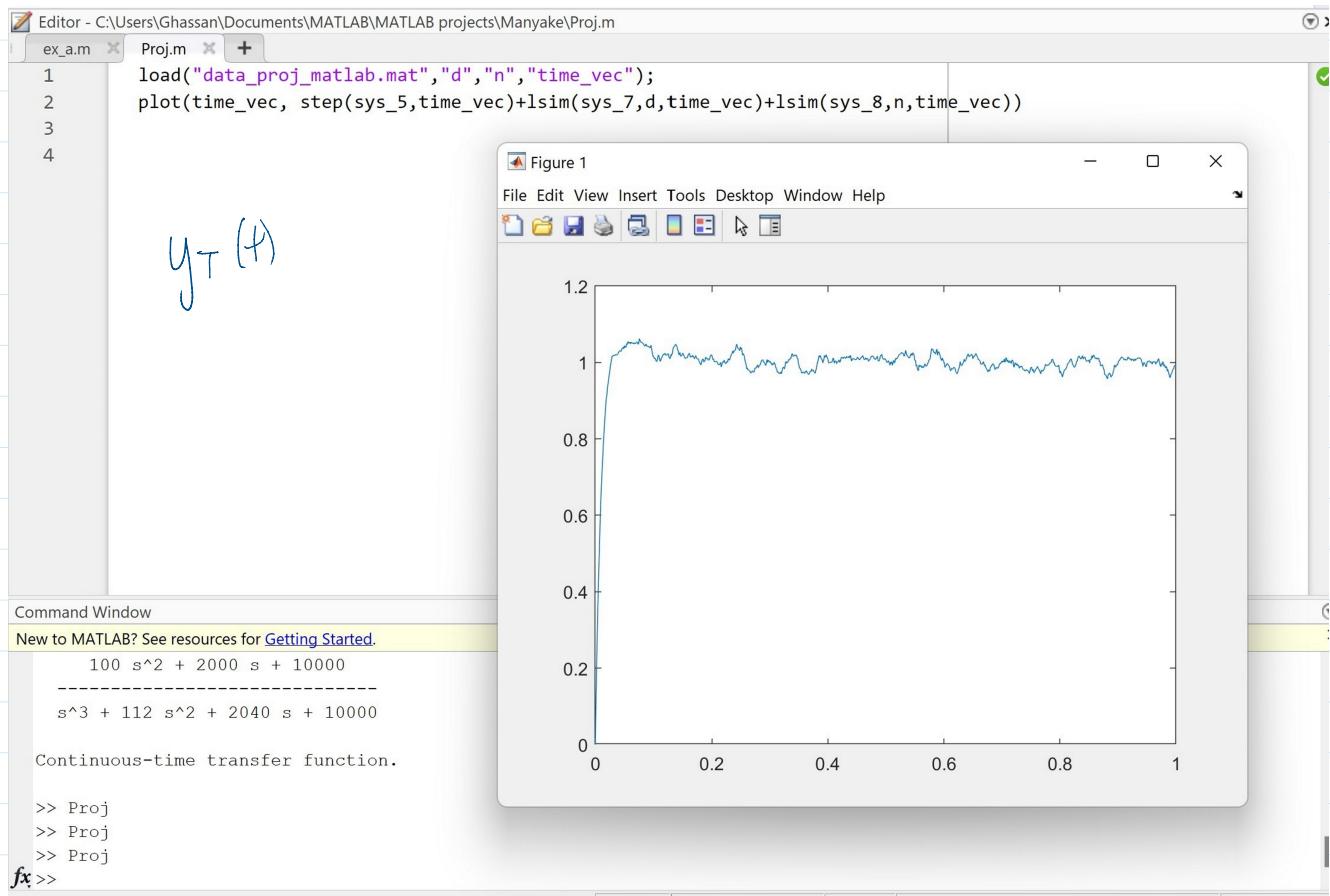
Continuous-time transfer function.
```



e-3)







$$e-4) G(s)P(s) = \frac{8(1+0.05s+s(s) \times 25)}{s^2 + 12s + 40} = \frac{(0s^2 + 200s + 1000)}{s^3 + 12s^2 + 40s}$$

$$- \frac{Y(s)}{R(s)} = \frac{G(s)P(s)}{1+G(s)P(s)} = \frac{10s^2 + 200s + 1000}{s^3 + 12s^2 + 40s}$$

The overshoot and rise time are much bigger than the ones obtained with $K=80$, which is bad because this means it reaches a higher peak and takes longer to stabilize.

$$- \frac{Y(s)}{D(s)} = \frac{s^3 + 12s^2 + 40s}{s^3 + 22s^2 + 240s + 1000}$$

$$\frac{Y(s)}{N(s)} = - \frac{G(s)P(s)}{1+G(s)P(s)} = - \frac{Y(s)}{R(s)}$$

