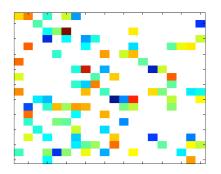
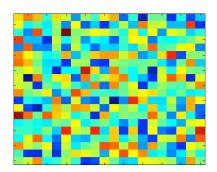


Unsupervised Learning - Matrix Completion

Creating our own truth





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Your Desktop

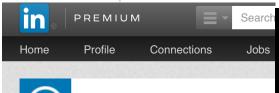
People You May Know



Google Apps
Administrator Guide: A
Private-Label Web
Workspace

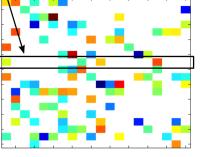


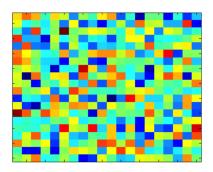
Googlepedia: The Ultimate Google Resource (3rd Edition)





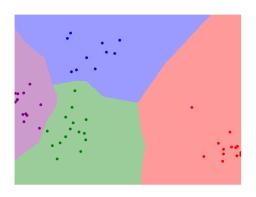
You (a single row)





How Do I Want to Fill in the Missing Entries?

k-Nearest Neighbors



Assumption: My "suggested friends" should be an average of my k closest friends' lists.

k-Nearest Neighbors

$$D_{\text{euclidian}} = \left(\sum_{i=i}^{n} |x_i - y_i|^2\right)^{1/2}$$

$$D_{manhattan} = \sum_{i=i}^{n} |x_i - y_i|$$

$$D_{\text{euclidian}} = \sqrt{1^2 + 1^2 + 1^2 + 1^2 + 10^2} = \sqrt{104} \approx 10$$

$$D_{\text{manhattan}} = 1 + 1 + 1 + 1 + 10 = 14$$

Low Rank Matrix Completion

$$\min_{M} \quad \text{rank}(M)$$
subject to
$$M_{ij} = X_{ij}$$

The *rank* of a matrix is defined as the number of linearly independent column vectors in a matrix.

Assumption: Our data matrix X can be represented by a small number of characteristics.

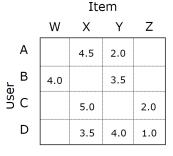
Low Rank Matrix Completion

		-1		
			1	
1	1	-1	1	-1
1				-1
		-1		

1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1
1	1	-1	1	-1

Matrix Factorization

$$\begin{aligned} & \underset{P,Q}{\text{min}} & & ||X-PQ||_F^2 \\ & \text{where} & & X \in \mathbb{R}^{n \times d}, \; P \in \mathbb{R}^{n \times r}, \; Q \in \mathbb{R}^{r \times d}, \\ & & & & r << d \end{aligned}$$



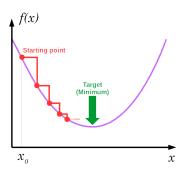
Α	1.2	0.8
В	1.4	0.9
С	1.5	1.0
D	1.2	0.8

W	Χ	Υ	Z
1.5	1.2	1.0	0.8
1.7	0.6	1.1	0.4

Rating Matrix

User Matrix Item Matrix

Matrix Factorization - Gradient Descent



Drawback: Since $P \in \mathbb{R}^{n \times r}$ and $Q \in \mathbb{R}^{r \times d}$ there are $(n \times r) + (r \times d)$ variables to optimize.

Matrix Factorization - Alternating Minimization

The Idea:

- 1. Fix Q and update P.
- 2. Then, fix P and update Q.
- 3. Rinse and repeat.

$$\min_{P,Q} ||X - PQ||_F^2$$

Derived Updates:

$$P^{t+1} = XQ^{T}(QQ^{T})^{-1}$$

 $Q^{t+1} = (P^{T}P)^{-1}P^{T}X$

Questions

These slides are designed for educational purposes, specifically the CSCI-470 Introduction to Machine Learning course at the Colorado School of Mines as part of the Department of Computer Science.

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