GPGN 552 Seismology I - Project
Point-Force Radiation in a Homogeneous Medium
Due: December 4, 2018

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I. Problem Statement

The purpose of the project is to study the wavefield generated by a point source in an unbounded homogeneous isotropic medium. The following is the exact expression for the displacement u(x,t) derived in class:

$$
\begin{aligned}
u_i(\mathbf{x}, t) = &\frac{1}{4\pi\rho}\left(3\gamma_i\gamma_j - \delta_{ij}\right)\frac{1}{R^3}\int_{R/\alpha}^{R/\beta}\tau X_0(t-\tau)d\tau \\
&+ \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{R}X_0\left(t-\frac{R}{\alpha}\right) \\
&+ \frac{1}{4\pi\rho\beta^2}\left(\delta_{ij} - \gamma_i\gamma_j\right)\frac{1}{R}X_0\left(t-\frac{R}{\beta}\right). \qquad (1)
\end{aligned}
$$

The first term represents the near field (NP), the second term represents the far field p-wave (FFP), and the third term represents the far field s-wave (FFS).

Any programming language or symbolic software (e.g., MATLAB or Mathematica) you are comfortable with is acceptable. Use one period of the sine function with a frequency of 25 Hz as the source wavelet X0(t). This will allow you to obtain the time dependence of the near-field term analytically; if you wish, you can evaluate this integral numerically. The velocity and density values are $\alpha = 2000$ m/s, $\beta = 1000$ m/s, and $\rho = 2.5$ g/cm^3.

Align the force with the x1 direction (j = 1) and calculate the seismograms of the two displacement components in the [x1, x3]-plane [u1(t) and u3(t)] for two source receiver distances: R = 80 m (near field) and R = 1000 m (far field). Rotate the source-receiver line from the horizontal (x1) to the vertical (x3) direction in 10◦ increments; this will give you 10 receiver locations for each distance R. In addition to the seismograms, compute the corresponding hodograms [u3(t) as a function of u1(t)], which represent the time-dependent particle motion. Please submit your code along with the seismograms and particle-motion hodograms.

II. Figures of Seismograms and Particle Motion Hodograms

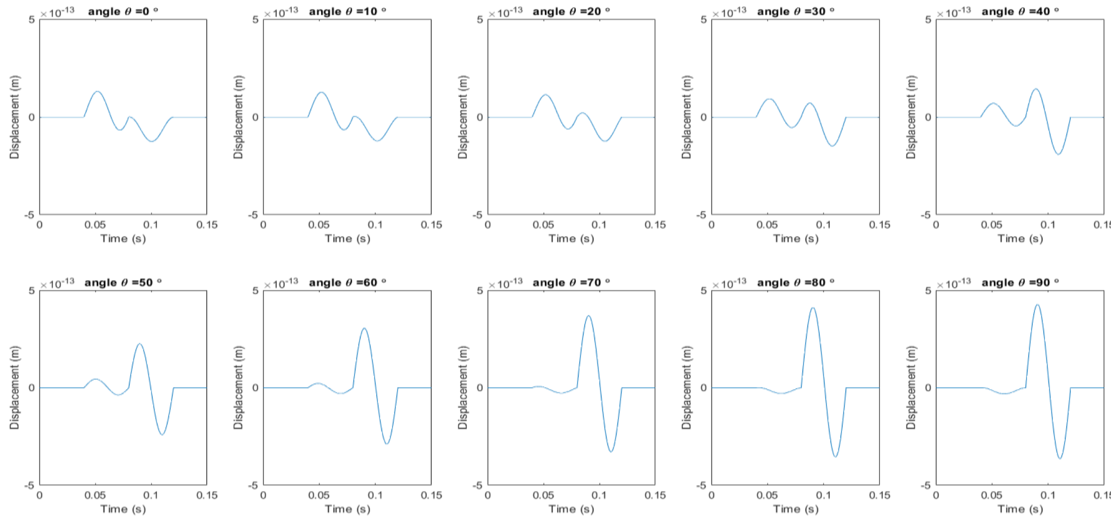

Figure 1: Seismograms for S-R Distance of 80 m in the U1 Direction

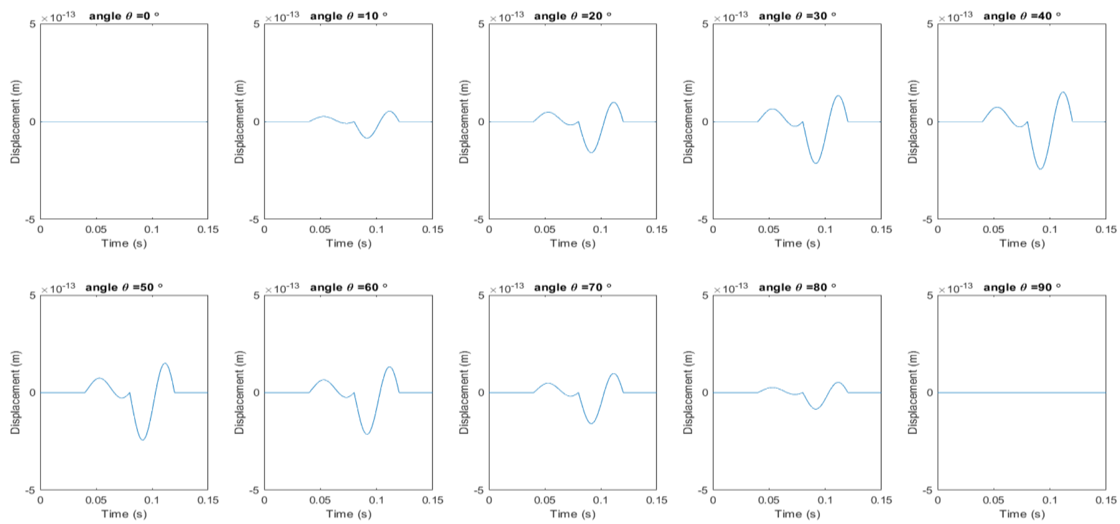

Figure 2: Seismograms for S-R Distance of 80 m in the U3 Direction

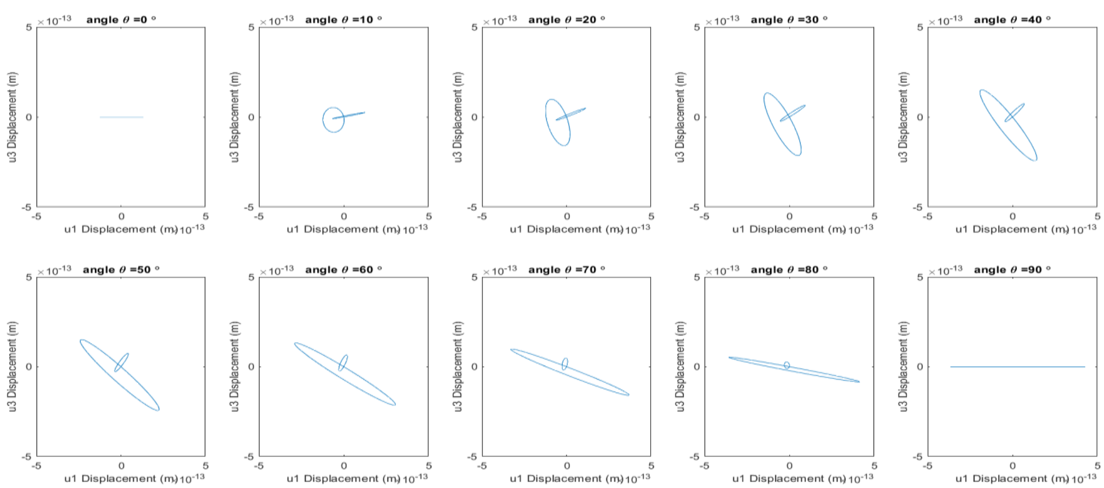

Figure 3: Particle Motion Hodograms for S-R Distance of 80 m

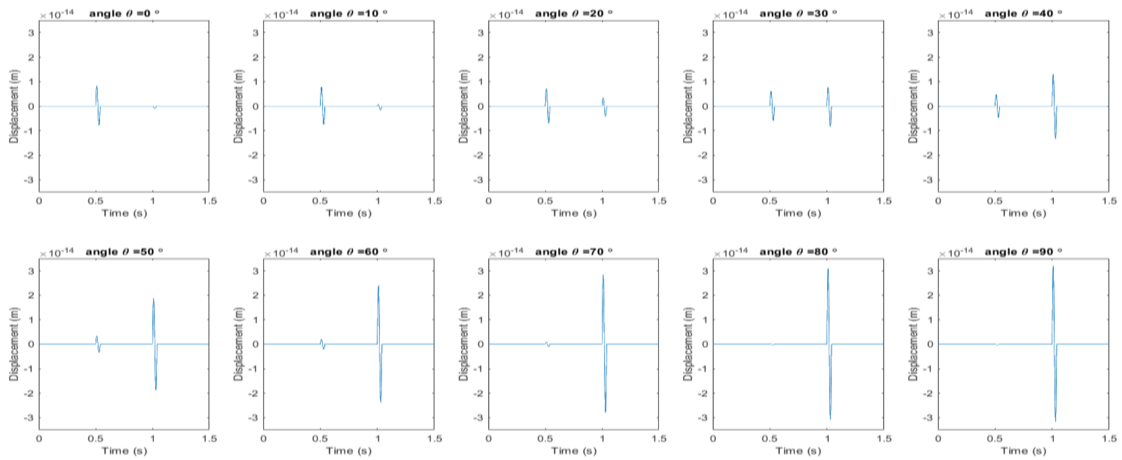

Figure 4: Seismograms for S-R Distance of 1000 m in the U1 Direction

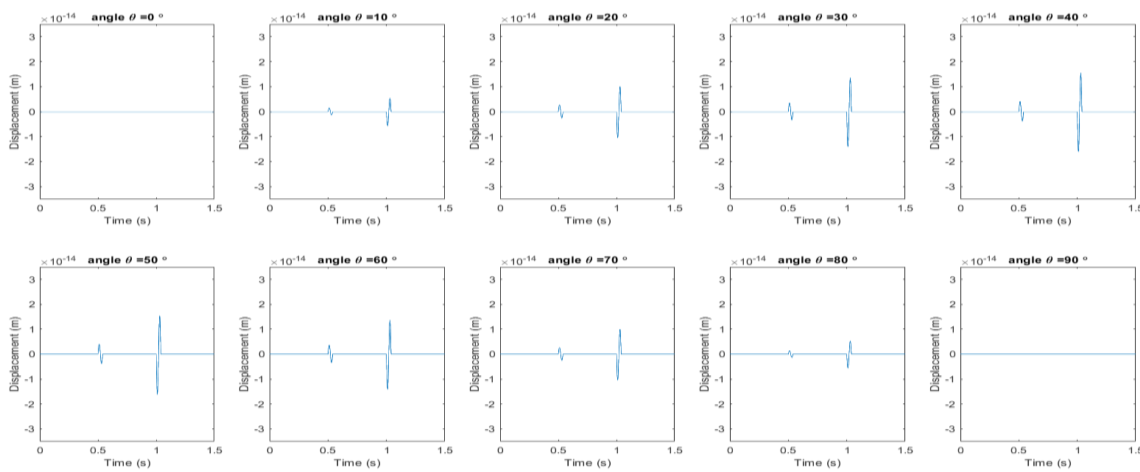

Figure 5: Seismograms for S-R Distance of 1000 m in the U3 Direction

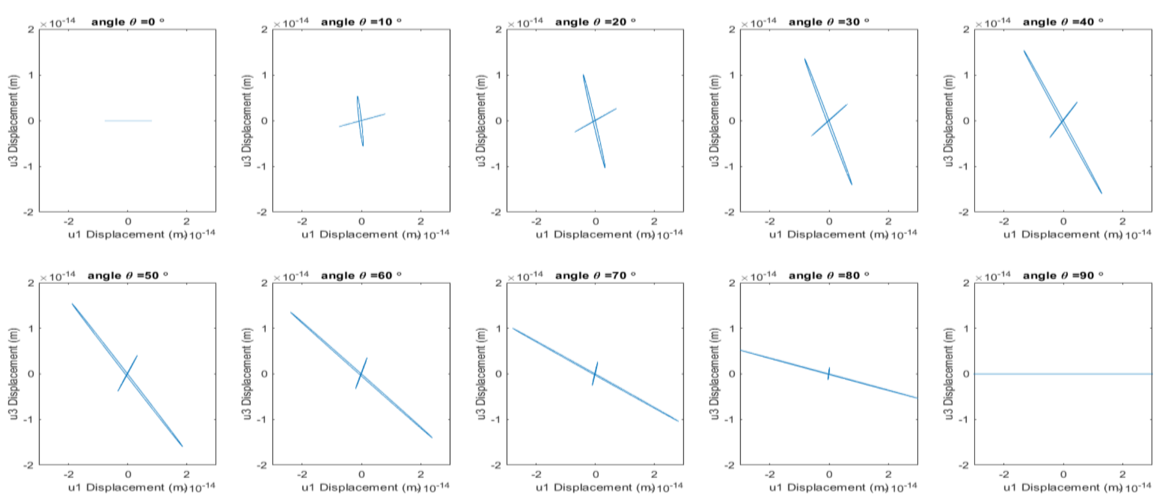

Figure 6: Particle Motion Hodograms for S-R Distance of 1000 m

III. Discussion

1. **The change in the amplitudes of the far-field P- and S- waves with angle and the angle-dependent P-to-S amplitude ratio (only for R = 1000 m).**

The Figure 7 below represents the change in amplitude of far field P-wave and S-wave with angles in the U1 direction, and the Figure 8 below represents the change in amplitudes of far field P-wave and S-wave with angles in the u3 direction.

In the U1 direction, Figure 7, the far field P-waves decrease in amplitude with increase in angle, and the far field S-waves increase in amplitude with increase in angle. In the U3 direction, Figure 8, both far field P-wave and S-wave increase in amplitude with increase in angle up until 45 degrees and then decrease in amplitude to zero until 90 degrees.

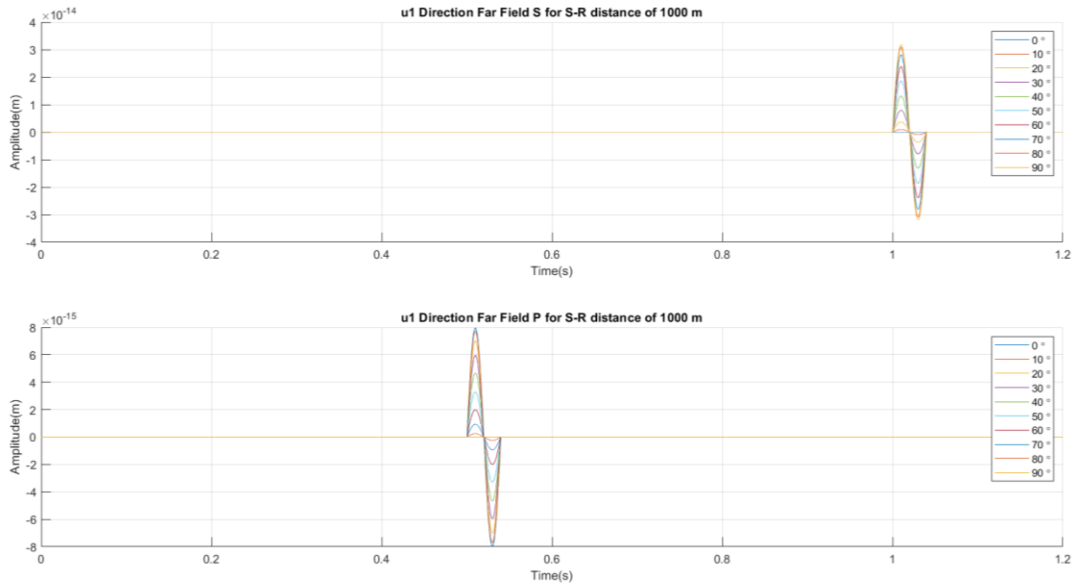

Figure 7: U1 Direction Seismogram for Far Field P- and S-Waves with Angles

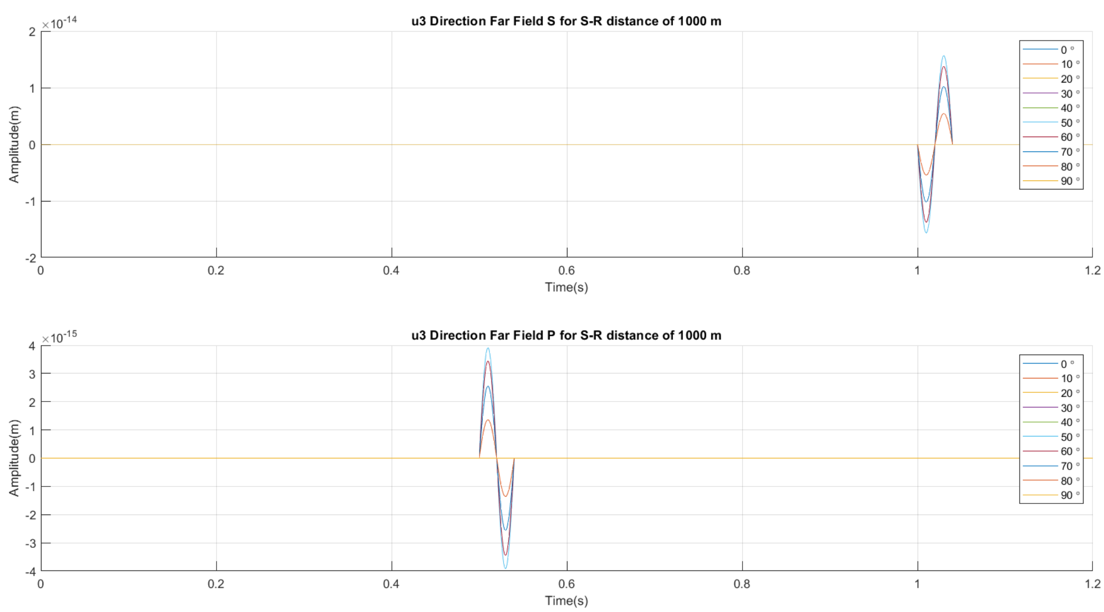

Figure 8: U3 Direction Seismogram for Far Field P- and S-Waves with Angles

For P/S Ratio in U1 Direction:

$$\frac{U1_{ffp}}{U1_{ffs}} = \frac{\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{R}x_o(t-R/\alpha)}{\frac{1}{4\pi\rho\beta^2}(\delta_{ij}-\gamma_i\gamma_j)\frac{1}{R}x_o(t-R/\beta)} = \frac{\frac{1}{4\pi\rho\alpha^2}\cos^2(\theta)\frac{1}{R}x_o(t-R/\alpha)}{\frac{1}{4\pi\rho\beta^2}(1-\cos^2(\theta))\frac{1}{R}x_o(t-R/\beta)} = \frac{\beta^2\cos^2(\theta)\frac{1}{R}x_o(t-R/\alpha)}{\alpha^2(1-\cos^2(\theta))\frac{1}{R}x_o(t-R/\beta)}$$

$$= \frac{1\cos^2(\theta)\frac{1}{R}x_o(t-R/\alpha)}{4\left(\sin^2(\theta)\right)\frac{1}{R}x_o(t-R/\beta)} = \frac{1}{4\left(\tan^2(\theta)\right)}$$

For P/S Ratio in U3 Direction:

$$\frac{U3_{ffp}}{U3_{ffs}} = \frac{\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{R}x_o(t-R/\alpha)}{\frac{1}{4\pi\rho\beta^2}(\delta_{ij}-\gamma_i\gamma_j)\frac{1}{R}x_o(t-R/\beta)} = \frac{\frac{1}{4\pi\rho\alpha^2}\cos(\theta)\sin(\theta)\frac{1}{R}x_o(t-R/\alpha)}{\frac{1}{4\pi\rho\beta^2}(-\cos(\theta)\sin(\theta))\frac{1}{R}x_o(t-R/\beta)}$$

$$= \frac{\beta^2\cos(\theta)\sin(\theta)\frac{1}{R}x_o(t-R/\alpha)}{\alpha^2\left(-\cos(\theta)\sin(\theta)\right)\frac{1}{R}x_o(t-R/\beta)} = \frac{1}{4}$$

The Figure 9 below shows the angle-dependent P-to-S amplitude ratio (R=1000 m) for the U1 and U3 directions. In the U1 direction, the P-to-S wave amplitude ratio decreases as the angle increases. The P-wave arrival is maximum at 0 degrees when parallel to the source and the S-wave arrival is zero. As the angle reaches 90 degrees, the amplitude of the P-wave decreases, and the S-wave decreases to zero. In the U3 direction, the P-wave arrivals are always ¼ the amplitude of the S-wave arrival and the shapes of both arrivals are opposite. This results in the flat line of the P-to-S wave constant amplitude ratio of ¼ throughout all angles in the U3 direction.

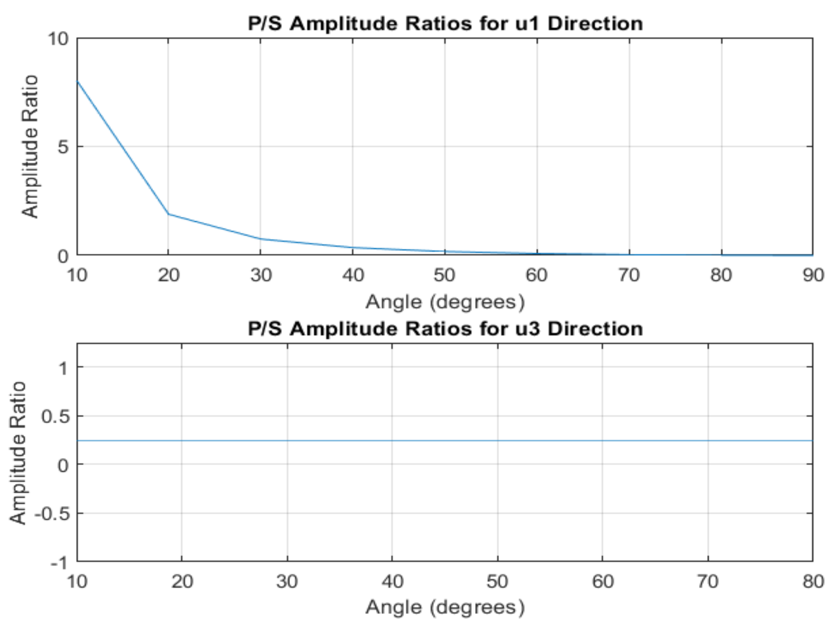

Figure 9: P- to S- Waves Amplitude Ratios with Angles for Far Field for R=1000 M in the U1 and U3 Directions

2. **The presence of energy at the S-wave arrival time in the direction parallel to the force, as well as energy at the P-wave arrival time in the direction perpendicular to the force. What is the polarization of these anomalous arrivals? Can you see them well in the far field?**

   The near field has a mixed polarization because the near field polarization is the sum of both the far field P-wave and far field S-wave polarization terms. In the direction parallel to the source, the energy at the S-wave arrival time (R/β) is the mixed polarization of the near field term that creates the S-wave energy with the P-wave polarization term. Similarly, in the direction perpendicular to the source, the energy at the P-wave arrival time (R/α) is the mixed polarization of the of the near field term that generates the P-wave with the S-wave polarization term.

   The anomalous arrivals can be seen in the Figures 1 to 5 above. For example, in Figure 1, at an angle of 0 degree and similarly at 90 degrees, the S-wave got influenced by the P-wave and becomes nonzero. In Figure 4, parallel to the force, there should only be P-wave arrival at an angle of 0 degree, but there is an additional anomalous wave after the P-wave. The anomalous wave has a small negative arrival with S-wave velocity and P-wave polarization, due to the mixed polarization with the near field term. Similarly, in the same Figure 4 at 90 degrees, there is a small negative anomalous arrival due to the mixed polarization affect.

   The Figure 10 below represents the near field signal for an angle at 0 degrees and 90 degrees for S-R distance of 80 m in the U1 direction. The Figure 11 below represents the near field for S-R distance of 1000 m in the U1 direction. Overall, the anomalous near field arrivals decays at a rate of $1/R^2$, thus can be seen in the far field but are small in amplitude. For example, the magnitude of the wave amplitude (order of 10^{-16}) in the far field, in Figure 11, is much smaller than the magnitude of the wave amplitude (order of 10^{-13}) in the near field, in Figure 10.

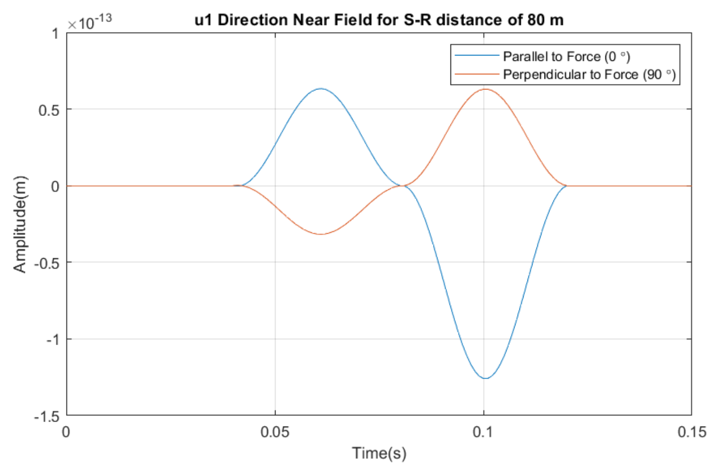

Figure 10: Near Field Parallel to Force (0 degree) and Perpendicular to Force (90 degree) for S-R Distance of 80 m in the U1 Direction. P-wave arrival at 0.04 s and S-wave arrival at 0.08 s.

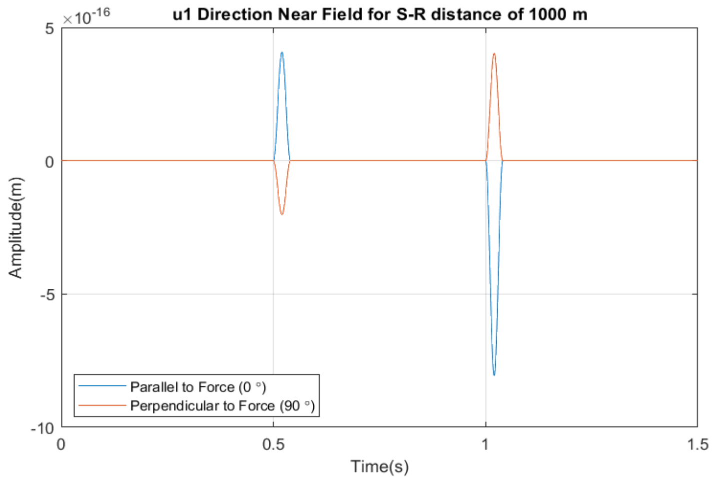

Figure 11: Near Field Parallel to Force (0 degree) and Perpendicular to Force (90 degree) for S-R Distance of 1000 m in the U1 Direction. P-wave arrival at 0.05 s and S-wave arrival at 1 s.

3. **The nonlinearity of the particle motion (polarization) for the events arriving at the P-wave travel time R/alpha and S-wave travel time R/beta. What is the range of source receiver directions for which the polarization of each event becomes visibly nonlinear in the near field (R=80 m) and far field (R = 1000 m)? Describe the waveforms of nonlinearly polarized arrivals.**

The Figure 3 above shows the particle motion hodograms in the near field (R = 80 m) and the Figure 6 above shows the particle motion hodograms in the far field (R = 1000 m).

The hodogram shows a straight line when the polarized particle motion is linear. This occurs when the displacements are either parallel or perpendicular to the force direction which happens for both the near field and the far field, at 0 degree and 90 degrees. In this case, there is either S-wave polarization which is orthogonal to the plane of the source-receiver line, or P-wave polariton which is parallel to the plane of the source-receiver line.

On the other hand, the nonlinear motion is the due to the near field mixed mode polarizations. For example, the non-linear particle motions are present at angles 10 to 80 degrees at the near field (R = 80 m) by the mixed polarizations. Similarly, the non-linear particle motions are present at angles 10 to 50 degrees at the far field (R = 1000 m). In this far field, at angles greater 50 degrees, the particle motion is dominated by only one mode of polarization which results in the linear motion.

IV. MATLAB Code

Far Field Source Wavelet Function

```
% Function for Far Field Source Wavelet
function[x_o]=x_o(R,beta,w,T,t)

x_o = zeros(1,length(t));
for i = 1:length(t)
    if t(i) < (R/beta)
        x_o(i) = 0;
    elseif (R/beta) <= t(i) && t(i) <= ((R/beta)+T)
        x_o(i) = sin(w*(t(i)-(R/beta)));
    else
        x_o(i) = 0;
    end
end
```

Near Field Source Wavelet Function

```
% Function for Near Field Source Wavelet
function [x_o_nf] = x_o_nf(R,alpha,beta,w,T,t)

x_o_nf = zeros(1,length(t));
for i = 1:length(t)
    if t(i) < (R/alpha)
        x_o_nf(i) = 0;
    elseif (R/alpha) <= t(i) && t(i) <= (R/alpha + T)
        upper_bound(i) = ((R/alpha*cos(w*(t(i)-R/alpha)))/w) +
((sin(w*(t(i))-R/alpha))/(w^2));
        lower_bound(i) = ((R/alpha*cos(w*(R/alpha-R/alpha)))/w) +
((sin(w*(R/alpha)-R/alpha))/(w^2));
        x_o_nf(i) = -(upper_bound(i) - lower_bound(i));
    elseif (R/beta) <= t(i)&& t(i) <= ((R/beta)+T)
        upper_bound(i) = ((R/beta*cos(w*(t(i)-R/beta)))/w) +
((sin(w*(t(i))-R/beta))/(w^2));
        lower_bound(i) = ((R/beta*cos(w*(R/beta-R/beta)))/w) +
((sin(w*(R/beta)-R/beta))/(w^2));
        x_o_nf(i) = (upper_bound(i) - lower_bound(i));
    else
        x_o_nf(i) = 0;
    end
end
```

MATLAB Code for S-R = 80 m

```
% GPGN 552 Introduction to Seismology I
% Point-Force Radiation in a Homogenous Medium Project
% Nadima Dwihusna

%% Variables
alpha = 2000; %m/s P wave velocity
beta = 1000; %m/s S wave velocity
rho = 2500; %kg/m3 density
fs = 25; %Hz source frequency
w = 2*pi*fs; %angular frequency
T = 1/fs; %time period
```

```
% Time and Angle Vector
t = 0:0.001:1.5; %time vector go to maximum R/beta of 1.5 s
theta = 0:10:90; %angle in 10 deg increments

% Receiver Distances (change R value if nf or ff)
%R_nf = 80; %m for near field
%R_ff = 1000; %m for far field
R = 80;

% Functions are defined for source wavelets
% x_o is for Far Field
% x_o_nf is for Near Field

%% Initialize Vectors
% Far Field
u1_ffs = zeros(length(theta),length(t));
u1_ffp = zeros(length(theta),length(t));
u3_ffs = zeros(length(theta),length(t));
u3_ffp = zeros(length(theta),length(t));
% Near Field
u1_nf = zeros(length(theta),length(t));
u3_nf = zeros(length(theta),length(t));
% Total Displacement
u1 = zeros(length(theta),length(t));
u3 = zeros(length(theta),length(t));
% P/S Ratio
u1_ps = zeros(1,length(theta));
u3_ps = zeros(1,length(theta));

%% Far Field Displacement Vectors
% u1 Far Field S
for j=1:length(theta)
    u1_ffs(j,:)=(1-
(cosd(theta(j))^2))*(1/R)*x_o(R,beta,w,T,t)*(1/(4*pi*rho*(beta^2)));
end

% u1 Far Field P
for i=1:10

u1_ffp(i,:)=(cosd(theta(i))^2)*(1/R)*x_o(R,alpha,w,T,t)*(1/(4*pi*rho*a
lpha^2));
end

% u3 Far Field S
for i=1:length(theta)
    u3_ffs(i,:)=(-
cosd(theta(i))*sind(theta(i)))*(1/R)*x_o(R,beta,w,T,t)*(1/(4*pi*rho*be
ta^2));
end

% u3 Far Field P
for i= 1:10

u3_ffp(i,:)=(cosd(theta(i))*sind(theta(i)))*(1/R)*x_o(R,alpha,w,T,t)*(
1/(4*pi*rho*alpha^2));
```

```
end

%% Near Field Displacement Vectors
% u1 Near Field (increment over theta)
for i=1:10
    u1_nf(i,:)=((3*(cosd(theta(i)))^2)-1)*((1/R^3)*
x_o_nf(R,alpha,beta,w,T,t))*(1/(4*pi*rho));
end

figure;
plot(t,u1_nf(1,:),t,u1_nf(10,:))
title('u1 Direction Near Field for S-R distance of 80 m');
legend('Parallel to Force (0 \circ)', 'Perpendicular to Force (90
\circ)');
xlabel('Time(s)');
ylabel('Amplitude(m)');
xlim([0,0.15]);
grid on;


% u3 Near Field (increment over theta)
for i=1:10

u3_nf(i,:)=3*cosd(theta(i))*sind(theta(i))*1/(R^3)*x_o_nf(R,alpha,beta
,w,T,t)*1/(4*pi*rho);
end

figure;
plot(t,u3_nf(1,:),t,u3_nf(10,:))
title('u3 Direction Near Field for S-R distance of 80 m');
legend('0 \circ', '90 \circ');
xlabel('Time(s)');
ylabel('Amplitude(m)');
xlim([0,0.15]);

%% Total Displacements
% u1 Total Displacement
u1 = u1_ffs + u1_ffp + u1_nf;

figure;
title('u1 Displacement')
for i=1:length(theta) % increment over theta
    subplot(2,5,i);
    plot(t,u1(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    xlim([0,0.15]);
    ylim([-5e-13,5e-13]);
end

% u3 Total Displacement
u3 = u3_ffs + u3_ffp + u3_nf;

figure;
```

```
title('u3 Displacement');
for i=1:length(theta) % increment over theta
    subplot(2,5,i);
    plot(t,u3(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    xlim([0,0.15]);
    ylim([-5e-13,5e-13]);
end

%% Particle Motions Hodograms
figure;
title('Particle Motions Hodograms');
for i=1:length(theta) % increment over theta
    subplot(2,5,i)
    plot(u1(i,:),u3(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('u1 Displacement (m)');
    ylabel('u3 Displacement (m)');
    xlim([-5e-13,5e-13]);
    ylim([-5e-13,5e-13]);
end
```

MATLAB Code for S-R = 1000 m

```
% Far Field

%% Variables
alpha = 2000; %m/s P wave velocity
beta = 1000; %m/s S wave velocity
rho = 2500; %kg/m3 density
fs = 25; %Hz source frequency
w = 2*pi*fs; %angular frequency
T = 1/fs; %time period

% Time and Angle Vector
t = 0:0.001:1.5; %time vector go to maximum R/beta of 1.5 s
theta = 0:10:90; %angle in 10 deg increments

% Receiver Distances (change R value if nf or ff)
%R_nf = 80; %m for near field
%R_ff = 1000; %m for far field
R = 1000;

% Functions are defined for source wavelets
% x_o is for Far Field
% x_o_nf is for Near Field

%% Initialize Vectors
% Far Field
u1_ffs = zeros(length(theta),length(t));
u1_ffp = zeros(length(theta),length(t));
u3_ffs = zeros(length(theta),length(t));
```

```
u3_ffp = zeros(length(theta),length(t));
% Near Field
u1_nf = zeros(length(theta),length(t));
u3_nf = zeros(length(theta),length(t));
% Total Displacement
u1 = zeros(length(theta),length(t));
u3 = zeros(length(theta),length(t));
% P/S Ratio
u1_ps = zeros(1,length(theta));
u3_ps = zeros(1,length(theta));

%% Far Field Displacement Vectors
% u1 Far Field S
for j=1:length(theta)
    u1_ffs(j,:)=(1-
(cosd(theta(j))^2))*(1/R)*x_o(R,beta,w,T,t)*(1/(4*pi*rho*(beta^2)));
end

% u1 Far Field P
for i=1:10

u1_ffp(i,:)=(cosd(theta(i))^2)*(1/R)*x_o(R,alpha,w,T,t)*(1/(4*pi*rho*a
lpha^2));
end

figure;
subplot(2,1,1);
for i=1:length(theta)
    hold on;
    plot(t,u1_ffs(i,:));
end
title('u1 Direction Far Field S for S-R distance of 1000 m')
xlabel('Time(s)');
ylabel('Amplitude(m)');
legend('0 \circ', '10 \circ','20 \circ','30 \circ','40 \circ','50
\circ','60 \circ','70 \circ','80 \circ', '90 \circ');
grid on;
xlim([0,1.2])
subplot(2,1,2);
for i=1:length(theta)
    hold on;
    plot(t,u1_ffp(i,:));
end
title('u1 Direction Far Field P for S-R distance of 1000 m')
xlabel('Time(s)');
ylabel('Amplitude(m)');
legend('0 \circ', '10 \circ','20 \circ','30 \circ','40 \circ','50
\circ','60 \circ','70 \circ','80 \circ', '90 \circ');
grid on;
xlim([0,1.2])

% u3 Far Field S
for i=1:length(theta)
```

```
    u3_ffs(i,:)=(-
cosd(theta(i))*sind(theta(i)))*(1/R)*x_o(R,beta,w,T,t)*(1/(4*pi*rho*be
ta^2));
end

% u3 Far Field P
for i= 1:10

u3_ffp(i,:)=(cosd(theta(i))*sind(theta(i)))*(1/R)*x_o(R,alpha,w,T,t)*(
1/(4*pi*rho*alpha^2));
end

figure;
subplot(2,1,1);
for i=1:length(theta)
    hold on;
    plot(t,u3_ffs(i,:));
end
title('u3 Direction Far Field S for S-R distance of 1000 m')
xlabel('Time(s)');
ylabel('Amplitude(m)');
legend('0 \circ', '10 \circ','20 \circ','30 \circ','40 \circ','50
\circ','60 \circ','70 \circ','80 \circ', '90 \circ');
grid on;
xlim([0,1.2])
subplot(2,1,2);
for i=1:length(theta)
    hold on;
    plot(t,u3_ffp(i,:));
end
title('u3 Direction Far Field P for S-R distance of 1000 m')
xlabel('Time(s)');
ylabel('Amplitude(m)');
legend('0 \circ', '10 \circ','20 \circ','30 \circ','40 \circ','50
\circ','60 \circ','70 \circ','80 \circ', '90 \circ');
grid on;
xlim([0,1.2])


%% Near Field Displacement Vectors
% u1 Near Field (increment over theta)
for i=1:10
    u1_nf(i,:)=((3*(cosd(theta(i)))^2)-1)*((1/R^3)*
x_o_nf(R,alpha,beta,w,T,t))*(1/(4*pi*rho));
end

figure;
plot(t,u1_nf(1,:),t,u1_nf(10,:))
title('u1 Direction Near Field for S-R distance of 1000 m');
legend('Parallel to Force (0 \circ)', 'Perpendicular to Force (90
\circ)');
xlabel('Time(s)');
ylabel('Amplitude(m)');
grid on;
```

```
% u3 Near Field (increment over theta)
for i=1:10

u3_nf(i,:)=3*cosd(theta(i))*sind(theta(i))*1/(R^3)*x_o_nf(R,alpha,beta
,w,T,t)*1/(4*pi*rho);
end

figure;
plot(t,u1_nf(1,:),t,u1_nf(10,:))
title('u3 Direction Near Field for S-R distance of 80 m');
legend('0 \circ', '90 \circ');
xlabel('Time(s)');
ylabel('Amplitude(m)');

%% Total Displacements
% u1 Total Displacement
u1 = u1_ffs + u1_ffp + u1_nf;

figure;
title('u1 Displacement')
for i=1:length(theta) % increment over theta
    subplot(2,5,i);
    plot(t,u1(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    xlim([0,1.5]);
    ylim([-3.5e-14,3.5e-14]);
end

% u3 Total Displacement
u3 = u3_ffs + u3_ffp + u3_nf;

figure;
title('u3 Displacement');
for i=1:length(theta) % increment over theta
    subplot(2,5,i);
    plot(t,u3(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('Time (s)');
    ylabel('Displacement (m)');
    xlim([0,1.5]);
    ylim([-3.5e-14,3.5e-14]);
end

%% Particle Motions Hodograms
figure;
title('Particle Motions Hodograms');
for i=1:length(theta) % increment over theta
    subplot(2,5,i)
    plot(u1(i,:),u3(i,:));
    title(['angle \theta =' num2str(theta(i)) ' \circ']);
    xlabel('u1 Displacement (m)');
    ylabel('u3 Displacement (m)');
```

```
    xlim([-3e-14,3e-14]);
    ylim([-2e-14,2e-14]);
end

%% P/S Ratio

for i=1:length(theta) % increment over theta
    u1_ps(i)=max(u1_ffp(i,:))/max(u1_ffs(i,:));
    u3_ps(i)=max(u3_ffp(i,:))/max(u3_ffs(i,:));
end

figure;
subplot(2,1,1);
plot(theta, u1_ps);
title('P/S Amplitude Ratios for u1 Direction')
xlabel('Angle (degrees)');
ylabel('Amplitude Ratio');
grid on;

subplot(2,1,2);
plot(theta, u3_ps);
hold on;
title('P/S Amplitude Ratios for u3 Direction')
xlabel('Angle (degrees)');
ylabel('Amplitude Ratio');
grid on;
```