CSCI 104
Backtracking Search

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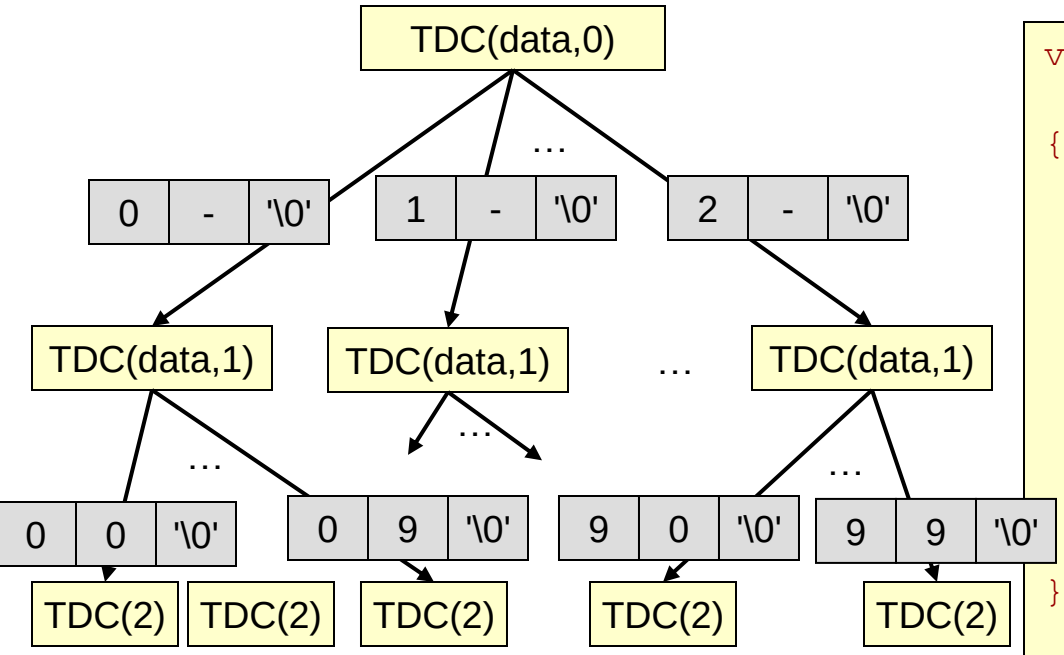
BACKTRACK SEARCH ALGORITHMS

Generating All Combinations

- Recursion offers a simple way to generate all combinations of N items from a set of options, S
  - Example: Generate all 2-digit decimal numbers (N=2, S={0,1,…,9})




```
void TwoDigCombos(char data[3],
                  int curr)
{
  if(curr == 2 )
    cout << data;
  else {
    for(int i=0; i < 10; i++){
      // set to 0
      data[curr] = '0'+i;
      // recurse
      TwoDigCombos(data, curr+1);
  }
}
```

Get the Code

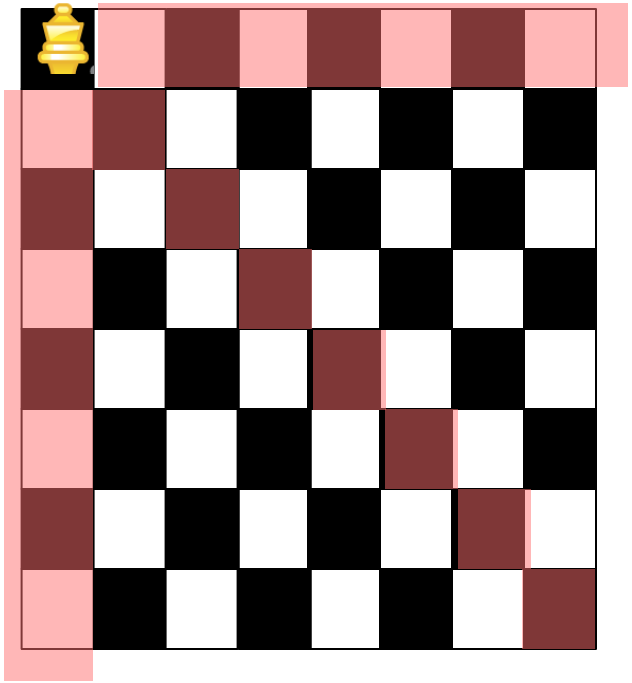
- On your VM
  - $ mkdir nqueens
  - $ cd nqueens
  - $ wget http://ee.usc.edu/~redekopp/cs104/nqueens.tar
  - $ tar xvf nqueens.tar

Recursive Backtracking Search

- Recursion allows us to "easily" enumerate all solutions to some problem
- Backtracking algorithms…
  - Are often used to solve constraint satisfaction problem or optimization problems
    - Several items that can be set to 1 of N values under some constraints
  - Stop searching down a path at the first indication that constraints won't lead to a solution
- Some common and important problems can be solved with backtracking
- Knapsack problem
  - You have a set of objects with a given weight and value. Suppose you have a knapsack that can hold N pounds, which subset of objects can you pack that maximizes the value.
  - Example:
    - Knapsack can hold 35 pounds
    - Object A: 7 pounds, $12 ea.
    - Object B: 10 pounds, $18 ea.
    - Object C: 4 pounds, $7 ea.
    - Object D: 2.4 pounds, $4 ea.
- Other examples:
  - Map Coloring
  - Traveling Salesman Problem
  - Sudoku
  - N-Queens

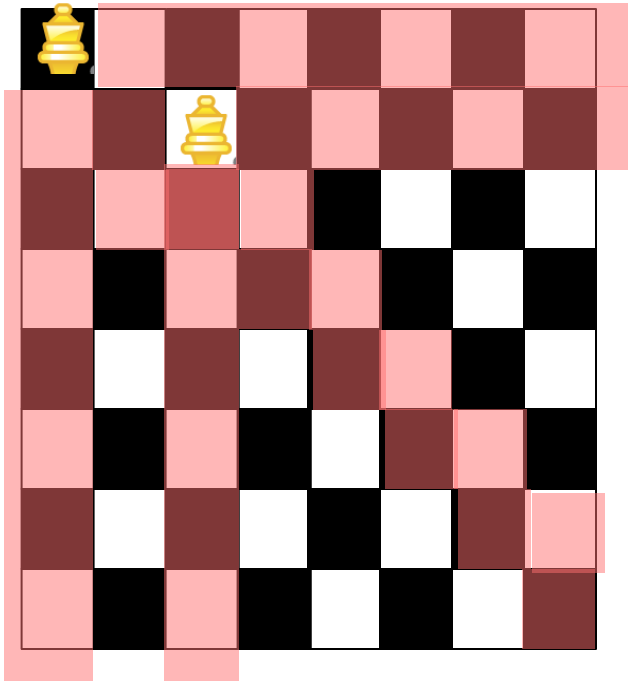
N-Queens Problem

- Problem: How to place N queens on an NxN chess board such that no queens may attack each other
- Fact: Queens can attack at any distance vertically, horizontally, or diagonally
- Observation: Different queen in each row and each column
- Backtrack search approach:
  - Place 1st queen in a viable option then, then try to place 2nd queen, etc.
  - If we reach a point where no queen can be placed in row i or we've exhausted all options in row i, then we return and change row i-1



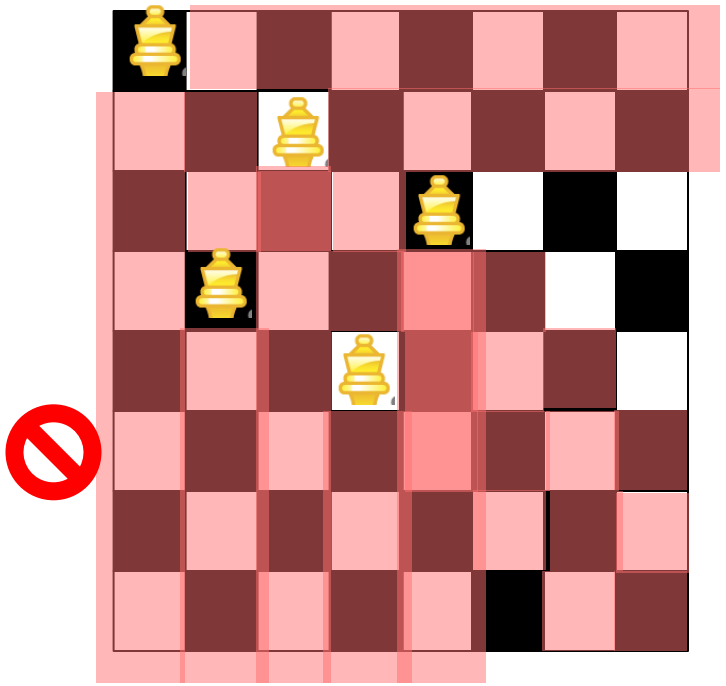
8x8 Example of N-Queens

- Now place 2nd queen



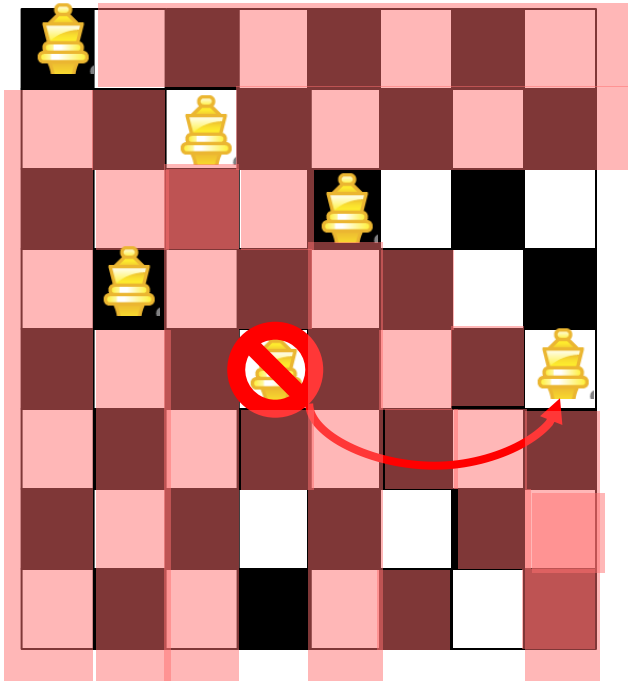
8x8 Example of N-Queens

- Now place others as viable
- After this configuration here, there are no locations in row 6 that are not under attack from the previous 5
- BACKTRACK!!!



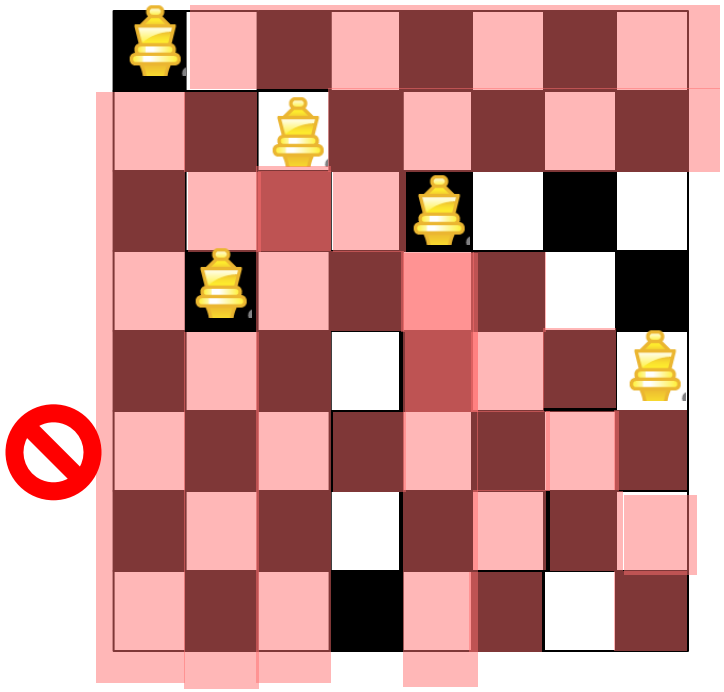
8x8 Example of N-Queens

- Now place others as viable
- After this configuration here, there are no locations in row 6 that is not under attack from the previous 5
- So go back to row 5 and switch assignment to next viable option and progress back to row 6



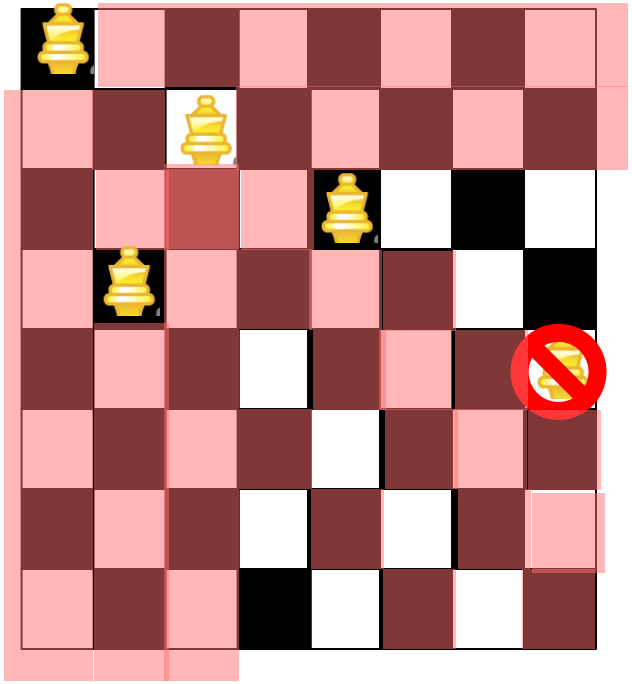
8x8 Example of N-Queens

- Now place others as viable
- After this configuration here, there are no locations in row 6 that is not under attack from the previous 5
- Now go back to row 5 and switch assignment to next viable option and progress back to row 6
- But still no location available so return back to row 5



8x8 Example of N-Queens

- Now place others as viable
- After this configuration here, there are no locations in row 6 that is not under attack from the previous 5
- Now go back to row 5 and switch assignment to next viable option and progress back to row 6
- But still no location available so return back to row 5
- But now no more options for row 5 so return back to row 4
- BACKTRACK!!!!



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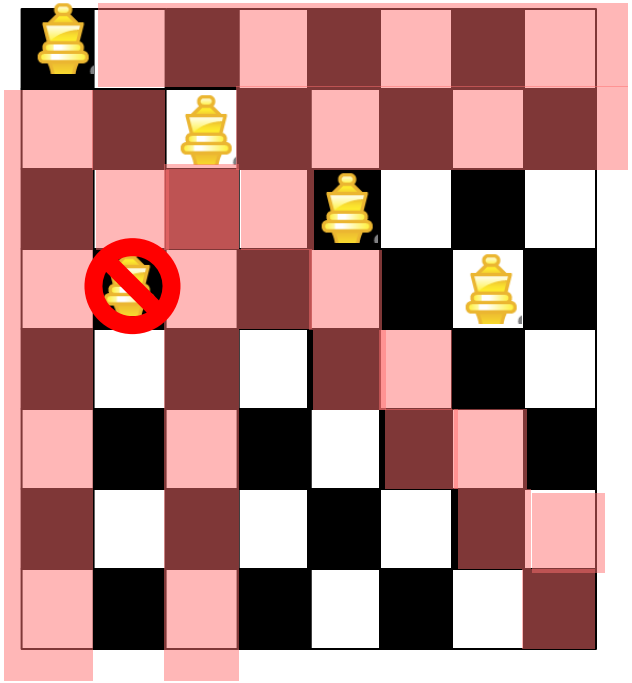
- Now place others as viable
- After this configuration here, there are no locations in row 6 that is not under attack from the previous 5
- Now go back to row 5 and switch assignment to next viable option and progress back to row 6
- But still no location available so return back to row 5
- But now no more options for row 5 so return back to row 4
- Move to another place in row 4 and restart row 5 exploration



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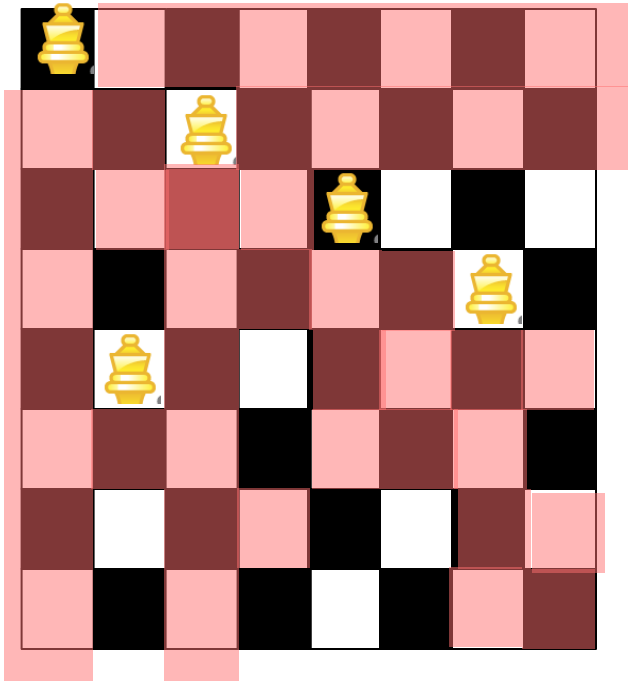
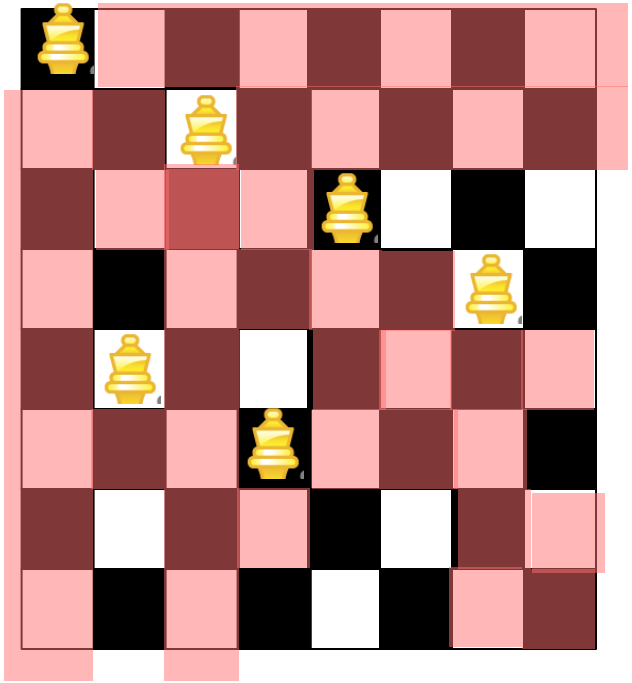


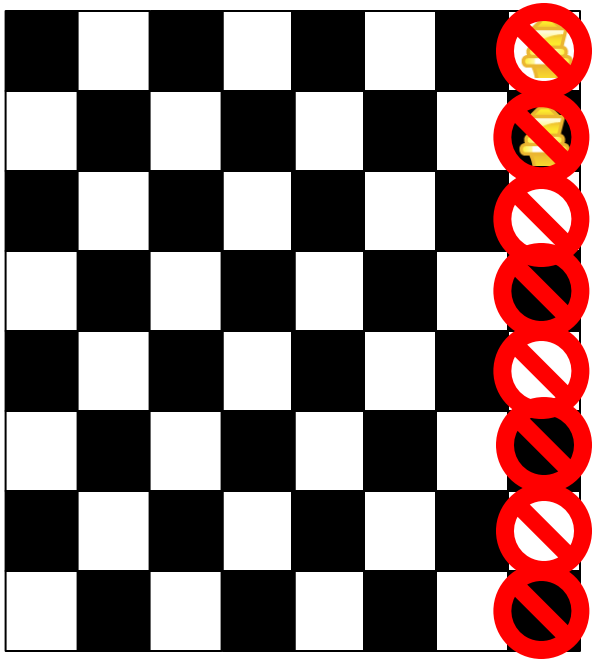
8x8 Example of N-Queens

- Now a viable option exists for row 6
- Keep going until you successfully place row 8 in which case you can return your solution
- What if no solution exists?



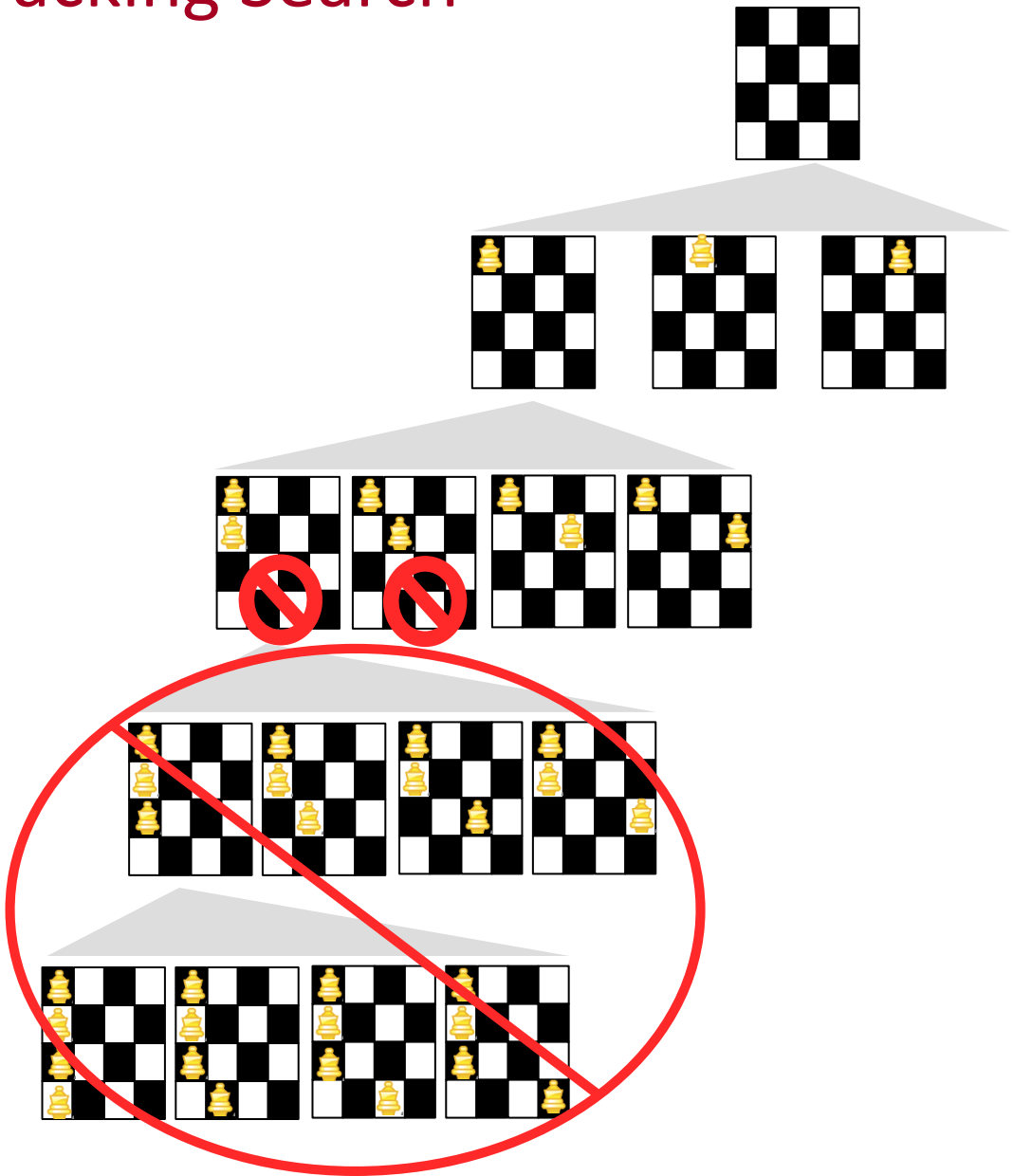
8x8 Example of N-Queens

- Now a viable option exists for row 6
- Keep going until you successfully place row 8 in which case you can return your solution
- What if no solution exists?
  - Row 1 queen would have exhausted all her options and still not find a solution



Backtracking Search

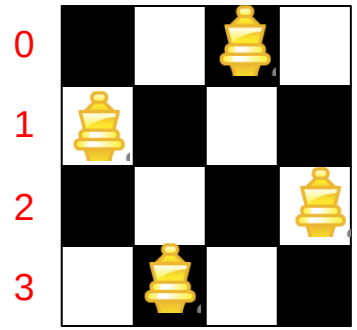
- Recursion can be used to generate all options
  - 'brute force' / test all options approach
  - Test for constraint satisfaction only at the bottom of the 'tree'
- But backtrack search attempts to 'prune' the search space
  - Rule out options at the partial assignment level



Brute force enumeration might test only once a possible complete assignment is made (i.e. all 4 queens on the board)

N-Queens Solution Development

- Let's develop the code
- 1 queen per row
  - Use an array where index represents the queen (and the row) and value is the column
- Start at row 0 and initiate the search [i.e. search(0)]
- Base case:
  - Rows range from 0 to n-1 so STOP when row == n
  - Means we found a solution
- Recursive case
  - Recursively try all column options for that queen
  - But haven't implemented check of viable configuration…



Index = Queen i in row i

q[i] = column of queen i

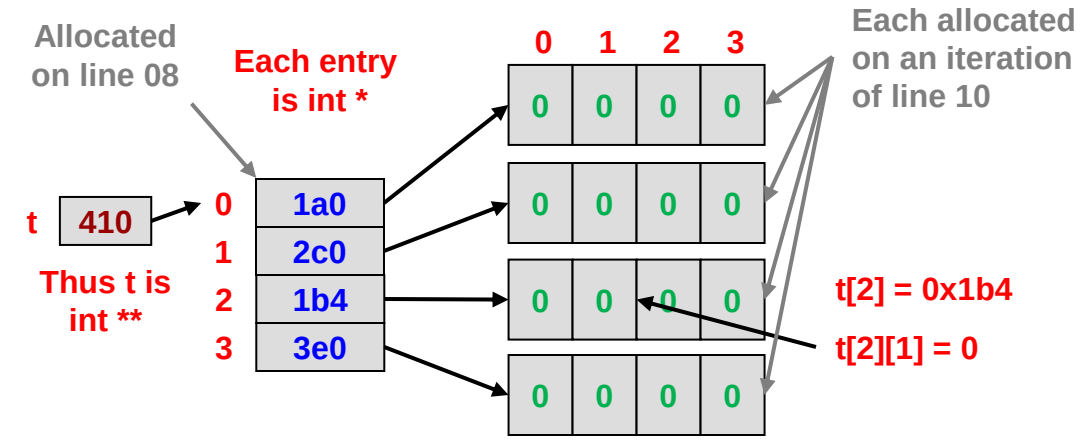
0	1	2	3
2	0	3	1

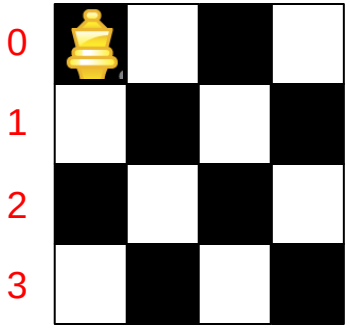
```
int *q;  // pointer to array storing
         // each queens location
int n;   // number of board / size

void search(int row)
{
  if(row == n)
    printSolution(); // solved!
  else {
   for(q[row]=0; q[row]<n; q[row]++){
     search(row+1);
  }
}
```

N-Queens Solution Development

- To check whether it is safe to place a queen in a particular column, let's keep a "threat" 2-D array indicating the threat level at each square on the board
  - Threat level of 0 means SAFE
  - When we place a queen we'll update squares that are now under threat
  - Let's name the array 't'
- Dynamically allocating 2D arrays in C/C++ doesn't really work
  - Instead conceive of 2D array as an "array of arrays" which boils down to a pointer to a pointer





0	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1

Index = Queen i in row i

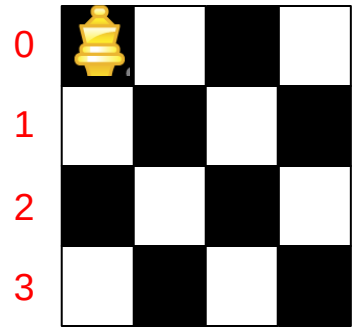
q[i] = column of queen i

0	1	2	3
0			

```
00  int *q;  // pointer to array storing
01           // each queens location
02  int n;   // number of board / size
03  int **t; // thread 2D array
04
05  int main()
06  {
07    q = new int[n];
08    t = new int*[n];
09    for(int i=0; i < n; i++){
10      t[i] = new int[n];
11      for(int j = 0; j < n; j++){
12        t[i][j] = 0;
13      }
14    }
15    search(0); // start search
16    // deallocate arrays
17    return 0;
18  }
```

N-Queens Solution Development

- After we place a queen in a location, let's check that it has no threats
- If it's safe then we update the threats (+1) due to this new queen placement
- Now recurse to next row
- If we return, it means the problem was either solved or more often, that no solution existed given our placement so we remove the threats (-1)
- Then we iterate to try the next location for this queen



Index = Queen i in row i

q[i] = column of queen i

0	1	2	3
0			

```
int *q;  // pointer to array storing
         // each queens location
int n;   // number of board / size
int **t; // n x n threat array
void search(int row)
{
  if(row == n)
    printSolution(); // solved!
  else {
   for(q[row]=0; q[row]<n; q[row]++){
     // check that col: q[row] is safe
     if(t[row][q[row]] == 0){
       // if safe place and continue
       addToThreats(row, q[row], 1);
       search(row+1);
       // if return, remove placement
       addToThreats(row, q[row], -1);
} } }
```

t	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Safe to place queen in upper left

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	0	1	0
3	1	0	0	1

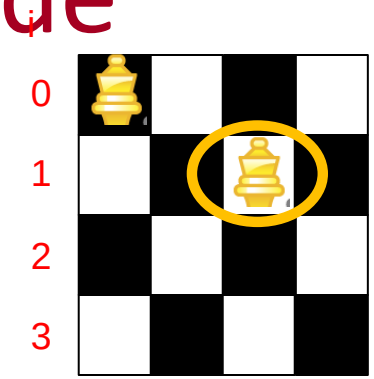
Now add threats

t	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Upon return, remove threat and iterate to next option

addToThreats Code

- Observations
  - Already a queen in every higher row so addToThreats only needs to deal with positions lower on the board
    - Iterate row+1 to n-1
  - Enumerate all locations further down in the same column, left diagonal and right diagonal
  - Can use same code to add or remove a threat by passing in change
- Can't just use 2D array of booleans as a square might be under threat from two places and if we remove 1 piece we want to make sure we still maintain the threat



Index = Queen i in row i

q[i] = column of queen i

0	1	2	3
0			

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	0	1	0
3	1	0	0	1

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	1	2	1
3	2	0	1	1

```
void addToThreats(int row, int col, int change)
{
  for(int j = row+1; j < n; j++){
    // go down column
    t[j][col] += change;
    // go down right diagonal
    if( col+(j-row) < n )
       t[j][col+(j-row)] += change;
    // go down left diagonal
    if( col-(j-row) >= 0)
       t[j][col-(j-row)] += change;
  }
}
```

N-Queens Solution

```
00  int *q;  // queen location array
01  int n;   // number of board / size
02  int **t; // n x n threat array
03
04  int main()
05  {
06    q = new int[n];
07    t = new int*[n];
08    for(int i=0; i < n; i++){
09      t[i] = new int[n];
10      for(int j = 0; j < n; j++){
11        t[i][j] = 0;
12      }
13    }
14    // do search
15    if( ! search(0) )
16       cout << "No sol!" << endl;
17    // deallocate arrays
18    return 0;
19  }
```

```
20  void addToThreats(int row, int col, int change)
21  {
22    for(int j = row+1; j < n; j++){
23      // go down column
24      t[j][col] += change;
25      // go down right diagonal
26      if( col+(j-row) < n )
27         t[j][col+(j-row)] += change;
28      // go down left diagonal
29      if( col-(j-row) >= 0)
30         t[j][col-(j-row)] += change;
31    }
32  }
33
34  bool search(int row)
35  {
36    if(row == n){
37      printSolution(); // solved!
38      return true;
39    }
40    else {
41     for(q[row]=0; q[row]<n; q[row]++){
42       // check that col: q[row] is safe
43       if(t[row][q[row]] == 0){
44         // if safe place and continue
45         addToThreats(row, q[row], 1);
46         bool status = search(row+1);
47         if(status) return true;
48         // if return, remove placement
49         addToThreats(row, q[row], -1);
50       }
51     }
52     return false;
53  } }
```

General Backtrack Search Approach

- Select an item and set it to one of its options such that it meets current constraints
- Recursively try to set next item
- If you reach a point where all items are assigned and meet constraints, done…return through recursion stack with solution
- If no viable value for an item exists, backtrack to previous item and repeat from the top
- If viable options for the 1st item are exhausted, no solution exists
- Phrase:
  - Assign, recurse, unassign

General Outline of Backtracking Sudoku Solver

```
00  bool sudoku(int **grid, int r, int c)
01  {
02    if( allSquaresComplete(grid) )
03      return true;
04    }
05    // iterate through all options
06    for(int i=1; i <= 9; i++){
07      grid[r][c] = i;
08      if( isValid(grid) ){
09        bool status = sudoku(...);
10        if(status) return true;
11      }
12    }
13    return false;
14  }
15
16
17
18
19
```

Assume r,c is current square to set and grid is the 2D array of values