

Queremos probar que la matriz planteada en la matriz A coincide con la ecuación (1)

$$[a]_i = \begin{cases} p * \frac{1-p}{n} & \text{si } c_j \neq 0 \\ \frac{1}{n} & \text{si } c_j = 0 \end{cases}$$

$$A = pWD + eZ^t(1)$$

$$pWD = p \cdot \begin{bmatrix} 0 & w_{12} & \cdots & w_{1n} \\ w_{21} & 0 & \ddots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & 0 \end{bmatrix} \cdot \begin{bmatrix} 1/c_1 & 0 & \cdots & 0 \\ 0 & 1/c_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1/c_n \end{bmatrix} =$$

$$\begin{bmatrix} 0 & p * w_{12}/c_2 & \cdots & p * w_{1n}/c_n \\ p * w_{21}/c_1 & 0 & \ddots & p * w_{2n}/c_n \\ \vdots & \vdots & \ddots & \vdots \\ p * w_{n1}/c_1 & w_{n2}/c_2 & \cdots & 0 \end{bmatrix}$$

$$\text{donde } [pWD]_{ij} = \begin{cases} p * \frac{w_{ij}}{c_j} & \text{si } c_j \neq 0 \\ 0 & \text{si } c_j = 0 \end{cases}$$

$$eZ^t = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \cdot \begin{bmatrix} z_1 & \cdots & z_n \end{bmatrix} = \begin{bmatrix} z_1 & \cdots & z_n \\ z_1 & \cdots & z_n \\ \vdots & \cdots & \vdots \\ z_1 & \cdots & z_n \end{bmatrix}$$

Entonces...

$$pWD + eZ^t =$$

$$\begin{bmatrix} 0 & p * w_{12}/c_2 & \cdots & p * w_{1n}/c_n \\ p * w_{21}/c_1 & 0 & \ddots & p * w_{2n}/c_n \\ \vdots & \vdots & \ddots & \vdots \\ p * w_{n1}/c_1 & w_{n2}/c_2 & \cdots & 0 \end{bmatrix} + \begin{bmatrix} z_1 & \cdots & z_n \\ z_1 & \cdots & z_n \\ \vdots & \cdots & \vdots \\ z_1 & \cdots & z_n \end{bmatrix} =$$

$$\begin{bmatrix} z_1 & \cdots & p * w_{1n} / \frac{p * w_{1n} c_n + z_n}{c_1} \\ \vdots & \cdots & \vdots \\ \frac{p * w_{n1}}{c_1} + z_1 & \cdots & z_n \end{bmatrix}$$

$$\text{donde } [Z]_i = \begin{cases} p * \frac{1-p}{n} & \text{si } c_j \neq 0 \\ 1/n & \text{si } c_j = 0 \end{cases}$$

Con esto podemos ver que la matriz resultante corresponde a

$$[pWD + eZ^t]_{ij} = \begin{cases} p * \frac{1-p}{n} & \text{si } c_j \neq 0 \\ \frac{1}{n} & \text{si } c_j = 0 \end{cases}$$

Finalmente, la matriz obtenida mediante la ecuación (1) coincide con la matriz A.