Point Reconstruction

Given two images I_1 and I_2, for any pair of 2-D points $m_1 \in I_1$ and $m_2 \in I_2$, there exist two 3-D lines L_1 and L_2 such that L_k passes through points m_k and C_k. L_1 and L_2 are the projection lines of points m_1 and m_2, respectively. Each projection lines $L_k(k = 1, 2)$ is defined as

$$\frac{x - C_{xk}}{u_{xk}} = \frac{y - C_{yk}}{u_{yk}} = \frac{z - C_{zk}}{u_{zk}} \tag{1}$$

where $C_k = (C_{xk}, C_{yk}, C_{zk})$ and the line unit direction vector $u_k = (u_{xk}, u_{yk}, u_{zk})^T$ is defined as

$$u_k = \frac{D_k - C_k}{|D_k - C_k|}, \tag{2}$$

$$D_k = B_k^{-1}(-b_k + m_k), \tag{3}$$

$$P_k = [Bb] \tag{4}$$

where P_k is the projection matrix of image I_k, B is the left 3x3 submatrix of P and b is the right 3-vector of P.

Assuming that m_1 and m_2 correspond to the same 3-D point M, then $M = (x, y, z)$ is the 3-D pseudo-intersection point. M is computed by minimizing the sum of distances from M to the lines L_1 and L_2. The error function [1] is defined as

$$E = [(x - C_{xk})u_{yk} - (y - C_{yk})u_{xk}]^2 + [(x - C_{xk})u_{zk} - (z - C_{zk})u_{xk}]^2 + [(y - C_{yk})u_{zk} - (z - C_{zk})u_{yk}]^2 \tag{5}$$

Setting $\frac{\delta E}{\delta x} = \frac{\delta E}{\delta y} = \frac{\delta E}{\delta z} = 0$, we compute M as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left[\sum_{k=1}^{2} A_k\right]^{-1} \left[\sum_{k=1}^{2} \left(A_k \begin{bmatrix} C_{xk} \\ C_{yk} \\ C_{zk} \end{bmatrix} \right)\right] \tag{6}$$

where

$$A_k = \begin{bmatrix} u_{yk}^2 + u_{zk}^2 & -u_{xk}u_{yk} & -u_{xk}u_{zk} \\ -u_{xk}u_{yk} & u_{xk}^2 + u_{zk}^2 & -u_{yk}u_{zk} \\ -u_{xk}u_{zk} & -u_{yk}u_{zk} & u_{xk}^2 + u_{yk}^2 \end{bmatrix} \tag{7}$$

References

[1] D. B. Goldgof, H. Lee and T. S. Huang, "Matching and motion estimation of three-dimensional point and line sets using eigenstrucute without correspondence", *Pattern Recognition,* Vol. 25(3), pp. 271-286, 1992.