Zewail City for Science & Technology
Faculty of Engineering
Space Department
3rd Year

SPC 403: Orbital & Space Flight
Project Progress Report

Submitted to:

Prof.: Dr. | Mohammed K. Ibrahim
TA: Engineer| Ahmed A. Hamada

Name: Nadine Amr Mahmoud
ID: 201501328

Submission Date: 16/05/2018
Due Date: 16/05/2018

Table of Content

Abstract: In this report, eight different algorithms for plotting the satellite's 2D and 3D ground track are explained and the results shown. Those methods are: given the initial state vector and using Range-Kutta 4, the Lagrange coefficients or the universal Kepler's equation, given the two-orbital elements, given the classical orbital elements, using Gibbs method, using Lambert's method, and using Gauss Method. A GUI was also designed to help the user easily choose between the different methods.

.

I. Introduction

Ground tracks are very crucial in any space mission. Ground measurements need to be well deployed in order to be able to fully describe the satellite's orbit and thus conclude its ground track. Ground measurements can vary significantly from one mission to the other. As a result, many methods that lead us to calculating the ground track are employed. Eight of those methods are discussed below.

II. Ground Track from Initial State Vector Using Range Kutta-4

This code plots the satellite's ground track throughout a user-defined period of time using Range-Kutta4. Its inputs are the satellite's initial state vector as well as the period of time over which the ground track is to be plotted.

A. Algorithm

1. The initial classical orbital elements are calculated from the first position and velocity vectors as follows:
   - Specific angular momentum:
     $$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$
     $$h = \sqrt{\mathbf{h} \cdot \mathbf{h}}$$
   - Inclination:
     $$i = \cos^{-1}\frac{h_Z}{h}$$
   - Right ascension of ascending node:
     $$\mathbf{N} = \hat{\mathbf{K}} \times \mathbf{h}$$
     $$N = \sqrt{\mathbf{N} \cdot \mathbf{N}}$$
     $$\Omega = \cos^{-1}\frac{N_X}{N}$$
   - Eccentricity:
     $$v_r = \frac{\mathbf{v} \cdot \mathbf{r}}{r}$$
     $$\mathbf{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\mathbf{r} - r v_r \mathbf{v}\right]$$
     $$e = \sqrt{\mathbf{e} \cdot \mathbf{e}}$$
   - Argument of perigee:
     $$\omega = \cos^{-1}\frac{\mathbf{N} \cdot \mathbf{e}}{Ne}$$
   - True anomaly:
     $$\theta = \cos^{-1}\left(\frac{\mathbf{e} \cdot \mathbf{r}}{er}\right)$$
   - Semi-major axis:
     $$\mathrm{a} = \frac{h^2}{\mu(1 - e^2)}$$
2. The orbital period is calculated from:
   $$T = \frac{2\pi a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}}$$
3. The initial eccentric anomaly is calculated from:
   $$E_0 = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_0}{2}\right)\right)$$
4. The initial mean anomaly is calculated from:
   $$M_0 = E_0 - e\sin(E_0)$$

5. The initial time is calculated from:

$$t_0 = \frac{M_0 T}{2\pi}$$

6. The coefficients of RK4 are calculated from:

$$\{a\} = \begin{Bmatrix} 0 \\ 1/2 \\ 1/2 \\ 1 \end{Bmatrix} \quad [b] = \begin{bmatrix} 0 & 0 & 0 \\ 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{c\} = \begin{Bmatrix} 1/6 \\ 1/3 \\ 1/3 \\ 1/6 \end{Bmatrix}$$

7. A column vector from the initial position and velocity components is constructed as follows:

$$y = [x_0\ y_0\ \ z_0\ \ \dot{x}_0\ \ \dot{y}_0\ \ \dot{z}_0]'$$

8. Using this initial vector, a similar vector is calculated at time + delta_time (h) by:
   - Calculating

$$\bar{\mathbf{f}}_1 = \mathbf{f}(t_i, \mathbf{y}_i) \quad \bar{\mathbf{f}}_2 = \mathbf{f}\left(t_i + \frac{1}{2}h, \mathbf{y}_i + \frac{1}{2}h\bar{\mathbf{f}}_1\right)$$

$$\bar{\mathbf{f}}_3 = \mathbf{f}\left(t_i + \frac{1}{2}h, \mathbf{y}_i + \frac{1}{2}h\bar{\mathbf{f}}_2\right) \quad \bar{\mathbf{f}}_4 = \mathbf{f}(t_i + h, \mathbf{y}_i + h\bar{\mathbf{f}}_3)$$

   Where **f** is a function that calculates the first and second derivatives of the satellite's radius such that the resulting vector becomes

$$[\,\dot{x}_0\ \ \dot{y}_0\ \ \dot{z}_0\ \ \frac{-\mu x}{r^3}\ \frac{-\mu y}{r^3}\ \frac{-\mu z}{r^3}]'$$

   - The vector after one time step thus becomes

$$\mathbf{y}_{i+1} = \mathbf{y}_i + h\left(\frac{1}{6}\bar{\mathbf{f}}_1 + \frac{1}{3}\bar{\mathbf{f}}_2 + \frac{1}{3}\bar{\mathbf{f}}_3 + \frac{1}{6}\bar{\mathbf{f}}_4\right)$$

9. The process is repeated so as to compute this vector (new position and velocity vectors) at delta_time (h) intervals throughout the plotting period specified by the user.
10. In order to take the **oblateness** into consideration, for each of the **y** vectors evaluated at each of the different times, the classical orbital elements are calculated.
11. The calculated right ascension of ascending node is replaced by the right ascension of ascending node associated with the position and velocity vectors at the previous time step added to the rate of change of the right ascension of ascending node multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\cos i$$

12. The calculated argument of perigee is replaced by the argument of perigee associated with the position and velocity vectors at the previous time step added to the rate of change of the argument of perigee multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$

13. The state vectors are then recalculated from the updated classical orbital elements (now considering the **oblateness**).
14. To account for the Earth's **rotation**, the transformation matrix from the geocentric equatorial frame to the rotating Earth-fixed frame is calculated from:

$$[\mathbf{R}_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   where

$$\theta = \omega_E(t - t_0)$$

   with $w_E = 4.178 * 10^{-3}$

15. The position vector in the rotating Earth-fixed frame is calculated from:

$$\{\mathbf{r}\}_{x'} = [\mathbf{R}_3(\theta)]\{\mathbf{r}\}_X$$

16. The direction cosines of $\{r\}_{x'}$ are calculated from:

$$l = \frac{(\{r\}_{x'})_x}{|\{r\}_{x'}|}, m = \frac{(\{r\}_{x'})_y}{|\{r\}_{x'}|},$$

$$n = \frac{(\{r\}_{x'})_z}{|\{r\}_{x'}|}$$

17. The declination is calculated from:

$$\delta = \sin^{-1} n$$

18. If m > 0, the right ascension is calculated from:

$$\alpha = \cos^{-1}\frac{l}{\cos(\delta)}$$

Otherwise it is calculated from:

$$\alpha = 360 - \cos^{-1}\frac{l}{\cos(\delta)}$$

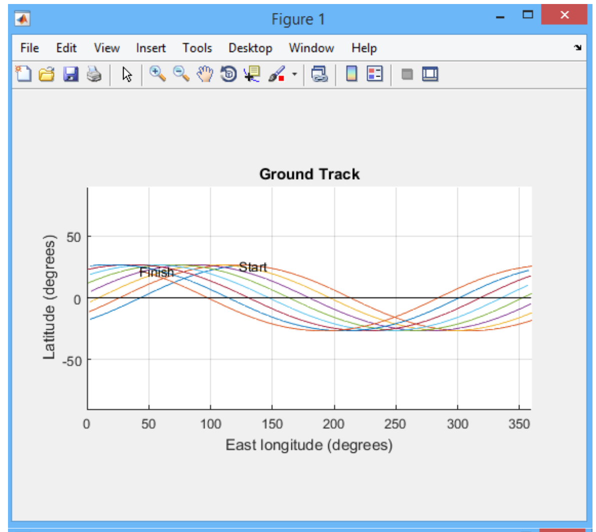
19. The right ascension is plotted on the x-axis against the declination on the y-axis, and the process is repeated by incrementing the time with delta t, until the desired period specified by the user ends.
20. The components of the position vector in the geocentric equatorial frame are plotted in 3D.

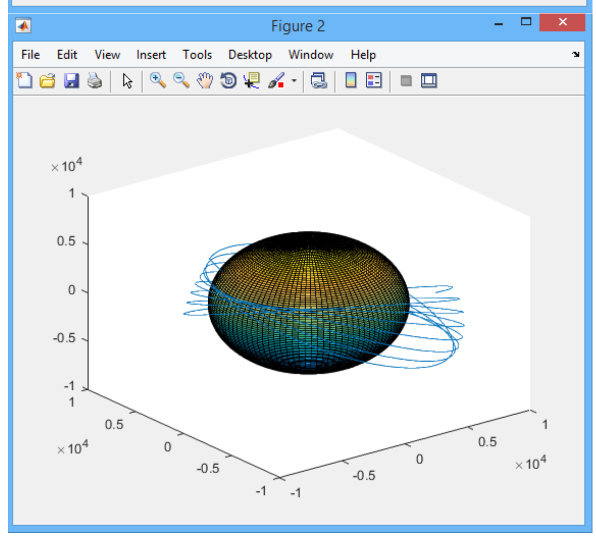
B. Required Functions:

- Main Function: GT_from_R0V0_RK4.m
- coe_from_sv.m
- rk4.m
- rates.m
- sv_from_coe.m
- ra_and_dec_from_r.m
- plot_3D.m

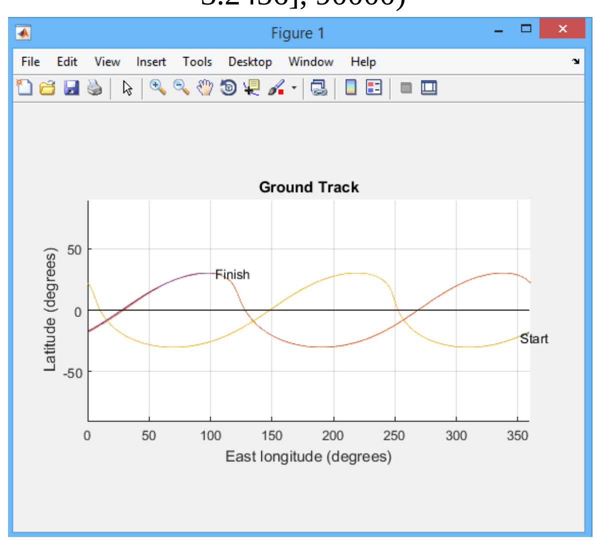
C. Results

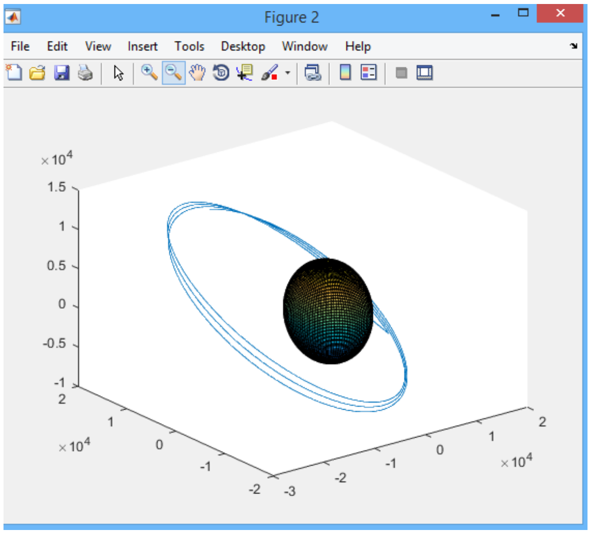
- GT_from_R0V0_RK4([-6045 -3490 2500], [-3.457, 6.618, 2.533], 60000)





- GT_from_R0V0_RK4([5000 10000 2100], [-5.9925 1.9254 3.2456], 90000)





III. Ground Track from Initial State Vector Using Lagrange Coefficients

This code plots the satellite's ground track throughout a user-defined period of time using the Lagrange coefficients. Its inputs are the satellite's initial state vector as well as the period of time over which the ground track is to be plotted.

A. Algorithm

1. The initial classical orbital elements are calculated from the initial state vectors using the same method illustrated in section II.
2. The orbital period is calculated from:

$$T = \frac{2\pi a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}}$$

3. The initial eccentric anomaly is calculated from:

$$E_0 = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_0}{2}\right)\right)$$

4. The initial mean anomaly is calculated from:

$$M_0 = E_0 - e\,sin(E_0)$$

5. The initial time is calculated from:

$$t_0 = \frac{M_0 T}{2\pi}$$

6. The transformation matrix from the geocentric equatorial coordinates to the perifocal coordinates is calculated from:

$$[\mathbf{Q}]_{X\bar{x}} = \begin{bmatrix} -\sin\Omega\cos i\sin\omega + \cos\Omega\cos\omega & \cos\Omega\cos i\sin\omega + \sin\Omega\cos\omega & \sin i\sin\omega \\ -\sin\Omega\cos i\cos\omega - \cos\Omega\sin\omega & \cos\Omega\cos i\cos\omega - \sin\Omega\sin\omega & \sin i\cos\omega \\ \sin\Omega\sin i & -\cos\Omega\sin i & \cos i \end{bmatrix}$$

7. The transformation matrix from the perifocal coordinates to the geocentric equatorial coordinates is calculated from:

$$[\mathbf{Q}]_{\bar{x}X} = \left([\mathbf{Q}]_{X\bar{x}}\right)^T$$

8. The initial position and velocity vectors in the perifocal coordinates are calculated from:

$$\{\mathbf{r}\}_{\bar{x}} = \begin{Bmatrix} \bar{x} \\ \bar{y} \\ 0 \end{Bmatrix} = [\mathbf{Q}]_{X\bar{x}}\{\mathbf{r}\}_X \quad \{\mathbf{v}\}_{\bar{x}} = \begin{Bmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \\ 0 \end{Bmatrix} = [\mathbf{Q}]_{X\bar{x}}\{\mathbf{v}\}_X$$

9. The norms of the initial position and velocity vectors are calculated (r, v).
10. The initial radial velocity is calculated using:

$$v_r = \frac{R_0 . V_0}{r}$$

11. The specific angular momentum is found as the norm of:

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

12. The Stump functions are calculated from:

$$S(z) = \begin{cases} \dfrac{\sqrt{z} - \sin\sqrt{z}}{(\sqrt{z})^3} & (z > 0) \\ \dfrac{\sinh\sqrt{-z} - \sqrt{-z}}{(\sqrt{-z})^3} & (z < 0) \\ \dfrac{1}{6} & (z = 0) \end{cases} \quad (z = \alpha\chi^2)$$

$$C(z) = \begin{cases} \dfrac{1 - \cos\sqrt{z}}{z} & (z > 0) \\ \dfrac{\cosh\sqrt{-z} - 1}{-z} & (z < 0) \\ \dfrac{1}{2} & (z = 0) \end{cases} \quad (z = \alpha\chi^2)$$

13. Lagrange Coefficients

$$f = 1 - \frac{\chi^2}{r_0} C(\alpha\chi^2)$$

$$g = \Delta t - \frac{1}{\sqrt{\mu}} \chi^3 S(\alpha\chi^2)$$

$$\dot{f} = \frac{\sqrt{\mu}}{r r_0} [\alpha\chi^3 S(\alpha\chi^2) - \chi]$$

$$\dot{g} = 1 - \frac{\chi^2}{r} C(\alpha\chi^2)$$

14. The new position and velocity vectors after dt time has passed are calculated using:

$$\mathbf{r} = f\mathbf{r}_0 + g\mathbf{v}_0$$

$$\mathbf{v} = \dot{f}\mathbf{r}_0 + \dot{g}\mathbf{v}_0$$

15. Those are in the perifocal frame, the ones in the geocentric equatorial frame are calculated from:

$$\{\mathbf{r}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{r}\}_{\bar{x}} \quad \{\mathbf{v}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{v}\}_{\bar{x}}$$

16. In order to take the **oblateness** into consideration, for each of the **y** vectors evaluated at each of the different times, the classical orbital elements are calculated.
17. The calculated right ascension of ascending node is replaced by the right ascension of ascending node associated with the position and velocity vectors at the previous time step added to the rate of change of the right ascension of ascending node multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\cos i$$

18. The calculated argument of perigee is replaced by the argument of perigee associated with the position and velocity vectors at the previous time step added to the rate of change of the argument of perigee multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$

19. The state vectors are then recalculated from the updated classical orbital elements (now considering the **oblateness**).
20. To account for the Earth's **rotation**, the transformation matrix from the geocentric equatorial frame to the rotating Earth-fixed frame is calculated from:

$$[\mathbf{R}_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$$\theta = \omega_E(t - t_0)$$

with $w_E = 4.178 * 10^{-3}$

21. The position vector in the rotating Earth-fixed frame is calculated from:

$$\{\mathbf{r}\}_{x'} = [\mathbf{R}_3(\theta)]\{\mathbf{r}\}_X$$

22. The direction cosines of $\{r\}_{x'}$ are calculated from:

$$l = \frac{(\{r\}_{x'})_x}{|\{r\}_{x'}|}, m = \frac{(\{r\}_{x'})_y}{|\{r\}_{x'}|},$$

$$n = \frac{(\{r\}_{x'})_z}{|\{r\}_{x'}|}$$

23. The declination is calculated from:

$$\delta = \sin^{-1} n$$

24. If m > 0, the right ascension is calculated from:

$$\alpha = \cos^{-1} \frac{l}{\cos(\delta)}$$

Otherwise it is calculated from:

$$\alpha = 360 - \cos^{-1} \frac{l}{\cos(\delta)}$$

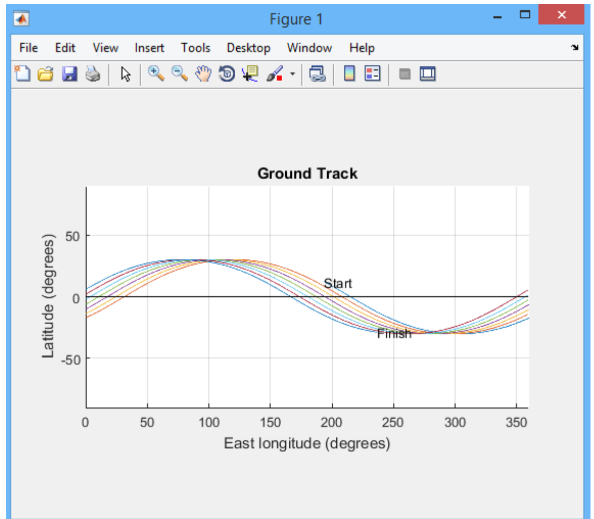
25. The whole process from step 9 is repeated with the time increased by delta_time each iteration.
26. The right ascension is plotted on the x-axis against the declination on the y-axis, and the process is repeated by incrementing the time with delta t, until the desired period specified by the user ends.
27. The components of the position vector in the geocentric equatorial frame are plotted in 3D.

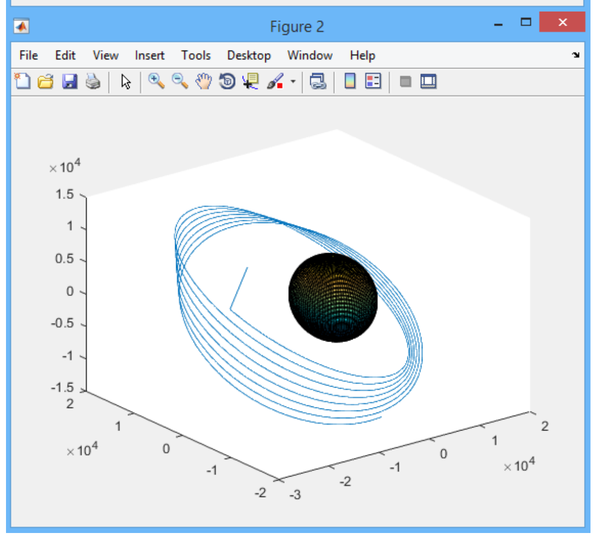
B. Required Functions

- Main Function: GT_from_R0V0_Lagrange.m
- coe_from_sv.m
- rv_from_r0v0_ta.m
- f_and_g_ta.m
- fDot_and_gDot_ta.m
- stumpC.m
- stumpS.m
- sv_from_coe.m
- ra_and_dec_from_r.m
- plot_3D

C. Results

- GT_from_R0V0_Lagrange([-14600 2500 7000], [-3.3125 -4.1966 -0.3853], 200000)





IV. Ground Track from Initial State Vector Using Universal Kepler's Equation

This code plots the satellite's ground track throughout a user-defined period of time using the universal Kepler's equations. Its inputs are the satellite's initial state vector as well as the period of time over which the ground track is to be plotted.

A. Algorithm

1. The initial classical orbital elements are calculated from the initial state vectors using the same method illustrated in section II.

2. The orbital period is calculated from:

$$T = \frac{2\pi a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}}$$

3. The initial eccentric anomaly is calculated from:

$$E_0 = 2\arctan(\sqrt{\frac{1-e}{1+e}}\tan(\frac{\theta_0}{2}))$$

4. The initial mean anomaly is calculated from:

$$M_0 = E_0 - e\,sin(E_0)$$

5. The initial time is calculated from:

$$t_0 = \frac{M_0 T}{2\pi}$$

6. The norms of the initial position and velocity vectors are calculated (r, v).
7. The initial radial velocity is calculated using:

$$v_r = \frac{R_0.V_0}{r}$$

8. The reciprocal of the semi-major axis is calculated from:

$$\alpha = \frac{2}{r} - \frac{v^2}{\mu}$$

9. The initial value for the universal anomaly is calculated from:

$$\chi_0 = \sqrt{\mu} * |\alpha| * dt$$

Where dt is the time since $\chi = 0$

10. The Stump functions are calculated from:

$$S(z) = \begin{cases} \dfrac{\sqrt{z} - \sin\sqrt{z}}{(\sqrt{z})^3} & (z > 0) \\ \dfrac{\sinh\sqrt{-z} - \sqrt{-z}}{(\sqrt{-z})^3} & (z < 0) \quad (z = \alpha\chi^2) \\ \dfrac{1}{6} & (z = 0) \end{cases}$$

$$C(z) = \begin{cases} \dfrac{1 - \cos\sqrt{z}}{z} & (z > 0) \\ \dfrac{\cosh\sqrt{-z} - 1}{-z} & (z < 0) \quad (z = \alpha\chi^2) \\ \dfrac{1}{2} & (z = 0) \end{cases}$$

11. The following equation is used to get the current universal anomaly by iteration

$$\chi_{i+1} = \chi_i - \frac{\frac{r_0 v_{r0}}{\sqrt{\mu}}\chi_i^2 C(z_i) + (1 - \alpha r_0)\chi_i^3 S(z_i) + r_0\chi_i - \sqrt{\mu}\Delta t}{\frac{r_0 v_{r0}}{\sqrt{\mu}}\chi_i\left[1 - \alpha\chi_i^2 S(z_i)\right] + (1 - \alpha r_0)\chi_i^2 C(z_i) + r_0}$$

12. The z is calculated such that:

$$z = \alpha * \chi^2$$

13. The Lagrange coefficients are calculated from:

$$f = 1 - \frac{\chi^2}{r_0}C(\alpha\chi^2)$$

$$g = \Delta t - \frac{1}{\sqrt{\mu}}\chi^3 S(\alpha\chi^2)$$

14. The derivatives of the Lagrange coefficients are calculated from:

$$\dot{f} = \frac{\sqrt{\mu}}{r r_0}\left[\alpha\chi^3 S(\alpha\chi^2) - \chi\right]$$

$$\dot{g} = 1 - \frac{\chi^2}{r}C(\alpha\chi^2)$$

15. The new position and velocity vectors after dt time has passed are calculated using:

$$\mathbf{r} = f\mathbf{r}_0 + g\mathbf{v}_0$$

$$\mathbf{v} = \dot{f}\mathbf{r}_0 + \dot{g}\mathbf{v}_0$$

16. In order to take the **oblateness** into

consideration, for each of the **y** vectors evaluated at each of the different times, the classical orbital elements are calculated.

17. The calculated right ascension of ascending node is replaced by the right ascension of ascending node associated with the position and velocity vectors at the previous time step added to the rate of change of the right ascension of ascending node multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2a^{\frac{7}{2}}}\right]\cos i$$

18. The calculated argument of perigee is replaced by the argument of perigee associated with the position and velocity vectors at the previous time step added to the rate of change of the argument of perigee multiplied by the time step. This time rate of change in terms of Earth's radius and $J_2 = 0.0010836$, is:

$$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2a^{\frac{7}{2}}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$

19. The state vectors are then recalculated from the updated classical orbital elements (now considering the **oblateness**).
20. To account for the Earth's **rotation**, the transformation matrix from the geocentric equatorial frame to the rotating Earth-fixed frame is calculated from:

$$[\mathbf{R}_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$$\theta = \omega_E(t - t_0)$$

with $w_E = 4.178 * 10^{-3}$

21. The position vector in the rotating Earth-fixed frame is calculated from:

$$\{\mathbf{r}\}_{x'} = [\mathbf{R}_3(\theta)]\{\mathbf{r}\}_X$$

22. The direction cosines of $\{r\}_{x'}$ are calculated from:

$$l = \frac{(\{r\}_{x'})_x}{|\{r\}_{x'}|}, m = \frac{(\{r\}_{x'})_y}{|\{r\}_{x'}|},$$
$$n = \frac{(\{r\}_{x'})_z}{|\{r\}_{x'}|}$$

23. The declination is calculated from:

$$\delta = \sin^{-1} n$$

24. If m > 0, the right ascension is calculated from:

$$\alpha = \cos^{-1}\frac{l}{\cos(\delta)}$$

Otherwise it is calculated from:

$$\alpha = 360 - \cos^{-1}\frac{l}{\cos(\delta)}$$

25. The whole process from step 9 is repeated with the time increased by delta_time each iteration.
26. The right ascension is plotted on the x-axis against the declination on the y-axis, and the process is repeated by incrementing the time with delta t, until the desired period specified by the user ends.

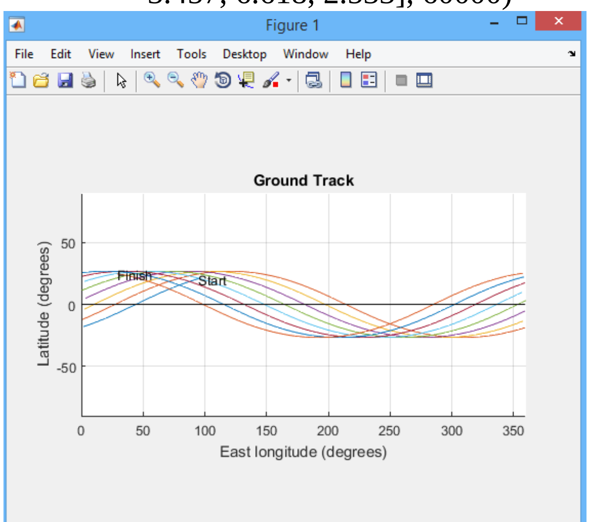
28. The components of the position vector in the geocentric equatorial frame are plotted in 3D.

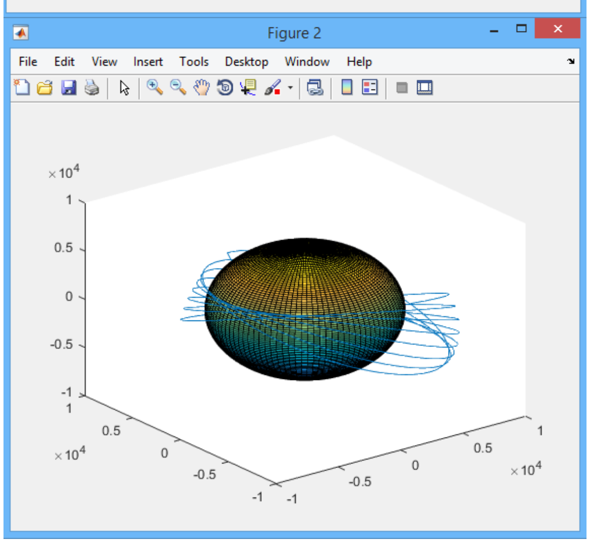
B. Required Functions

- Main Function: GT_from_R0V0_UniversalKepler.m
- coe_from_sv.m
- rv_from_r0v0.m
- kepler_U.m
- f_and_g.m
- stumpC.m
- stumpS.m
- fDot_and_gDot.m
- sv_from_coe.m
- ra_and_dec_from_r.m
- plot_3D.m

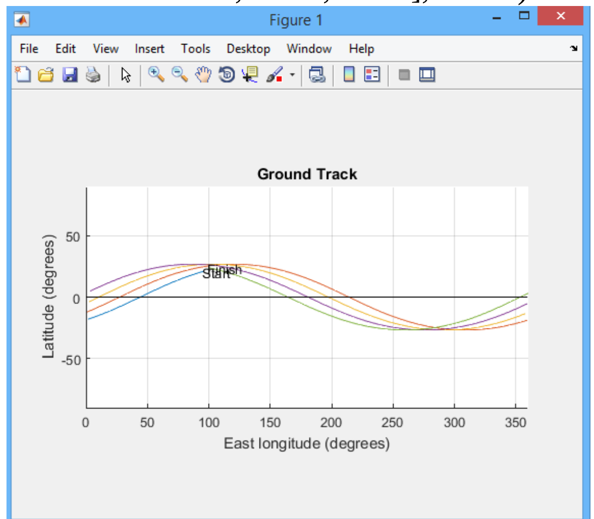
C. Results

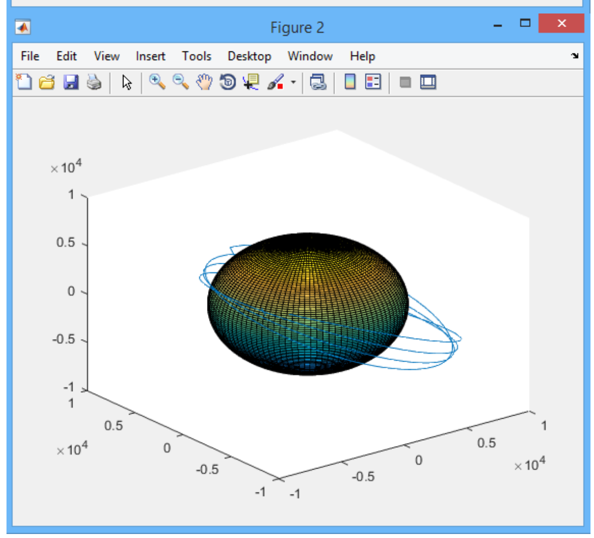
- GT_from_R0V0_UniversalKepler([-6045 -3490 2500], [-3.457, 6.618, 2.533], 60000)





- GT_from_R0V0_UniversalKepler([-6045 -3490 2500], [-3.457, 6.618, 2.533], 30000)





V. Ground Track from Classical Orbital Elements

The satellite's ground track is plotted throughout a user-defined period of time where the inputs provided by the user are the satellite's orbital elements (initial true anomaly, orbit's semi-major axis, eccentricity, initial right ascension of ascending node, initial argument of perigee and inclination) as well as the period of time over which the ground track is to be plotted. The Earth's oblateness and rotation are taken into consideration.

A. Algorithm

1. All the angles provided by the user as inputs are changed from degrees to radians.
2. The delta time every which the position of the satellite relative to Earth is to be calculated is set to be 60 seconds.
3. The specific angular momentum is calculated using the equation:
   $h = \sqrt{\mu a(1-e^2)}$
4. The orbital period is calculated using the equation:
   $$T = \frac{2\pi a^{\frac{3}{2}}}{\mu^{\frac{1}{2}}}$$
5. To account for the Earth's **oblateness**, the rate of change of the right ascension of ascending node is calculated from:
   $$\dot{\Omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\cos i$$

where $J_2 = 0.0010836$ and R is Earth's radius.

6. To account for the Earth's **oblateness**, the rate of change of the argument of perigee is calculated from:
   $$\dot{\omega} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2 a^{\frac{7}{2}}}\right]\left(\frac{5}{2}\sin^2 i - 2\right)$$
7. The initial eccentric anomaly is calculated from:
   $$E_0 = 2\arctan\left(\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta_0}{2}\right)\right)$$
8. The initial mean anomaly is calculated from:
   $$M_0 = E_0 - e\,sin(E_0)$$
9. The initial time is calculated from:
   $$t_0 = \frac{M_0 T}{2\pi}$$
10. The delta time is added to t_0, and the new mean anomaly is calculated from:
    $$M = \frac{2\pi t}{T}$$
11. If M < π, an initial guess of (M+e/2) is given for E, while if M > π, then an initial guess of (M-e/2) is given instead.
12. Iterations are carried out to calculate the correct E by using the transcendental equation:
    $$E_{i+1} = M + e\,sin(E_i)$$
13. The new true anomaly is calculated from:
    $$\theta = 2\arctan\left(\sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)\right)$$
14. The argument of perigee and the right ascension of ascending node are updated using:
    $$\Omega = \Omega_0 + \dot{\Omega}\,\Delta t$$
    $$w = w_0 + \dot{w}\,\Delta t$$
15. The current position and velocity vectors in the perifocal frame are calculated from:
    $$\{\mathbf{r}\}_{\bar{x}} = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta}\begin{Bmatrix}\cos\theta\\ \sin\theta\\ 0\end{Bmatrix}$$
    $$\{\mathbf{v}\}_{\bar{x}} = \frac{\mu}{h}\begin{Bmatrix}-\sin\theta\\ e+\cos\theta\\ 0\end{Bmatrix}$$
    (the velocity vector is unnecessary)
16. The transformation matrix from the perifocal

frame to the geocentric equatorial frame is calculated from:

$$[\mathbf{Q}]_{\bar{x}X} = \begin{bmatrix} -\sin\Omega\cos i\sin\omega + \cos\Omega\cos\omega & -\sin\Omega\cos i\cos\omega - \cos\Omega\sin\omega & \sin\Omega\sin i \\ \cos\Omega\cos i\sin\omega + \sin\Omega\cos\omega & \cos\Omega\cos i\cos\omega - \sin\Omega\sin\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix}$$

17. The current position and velocity vectors in the geocentric equatorial frame are calculated from:

$$\{\mathbf{r}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{r}\}_{\bar{x}} \quad \{\mathbf{v}\}_X = [\mathbf{Q}]_{\bar{x}X}\{\mathbf{v}\}_{\bar{x}}$$

(the velocity vector is unnecessary)

18. To account for the Earth's **rotation**, the transformation matrix from the geocentric equatorial frame to the rotating Earth-fixed frame is calculated from:

$$[\mathbf{R}_3(\theta)] = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$$\theta = \omega_E(t - t_0)$$

with $w_E = 4.178 * 10^{-3}$

19. The position vector in the rotating Earth-fixed frame is calculated from:

$$\{\mathbf{r}\}_{X'} = [\mathbf{R}_3(\theta)]\{\mathbf{r}\}_X$$

20. The direction cosines of $\{r\}_{x'}$ are calculated from:

$$l = \frac{(\{r\}_{x'})_x}{|\{r\}_{x'}|}, m = \frac{(\{r\}_{x'})_y}{|\{r\}_{x'}|},$$

$$n = \frac{(\{r\}_{x'})_z}{|\{r\}_{x'}|}$$

21. The declination is calculated from:

$$\delta = \sin^{-1} n$$

22. If m > 0, the right ascension is calculated from:

$$\alpha = \cos^{-1}\frac{l}{\cos(\delta)}$$

Otherwise it is calculated from:

$$\alpha = 360 - \cos^{-1}\frac{l}{\cos(\delta)}$$

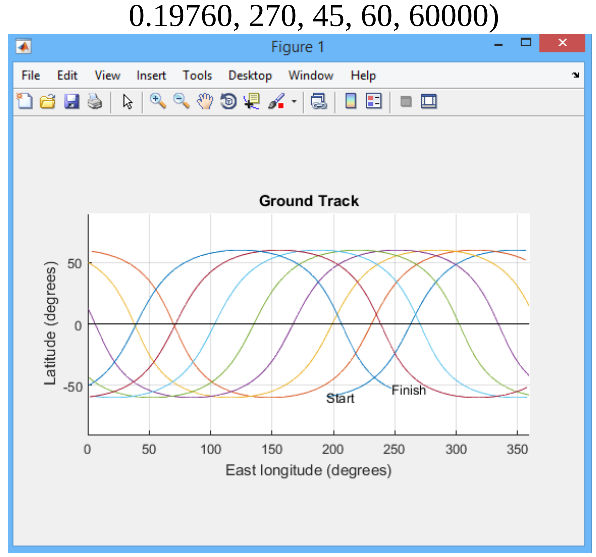
23. The right ascension is plotted on the x-axis against the declination on the y-axis, and the process is repeated by incrementing the time with delta t, until the desired period specified by the user ends.
24. The components of the position vector in the geocentric equatorial frame are plotted in 3D.

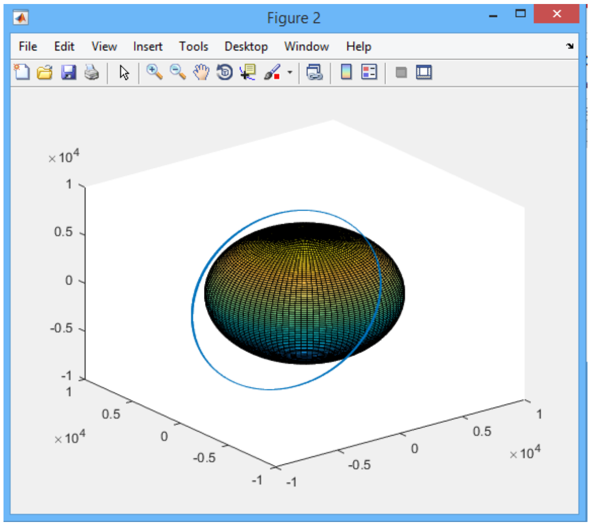
B. Required Functions:

- Main Function: GT_from_COE.m
- sv_from_coe.m
- ra_and_dec_from_r.m
- plot_3D.m

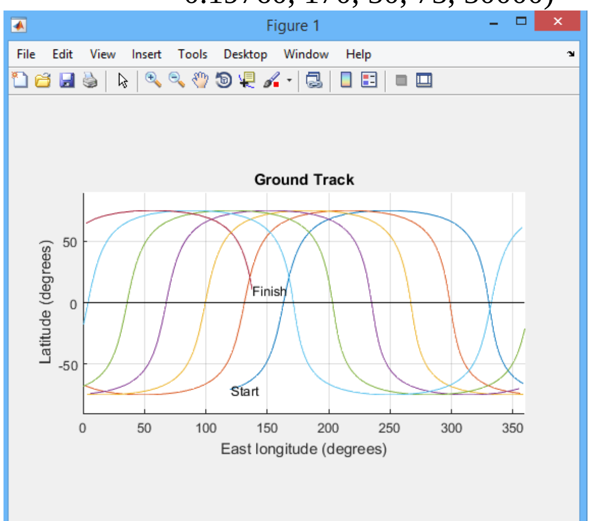
C. Results

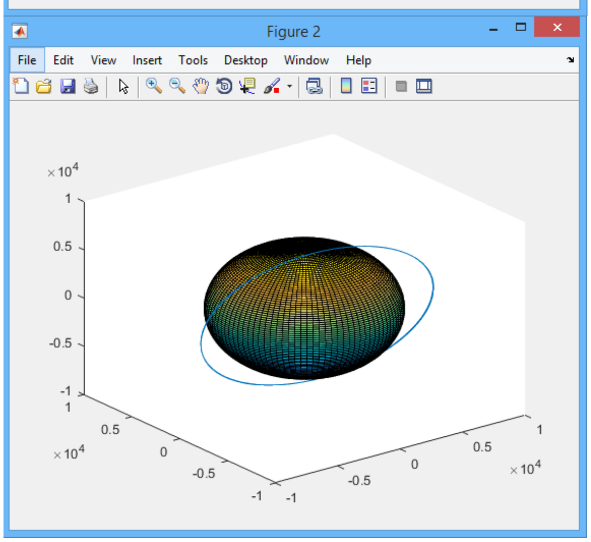
- GT_from_COE(230, 8350, 0.19760, 270, 45, 60, 60000)





- GT_from_COE(230, 8350, 0.19760, 170, 50, 75, 50000)





VI. Ground Track from Two Line Element (TLE)

The satellite's ground track is plotted throughout a user-defined period of time where the inputs provided by the user are the name of the satellite's two-line element file as well as the period of time over which the ground track is to be plotted. The Earth's oblateness and rotation are taken into consideration.

A. Algorithm

1. The two-line elements file specified by the user is opened.
2. The second line is extracted.
3. The inclination of the satellite's orbit is extracted from the 9^{th} to the 16^{th} columns of the second line in degrees.
4. The right ascension of the ascending node is extracted from the 18^{th} to the 25^{th} columns of the second line in degrees.
5. The eccentricity (with a decimal point assumed at the beginning) is extracted from the 27^{th} to the 33^{rd} columns of the second line.
6. The argument of perigee is extracted from the 35^{th} to the 42^{nd} columns of the second line in degrees.
7. The mean motion (n) is extracted from the 53^{rd} to the 63^{rd} columns of the second line in revolutions per day, and then changed to radians per second.
8. The semi-major axis is calculated using:

$$a = (\frac{\mu}{n^2})^{\frac{1}{3}}$$

9. The specific angular momentum is calculated from:

$$h = \sqrt{a\mu(1-e^2)}$$

10. The mean anomaly is extracted from the 44^{th} to the 51^{st} columns of the second line in degrees and changed to radians.
11. If M < πan initial guess of (M+e/2) is given for E, while if M > π, then an initial guess of (Me/2) is given instead.
12. Iterations are carried out to calculate the correct E by using the transcendental equation:

$$E_{i+1} = M + e\,sin(E_i)$$

13. The true anomaly is calculated from:

$$\theta = 2\arctan(\sqrt{\tfrac{1+e}{1-e}}\tan(\tfrac{E}{2})),$$

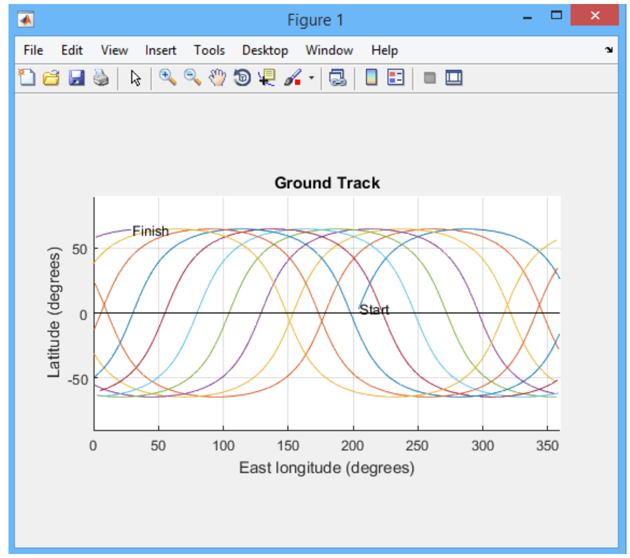
 and changed from radians to degrees.
14. Given those orbital elements, the algorithm explained in section (I) is used to plot the ground track of the satellite for the period defined by the user, taking into account the Earth's oblateness and rotation.

B. Required Functions/Files

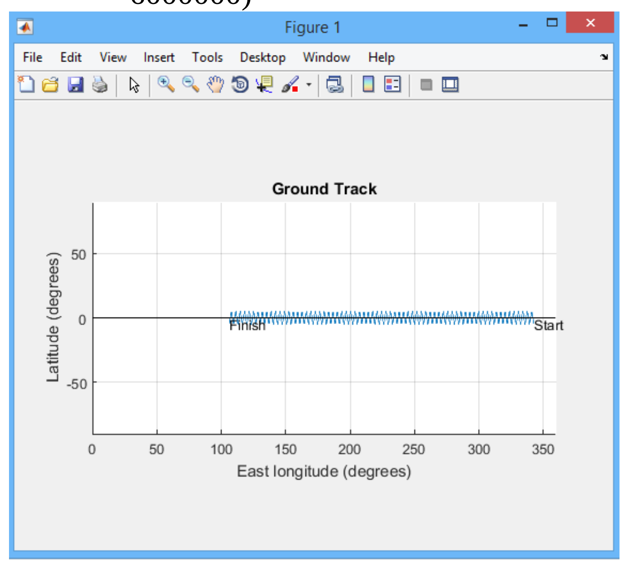
- Main Function: GT_from_TLE.m
- TLE1.txt, TLE2.txt, …
- kepler_E.m
- GT_from_COE.m (and its required functions mentioned earlier)

C. Results

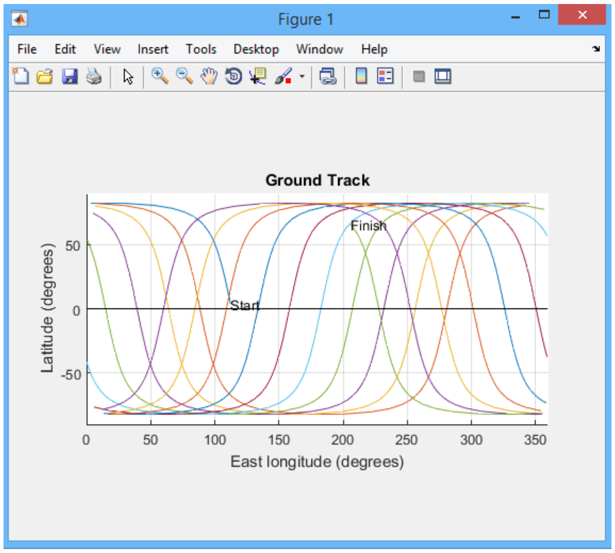
- GT_from_TLE('TLE1.txt', 60000)



- GT_from_TLE('TLE2.txt', 6000000)



- GT_from_TLE('TLE3.txt', 60000)



VII. Ground Track from Three Position Vectors (Gibbs Method)

The satellite's ground track is plotted throughout a user-defined period of time where the inputs provided by the user are three position vectors and the period of time over which the ground track is to be plotted. The Earth's oblateness and rotation are taken into consideration.

A. Algorithm

1. The magnitudes of the three position vectors are calculated.
2. The cross products between the three vectors are calculated such that:

$$C_{12} = r_1 \times r_2, C_{23} = r_2 \times r_3, C_{31} = r_3 \times r_1$$

3. Check whether the three position vectors are coplanar by verifying that

$$\frac{r_1}{||r_1||} \cdot \frac{C_{23}}{||C_{23}||} = 0.$$

   If they are not, an error message is displayed and the program terminated.
4. Calculate N using:

$$\mathbf{N} = r_1(\mathbf{r}_2 \times \mathbf{r}_3) + r_2(\mathbf{r}_3 \times \mathbf{r}_1) + r_3(\mathbf{r}_1 \times \mathbf{r}_2)$$

5. Calculate D using:

$$\mathbf{D} = \mathbf{r}_1 \times \mathbf{r}_2 + \mathbf{r}_2 \times \mathbf{r}_3 + \mathbf{r}_3 \times \mathbf{r}_1$$

6. Calculate S using:

$$\mathbf{S} = \mathbf{r}_1(r_2 - r_3) + \mathbf{r}_2(r_3 - r_1) + \mathbf{r}_3(r_1 - r_2)$$

7. Calculate v_2 by substituting r_1 in

$$\mathbf{v} = \sqrt{\frac{\mu}{ND}}\left(\frac{\mathbf{D} \times \mathbf{r}}{r} + \mathbf{S}\right)$$

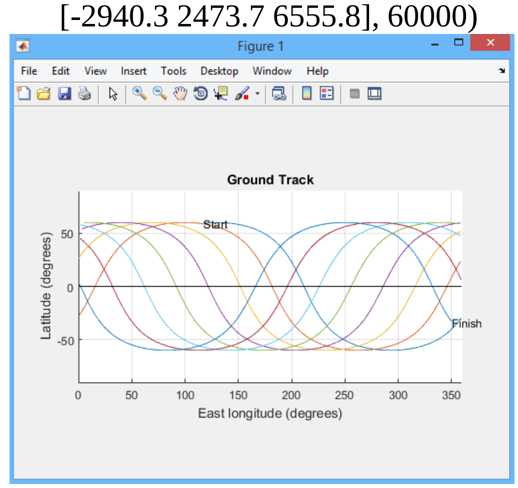
8. The classical orbital elements are calculated using the algorithm in Ground-Track-from-COE section.
9. Given those orbital elements, the algorithm explained in section V is used to plot the 2D & 3D ground track of the satellite for the period defined by the user, taking into account the Earth's oblateness and rotation.

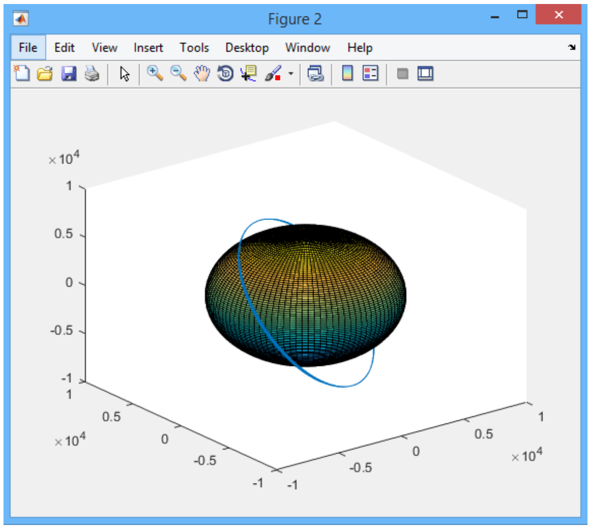
B. Required Functions

- Main Function: GT_from_Gibbs.m
- coe_from_sv.m
- GT_from_COE.m and its required functions mentioned earlier

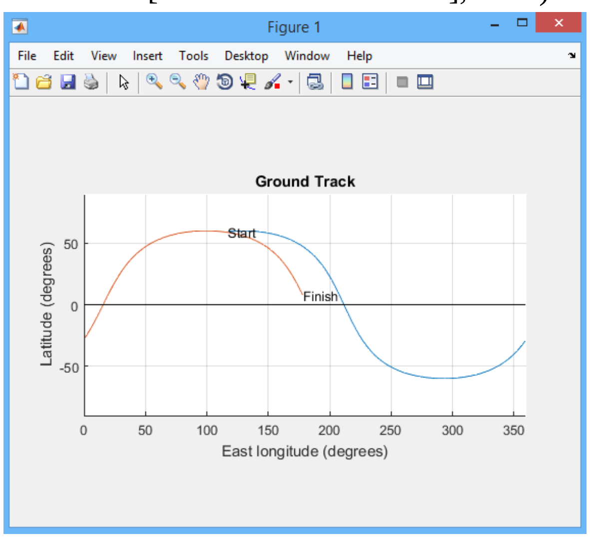
C. Results:

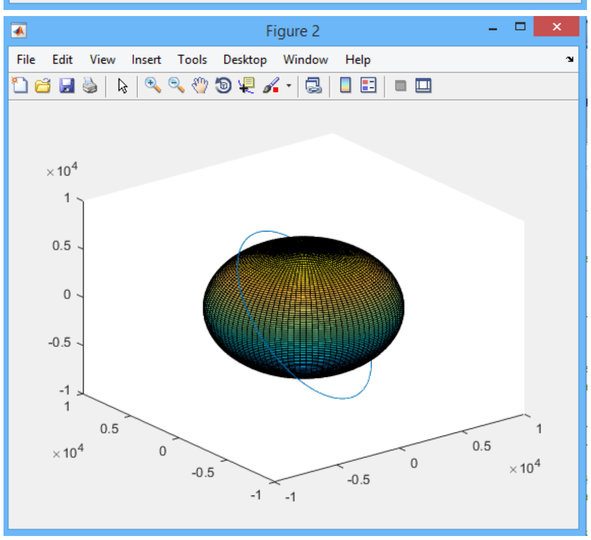
- GT_from_Gibbs([-294.32 4265.1 5986.7], [-1365.5 3637.6 6346.8], [-2940.3 2473.7 6555.8], 60000)





- GT_from_Gibbs([-294.32 4265.1 5986.7], [-1365.5 3637.6 6346.8], [-2940.3 2473.7 6555.8], 9000)





VIII. Ground Track from Two Position Vectors & the Time Interval Between Them (Lambert Problem)

The satellite's ground track is plotted throughout a user-defined period of time where the inputs provided by the user are two position vectors, the time interval between them, and the period of time over which the ground track is to be plotted. The Earth's oblateness and rotation are taken into consideration.

A. Algorithm

1. The magnitudes of the two position vectors are calculated.
2. The angle (delta theta) between the two position vectors is calculated depending on whether the orbit is prograde or retrograde, and according to the z^{th} component of the cross product of the two position vectors such that:

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if} \quad (\mathbf{r}_1\times\mathbf{r}_2)_Z \geq 0 \\ & \qquad\qquad \text{prograde trajectory} \\ 360^\circ - \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if} \quad (\mathbf{r}_1\times\mathbf{r}_2)_Z < 0 \\ \hline \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if} \quad (\mathbf{r}_1\times\mathbf{r}_2)_Z < 0 \\ & \qquad\qquad \text{retrograde trajectory} \\ 360^\circ - \cos^{-1}\left(\frac{\mathbf{r}_1\cdot\mathbf{r}_2}{r_1 r_2}\right) & \text{if} \quad (\mathbf{r}_1\times\mathbf{r}_2)_Z \geq 0 \end{cases}$$

3. The constant A is calculated using:

$$A = \sin\Delta\theta\sqrt{\frac{r_1 r_2}{1-\cos\Delta\theta}}$$

4. We obtain an initial guess of z in the equation:

$$F(z) = \left[\frac{y(z)}{C(z)}\right]^{\frac{3}{2}} S(z) + A\sqrt{y(z)} - \sqrt{\mu}\Delta t$$

by plugging in any value for z and keep incrementing/ decrementing it until the value of F(z) changes sign; the last calculated z is our initial guess.
The functions S(z), C(z) and y(z) are calculated from:

$$S(z) = \begin{cases} \dfrac{\sqrt{z} - \sin\sqrt{z}}{(\sqrt{z})^3} & (z > 0) \\ \dfrac{\sinh\sqrt{-z} - \sqrt{-z}}{(\sqrt{-z})^3} & (z < 0) \\ \dfrac{1}{6} & (z = 0) \end{cases}$$

$$C(z) = \begin{cases} \dfrac{1 - \cos\sqrt{z}}{z} & (z > 0) \\ \dfrac{\cosh\sqrt{-z} - 1}{-z} & (z < 0) \\ \dfrac{1}{2} & (z = 0) \end{cases}$$

$$y(z) = r_1 + r_2 + A\frac{zS(z) - 1}{\sqrt{C(z)}}$$

6. We then iterate to find z from the equation:

$$z_{i+1} = z_i - \frac{F(z_i)}{F'(z_i)}$$

where F'(z) is calculated from:

$$F'(z) = \begin{cases} \left[\dfrac{y(z)}{C(z)}\right]^{\frac{3}{2}}\left\{\dfrac{1}{2z}\left[C(z) - \dfrac{3}{2}\dfrac{S(z)}{C(z)}\right] + \dfrac{3}{4}\dfrac{S(z)^2}{C(z)}\right\} + \dfrac{A}{8}\left[3\dfrac{S(z)}{C(z)}\sqrt{y(z)} + A\sqrt{\dfrac{C(z)}{y(z)}}\right] & (z \neq \\ \dfrac{\sqrt{2}}{40}y(0)^{\frac{3}{2}} + \dfrac{A}{8}\left[\sqrt{y(0)} + A\sqrt{\dfrac{1}{2y(0)}}\right] & (z = \end{cases}$$

7. We calculate the Lagrange coefficients from the equations:

$$f = 1 - \frac{y(z)}{r_1}$$

$$g = A\sqrt{\frac{y(z)}{\mu}}$$

8. The velocity vector corresponding to the first position vector is calculated from:

$$\mathbf{v}_1 = \frac{1}{g}(\mathbf{r}_2 - f\mathbf{r}_1)$$

9. The classical orbital elements are calculated from the first position and velocity vectors as follows:

- Specific angular momentum:

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

$$h = \sqrt{\mathbf{h} \cdot \mathbf{h}}$$

- Inclination:

$$i = \cos^{-1}\frac{h_Z}{h}$$

- Right ascension of ascending node:

$$\mathbf{N} = \hat{\mathbf{K}} \times \mathbf{h}$$

$$N = \sqrt{\mathbf{N} \cdot \mathbf{N}}$$

$$\Omega = \cos^{-1}\frac{N_X}{N}$$

- Eccentricity:

$$v_r = \frac{\mathbf{v} \cdot \mathbf{r}}{r}$$

$$\mathbf{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\mathbf{r} - r v_r \mathbf{v}\right]$$

$$e = \sqrt{\mathbf{e} \cdot \mathbf{e}}$$

- Argument of perigee:

$$\omega = \cos^{-1}\frac{\mathbf{N} \cdot \mathbf{e}}{Ne}$$

- True anomaly:

$$\theta = \cos^{-1}\left(\frac{\mathbf{e} \cdot \mathbf{r}}{er}\right)$$

- Semi-major axis:

$$a = \frac{h^2}{\mu(1 - e^2)}$$

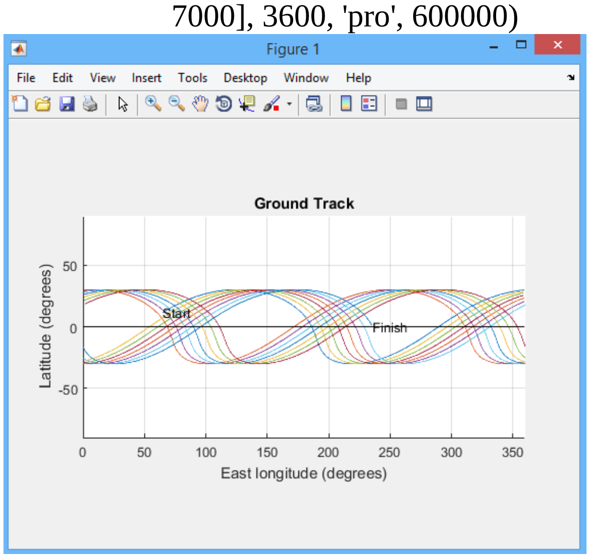
10. Given those orbital elements, the algorithm explained in section V is used to plot the ground track of the satellite for the period defined by the user, taking into account the Earth's oblateness and rotation.

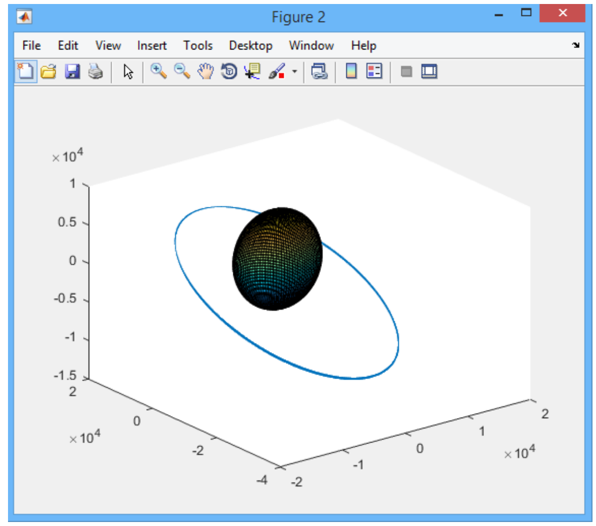
B. Required Functions

- Main Function: GT_from_Lambert.m
- coe_from_sv.m
- GT_from_COE.m (and its required functions mentioned earlier)

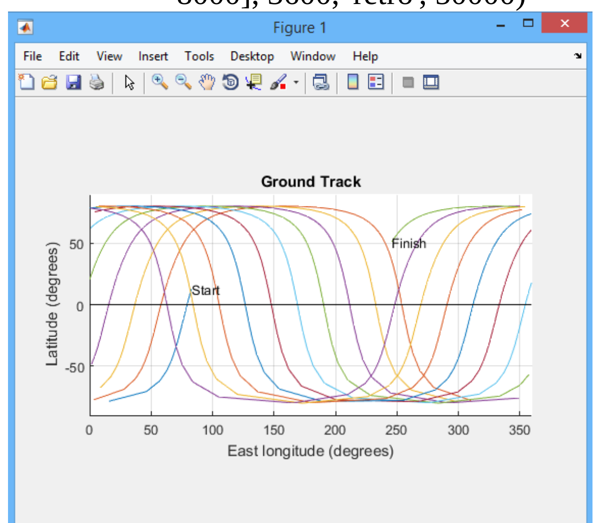
C. Results

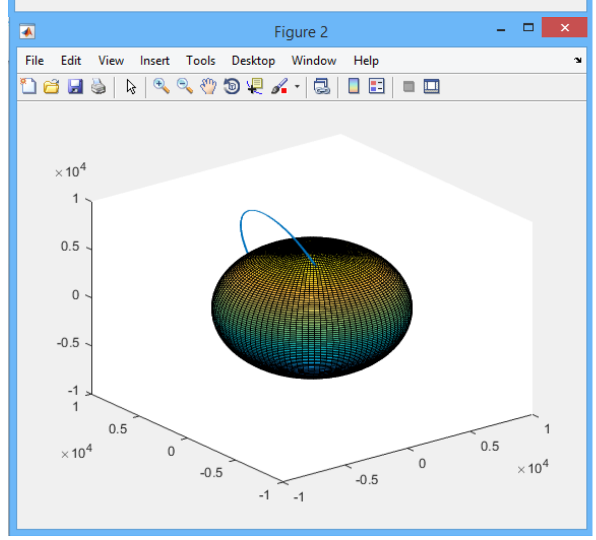
- GT_from_Lambert([5000 10000 2100], [-14600 2500 7000], 3600, 'pro', 600000)





- GT_from_Lambert([900, 8000, 2200], [-600, 5000, 8000], 3600, 'retro', 50000)





-

IX. Ground Track from Three Ground Tracking Locations & Angles of Observation (Gauss Method)

The satellite's ground track is plotted throughout a user-defined period of time where the inputs provided by the user are three ground track locations, the angles of observation, and the period of time over which the ground track is to be plotted. The Earth's oblateness and rotation are taken into consideration.

A. Algorithm

1. Calculate the time intervals between the observations:

$$\tau_1 = t_1 - t_2$$
$$\tau_3 = t_3 - t_2$$
$$\tau = \tau_3 - \tau_1$$

2. Calculate the direction cosine vectors at each of the ground tracking sites in the topocentric horizon coordinates using the equation:

$$\begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix} = \begin{Bmatrix} \sin A \cos a \\ \cos A \cos a \\ \sin a \end{Bmatrix}$$

3. Calculate the rotation matrix (QxX) from the topocentric horizon coordinates to the topocentric equatorial coordinates at each of the ground tracking sites:

$$\begin{bmatrix} -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \\ \cos\theta & -\sin\theta\sin\phi & \sin\theta\cos\phi \\ 0 & \cos\phi & \sin\phi \end{bmatrix}$$

4. Calculate the direction cosine vectors at each of the ground tracking sites in the topocentric equatorial coordinates using the equation:

$$\begin{Bmatrix} L_X \\ L_Y \\ L_Z \end{Bmatrix} = [\mathbf{Q}]_{xX} \begin{Bmatrix} l_x \\ l_y \\ l_z \end{Bmatrix}$$

5. Construct the direction cosine vectors from their components such that:

$$\hat{\boldsymbol{\rho}} = L_X\hat{\mathbf{I}} + L_Y\hat{\mathbf{J}} + L_Z\hat{\mathbf{K}}$$

6. Calculate the cross products among the direction cosine vectors:

$$p_1 = \widehat{\rho_2} \times \widehat{\rho_3}$$
$$p_2 = \widehat{\rho_1} \times \widehat{\rho_3}$$
$$p_3 = \widehat{\rho_1} \times \widehat{\rho_2}$$

7. Calculate the scalar triple product of $\widehat{\rho_1}$, $\widehat{\rho_2}$, and $\widehat{\rho_3}$:

$$D_0 = \hat{\boldsymbol{\rho}}_1 \cdot (\hat{\boldsymbol{\rho}}_2 \times \hat{\boldsymbol{\rho}}_3)$$

8. Calculate the following scalar triple product:

$$D_{11} = \mathbf{R}_1 \cdot (\hat{\boldsymbol{\rho}}_2 \times \hat{\boldsymbol{\rho}}_3) \quad D_{21} = \mathbf{R}_2 \cdot (\hat{\boldsymbol{\rho}}_2 \times \hat{\boldsymbol{\rho}}_3) \quad D_{31} = \mathbf{R}_3 \cdot (\hat{\boldsymbol{\rho}}_2 \times \hat{\boldsymbol{\rho}}_3)$$
$$D_{12} = \mathbf{R}_1 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_3) \quad D_{22} = \mathbf{R}_2 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_3) \quad D_{32} = \mathbf{R}_3 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_3)$$
$$D_{13} = \mathbf{R}_1 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_2) \quad D_{23} = \mathbf{R}_2 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_2) \quad D_{33} = \mathbf{R}_3 \cdot (\hat{\boldsymbol{\rho}}_1 \times \hat{\boldsymbol{\rho}}_2)$$

9. Calculate the values of A and B from:

$$A = \frac{1}{D_0}\left(-D_{12}\frac{\tau_3}{\tau} + D_{22} + D_{32}\frac{\tau_1}{\tau}\right)$$

$$B = \frac{1}{6D_0}\left[D_{12}\left(\tau_3^2 - \tau^2\right)\frac{\tau_3}{\tau} + D_{32}\left(\tau^2 - \tau_1^2\right)\frac{\tau_1}{\tau}\right]$$

10. Calculate E from:

$$E = \mathbf{R}_2 \cdot \hat{\boldsymbol{\rho}}_2$$

11. Calculate $R_2^2 = R_2 \cdot R_2$
12. Calculate the coefficients of the 8th order polynomial in the estimated geocentric radius x from:

$$a = -\left(A^2 + 2AE + R_2^2\right) \quad b = -2\mu B(A + E) \quad c = -\mu^2 B^2$$

13. We obtain an initial guess of x in the equation:

$$F = x^8 + ax^6 + bx^3 + c$$

by plugging in any value for x and keep incrementing/ decrementing it until the value of F changes sign; the last calculated x is our initial guess.

14. We then iterate to find x from the equation:

$$x_{i+1} = x_i - \frac{x_i^8 + \mathrm{a}x_i^6 + bx_i^3 + c}{8x_i^7 + 6\mathrm{a}x_i^5 + 3bx_i^2}$$

15. Calculate the slant ranges in terms of the calculated $r_2(x)$:

$$\rho_1 = \frac{1}{D_0}\left[\frac{6\left(D_{31}\frac{\tau_1}{\tau_3} + D_{21}\frac{\tau}{\tau_3}\right)r_2^3 + \mu D_{31}\left(\tau^2 - \tau_1^2\right)\frac{\tau_1}{\tau_3}}{6r_2^3 + \mu\left(\tau^2 - \tau_3^2\right)} - \right.$$

$$\rho_2 = A + \frac{\mu B}{r_2^3}$$

$$\rho_3 = \frac{1}{D_0}\left[\frac{6\left(D_{13}\frac{\tau_3}{\tau_1} - D_{23}\frac{\tau}{\tau_1}\right)r_2^3 + \mu D_{13}\left(\tau^2 - \tau_3^2\right)\frac{\tau_3}{\tau_1}}{6r_2^3 + \mu\left(\tau^2 - \tau_1^2\right)} - \right.$$

16. Calculate r_1, r_2 and r_3 from:

$$\mathbf{r}_1 = \mathbf{R}_1 + \rho_1\hat{\boldsymbol{\rho}}_1$$

$$\mathbf{r}_2 = \mathbf{R}_2 + \rho_2\hat{\boldsymbol{\rho}}_2$$

$$\mathbf{r}_3 = \mathbf{R}_3 + \rho_3\hat{\boldsymbol{\rho}}_3$$

17. Calculate the Lagrange coefficients from:

$$f_1 \approx 1 - \frac{1}{2}\frac{\mu}{r_2^3}\tau_1^2$$

$$f_3 \approx 1 - \frac{1}{2}\frac{\mu}{r_2^3}\tau_3^2$$

$$g_1 \approx \tau_1 - \frac{1}{6}\frac{\mu}{r_2^3}\tau_1^3$$

$$g_3 \approx \tau_3 - \frac{1}{6}\frac{\mu}{r_2^3}\tau_3^3$$

18. Calculate v_2 from:

$$\mathbf{v}_2 = \frac{1}{f_1 g_3 - f_3 g_1}\left(-f_3\mathbf{r}_1 + f_1\mathbf{r}_3\right)$$

19. Calculate the magnitudes of r_2 and v_2.
20. Calculate the reciprocal of the semi-major axis:

$$\alpha = \frac{2}{r_2} - \frac{v_2^2}{\mu}$$

21. Calculate the radial component of v_2:

$$v_{r2} = \frac{v_2 . r_2}{r_2}$$

22. Iterate to find the solution of the transcendental equation:

$$\sqrt{\mu}\tau_1 = \frac{r_2 v_{r2}}{\sqrt{\mu}}\chi_1^2 C\left(\alpha\chi_1^2\right) + \left(1 - \alpha r_2\right)\chi_1^3 S\left(\alpha\chi_1^2\right) + r_2\chi_1$$

$$\sqrt{\mu}\tau_3 = \frac{r_2 v_{r2}}{\sqrt{\mu}}\chi_3^2 C\left(\alpha\chi_3^2\right) + \left(1 - \alpha r_2\right)\chi_3^3 S\left(\alpha\chi_3^2\right) + r_2\chi_3$$

23. Use the calculated χ_1 and χ_3 to update the Lagrange coefficients:

$$f_1 = 1 - \frac{\chi_1^2}{r_2}C\left(\alpha\chi_1^2\right) \quad g_1 = \tau_1 - \frac{1}{\sqrt{\mu}}\chi_1^3 S\left(\alpha\chi_1^2\right)$$

$$f_3 = 1 - \frac{\chi_3^2}{r_2}C\left(\alpha\chi_3^2\right) \quad g_3 = \tau_3 - \frac{1}{\sqrt{\mu}}\chi_3^3 S\left(\alpha\chi_3^2\right)$$

24. Find c_1 & c_2 from:

$$c_1 = \frac{g_3}{f_1 g_3 - f_3 g_1}$$

$$c_3 = -\frac{g_1}{f_1 g_3 - f_3 g_1}$$

25. Update the values of slant ranges using:

$$\rho_1 = \frac{1}{D_0}\left(-D_{11} + \frac{1}{c_1}D_{21} - \frac{c_3}{c_1}D_{31}\right)$$

$$\rho_2 = \frac{1}{D_0}\left(-c_1 D_{12} + D_{22} - c_3 D_{32}\right)$$

$$\rho_3 = \frac{1}{D_0}\left(-\frac{c_1}{c_3}D_{13} + \frac{1}{c_3}D_{23} - D_{33}\right)$$

26. Update the values of r_1, r_2 and r_3 from:

$$\mathbf{r}_1 = \mathbf{R}_1 + \rho_1\hat{\boldsymbol{\rho}}_1$$

$$\mathbf{r}_2 = \mathbf{R}_2 + \rho_2\hat{\boldsymbol{\rho}}_2$$

$$\mathbf{r}_3 = \mathbf{R}_3 + \rho_3\hat{\boldsymbol{\rho}}_3$$

27. Recalculate v_2 from the new Lagrange coefficients using:

$$\mathbf{v}_2 = \frac{1}{f_1 g_3 - f_3 g_1}\left(-f_3\mathbf{r}_1 + f_1\mathbf{r}_3\right)$$

28. The process is repeated until the required precision is reached.
29. Calculate the classical orbital elements from r_2 and v_2 using the algorithm described in the previous section.

30. Given those orbital elements, the algorithm explained in the first section is used to plot the 2D & 3D ground track of the satellite for the period defined by the user, taking into account the Earth's oblateness and rotation.

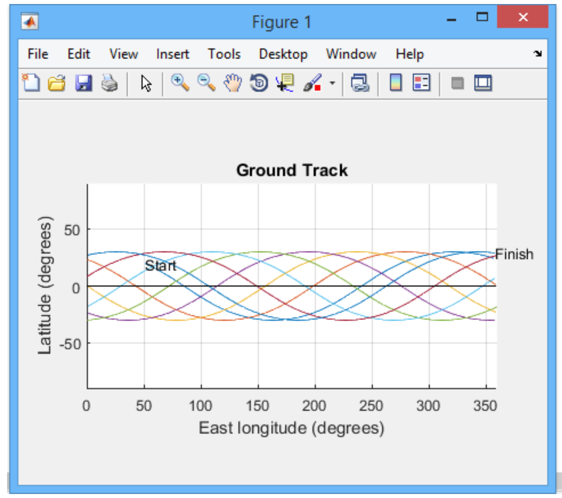
B. Required Functions:

- Main Function: GT_from_Gauss.m
- posroot.m
- kepler_U.m
- f_and_g.m
- coe_from_sv.m
- GT_from_COE.m (and its required functions mentioned earlier)

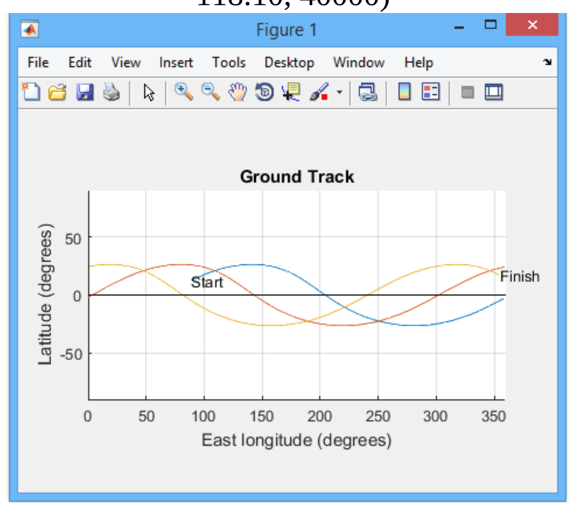
C. Results

Note: the direction cosines were inputted directly because sample test data were not found for azimuth and elevation. However, if they were found, the sent GT_from_Gauss.m function takes them in as input normally as presented in the algorithm.

- GT_from_Gauss([0.7164 0.6807 -0.1527], [0.5690 0.7953 -0.2092], [0.4184 0.8701 -0.2606], [3489.3 3429.6 4077.9], [3459.6 3459.6 4077.9], [3429.3 3489.6 4077.9], 0, 118.10, 237.58, 90000)

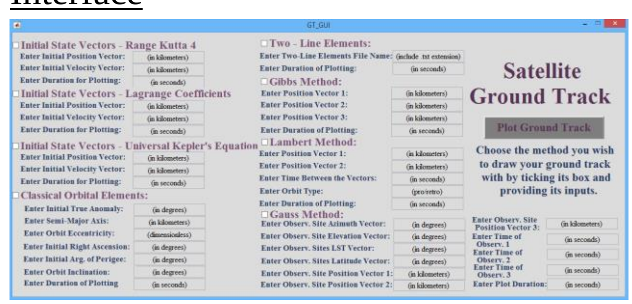


- GT_from_Gauss([0.7164 0.6807 -0.1527], [0.5690 0.7953 -0.2092], [0.4184 0.8701 -0.2606], [3489.3 3429.6 4077.9], [3459.6 3459.6 4077.9], [3429.3 3489.6 4077.9], 0, 50.2, 118.10, 40000)



X. Graphical User Interface

A. Interface



B. Required Functions:

- GT_GUI
- string_to_double_vector

XI. Conclusion

In fact, it is very impressing to be able to plot the ground track with so many different methods. Further improvements in each of the presented algorithms would be a great project to work on.

XII. References

[1] Curtis, D. Howard.*"Orbital Mechanics for Engineering Students"*. Third Edition. 2010.