Holo School

# Orthogonality in space

## I-Definition:

**Orthogonality in space** refers to the condition where two objects, such as **lines** or **planes**, are **perpendicular** to each other in **3D space**. This means they intersect at a **right angle** (90 degrees). When two lines or a line and a plane are orthogonal, the angle between them is exactly 90 degrees, indicating that they are independent in three-dimensional space

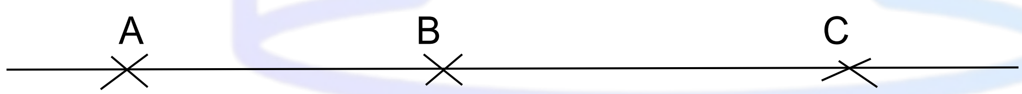
## II-Properties:

1-Three points that are not on the same line determine a plane (they form a triangle):

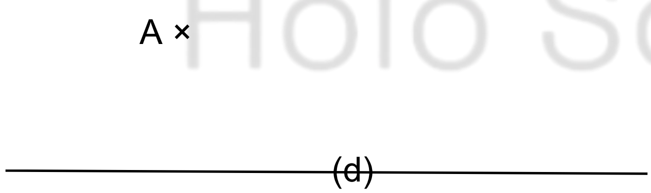
× A

× B × C

2-Three points that are on the same line do not form a unique plane; there are infinitely many planes that pass through a line,



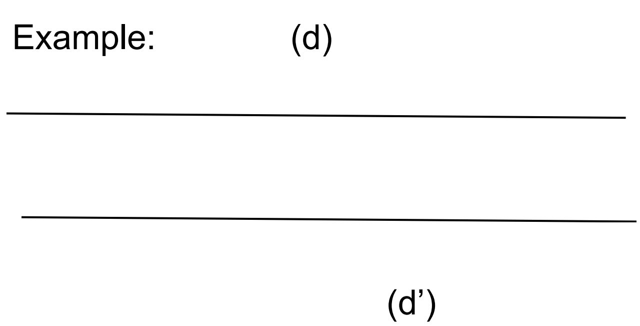

3- a point A and a line (d), such that A is outside the line (d), determine a single plane.






4- 2 parallel lines (d) and (d’) determine a single plane.

Example:

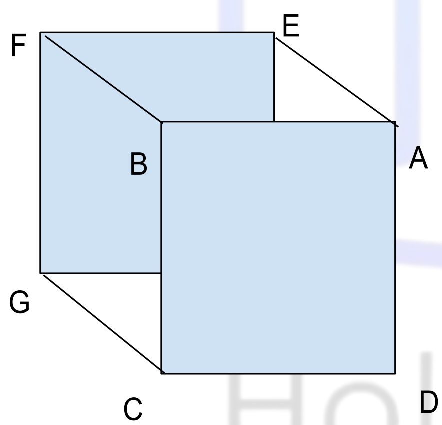


5-Relative positions of two planes:
a- parallel planes
b-intersecting planes
c-coincident planes

6- a-if two lines are orthogonal, any line parallel to one is orthogonal to the other
b-If two lines are parallel, any line orthogonal to one is orthogonal to the other

7-Very important: A line is perpendicular to a plane (P) ,if and only if it is orthogonal to two intersecting lines in (P).

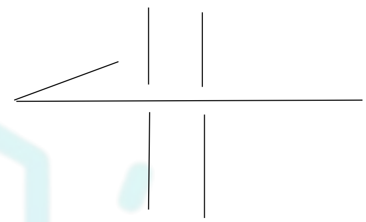
Ex:Prove that AB is perpendicular to (BCG):




(AB) perp (BF)(ABFE square)
(AB) perp (BC)(ABCD square)
=> (AB) perp (BFC)=(BCG)

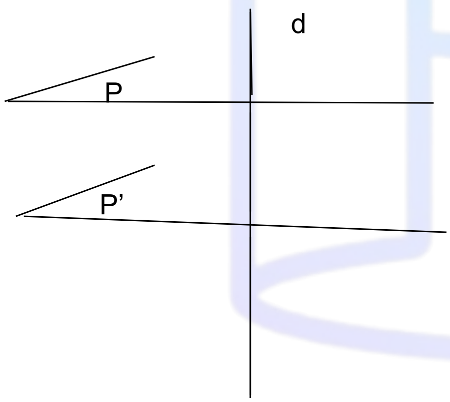
8- If a line (d) is perpendicular to a plane (P), then it is orthogonal to all the lines of that plane.
Ex: (CD)perp(CG) and (CD)perp(BC)=> (CD) perp (BCG)=(BCF)

9- If two lines are parallel, any line perpendicular to one is perpendicular to the other . ex:



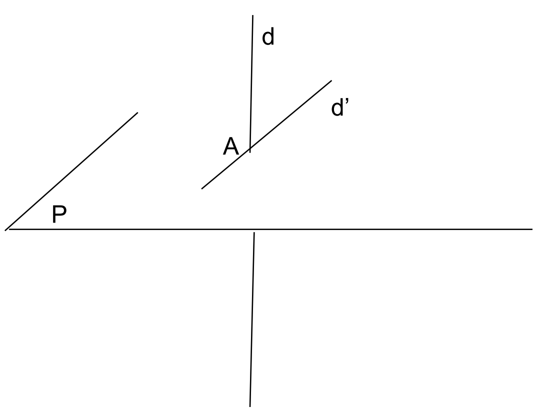
10-Two lines perpendicular to the same plane are parallel to each other

11- If two planes are parallel, any line perpendicular to one is perpendicular to the other

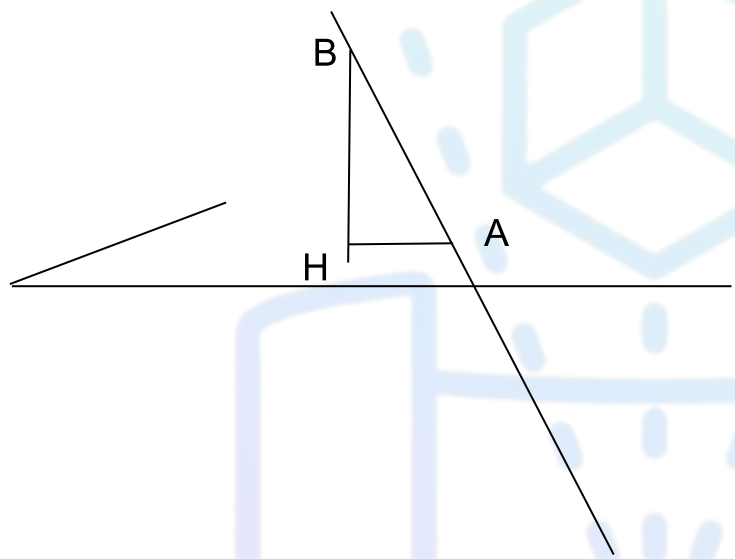





12- If a line (d) is perpendicular to a plane (P) at point A, then any line (d') passing through A and perpendicular to (d) is contained in P.

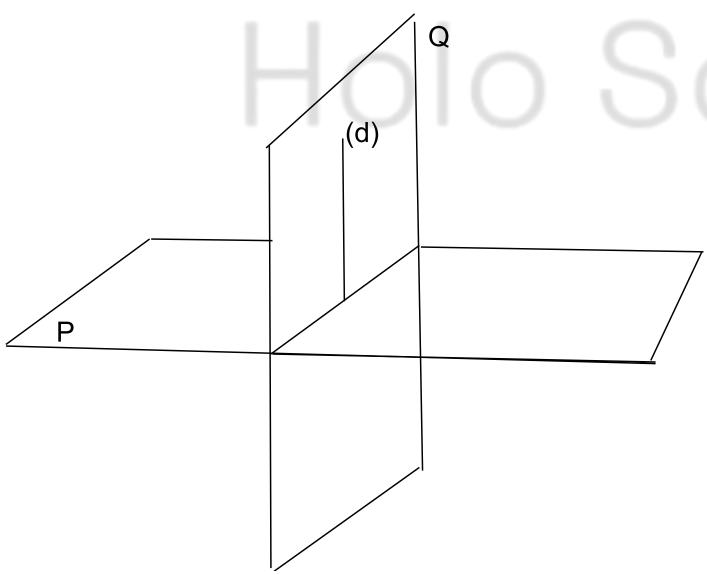


13- Angle between a line and a plane: Given a line (d) and a plane (P), the line (d) intersects (P) at point A. The angle between (d) and (P) is defined as the angle between (d) and the projection of a point B (any point on (d)) onto the plane (P). Let H be the orthonormal projection of B onto (P)




-BAH=angle between (d) and (P)

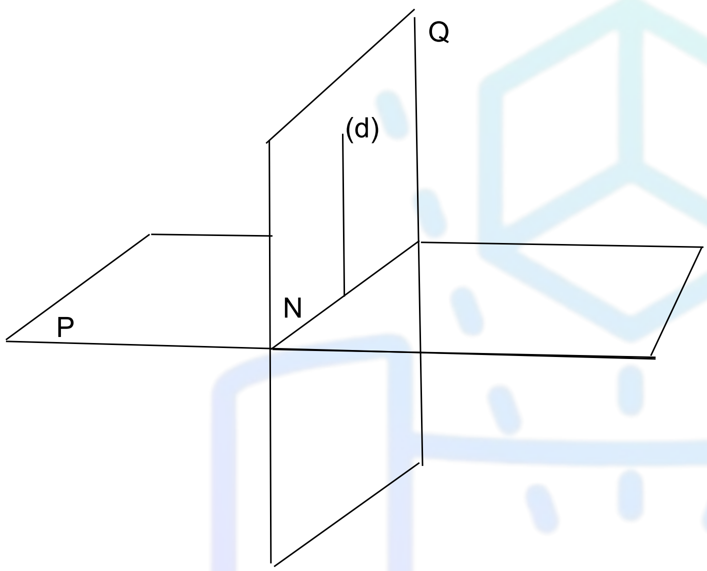
14-Two planes are perpendicular if the angle between them is a right angle (90 degrees)




Two planes are perpendicular if one of them contains a line (d) perpendicular to the other

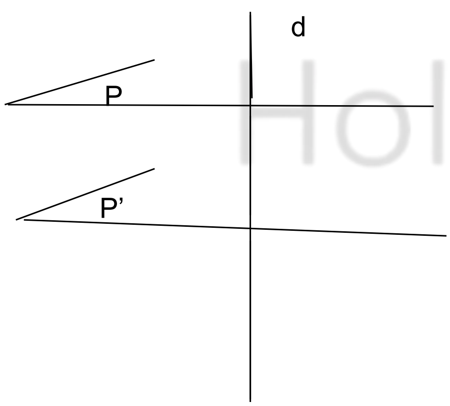
ex:(d) included in (Q) and (d) perp (P) => (P) perp (Q)

15- If two planes (P) and (Q) are perpendicular, then any line (d) in plane (P) that is perpendicular to their line of intersection (N) is perpendicular to plane (Q).



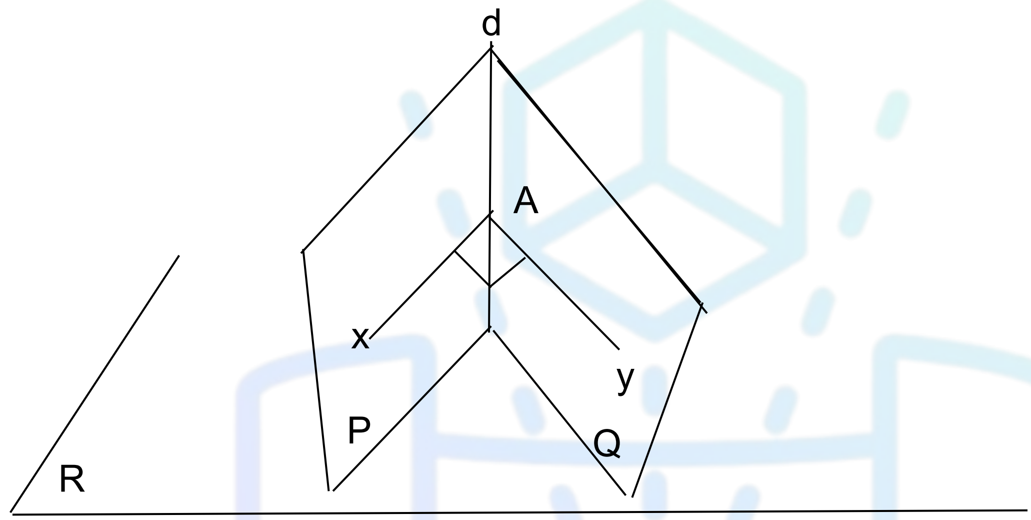

(d) included in(Q) and (d) perp (N) => (d) perp (P)

16-Two planes perpendicular to the same line are parallel to each other




(d) perp (P) and (d) perp (P') => (P) parallel (Q)

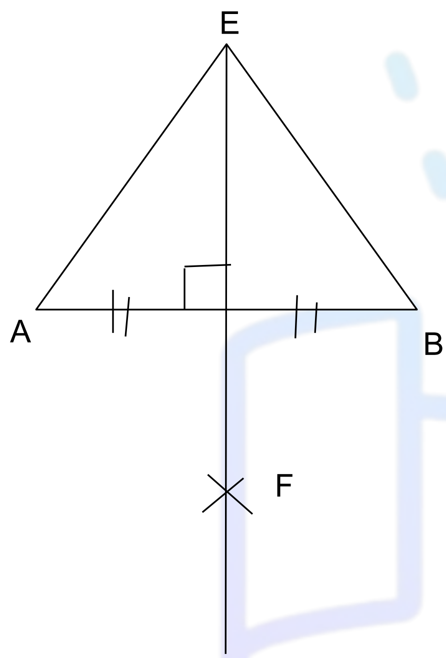
17-If two planes are perpendicular to a third plane (R), their line of intersection (d) is perpendicular to (R)

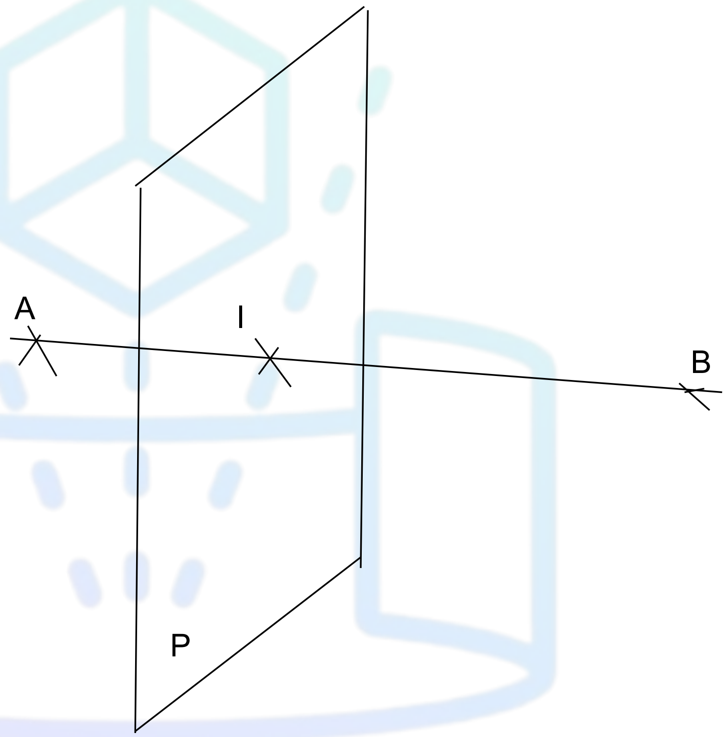



(P) perp ( R )and (Q) perp ( R ) => (P)inter(Q) perp ( R ) => (d) perp ( R )

18-Angle between two planes (P) and (Q): Angle of the dihedral: xAy where (Ax) perp (d) and (Ay) perp (d)

19- The plane (P) perpendicular to [AB] at its midpoint I is called the perpendicular bisector plane of [AB]. (P) is the set of all points equidistant (at the same distance from the two endpoints A and B of the segment [AB])




20-Positions of 2 lines in space.

- 2 intersecting lines meet at 1 point.
- 2 parallel lines never meet.
- 2 lines are neither intersecting nor parallel.
- 2 lines coincide. ( فوق بعضهم)
- 2 coplanar lines <==> 2 lines lie in the same plane.
- If the angle between 2 lines = 90°, then they are orthogonal.
- A line is perpendicular to the plane (P), if and only if the line is perpendicular to 2 intersecting lines in the plane (P).

## Properties of:

## Cube:

1. **Faces**: 6 square faces.
2. **Edges**: 12 edges.
3. **Vertices**: 8 vertices.
4. **Angle between faces**: All angles between faces are 90° (right angles).
5. **Symmetry**: Highly symmetrical (all faces, edges, and angles are identical).
6. **Diagonals**: 4 face diagonals and 4 space diagonals (connecting opposite vertices).
7. **Volume**: V=a3V = a^3V=a3 (where aaa is the length of an edge).
8. **Surface Area**: A=6a2A = 6a^2A=6a2.

## Tetrahedron:

1. **Faces**: 4 triangular faces.
2. **Edges**: 6 edges.
3. **Vertices**: 4 vertices.
4. **Angle between faces**: The dihedral angle between any two faces is the same.
5. **Symmetry**: It has 12 rotational symmetries.
6. **Volume**: V=a362V = \frac{a^3}{6\sqrt{2}}V=62



## Cuboid:

1. **Faces**: 6 rectangular faces.
2. **Edges**: 12 edges.
3. **Vertices**: 8 vertices.
4. **Angle between faces**: All angles between faces are 90° (right angles).
5. **Symmetry**: Fewer symmetries compared to a cube (depends on the dimensions of the cuboid).
6. **Volume**: V=l×w×hV = l \times w \times hV=l×w×h (where lll, www, and hhh are the length, width, and height).
7. **Surface Area**: A=2(lw+lh+wh)A = 2(lw + lh + wh)A=2(lw+lh+wh).

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