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Extra Credit 2

The cross product of r & s is $r \times s$

$$r(ABC) \times s(CD) = r \times s(ABCCD)$$

a_1	b_1	c_1
a_2	b_2	c_2
a_2	b_3	c_3
a_3	b_3	c_3

c_1	d_1
c_1	d_2
c_2	d_3
c_3	d_4
c_4	d_5

a_1	b_1	c_1	c_1	d_1
a_1	b_1	c_1	c_1	d_2
a_1	b_1	c_1	c_2	d_3
a_1	b_1	c_1	c_3	d_4
a_1	b_1	c_1	c_4	d_5
a_2	b_2	c_2	c_1	d_1
a_2	b_2	c_2	c_1	d_2
a_2	b_2	c_2	c_2	d_3
a_2	b_2	c_2	c_3	d_4
a_2	b_2	c_2	c_4	d_5
a_2	b_3	c_3	c_1	d_1
a_2	b_3	c_3	c_1	d_2
a_2	b_3	c_3	c_2	d_3
a_2	b_3	c_3	c_3	d_4
a_2	b_3	c_3	c_4	d_5
a_3	b_3	c_3	c_1	d_1
a_3	b_3	c_3	c_1	d_2
a_3	b_3	c_3	c_2	d_3
a_3	b_3	c_3	c_3	d_4
a_3	b_3	c_3	c_4	d_5

$$\text{arity} = \text{arity of } r + \text{arity of } s = 2 + 3 = 5$$

$$\# \text{ rows} = n_r \times n_s = 4 \times 5 = 20$$

now, use cross product to combine tables using matching columns (join):

$$r \bowtie s = \prod_{A, B, r.C, D} \left(\sigma_{r.C = s.C} (r \times s) \right)$$

The σ is what is implemented first as:

A	B	r.C	s.C	D
a ₁	b ₁	c ₁	c ₁	d ₁
a ₁	b ₁	c ₁	c ₁	d ₂
a ₂	b ₂	c ₂	c ₂	d ₃
a ₂	b ₃	c ₃	c ₃	d ₄
a ₃	b ₃	c ₃	c ₃	d ₄

Now if we move this through \prod , these collapse into one

The result:

A	B	C	D
a ₁	b ₁	c ₁	d ₁
a ₁	b ₁	c ₁	d ₂
a ₂	b ₂	c ₂	d ₃
a ₂	b ₃	c ₃	d ₄
a ₃	b ₃	c ₃	d ₄