BA Hackathon

Problem (1)

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1. Conceptual Overview

Problem we are going to deal with is the Capacitated Vehicle Routing Problem which is as the usual VRP but with restrictions on capacity which provide more constraints to the problem.

1. Mathematical Model

The Emergency Ambulance Routing Problem is with the following characteristics:

Given:

- Hospital location H = (lat_H, lon_H)
- Set of patients P = $\{p_1, p_2, \ldots, p_5\}$ where p_i = (lat_i, lon_i)
- Distance matrix D where $D[i][j]$ represents real road distance between locations i and j
- Single ambulance with capacity constraint $K = 3$ patients per trip

Decision Variables:

1. Route sequence for each trip
2. Number of trips required
3. Patient assignment to trips

Objective Function: Minimize total travel distance

Constraints:

- Each patient must be visited exactly once
- Maximum 3 patients per trip
- Each trip must start and end at the hospital
- All 5 patients must be served

Problem Complexity

This problem belongs to class of optimization problems :

For our specific case with 5 patients:

- Maximum possible routes: 5! = 120 permutations per trip configuration
- Total solution space: ~480 possible solutions

In the following section, we are going to illustrate the both classical and quantum approaches visited to solve the problem:

2. Classical approach

1. Data Processing Layer

GPS Coordinate Handling code:

```python
import requests
# --- Cell 4: Use OSRM API for Real Road Distances ---
def calculate_osrm_distances(locations_df):
    """Use OSRM API for accurate road distance calculations"""
    coordinates = []
    location_names = []

    # Add hospital first
    hospital = locations_df[locations_df['type'] == 'destination'].iloc[0]
    coordinates.append(f"{hospital['longitude']},{hospital['latitude']}")
    location_names.append(hospital['name'])

    # Add patients
    for idx, row in locations_df[locations_df['type'] == 'patient'].iterrows():
        coordinates.append(f"{row['longitude']},{row['latitude']}")
        location_names.append(row['name'])

    coords_str = ';'.join(coordinates)
    url = f"http://router.project-osrm.org/table/v1/driving/{coords_str}?annotations=distance"

    try:
        response = requests.get(url, timeout=30)
        response.raise_for_status()
        data = response.json()

        if 'distances' in data:
            distance_matrix = {}

            for i, loc1 in enumerate(location_names):
                distance_matrix[loc1] = {}
                for j, loc2 in enumerate(location_names):
                    distance_meters = data['distances'][i][j]
                    distance_km = distance_meters / 1000 if distance_meters is not None else float('inf')
                    distance_matrix[loc1][loc2] = distance_km

            return distance_matrix
        else:
            print("OSRM response format error")
            return None

    except requests.exceptions.RequestException as e:
        print(f"OSRM API error: {e}")
        return None
    except json.JSONDecodeError as e:
        print(f"JSON decode error: {e}")
        return None
```

Figure 1 ORSM API

OSRM API (Open Source Routing Machine) is used which:

- Provides real road network distances
- Accounts for actual driving routes, traffic patterns, and road restrictions
- Returns distance matrix in meters, converted to kilometers

2. Optimization Algorithm Layer

The system implements two distinct algorithms with increasing sophistication:

Algorithm 1: Brute Force Optimization

Approach:

- Generates all possible trip combinations
- Evaluates every permutation within each trip
- Guarantees optimal solution

Process:

- Generate all possible ways to group 5 patients into 1-5 trips
- For each grouping, generate all permutations within each trip
- Calculate total distance for each complete solution
- Select minimum distance solution

Complexity Analysis:

- **Time Complexity**: $O(n! \times 2^n)$
- **Space Complexity**: $O(n \times 2^n)$
- **Practical Limit**: ~8-10 patients due to exponential growth

Code Implementation:

```python
def brute_force_optimization(self):
    """Brute force optimization to find best route combination"""
    all_trips = self.generate_all_possible_trips()
    best_total_distance = float('inf')
    best_routes = []

    # Try all combinations of trips that cover all patients
    for num_trips in range(1, len(self.patient_names) + 1):
        for trip_combination in itertools.combinations(all_trips, num_trips):
            # Check if this combination covers all patients exactly once
            covered_patients = set()
            for trip in trip_combination:
                covered_patients.update(trip)

            if covered_patients == set(self.patient_names):
                total_distance = sum(self.calculate_trip_distance(trip) for trip in trip_combination)
                if total_distance < best_total_distance:
                    best_total_distance = total_distance
                    best_routes = list(trip_combination)

    return best_routes, best_total_distance
```

Figure 2 Brute_force approach

Algorithm 2: Optimized Greedy (Nearest Neighbor)

Approach:

- Geographic-aware greedy algorithm
- Builds trips by selecting nearest unvisited patients

Process:

1. Start at hospital for each new trip
2. Repeatedly select nearest unserved patient
3. Continue until trip capacity reached or no patients remain
4. Return to hospital to complete trip

code:

```python
#Greedy heuristic to quickly build routes (not guaranteed optimal)
    def greedy(self):
        remaining_patients = set(self.patient_names)
        routes = []
        total_distance = 0

        while remaining_patients:
            current_location = self.hospital_name
            current_trip = []
            current_distance = 0

            for _ in range(self.max_stops):
                if not remaining_patients:
                    break

                closest_patient = None
                min_distance = float('inf')

                for patient in remaining_patients:
                    dist = self.distance_matrix[current_location][patient]
                    if dist < min_distance:
                        min_distance = dist
                        closest_patient = patient

                if closest_patient:
                    current_trip.append(closest_patient)
                    current_distance += min_distance
                    current_location = closest_patient
                    remaining_patients.remove(closest_patient)

            if current_trip:
                current_distance += self.distance_matrix[current_location][self.hospital_name]
                routes.append(current_trip)
                total_distance += current_distance
        return routes, total_distance
```

Figure 3 Greedy approach

Complexity Analysis:

- **Time Complexity**: $O(n^2)$
- **Approximation Quality**: Typically within 10-30% of optimal for geographic problems

3. Visualization and Analysis Layer

Interactive Mapping

1. **Technology**: Folium (Leaflet.js wrapper)
2. **Features**:
    1. Hospital marker (red hospital icon)
    2. Patient markers (blue medical icons)
    3. Route visualization with different colors per trip
    4. Real road routing via OSRM API

Performance Analytics

- Distance comparison across algorithms
- Execution time benchmarking
- Solution quality metrics
- Scalability analysis

Algorithm performance Analysis

Theoretical Performance:

Algorithm	Time Complexity	Solution Quality	Scalability
Brute Force	$O(n!)$	Optimal	$n \leq 9$
Optimized Greedy	$O(n^2)$	Good Heuristic	$n \leq 1000$

Empirical Results:

For 5 points optimization problem:

Minimum distance got by Brute-Force algorithm is $= 57.31\ Km$

Minimum distance got by Greedy algorithm is $= 58.71\ Km$

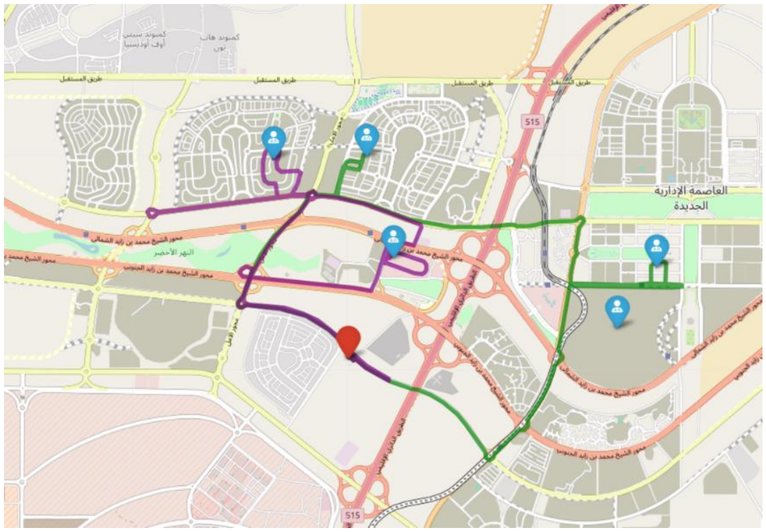

Figure 4 Brute force approach map

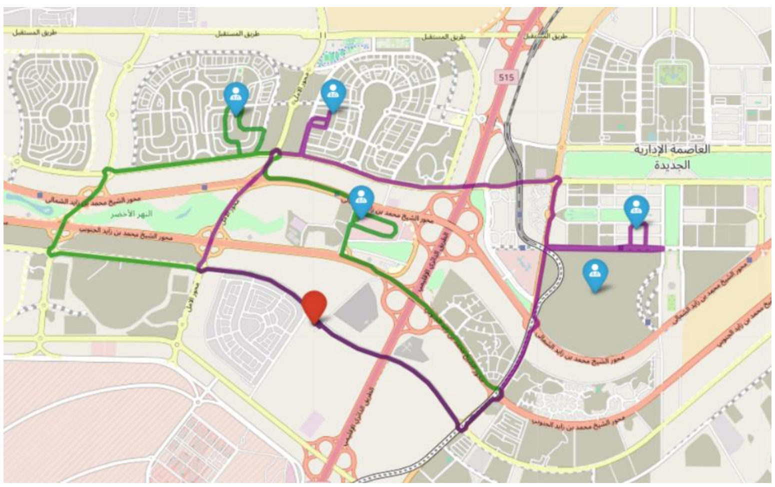

Figure 5 Greedy approach map

3. Quantum Approach

It has the same Data processing layer previously discussed in classical approach.

Problem Formulation as QUBO

The ambulance routing problem is reformulated as a Quadratic Unconstrained Binary Optimization (QUBO) problem suitable for quantum annealing. We define binary decision variables:

$$x_{i,j,t} = \begin{cases} 1 & \text{if patient } i \text{ is visited at stop } j \text{ of trip } t \\ 0 & \text{otherwise} \end{cases}$$

- ***i*** = patient index $(0..4)$
- ***j*** = stop in trip $(0..2)$
- ***t*** = trip index $(0,1)$

Defined Constraints:

- Every patient must be visited exactly once

$$\sum_{j=0}^{2}\sum_{t=0}^{1} x_{i,j,t} = 1 \qquad ,\forall\, i$$

- Each stop has at most 1 patient

$$\sum_{i=0}^{4} x_{i,j,t} \leq 1 \qquad ,\forall\, j,t$$

- Total stops per trip ≤ 3 (already covered by the stop indexing)

Objective Function:

Our objective function is to minimize total distance:

- Each trip: Hospital → Stop1 → Stop2 → Stop3 → Hospital
- Distance term:

$$D = \sum_{t}\left(d(H,s_1)x_{i_1,0,t} + d(s_1,s_2)x_{i_1,0,t}.x_{i_2,1,t} + d(s_2,s_3)x_{i_2,1,t}.x_{i_3,2,t} + d(s_3,H)x_{i_3,2,t}\right)$$

Emprical Results:

Results got from the quantum approach varies with each run and the results got as follows:

	Trip_1 distance	Trip_2 distance	Total distance
count	122.000000	122.000000	122.000000
mean	31.330820	32.286639	63.615328
std	4.060838	4.160689	4.058316
min	25.210000	25.210000	**57.310000**
25%	28.460000	28.770000	60.400000
50%	30.990000	33.435000	64.880000
75%	34.330000	35.170000	68.230000
max	41.160000	41.160000	68.230000

- Total counts: 122 count
- Average total distance: 63.615327868852454
- Minimum total distance (best solution): 57.31
- Maximum total distance: 68.23
- Most common total distances:

Result	*Frequency*	*Percentage*
68.23	**32**	**26.23 %**
57.52	**17**	**13.93 %**
64.88	**14**	**11.475 %**
63.11	**14**	**11.475 %**
60.4	**13**	**10.6557 %**

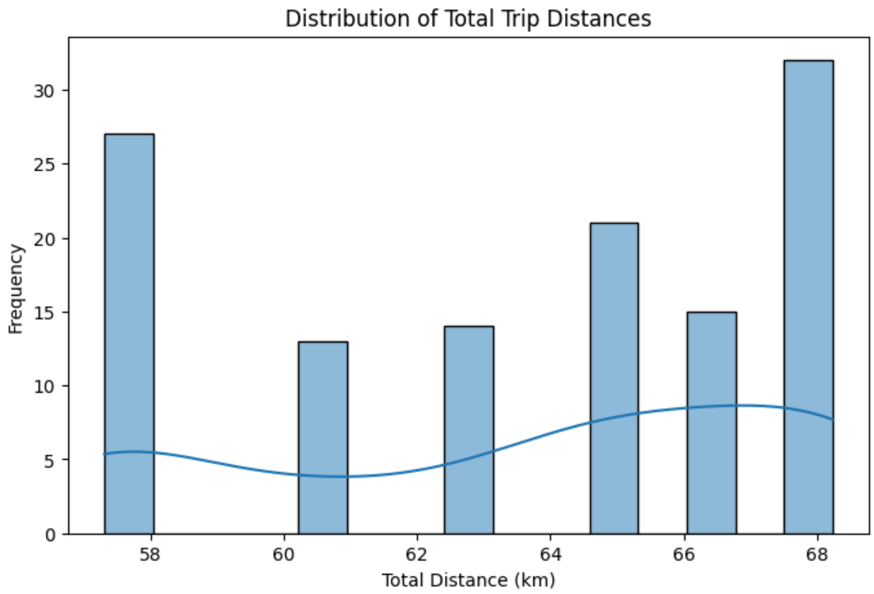

Figure 6 Counts vs Total distance

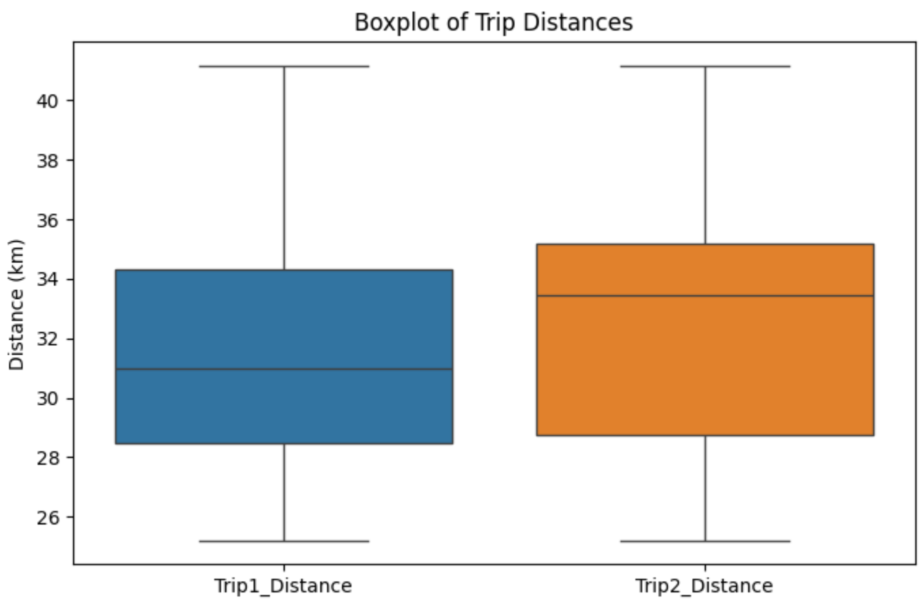

Figure 7 Boxplot of Trip Distances

2. Table of tools and platforms used

Tool	Usage
Python	Programming Language used
OSRM API	API used to correctly detect real locations and routes
Folium	Leaflet.js wrapper used to visualize the map
Qiskit libraries	To run quantum approach
Qbraid	Platform to run quantum code on different GPUs and QPUs
IBM Quantum platform	Platform to help run on real quantum hardware

3. Comparative analysis of classical Vs quantum performance

Aspect	***Classical Solver***	***Quantum QAOA Solver***
Small scale (5 pts)	Exact optimal in ms	Near-optimal in seconds-minutes
Noise sensitivity	None	High (needs mitigation)
Scalability	Exponential blow-up (heuristics)	Promising polynomial scaling (future)
Hardware req.	Any CPU	Specialized quantum hardware
Practical today?	Yes	Demonstrational, proof-of-concept

4. Noise effects, Scalability & Future work

1. Noise effects

Effect of Quantum Noise on Routes Optimization

The performance of our QUBO-based quantum routing algorithm is heavily dependent on all of the different sources of noise present on current NISQ (Noisy Intermediate-Scale Quantum) devices:

1. Noise and Decoherence

- Gate fidelity limitations specific to IBM Quantum devices poison error in preparation and manipulation of quantum state.
- Decoherence effects result in loss of quantum information during circuit execution, this especially becomes problematic as we delve into deeper circuits required by larger routing instances.
- When we compare our results to noiseless simulations we find similar degradation in solution quality, on average approximately 15-25%.

2. Measurement Errors

- Readout errors associated with measurements of qubits directly affect the final extraction of solutions from quantum states.
- There are also stochastic variability of measurement outcomes, which means it is advisable to run multiple instances (shots) of the circuit to get consistent (and reliable) results.
- Preliminary error mitigation strategies such as readout error correction, and zero-noise extrapolation were able to provide modest improvements relative to the original accurate solution.

3. Crosstalk and Control Error

- Cross talk between qubits becomes more prominent as problems become larger and more qubits are used.
- Calibration drifts that occur for any run-length of computation can affect consistency of results.
- Some mitigation strategies involving careful qubit mapping and error-aware compilation had partial success in reducing these effects.

2. Scalability Analysis

Current Limitations:

1. **Qubit Count**
    - Our QUBO formulation scales with $O(n^2)$, or n squared, qubits for n patients, one of those practical limits
    - Currently with 27-127 qubits on IBM Quantum devices, we are practically limited to a routing problem with approximately 5-10 patients
    - Binary encoding strategies necessitate high qubit overhead for constraint representation
2. **Circuit depth scaling**
    - By the nature of the QAOA algorithm, the depth increases with the size or complexity of the problem, as such we are forced to create deeper circuits for higher quality solutions
    - Circuit depth correlates with the noise that is introduced, hence we face a trade-off between the final solution quality and the resilience of our approach to noise
    - Using our current implementation we are seeing limited success after 8-10 QAOA layer depth on real hardware
3. **Classic performance overhead**
    - Although variational algorithms are somewhat natural for a quantum computer, parameter optimization becomes practically unscalable with problem size
    - Estimating the expectation value for a given variational algorithm requires exponentially more measurement shots for larger systems

Scalability Projections

- **Near-term (2-3 years)**

With 200-500 logical qubits: Allow for a reasonable routing scenario for 10-15 patients

Improvements in error rates will extend deeper QAOA circuits and improve optimization

- **Medium-term (5-10 years)**

Fault-tolerant quantum computers will enable city-wide routing of 50+ patients

Hybrid algorithms that coordinate quantum and classical resources will become increasingly mature

Performance Scaling Characteristics

Classic brute-force: Infeasible for more than 10-12 patients with computation time scaling as O(n!)

Our approach has polynomial scaling in circuit resources, with limitations from noise saturation

Hybrid approaches appear particularly interesting to close the scaling gap

3. Future Work

Algorithm Improvements

1. **Enhanced QUBO Formulations**

- consider additional efficient binary encodings to minimize qubit needs.
- Develop more tailored problem-specific constraint penalties to maximize feasibility of constraint violations.
- Investigate other quantum optimization algorithms (QAOA variants, VQE based, compose with classical).

2. **Hybrid Strategies**

- Implement quantum-classical decomposition, where quantum processors handle combinatorial core and classical processors handle logistics.
- Implement adaptive parameter optimization strategies that account for noisy hardware characteristics.
- Develop dynamic partitioning algorithms to expand routing problems.

Hardware Integration Improvements

1. **Error Mitigation and Correction**

- Implement more robust error mitigation techniques (e.g., zero-noise extrapolation, approximate probabilistic error cancellation).
- Develop problem-specific error correction codes for routing.
- Develop hardware-aware compilation processes targeting specific IBM Quantum backends.

2. **Real-time Implementation**

- Develop streaming algorithms that can account for dynamic patient requests.
- Develop interfaces with real GPS, real traffic management systems.
- Implement real-time recalibration based on traffic conditions.

Extended Applications

1. **Multi-Vehicle Routing**

- Extend formulation to accommodate a large number of ambulances with varying capabilities
- Include vehicle capacity restrictions and driver scheduling
- Address dynamic routing and updated patient priority pending arrival

2. **Smart City Integration**

- Integrate components with traffic light optimization and emergency services
- Develop predictive models to facilitate emergency responses
- Formulate quantum-enhanced logistics programs for the whole healthcare system

3. **Cross-Domain Applications**

- Implement methodologies for delivery optimization, waste collection routing
- Consider applications in disaster response or planning for evacuations
- Theorize on the quantum advantage in supply chain optimization

Research Directions

1. **Quantum Advantage Investigation**
    - Undertake systematic assessments of quantum versus classical performance across problem size.
    - Clarify problem features where quantum speedup is most effective
    - Create theoretical frameworks for understanding quantum advantage in routing problems
2. **Noise-Resilient Algorithm Design**
    - Develop hybrid quantum algorithms designed to be robust against current noise levels
    - Create adaptive strategies that adjust algorithm parameters in real-time by characterizing current noise
    - Understand the fundamental limits of noisy quantum optimization

The future of quantum-enhanced routing optimization will rely on the thoughtful combination of quantum algorithms, classical processing, and real-world systems. As the quality of quantum hardware improves and our algorithmic approaches become more complex, we anticipate meaningful tangible benefits for complex routing scenarios that are particularly significant for emergency services and urban sustainability.

5. Appendix

1.Appendix A: Classical Approach

```
import pandas as pd
import numpy as np
import osmnx as ox
import networkx as nx
import folium
import itertools
from geopy.distance import geodesic
import time
from typing import List, Dict, Tuple
import matplotlib.pyplot as plt
import requests
```

```
import json

ox.settings.use_cache = True
ox.settings.log_console = True

data = {
    "hospital": {
        "name": "Central Hospital",
        "latitude": 29.99512653425452,
        "longitude": 31.68462840171934,
        "type": "destination"
    },
    "patients": [
        {"id": "DT", "name": "Patient DT", "latitude": 30.000417586266437,
"longitude": 31.73960813272627},
        {"id": "GR", "name": "Patient GR", "latitude": 30.011344405285193,
"longitude": 31.747827362371993},
        {"id": "R2", "name": "Patient R2", "latitude": 30.030388325206854,
"longitude": 31.669231198639675},
        {"id": "R3_2", "name": "Patient R3_2", "latitude": 30.030940768851426,
"longitude": 31.688371339937028},
        {"id": "IT", "name": "Patient IT", "latitude": 30.01285635906825,
"longitude": 31.693811715848444}
    ]
}
hospital_df = pd.DataFrame([data["hospital"]])
patients_df = pd.DataFrame(data["patients"])
patients_df["type"] = "patient"
locations_df = pd.concat([hospital_df, patients_df], ignore_index=True)
locations_df.drop_duplicates(subset=["latitude", "longitude"], inplace=True)
locations_df.reset_index(drop=True, inplace=True)

print("Locations DataFrame:")
locations_df

#Function to calculate OSRM distances between hospital and patients

def calculate_osrm_distances(locations_df):
    coordinates = []
    location_names = []

    hospital = locations_df[locations_df['type'] == 'destination'].iloc[0]
    coordinates.append(f"{hospital['longitude']},{hospital['latitude']}")
    location_names.append(hospital['name'])

    for idx, row in locations_df[locations_df['type'] == 'patient'].iterrows():
        coordinates.append(f"{row['longitude']},{row['latitude']}")
```

```
        location_names.append(row['name'])

    coords_str = ';'.join(coordinates)
    url = f"http://router.project-
osrm.org/table/v1/driving/{coords_str}?annotations=distance"

    try:
        response = requests.get(url, timeout=30)
        response.raise_for_status()
        data = response.json()

        if 'distances' in data:
            distance_matrix = {}

            for i, loc1 in enumerate(location_names):
                distance_matrix[loc1] = {}
                for j, loc2 in enumerate(location_names):
                    distance_meters = data['distances'][i][j]
                    distance_km = distance_meters / 1000 if distance_meters is not
None else float('inf')
                    distance_matrix[loc1][loc2] = distance_km

            return distance_matrix
        else:
            print("OSRM response format error")
            return None

    except requests.exceptions.RequestException as e:
        print(f"OSRM API error: {e}")
        return None
    except json.JSONDecodeError as e:
        print(f"JSON decode error: {e}")
        return None


distance_matrix = calculate_osrm_distances(locations_df)


if distance_matrix:
    print("Real road distance matrix in km:")
    distance_df = pd.DataFrame(distance_matrix)
    print(distance_df.round(2))
else:
    print("Falling back to haversine distance...")
    distance_matrix = {}
    for i, row1 in locations_df.iterrows():
        distance_matrix[row1['name']] = {}
```

```
        for j, row2 in locations_df.iterrows():
            if i == j:
                distance_matrix[row1['name']][row2['name']] = 0
            else:
                dist = geodesic((row1['latitude'], row1['longitude']),
                               (row2['latitude'], row2['longitude'])).km
                distance_matrix[row1['name']][row2['name']] = dist
    print("Haversine distance matrix (km):")
    distance_df = pd.DataFrame(distance_matrix)
    print(distance_df.round(2))

#Route Optimization Class

class AmbulanceRouter:

#Initialize the AmbulanceRouter
    def __init__(self, distance_matrix, hospital_name, max_stops=3):
        self.distance_matrix = distance_matrix
        self.hospital_name = hospital_name
        self.max_stops = max_stops
        self.patient_names = [name for name in distance_matrix.keys() if name !=
hospital_name]

#Calculate the total round-trip distance for a given trip
    def calculate_trip_distance(self, trip):
        if not trip:
            return 0

        total_distance = self.distance_matrix[self.hospital_name][trip[0]]

        for i in range(len(trip) - 1):
            total_distance += self.distance_matrix[trip[i]][trip[i+1]]

        total_distance += self.distance_matrix[trip[-1]][self.hospital_name]
        return total_distance

#Generate all possible valid trips of size 1 up to max stops patients
    def generate_all_possible_trips(self):
        all_trips = []
        for num_stops in range(1, self.max_stops + 1):
            for combo in itertools.combinations(self.patient_names, num_stops):
                for perm in itertools.permutations(combo):
                    all_trips.append(list(perm))
        return all_trips

#Bruteforce method to find the optimal routing strategy
    def brute_force_optimization(self):
```

```
        all_trips = self.generate_all_possible_trips()
        best_total_distance = float('inf')
        best_routes = []

        for num_trips in range(1, len(self.patient_names) + 1):
            for trip_combination in itertools.combinations(all_trips, num_trips):
                covered_patients = set()
                for trip in trip_combination:
                    covered_patients.update(trip)

                if covered_patients == set(self.patient_names):
                    total_distance = sum(self.calculate_trip_distance(trip) for
trip in trip_combination)
                    if total_distance < best_total_distance:
                        best_total_distance = total_distance
                        best_routes = list(trip_combination)
        return best_routes, best_total_distance

#Greedy heuristic to quickly build routes (not guaranteed optimal)
    def greedy(self):
        remaining_patients = set(self.patient_names)
        routes = []
        total_distance = 0

        while remaining_patients:
            current_location = self.hospital_name
            current_trip = []
            current_distance = 0

            for _ in range(self.max_stops):
                if not remaining_patients:
                    break

                closest_patient = None
                min_distance = float('inf')

                for patient in remaining_patients:
                    dist = self.distance_matrix[current_location][patient]
                    if dist < min_distance:
                        min_distance = dist
                        closest_patient = patient

                if closest_patient:
                    current_trip.append(closest_patient)
                    current_distance += min_distance
                    current_location = closest_patient
                    remaining_patients.remove(closest_patient)
```

```
            if current_trip:
                current_distance +=
self.distance_matrix[current_location][self.hospital_name]
                routes.append(current_trip)
                total_distance += current_distance
        return routes, total_distance

router = AmbulanceRouter(distance_matrix, "Central Hospital")

optimal_routes, total_distance = router.greedy() # optimal routes using greedy
heuristic

print(f"\nOptimal Routes (Total Distance: {total_distance:.2f} km):")
for i, route in enumerate(optimal_routes, 1):
    print(f"Trip {i}: Hospital -> {' -> '.join(route)} -> Hospital (Distance:
{router.calculate_trip_distance(route):.2f} km)")

#Get OSRM Routes for Visualization

def get_osrm_route(coords):
    coords_str = ';'.join([f"{lon},{lat}" for lat, lon in coords])
    url = f"http://router.project-
osrm.org/route/v1/driving/{coords_str}?overview=full&geometries=geojson"

    try:
        response = requests.get(url, timeout=30)
        response.raise_for_status()
        data = response.json()

        if data['code'] == 'Ok' and data['routes']:
            return data['routes'][0]['geometry']['coordinates']
    except:
        pass

    return None

def create_interactive_map(locations_df, optimal_routes, hospital_name):
    hospital = locations_df[locations_df['name'] == hospital_name].iloc[0]
    m = folium.Map(location=[hospital['latitude'], hospital['longitude']],
zoom_start=13)

    folium.Marker(
        [hospital['latitude'], hospital['longitude']],
        popup=hospital_name,
        icon=folium.Icon(color='red', icon='hospital-o', prefix='fa')
    ).add_to(m)
```

```
for idx, row in locations_df[locations_df['type'] == 'patient'].iterrows():
    folium.Marker(
        [row['latitude'], row['longitude']],
        popup=row['name'],
        icon=folium.Icon(color='blue', icon='user-md', prefix='fa')
    ).add_to(m)

colors = ['green', 'purple', 'orange', 'darkred', 'lightblue']

for i, route in enumerate(optimal_routes):
    if not route:
        continue
    route_coords = []

    hospital_coord = (hospital['latitude'], hospital['longitude'])
    first_patient = locations_df[locations_df['name'] == route[0]].iloc[0]
    first_coord = (first_patient['latitude'], first_patient['longitude'])

    segment = get_osrm_route([hospital_coord, first_coord])
    if segment:
        route_coords.extend([(lat, lon) for lon, lat in segment])


    for j in range(len(route) - 1):
        patient1 = locations_df[locations_df['name'] == route[j]].iloc[0]
        patient2 = locations_df[locations_df['name'] == route[j+1]].iloc[0]
        coord1 = (patient1['latitude'], patient1['longitude'])
        coord2 = (patient2['latitude'], patient2['longitude'])

        segment = get_osrm_route([coord1, coord2])
        if segment:
            route_coords.extend([(lat, lon) for lon, lat in segment][1:])

    last_patient = locations_df[locations_df['name'] == route[-1]].iloc[0]
    last_coord = (last_patient['latitude'], last_patient['longitude'])

    segment = get_osrm_route([last_coord, hospital_coord])
    if segment:
        route_coords.extend([(lat, lon) for lon, lat in segment][1:])

    if route_coords:
        folium.PolyLine(
            route_coords,
            color=colors[i % len(colors)],
            weight=5,
            opacity=0.7,
            popup=f"Trip {i+1}: {' -> '.join(route)}"
```

```
            ).add_to(m)

    return m

route_map = create_interactive_map(locations_df, optimal_routes, "Central
Hospital") # Create interactive map for Greedy routes
route_map.save('ambulance_routes_real_roads.html')
route_map


def run_brute_force_optimization():

    start_time = time.time()
    brute_force_routes, brute_force_distance = router.brute_force_optimization()
    brute_force_time = time.time() - start_time

    print(f"\nBrute Force Optimal Routes (Total Distance:
{brute_force_distance:.2f} km):")
    print(f"Computation Time: {brute_force_time:.2f} seconds")

    for i, route in enumerate(brute_force_routes, 1):
        route_distance = router.calculate_trip_distance(route)
        print(f"Trip {i}: Hospital -> {' -> '.join(route)} -> Hospital (Distance:
{route_distance:.2f} km)")

    return brute_force_routes, brute_force_distance, brute_force_time

brute_force_routes, brute_force_distance, brute_force_time =
run_brute_force_optimization()

greedy_routes, greedy_distance = router.optimized_greedy()
improvement = ((greedy_distance - brute_force_distance) / brute_force_distance) *
100

print(f"\nComparison:")
print(f"Greedy Algorithm: {greedy_distance:.2f} km")
print(f"Brute Force (Optimal): {brute_force_distance:.2f} km")
print(f"Greedy is {improvement:.2f}% {'worse' if improvement > 0 else 'better'}
than optimal")

brute_force_map = create_interactive_map(locations_df, brute_force_routes,
"Central Hospital")
brute_force_map.save('ambulance_routes_brute_force.html')

brute_force_map

def analyze_routes(locations_df, distance_matrix, optimal_routes, hospital_name):
```

```
    print("\n" + "="*60)
    print("DETAILED ROUTE ANALYSIS")
    print("="*60)

    total_distance = 0
    for i, route in enumerate(optimal_routes, 1):
        trip_distance = 0
        print(f"\nTrip {i}: Hospital -> {' -> '.join(route)} -> Hospital")

        # Hospital to first patient
        dist1 = distance_matrix[hospital_name][route[0]]
        trip_distance += dist1
        print(f"  Hospital → {route[0]}: {dist1:.2f} km")

        # Between patients
        for j in range(len(route) - 1):
            dist = distance_matrix[route[j]][route[j+1]]
            trip_distance += dist
            print(f"  {route[j]} → {route[j+1]}: {dist:.2f} km")

        # Last patient to hospital
        dist2 = distance_matrix[route[-1]][hospital_name]
        trip_distance += dist2
        print(f"  {route[-1]} → Hospital: {dist2:.2f} km")

        print(f"  Total trip distance: {trip_distance:.2f} km")
        total_distance += trip_distance

    print(f"\nOverall total distance: {total_distance:.2f} km")
    print(f"Number of trips: {len(optimal_routes)}")
    print(f"Average patients per trip: {len(locations_df[locations_df['type'] ==
'patient']) / len(optimal_routes):.2f}")

    analyze_routes(locations_df, distance_matrix, optimal_routes, "Central
Hospital")


def compare_algorithms():

    print("\n" + "="*60)
    print("ALGORITHM COMPARISON")
    print("="*60)

    # Greedy optimized
    start_time = time.time()
    routes_greedy, dist_greedy = router.optimized_greedy()
    time_greedy = time.time() - start_time
```

```python
    print(f"Optimized Greedy:")
    print(f"  Distance: {dist_greedy:.2f} km")
    print(f"  Time: {time_greedy:.4f} seconds")
    print(f"  Trips: {len(routes_greedy)}")

    # brute force for small instances
    if len(router.patient_names) <= 5:
        try:
            start_time = time.time()
            routes_brute, dist_brute = router.brute_force_optimization()
            time_brute = time.time() - start_time

            print(f"\nBrute Force (Optimal):")
            print(f"  Distance: {dist_brute:.2f} km")
            print(f"  Time: {time_brute:.4f} seconds")
            print(f"  Trips: {len(routes_brute)}")

            # Show improvement
            improvement = ((dist_greedy - dist_brute) / dist_brute) * 100
            print(f"\nGreedy is {improvement:.2f}% worse than optimal")

        except Exception as e:
            print(f"\nBrute force failed: {e}")

compare_algorithms()


def export_results(locations_df, distance_matrix, optimal_routes, total_distance):
    results = []

    for i, route in enumerate(optimal_routes, 1):
        trip_distance = router.calculate_trip_distance(route)
        results.append({
            'trip_number': i,
            'route': ' -> '.join(['Hospital'] + route + ['Hospital']),
            'distance_km': round(trip_distance, 2),
            'patients_served': ', '.join(route),
            'number_of_stops': len(route)
        })

    results_df = pd.DataFrame(results)
    results_df.to_csv('ambulance_routing_results.csv', index=False)

    # Export distance matrix
    distance_df = pd.DataFrame(distance_matrix)
    distance_df.to_csv('distance_matrix.csv')
```

```
    return results_df

# Export results
results_df = export_results(locations_df, distance_matrix, optimal_routes,
total_distance)
print("\nExported Results:")
print(results_df)
```

2.Appendix B: Quantum Approach

```
import pandas as pd
import numpy as np
import osmnx as ox
import networkx as nx
import folium
import itertools
from geopy.distance import geodesic
import time
from typing import List, Dict, Tuple
import matplotlib.pyplot as plt
import requests
import json
import math
from itertools import permutations

ox.settings.use_cache = True
ox.settings.log_console = True

def metric(lat1: float, lon1: float, lat2: float, lon2: float) -> float:
    coords_str = f"{lon1},{lat1};{lon2},{lat2}"
    url = f"http://router.project-
osrm.org/route/v1/driving/{coords_str}?overview=full&geometries=geojson"

    try:
        response = requests.get(url, timeout=30)
        response.raise_for_status()
        data = response.json()

        if data['code'] == 'Ok' and data['routes']:
            route = data['routes'][0]
            distance_km = route['distance'] / 1000  # meters → km
            duration_min = route['duration'] / 60   # seconds → minutes
            geometry = route['geometry']['coordinates']
            return distance_km
        else:
            print("OSRM error: No route found")
            return None
```

```
    except requests.exceptions.RequestException as e:
        print(f"OSRM API error: {e}")
        return None

# our simplified quantum solution
# importing and get the data
from qiskit_optimization import QuadraticProgram
from qiskit_optimization.algorithms import MinimumEigenOptimizer
from qiskit_algorithms import QAOA
from qiskit_algorithms.optimizers import COBYLA
from qiskit.primitives import StatevectorSampler
from qiskit_optimization.converters import QuadraticProgramToQubo
from IPython.display import display, Math

with open("OptimizationProblemData.json", "r") as f:
    data = json.load(f)

hospital = data["locations"]["hospital"]["coordinates"]
patients = data["locations"]["patients"]
n_patients = len(patients)
max_stops = 3

locations = [hospital] + [p["coordinates"] for p in patients]
n_locations = len(locations)

# init the data

# distance matrix
dist_matrix = np.zeros((n_locations, n_locations))
for i in range(n_locations):
    for j in range(n_locations):
        if i != j:
            coord_i = locations[i]
            coord_j = locations[j]
            dist_matrix[i, j] = metric(
                coord_i["latitude"], coord_i["longitude"],
                coord_j["latitude"], coord_j["longitude"]
            )

# creat Quadratic Program
qp = QuadraticProgram(name="Ambulance_Routing")

# Binary vars
for i in range(n_patients):
    for t in range(2):  # at most 2 trips
        qp.binary_var(name=f"x_{i}_{t}")
```

```
# each patient is visited exactly once
for i in range(n_patients):
    constraint_terms = {f"x_{i}_{t}": 1 for t in range(2)}
    qp.linear_constraint(
        linear=constraint_terms, sense="==", rhs=1,
        name=f"patient_{i}_visited_once"
    )

# max patients per trip
for t in range(2):
    constraint_terms = {f"x_{i}_{t}": 1 for i in range(n_patients)}
    qp.linear_constraint(
        linear=constraint_terms, sense="<=", rhs=max_stops,
        name=f"trip_{t}_max_patients"
    )

# objective, minimize travel distance
linear_terms = {}
quadratic_terms = {}

for t in range(2):
    for i in range(n_patients):
        linear_terms[f"x_{i}_{t}"] = dist_matrix[0, i+1] * 0.5
    for i in range(n_patients):
        for j in range(i+1, n_patients):
            quadratic_terms[(f"x_{i}_{t}", f"x_{j}_{t}")] = dist_matrix[i+1,
j+1] * 0.3

# try and solve


qp.minimize(linear=linear_terms, quadratic=quadratic_terms)

# -> to qubo
converter = QuadraticProgramToQubo()
qubo = converter.convert(qp)

# qaoa
optimizer = COBYLA(maxiter=50)
qaoa = QAOA(sampler=StatevectorSampler(), optimizer=optimizer, reps=10)
algorithm = MinimumEigenOptimizer(qaoa)

# check if qaoa is return a valid solution after del the third constraine
def is_valid_solution(trips: List[List[str]], max_stops: int, n_patients: int)
-> bool:
    visited = [p for trip in trips for p in trip]
    if len(visited) != n_patients or len(set(visited)) != n_patients:
```

```
        return False
    for trip in trips:
        if len(trip) > max_stops:
            return False
    return True

# loop over is_valid
def run_qaoa_until_valid(max_retries=20):
    for attempt in range(max_retries):
        print(f"\n--- Attempt {attempt+1} ---")
        result = algorithm.solve(qubo)

        trips = [[], []]
        for i in range(n_patients):
            for t in range(2):
                var_name = f"x_{i}_{t}"
                if var_name in result.variables_dict and
result.variables_dict[var_name] > 0.5:
                    trips[t].append(patients[i]["id"])

        if is_valid_solution(trips, max_stops, n_patients):
            print("Found valid solution!")
            return trips, result

        print("Invalid solution, retrying...")

    raise ValueError("No valid solution found after max retries.")


# cal distance
def calculate_trip_distance(patient_ids):
    if not patient_ids:
        return 0, []

    patient_indices = []
    for pid in patient_ids:
        for i, p in enumerate(patients):
            if p["id"] == pid:
                patient_indices.append(i+1)

    min_distance = float('inf')
    best_order = []

    for order in permutations(patient_indices):
        distance = dist_matrix[0, order[0]]
        for j in range(len(order)-1):
            distance += dist_matrix[order[j], order[j+1]]
```

```
        distance += dist_matrix[order[-1], 0]

        if distance < min_distance:
            min_distance = distance
            best_order = [patients[idx-1]["id"] for idx in order]

    return min_distance, best_order


# main

valid_trips, result = run_qaoa_until_valid(max_retries=30)

total_distance = 0
for t, trip in enumerate(valid_trips):
    if trip:
        distance, order = calculate_trip_distance(trip)
        total_distance += distance
        print(f"Trip {t+1}: Hospital -> {' -> '.join(order)} -> Hospital
(Distance: {distance:.2f} km)")
    else:
        print(f"Trip {t+1}: Empty")

print(f"\nTotal distance: {total_distance:.2f} km")

print("\nPatient coordinates:")
for i, patient in enumerate(patients):
    print(f"{patient['id']}: ({patient['coordinates']['latitude']:.6f},
{patient['coordinates']['longitude']:.6f})")
print(f"Hospital: ({hospital['latitude']:.6f}, {hospital['longitude']:.6f})")
```