

## Section ①



Fuclids ( $\frac{64}{m}, \frac{24}{n}$ )

While  $n \neq 0$  do

$$r \leftarrow m \bmod n$$

$$m \leftarrow n$$

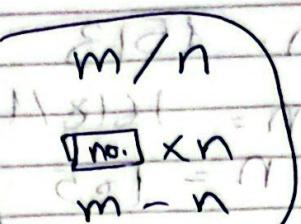
$$n \leftarrow r$$

return  $m$ .

$m/n$

$$\boxed{\begin{matrix} \text{no.} & x \\ m & = n \end{matrix}}$$

JKM  
sabeek



$$\boxed{\begin{matrix} m/n \\ \text{dec} \times n \end{matrix}} \text{ (dec)}$$

Ex 1.1

Basic op ( $E1, 201.$ ) do

$$:=, \text{if } =, \leq, \geq, <, >, =, >$$

arithmatic op:

$$+, -, \times, /, *$$

$$e1 - n$$

6a.

$$r \leftarrow 16$$

$$m \leftarrow 24$$

$$n \leftarrow 16$$

$$r \leftarrow \cancel{16} 8$$

$$m \leftarrow 16$$

$$n \leftarrow 8$$

$$r \leftarrow 0$$

$$m \leftarrow 8$$

$$n \leftarrow 0 \Rightarrow$$

Return 8

(2,1) GCD ( $31415, 14142$ )

$$m = 31415, n = 14142$$

~~$$D = N = 31415$$~~

(1,1)  $m = 14142$

~~$$N = 3131$$~~

(1,1) GCD ( $14142, 3131$ )

~~$$D = N = 14142$$~~

~~$$m = 3131$$~~

(0,1)  $n = 14142$

GCD ( $3131, 14142$ )

~~$$D = N = 14142$$~~

~~$$m = 14142$$~~

~~$$N = 1513$$~~

$\text{GCD}(1618, 1513)$

$$m = 1513$$

$$n = 1618 / 1513 \rightarrow 0.\square \times 1513$$

$$n = 105$$

$\text{GCD}(1513, 105)$

$$r = 1513 / 105$$

$$m = 105$$

$$n = 43$$

$\text{GCD}(105, 43)$

$$r = 105 / 43$$

$$m = 43$$

$$n = 19$$

$\text{GCD}(43, 19)$

$$r = 43 / 19$$

$$m = 19$$

$$n = 5$$

$\text{GCD}(19, 5)$

$$r = 19 / 5$$

$$m = 5$$

$$n = 4$$

$\text{GCD}(5, 4)$

$$r = 5 / 4$$

$$m = 4$$

$$n = 1$$

$\text{GCD}(4, 1)$

$$r = 4 / 1$$

$$m = 1$$

$$n = 0$$

$\text{GCD}(1, 0) \leftarrow \otimes \text{ Stop}$

Return ①

Ex 1.2



2.

$$\begin{aligned} p_1 &\leftarrow 1 \\ p_2 &\leftarrow \cancel{2} \\ p_3 &\leftarrow 5 \\ p_4 &\leftarrow 10 \end{aligned}$$

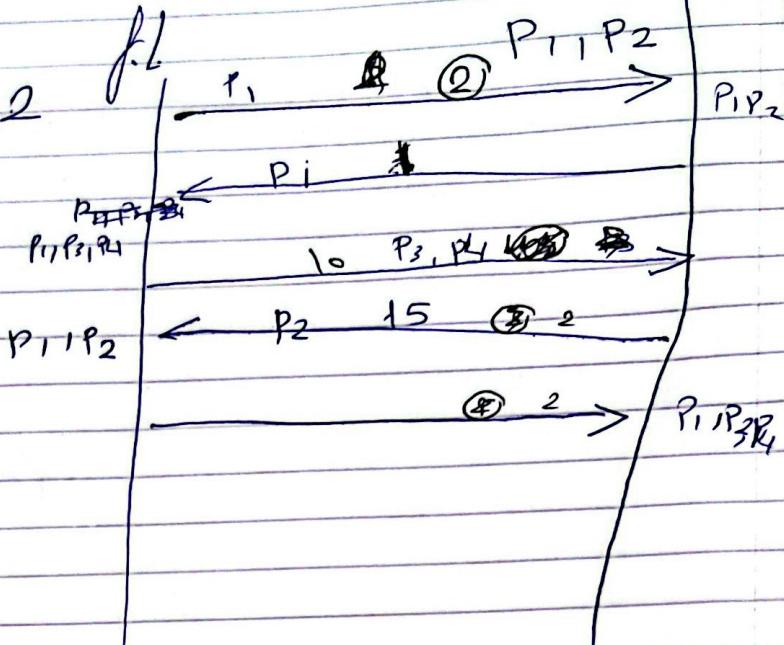
$p_1$  and  $p_2$  go first

then  $p_1$  return

$p_3, p_4$  goes

$p_2$  return back

$p_1, p_2$  goes



③

$d_{min} \leftarrow \infty$   
for  $i \leftarrow 0$  to  $n-1$  do

for  $j \leftarrow 0$  to  $n-1$  do

let this  
 $j = i+1$

remove this

[if  $i \neq j$ ]  $|A[i] - A[j]|$   $\leftarrow d_{min}$

$d_{min} \leftarrow |A[i] - A[j]|$

$i \neq j$  return  $d_{min}$ .

$i$	$j_1$	$j_2$	$j_3$
10	70	20	5

1.3 ①

for  $i \leftarrow 0$  to  $n-1$  do  $Count[i] \leftarrow 0$   
for  $i \leftarrow 0$  to  $n-2$  do  
    for  $j \leftarrow i+1$  to  $n-1$  do  
        if  $A[i] = A[j]$  :  
             $Count[j] \leftarrow Count[j] + 1$   
        else  
             $Count[i] \leftarrow Count[i] + 1$

for  $i \leftarrow 0$  to  $n-1$  do  $S[Count[i]] \leftarrow A[i]$   
return  $S$

①

$i$	0	1	2	3	4	5
A	60	35	81	98	14	47

نحوه

function  $\rightarrow$   $i \leftarrow 0$  to  $n-1$   $Count$   
    ②  $\rightarrow$   $i+1$  to  $n-1$

$i$	0	1	2	3	4	5
S	14	35	17	60	81	98

$i$	0	1	2	3	4	5
	0	0	0	0	0	0

1.3

2A and 3P

Count

0	0	2	2	4	5
3	1	4	5	0	2

(i)

0	-	-	-	-	-	-
1	-	-	-	-	-	-
2	✓	✓	-	-	-	-
3	✓	✓	✓	-	-	-
4	-	-	-	-	-	-
5	✓	✓	-	-	-	-

not in place

(j)

A	0	1	2	3	4	5
	60	35	81	98	14	47

S


$$S[3] = 60$$

$$S[1] = 35$$

$$S[4] = 81$$

$$S[5] = 98$$

$$S[0] = 14$$

$$S[2] = 47$$

Ex. 1.4

1. ①  $A[\alpha] = A[^\alpha_4]$

Δ

30	10	50	60	5
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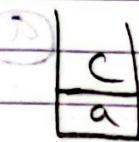
size ينقص كل 1 /  $n = n - 1$

3. ②

LIFO



1	2	3	15	10
1	2	3	18	28



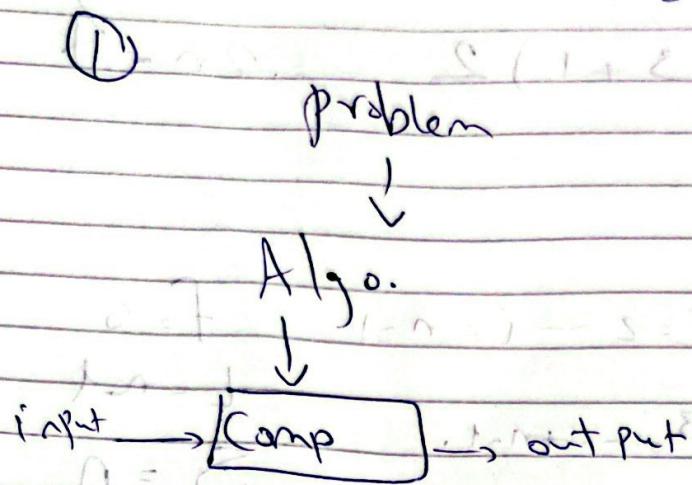
i) ④ FIFO

a	*	c	d
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Regression :  $y = f(x)$  (ال帰属方程式)

## Section ②

### Algorithm.



$$Q_2 @ 1+2+3+\dots+70$$

$$f=1 \quad i=1 \quad L=70 \quad S=0 \quad o=1$$

$$L=70 \quad S=70$$

$$\frac{(f+L)S}{2} = \frac{(1+70)70}{2}$$

$$1+3+5+\dots+99 \rightarrow f=1$$

$$L=99$$

$$\frac{(f+L)S}{2} = \frac{1+99(50)}{2} S = 500 \leftarrow \boxed{500}$$

$$\boxed{\text{Size} = \frac{\text{up}}{\text{down}} + 1}$$

⑤  $\sum_{i=3}^5 2$  |  $\sum_{i=5}^{1000} 5 (1000 - 5 + 1) 5$   
 $2 + 2 + 2 = 6$

⑥  $\sum_{i=3}^{10} C = (10 - 3 + 1) C = 8C$

⑦  $\sum_{i=3}^n 2 = (n - 3 + 1) 2 = 2n - 4$

⑧  $\sum_{i=0}^{n-1} i$   
 $i=0, i=1, i=2 - i=n-1$   $f=6$   
 $0+1+2+3+\dots+n-1$   $L=n$   
 $S=n$

$\sum_{i=0}^{n-1} i+1$   $\alpha F = +E + Cx + D$   
 $i=0 \Rightarrow i+1, i=n+1$   $f=1$   
 $i+1=2 \quad \alpha F = 1$   $L=n$   
 $\alpha F(i+1) = 2(i+1)$   $S = n+1-D$

⑨  $\sum_{i=0}^{n-1} n$   $i=1 \Rightarrow n+0+1+n = n^2$

$\sum_{i=0}^{n-1} i^2$   $\sum_{i=0}^{n-1} i^2 = 2(n+1) - 2(1+2)$   
 $\sum_{i=0}^{n-1} i^2 = n^2 + n = n^2 + n$   $= n^2$

$$\textcircled{1} \quad \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} j$$

$f = 0$   
 $L = n-1$   
 $S = n$

$$\sum_{i=0}^{n-1} \frac{(n-1+\sigma)n}{2} = \sum_{i=0}^{n-1} \frac{n^2 + n}{2}$$

$$= \frac{(n-1)(-o+n+1)n^2 - n}{2}$$

$$= \boxed{\frac{n^3 - n^2}{2}}$$

$$\textcircled{2} \quad \sum_{l=0}^{n-1} \sum_{j=0}^{n-1} l = \sum_{i=0}^{n-1} \text{mix-out}$$

$f = 0$   
 $L = n(n-1)$   
 $S = n(n-1)$

$$\sum_{i=0}^{n-1} i$$

$\text{or}$

$i = 0$	$f = 0$
$L = n-1$	$L = n-1$
$S = n$	<del><math>S = n</math></del>

$$= n \cancel{(0+1+...+n-1)}$$

$$= n(n-1) \cdot n$$


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$$\frac{2}{2} = \frac{(n^2 - n)n}{2} \quad \boxed{f = \frac{n^3 - n^2}{2}} \quad \frac{2}{2} = \frac{(n^3 - n^2)}{2}$$

$$\textcircled{K} \quad \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} k$$

$$f = 0 \\ L = n - 1 \\ S = n + 0 + 1$$

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \frac{n^2 - n}{2}$$

$$\frac{(n-1) \cancel{n+1} n}{2}$$

$$\sum_{i=0}^{n-1} (n + 0 + 1) \frac{n^2 - n}{2}$$

$$\sum_{i=0}^{n-1} \frac{n^3 - n^2}{2}$$

$$\frac{n + 0 + 1 (n^2 - n^2)}{2}$$

$$\boxed{\frac{n^4 - n^3}{2}}$$

$$\textcircled{L} \quad \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} i \cdot j$$

$$f = 0 \\ L = n - 1$$

$$S = n + 0 + 1$$

$$\sum_{i=0}^{n-1} \frac{n^2 - n}{2} \cdot i$$

$$\frac{n^2 - n}{2} \sum_{i=0}^{n-1} i =$$

$$f = 0 \\ L = n - 1 \\ S = n + 0 + 1$$

$$\frac{n^2 - n}{2} \cdot \frac{(0 + n - 1)n}{2} =$$

$$\left( \frac{n^2 - n}{2} \right) \left( \frac{n^2 - n}{2} \right) = \frac{1}{2} \left( \frac{n^2 - n}{2} \right)^2.$$

$$m \cdot \sum_{i=0}^{n-1} \sum_{j=0}^i 1$$

$$\sum_{i=0}^{n-1} (i+1) \cdot 1$$

$$\sum_{i=0}^{n-1} i + 1$$

$$f = 1$$

$$L = n$$

$$S = n + 0 + 1$$

$$(1+n)n$$

$$= \frac{(1+n)n}{2}$$

$$\boxed{\frac{n+n^2}{2}}$$

(1)

$$\sum_{i=0}^{n-1} \sum_{j=0}^i j$$

$$f = 0$$

$$L = i$$

$$S = i + 1$$

$$\sum_{i=0}^{n-1} (\cancel{0} + i) \cdot i + 1$$

$$\sum_{i=1}^n i = (n+1)T - 1$$

$$\sum_{i=0}^{n-1} \frac{i^2 + i}{2}$$

$$f = 0$$

$$L = \frac{(n-1)^2 + (n-1)}{2}$$

$$S = n + 0 + 1$$

$$\left( 0 + \left[ \frac{(n-1)^2 + (n-1)}{2} \right] \right) n - 1/2$$

Q3

basic op

$\ast, / \rightarrow$  more time

$+, - \rightarrow$  less time

$>, <, =, +, -, \ast, / \rightarrow$  less time  
 $: \rightarrow$  less time

Q3 (a)

$x < 0$

for  $i \in [1, n]$

$x \leftarrow x + 1$

end.

Basic op: add

$$T(N) = \sum_{i=1}^n 1 \rightarrow 1 \cdot f(1) \cdot p_i \cdot \Sigma$$

$$= (n - k++) \cdot 1 = n$$

(b)  $T(N) = \sum_{i=1}^n 1$

basic op: multiplication

$$= (n - k++) \cdot 1 = n$$

(c)  $T(N) = \sum_{i=1}^n 2 = 2n$

Basic op: Add

$$\sum_{i=1}^n 2 = 2n$$

Basic op: add,

Subtraction

$$2 \times n \left( (1 - \alpha)^2 + (1 - \alpha) \cdot \alpha \right)$$

$$\begin{aligned}
 e) \quad T(N) &= \sum_{i=1}^n \sum_{i=1}^n 2 \\
 &= \sum_{i=1}^n 2n \\
 &= (n+1) \cdot 2n \\
 &= 2n^2
 \end{aligned}$$

B. op.: add.

$$\begin{aligned}
 f) \quad \sum_{i=1}^n \sum_{i=1}^i 2
 \end{aligned}$$

B. op.: dev, multiply

$$\sum_{i=1}^n (i+1) 2$$

$$2 \cdot \sum_{i=1}^n i \rightarrow 2 \frac{(n+1)n}{2} = n^2 + n$$

$$\begin{aligned}
 L &= n \\
 S &= n+1
 \end{aligned}$$

### Section 3

$$Q1 \circ @ \xrightarrow{i} A \xrightarrow{j} \begin{bmatrix} 0 & \times \\ 0 & \Delta \end{bmatrix} + \begin{bmatrix} 0 & \times \\ 0 & \Delta \end{bmatrix} \rightarrow B$$

for  $i = 0$  to  $n-1$   
 for  $j = 0$  to  $n-1$

$$C[i, j] = A[i, j] + B[i, j]$$

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1$$

$$\sum_{i=0}^{n-1} (n - i - 1) + 1$$

$$T(N) = \sum_{i=0}^{n-1} n = (n - k - 1) n = \boxed{n^2}$$

$$\boxed{\text{input } = 2n^2}, \boxed{n^2 + n^2}$$

Q1

$$A \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} B \begin{bmatrix} 3 \\ 3 \end{bmatrix} ?$$

$C = A \cdot B$

Operations:

- $1 \times 3 + 1 \times 3$  (top-left)
- $1 \times 2 + 1 \times 1$  (top-right)
- $2 \times 2 + 1 \times 2$  (bottom-left)
- $2 \times 3 + 1 \times 3$  (bottom-right)

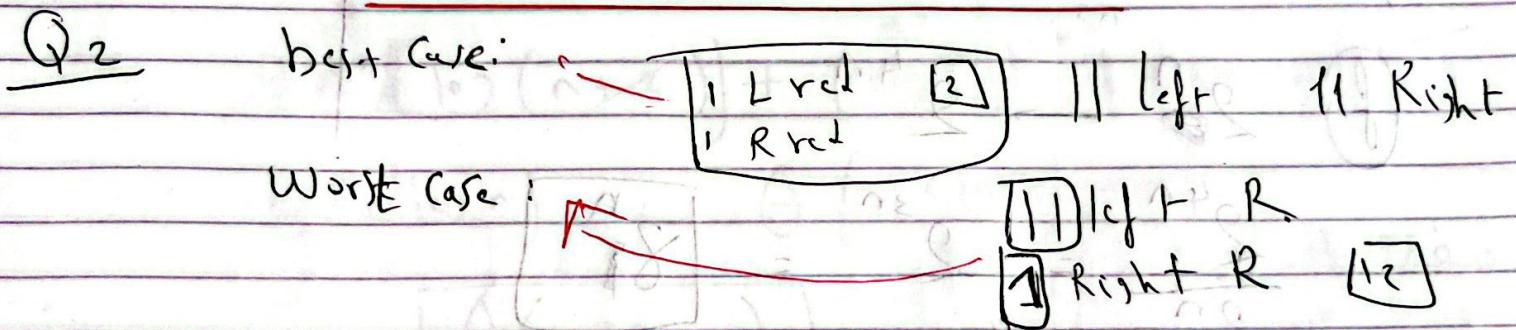
Time complexity:  $n^3$  times multiplication

$$T(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} n^2$$

$$\boxed{T(n) = n^3}$$

Inputs  $\approx 2n^2$

Inputs  $\Rightarrow$   $\oplus$   
بعض



~~$T(n) = n^3 + n^2 + n$~~

~~$T(1000) = 4(1000) + (1000) + (1000)$~~

Ignoring terms: We ignore constants  
to bring  $\rightarrow g(n^3)$  take highest power

Q3

$$\textcircled{c} \quad n \rightarrow 4n$$

$$\frac{4n}{n} = 4 \text{ time}$$

$$\textcircled{d} \quad n^2 \rightarrow (4n)^2$$

$$\frac{(4n)^2}{n^2} = \frac{16n^2}{n^2} = 16$$

$$\textcircled{e} \quad n^3 \rightarrow (4n^3)$$

$$\frac{4^3 n^3}{n^3} = 64 \text{ time}$$

\textcircled{f}

$$2^n \rightarrow 2^{4n}$$

$$\frac{2^{4n}}{2^n} = 2^{3n} = \boxed{8^n}$$

$$\textcircled{g} \quad \log_2 n \rightarrow \log_2 4n$$

$$\frac{\log_2(4n)}{\log_2(n)} =$$

$$\log_2 4 + \log_2 \frac{1}{n} =$$

$$= 2$$

$$\textcircled{h} \quad \sqrt{n} \rightarrow \sqrt{4n}$$

$$\frac{\sqrt{4n}}{\sqrt{n}} = 2$$

\textcircled{i}

$$\frac{n(n+1)}{2} \in O(n^3)$$

$$\frac{n^2 + n}{2} \rightarrow \text{it belongs to}$$

\textcircled{j} ~~Not belongs~~ True,  $\in O(n^2)$

G ~~below~~ false  
not belong  $\Rightarrow$  no exact

D True

Q4 a)  $T(n) = (n^2 + 1)^{10}$   $\boxed{n^{20}}$

$$\Theta(n^{20})$$

b)  $\boxed{\log n^2 + 7n + 3}$

$$\sqrt{10} \cdot n + \sqrt{7n} + \sqrt{3}$$

$$\Theta(n)$$

c)  $2n \log(n+2)^2 + (n+2)^2 \log\frac{n}{2}$

$$= 4n \log(n+2) + (n+2)^2 \log n - \cancel{\log 2}$$

$$\Theta(n \log n) + \Theta(n^2 \log n)$$

$$\boxed{\Theta(n^2 \log n)}$$

1. multiply &  
divide  $\Rightarrow$

d)  $2^{n+1} + 3^{n-1}$

$$2^n \cdot 2^1 + \frac{3^n}{3^1}$$

$$\Theta(2^n) + \Theta(3^n)$$

$$\textcircled{C} \quad | \log_2 n | \quad \Theta(\log_2 n)$$

Q5      ①  $\Theta(n!)$

$$\textcircled{2} \quad 5 \log(n+100)^{10} \sim \Theta(\log n)$$

$$\textcircled{3} \quad 2^{en} \rightarrow 4^n \rightarrow \Theta(4^n)$$

$$\textcircled{4} \quad 0.001n^4 + 3n^3 + 1 \rightarrow \Theta(n^4)$$

$$*\textcircled{5} \quad kn^2n = \log^2 n \quad \Theta(\log^2 n)$$

$$\textcircled{6} \quad \sqrt[3]{n} \rightarrow \Theta(n^{1/3})$$

$$\textcircled{7} \quad 3^n = \Theta(3^n)$$

②, ⑤, ⑥, ④, ⑦, ③, ①

Q6       $i \leftarrow 0 \text{ to } n-1$        $x \leftarrow 0 \text{ min} \leftarrow \infty$        $T(n) = n$ .

for  $j \leftarrow 0 \text{ to } n-1$

if  $A[i] > \max$   
 $\max \leftarrow A[i]$

if  $A[i] < \min$   
 $\min \leftarrow A[i]$

$$\text{(i)} T(n) = 1 \\ \Theta(1)$$

$$\text{(ii)} T(n) = N \\ \Theta(N)$$

$$\text{(iv)} T(n) = T(10^n) + \Theta(10^n)$$

### Section 4

$$\text{(i)} X(n) = X(n-1) + 5, \quad X(1) = 0$$

$$X(n-1) = X(n-2) + 5$$

$$X(n-2) = X(n-3) + 5$$

$$X(n) = X(n-2) + 5 + 5$$

$$X(n) = X(n-3) + 15$$

$$X(n) = X(n-k) + 5k$$

assume

$$X(n-k) = X(1) = 0$$

$$n-k=1$$

$$k=1-n$$

$$k=n-1$$

$$X(n) = X(n-k) + 5k$$

$$X(n) = 5(n-1) = 5n-5$$

$$\Theta(n)$$

$$\textcircled{b} \quad X(n) = 3 \times (n-1) \quad \boxed{X(1)=4}$$

$$X(n-1) = 3 \times (n-2)$$

$$X(n-2) = 3 \times (n-3)$$

$$X(n) = 3 \times 3 \times (n-2)$$

$$X(n) = 3 \times 3 \times 3 \times (n-3)$$

$$X(n) = 27 \times (n-3).$$

$$\boxed{X(n) = 3^k \times (n-k)}$$

$$X(n-k) = X(1) = 4$$

$$\Theta(3^n), \quad \boxed{\begin{matrix} n-k=1 \\ k=n-1 \end{matrix}} \quad X(n) = 3^{n-1} \cdot 4$$

$$\textcircled{c} \quad X(n) = X(n-1) + n. \quad X(0) = 0$$

$$X(n-1) = X(n-2) + (n-1)$$

$$X(n-2) = X(n-3) + (n-2)$$

$$X(n) = X(n-2) + n-1 + n$$

$$X(n) = X(n-3) + (n-2) + (n-1) + n$$

$$f \leftarrow 0 \quad \overbrace{n} \rightarrow \infty$$

$\Sigma$

$$S = n + 1$$

$$\frac{(0+n) n+1}{2} = \frac{n^2+n}{2} \boxed{\Theta(n^2)}$$

③  $X(n) = X(n/3) + 1$   $X(1) = 1$

$$X(n/3) = X(n/3) + 1$$

$$X(n/3) = X(n/3) + 1$$

$$X(n) = \underline{X(n/3)} + 1 + 1$$

$$X(n) = \underline{X(n/3)} + 1 + 1 + 1$$

$$X(n) = X(n/3^K) + K$$

$$\frac{n}{3^K} = 1$$

assume  $X\left(\frac{n}{3^K}\right) = X(1) = 1$

$$\frac{n}{3^K} = 1$$

$$n = 3^K$$

$$K = \log_3 n$$

$$X(n) = \cancel{X\left(\frac{n}{3^K}\right)} + K$$

$$1 + \log_3 n$$

$$\boxed{\Theta(\log n)}$$

$$\rightarrow 2^1 \times \left(\frac{n}{2^1}\right) + (1n)$$

⑥  $X(n) = 2 \times \left(\frac{n}{2}\right) + n \quad X(n-1)$

$$X\left(\frac{n}{2}\right) = 2 \times \left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)$$

$$X\left(\frac{n}{4}\right) = 2 \times \left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)$$

~~$X(n) = 2 \times \left(\frac{n}{2}\right) + n$~~

~~$X(n) = 2 \times \left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)$~~

$$X(n) = 4 \times \left(\frac{n}{8}\right) + n + n \quad X(n) = 4 \left[2 \times \left(\frac{n}{8}\right) + \frac{n}{4}\right]$$

$$\rightarrow X(n) = 8 \times \left(\frac{n}{16}\right) + n + n + n$$

$$X(n) = 2^k \times \left(\frac{n}{2^k}\right) + n \cdot k$$

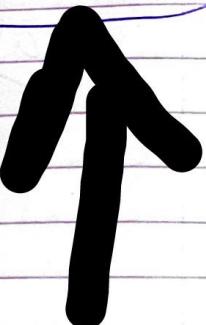
$$2^3 \times \left(\frac{n}{2^3}\right) + 3n$$

assume

$$X\left(\frac{n}{2^k}\right) = X(1) = 1$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$



$$k = \log_2 n$$

~~$$X(n) = 2^{\log_2 n} \times \left(\frac{n}{2^{\log_2 n}}\right) + n \cdot 1 \cdot 2^{\log_2 n}$$~~

$$X(n) = n \log_2 n$$

$\Theta(n \log n)$

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②  $X(n) = X\left(\frac{n}{2}\right) + 5$   $\xrightarrow{\text{recurrence relation}}$   $+ X(1) = 1$ .

$X\left(\frac{n}{2}\right) = X\left(\frac{n}{4}\right) + 5$

$X\left(\frac{n}{4}\right) = X\left(\frac{n}{8}\right) + 5$

$X(n) = X\left(\frac{n}{4}\right) + 5 + 5$

$X(n) = X\left(\frac{n}{8}\right) + 5 + 5 + 5$   $\xrightarrow{\text{from above}}$   $(X(n) = X\left(\frac{n}{2^k}\right) + 5_k)$

assume  $X\left(\frac{n}{2^k}\right) = X(1) = 1$

$K = \log_2 n$

$X(n) = 1 + 5 \log_2 n$

$\Theta(\log n)$ .

$$(b) \quad x(n) = \overbrace{x(n-3)}^{3-1} + 3 \quad x(1) = 0$$

$$x(n-3) = x(n-6) + 3$$

$$x(n-6) = x(n-9) + 3$$

$$x(n) = x(n-6) + 3 + 3$$

$$x(n) = x(n-9) + 3 + 3 + 3$$

$$\boxed{x(n) = x(n-3k) + 3k}$$

$$\text{assume } x(n-3k) = x(1) = 0$$

$$n-3k = 1$$

$$\boxed{3k = n-1}$$

$$\boxed{k = \frac{n-1}{3}}$$

$$x(n) = 0 - \Theta\left(\frac{n-1}{3}\right)$$

$$x(n) = n-1$$

$$\boxed{\Theta(n)}$$

Sheet 3 (Ch 2.3 → 2.5)

$$\textcircled{1} \quad \textcircled{b} \quad 2 + 4 + 8 + 16 + \dots = 2^{10}$$

$$2^1 + 2^2 + 2^3 + \dots + 2^{10}$$

first  $\xrightarrow{a}$   $\left( \frac{1 - r^m}{1 - r} \right)$  multiply  
size.

$$2 \left( \frac{1 - 2^{10}}{1 - 2} \right)$$

$$\textcircled{2} \quad \sum_{j=1}^n 3^{j+1}$$

$$3^2 + 3^3 + 3^4 + \dots + 3^{n+1}$$

$$\boxed{3^2 \left( \frac{1 - 3^n}{1 - 3} \right)}$$

Sheet 3

$$\textcircled{i} \quad X(n) = 2X(n-4) + 2$$

$$X(1) = 0$$

$$X(n-2) = 2X(n-4) + 2$$

$$X(n-4) = 2X(n-6) + 2$$

$$X(n) = 2[2X(n-4) + 2] + 2$$

$$X(n) = 2 \left[ 2 \times (n-4) + 2 \right] + 2$$

$$= 4 \times (n-4) + 4 + 2$$

$$X(n) = 4 \left[ 2X(n-6) + 2 \right] + 4 + 2$$

$$\frac{8X(n-6)}{2^3} + \frac{8}{(3 \cdot 2)} + \frac{4}{2^3} + \frac{2}{2^2}$$

~~$$X(n) = 2^K X(n-2^K) + 2^K + 2^{K-1} + 2^{K-2} + \dots + 2^1$$~~

assume  $X(n-2^K) - X(1) = 0$

$$n-2^K = 1$$

~~$$n-2^K = 1-2^K$$~~

$$\boxed{K = \frac{n-1}{2}}$$

$$\boxed{X(n) = 2^{\frac{n-1}{2}} + \dots + 2^1}$$

~~$$2^{\frac{n-1}{2}} \left( \frac{1-2^{\frac{n-1}{2}}}{1-2} \right) \approx 2^n$$~~

### Sheet 3 (Ch 2.3 → 1.5)

Q3

(B)  $Q(n) = Q(n-1) + 1$

$Q(1) = 0 \rightarrow$  no multiplication solved.

$\boxed{Q(n) = Q(n-k) + k}$

no multiplication

$$Q(n) = Q(n-2) + \underbrace{1}_{\frac{1}{2}} + 1$$

assume  $Q(n-k) = Q(1) = 0$

$$Q(n) = 0 + k$$

$\Theta(n)$

$$n-k = 1$$

$K = n-1$

(C)  $Q(n) = Q(n-1) + 2$

$Q(1) = 0 \rightarrow$  even  $\oplus$  or  $\ominus$

$$Q(n-1) = Q(n-2) + \underbrace{2+2}_{2^2}$$

$$Q(n-2) = Q(n-3) + \underbrace{(2+2+2)}_{2^3}$$