hypothesis test formula sheet

One sample tests

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
one sample z-test	μ unknown, σ^2 known	$H_0 : \mu = \mu_0$	$Z_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}}\rvert)$
one sample z-test for a proportion	π unknown	$H_0 : \pi = \pi_0$	$Z_{\text{obs}} = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}}\rvert)$
one sample t-test	μ, σ^2 unknown	$H_0 : \mu = \mu_0$	$T_{\text{obs}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$	t_{n-1}	$2 \cdot P(t_{n-1} < -\lvert T_{\text{obs}}\rvert)$
chi-square test for variance	μ, σ^2 unknown	$H_0 : \sigma^2 = \sigma_0^2$	$T = \frac{(n-1) \cdot s^2}{\sigma_0^2}$	χ^2_{n-1}	TODO
sign test	none	$H_0 : m = m_0$	$B_{\text{obs}} = \sum_i I_{x_i > m_0}$	Binomial($\sum_i I_{x_i \neq m_0}, \frac{1}{2}$)	$2 \cdot \min(P(B \geq B_{\text{obs}}), P(B \leq B_{\text{obs}}))$

Two sample tests

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
two sample z-test	μ_1, μ_2 unknown σ_1^2, σ_2^2 known	$H_0 : \mu_1 - \mu_2 = \delta_0$	$Z_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}}\rvert)$
two sample z-test for a proportion	π_1, π_2 unknown	$H_0 : \pi_1 = \pi_2$	$Z_{\text{obs}} = \frac{\hat{\pi}_1 - \hat{\pi}_2}{\sqrt{\hat{\pi}(1 - \hat{\pi})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}}\rvert)$
equal variance t-test	μ_1, μ_2 $\sigma_1^2 = \sigma_2^2$ unknown	$H_0 : \mu_1 - \mu_2 = \delta_0$	$T_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{s_{\text{pooled}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	$t_{n_1+n_2-2}$	$2 \cdot P(t_{n_1+n_2-2} < -\lvert T_{\text{obs}}\rvert)$
welch's t-test	μ_1, μ_2 $\sigma_1^2 \neq \sigma_2^2$ unknown	$H_0 : \mu_1 - \mu_2 = \delta_0$	$T_{\text{obs}} = \frac{(\bar{x} - \bar{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	t_ν	$2 \cdot P(t_\nu < -\lvert T_{\text{obs}}\rvert)$
f-test for equal variance	μ_1, μ_2 σ_1^2, σ_2^2 unknown	$H_0 : \sigma_1^2 = \sigma_2^2$	$F_{\text{obs}} = \frac{s_1^2}{s_2^2}$	F_{n_1-1, n_2-1}	TODO

where

$$s_{\text{pooled}} = \sqrt{\frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{n_1 + n_2 - 2}} \qquad \nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} \qquad \hat{\pi} = \frac{n_1\hat{\pi}_1 + n_2\hat{\pi}_2}{n_1 + n_2} \tag{1}$$

Paired two-sample tests (incomplete, don't trust these yet)

Suppose x_i is paired with y_i. Then let $d_i = x_i - y_i$.

name	known parameters	null hypothesis	test statistic	null distribution	p-value (two sided)
paired z-test	μ unknown, σ^2 known	$H_0 : \mu_1 - \mu_2 = \delta_0$	$Z_{\text{obs}} = \frac{\bar{d} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$	N(0, 1)	$2 \cdot P(Z < -\lvert Z_{\text{obs}}\rvert)$
paired t-test	μ, σ^2 unknown	$H_0 : \mu_1 - \mu_2 = \delta_0$	$T_{\text{obs}} = \frac{\bar{d} - \delta_0}{\frac{s_d}{\sqrt{n}}}$	t_{n-1}	$2 \cdot P(t_{n-1} < -\lvert T_{\text{obs}}\rvert)$
paired sign test	none	$H_0 : m_1 - m_2 = \delta_0$	$B_{\text{obs}} = \sum_i I_{d_i > \delta_0}$	$\text{Binomial}(\sum_i I_{d_i \neq \delta_0}, \frac{1}{2})$	$2 \cdot \min(P(B \geq B_{\text{obs}}), P(B \leq B_{\text{obs}}))$

Related to linear regression and ANOVA

A one-way ANOVA table looks like

source	df	SS	MS	F	p-value
treatment	$k-1$	SSTreatment	$\text{MSTreatment} = \frac{\text{SSTreatment}}{k-1}$	$\frac{\text{MSTreatment}}{\text{MSError}}$	$P(F_{k-1,N-k} > F_{\text{obs}})$
error	$N-k$	SSError	$\text{MSError} = \frac{\text{SSError}}{N-k}$		
total	$N-1$	SSTotal			

where

$$\text{SSTreatment} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{k} n_i(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2 \tag{2}$$

$$\text{SSError} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{i\cdot})^2 = \sum_{i=1}^{k}(n_i - 1)s_i^2 \tag{3}$$

$$\text{SSTotal} = \text{SSTreatment} + \text{SSError} = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_{\cdot\cdot})^2 \tag{4}$$

We use this to compare group means (i.e. categorical predictor). When we have continuous predictors we use simple linear regression which results in the following:

name	null hypothesis	test statistic	null distribution	p-value
t-test for β_0	$H_0 : \beta_0 = c$	$T_{\text{obs}} = \dfrac{\hat{\beta}_0 - c}{\hat{\sigma}\sqrt{\frac{1}{n} + \bar{x}^2 / \sum_{i=1}^n (x_i - \bar{x})^2}}$	t_{n-2}	$2 \cdot P(T < -\lvert T_{\text{obs}} \rvert)$
t-test for β_1	$H_0 : \beta_1 = c$	$T_{\text{obs}} = \dfrac{\hat{\beta}_1 - c}{\hat{\sigma} / \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$	t_{n-2}	$2 \cdot P(T < -\lvert T_{\text{obs}} \rvert)$
overall f-test for linear model	TODO	TODO	TODO	TODO

Miscelleanous simple linear regression formulas not from hypothesis tests

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i \qquad \varepsilon_i \sim \text{Normal}(0, \sigma^2) \tag{5}$$

$$\rho = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{s_{xy}}{s_x s_y} \tag{6}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \rho \cdot \frac{s_y}{s_x} \tag{7}$$

$$\hat{\beta}_0 = \hat{y} - \hat{\beta}_1 \bar{x} \tag{8}$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \tag{9}$$

$$e_i = y_i - \hat{y}_i \tag{10}$$

$$\text{SSE} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \tag{11}$$

$$\hat{\sigma}^2 = \text{MSE} = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2} \tag{12}$$

Confidence intervals for means and future predictions at x^* are given by

$$TODO \tag{13}$$

Post-hoc tests for ANOVA

name	null hypothesis	test statistic	null distribution	p-value
linear contrast on group means	TODO	TODO	TODO	TODO
tukey's honestly significant differences	TODO	TODO	TODO	TODO

Nonparametric tests

- Kruskal-Wallis
- Rank-Sum
- Signed-Rank

Bootstrap and permutation (parametric, one and two sample, paired)