# UNIT III - INVENTORY MANAGEMENT

## Dynamic Lot sizing Methods:

Dynamic lot sizing methods are essential in inventory management and production planning. These methods determine the optimal order quantity and timing for replenishing inventory to meet fluctuating demand over a planning horizon. Here are some widely used dynamic lot sizing methods:

### 1. Economic Order Quantity (EOQ)

Although traditionally static, EOQ can be adapted for dynamic environments by recalculating the optimal order quantity and timing based on changing demand and costs.

### 2. Silver-Meal Heuristic

The Silver-Meal heuristic aims to minimize the average cost per period by considering the trade-off between holding and setup costs over a specific number of periods. It determines the optimal lot size by incrementally adding periods until the average cost per period starts to increase.

### 3. Least Unit Cost (LUC)

The Least Unit Cost method seeks to minimize the cost per unit of product by comparing the unit cost of ordering different quantities. It includes setup costs and holding costs, choosing the order quantity that results in the lowest cost per unit.

### 4. Part-Period Balancing (PPB)

Part-Period Balancing balances the total holding costs against the setup costs over a planning horizon. It calculates the cumulative demand and selects the order quantity that equates the total holding cost with the setup cost.

### 5. Wagner-Whitin Algorithm

The Wagner-Whitin algorithm is an exact dynamic programming approach. It determines the optimal order quantity and timing by considering all possible ordering scenarios over the planning horizon and selecting the one that minimizes the total cost, including setup and holding costs.

### 6. Lot-for-Lot (LFL)

The Lot-for-Lot method orders exactly what is required for each period, resulting in no holding costs. This method is simple and useful when setup costs are low, or there are no significant economies of scale.

### 7. Period Order Quantity (POQ)

The Period Order Quantity method determines the order quantity by dividing the planning horizon into equal periods. It orders enough to cover the demand for a set number of periods, balancing setup and holding costs.

## 8. Dynamic Economic Lot Sizing (DELS)

The Dynamic Economic Lot Sizing method uses a more complex approach to account for fluctuating demand and varying costs. It dynamically adjusts order quantities and timing to minimize total costs over the planning horizon.

## 9. Rolling Horizon Planning

Rolling Horizon Planning involves continuously updating the lot-sizing plan at regular intervals (e.g., monthly or weekly). This method allows for adjustments based on the most recent demand forecasts and cost information.

## Practical Considerations

- **Demand Variability:** Accurate demand forecasting is crucial for effective lot sizing.
- **Lead Time:** Lead times should be considered, especially for methods like Lot-for-Lot where frequent ordering can lead to high costs if lead times are long.
- **Setup and Holding Costs:** Balancing setup and holding costs is key to finding the optimal solution.
- **Computational Complexity:** Methods like the Wagner-Whitin algorithm can be computationally intensive, making heuristics like Silver-Meal or PPB more practical for large problems.

## Conclusion

Selecting the right dynamic lot sizing method depends on the specific context, including demand patterns, cost structures, and computational resources. Often, a combination of methods and continuous refinement of strategies is needed to achieve optimal results in dynamic environments.

## 1. Economic Order Quantity (EOQ)

Adapted for dynamic demand, EOQ can be recalculated for each period:
$EOQ = \sqrt{\frac{2DS}{H}}$
where:

- $D$ = Demand per period
- $S$ = Setup cost per order
- $H$ = Holding cost per unit per period

## 2. Silver-Meal Heuristic

This heuristic minimizes the average cost per period over a planning horizon. For each period $t$, calculate:
$TC(t) = \frac{S+\sum_{k=0}^{t} H\cdot\sum_{j=0}^{k} d_j}{t+1}$
where:

- $TC(t)$ = Total cost for $t+1$ periods
- $d_j$ = Demand in period $j$
- $S$ = Setup cost
- $H$ = Holding cost per unit per period

Continue adding periods until $TC(t+1) > TC(t)$.

## 3. Least Unit Cost (LUC)

Calculate the unit cost for ordering various quantities:

$$LUC(t) = \frac{S+\sum_{j=0}^{t} H\cdot(t-j)\cdot d_j}{\sum_{j=0}^{t} d_j}$$

Choose the order quantity that minimizes $LUC(t)$.

## 4. Part-Period Balancing (PPB)

Balance the holding costs against setup costs:

$$\text{Order quantity at period } t \text{ if } \sum_{k=0}^{t} H \cdot \sum_{j=0}^{k} d_j \approx S$$

## 5. Wagner-Whitin Algorithm

This algorithm finds the optimal solution through dynamic programming. Define:

$$C(t) = \min_{k<t}\{C(k) + S + \sum_{j=k+1}^{t} H \cdot (t-j) \cdot d_j\}$$

where:

- $C(t)$ = Minimum cost up to period $t$
- $k$ = Previous period with an order

## 6. Lot-for-Lot (LFL)

Order exactly what is needed for each period:

$$Q_t = d_t$$

where $Q_t$ is the order quantity in period $t$.

## 7. Period Order Quantity (POQ)

Divide the planning horizon into $n$ periods and order enough to cover demand for $T$ periods:

$$Q_t = \sum_{j=t}^{t+T-1} d_j$$

where $T$ is the number of periods covered by each order.

### 8. Dynamic Economic Lot Sizing (DELS)

Minimize total costs considering demand fluctuations:

$$TC = \min \left\{ \sum_{t=1}^{N} (S \cdot I_t + H \cdot \sum_{j=1}^{t} (t-j) \cdot d_j) \right\}$$

where:

- $I_t$ = Indicator variable (1 if order is placed in period $t$, 0 otherwise)
- $N$ = Total number of periods

### 9. Rolling Horizon Planning

Recalculate lot sizes at each interval $\Delta t$:

$$Q_{t+\Delta t} = f(d_{t+\Delta t}, S, H)$$

where $f$ is the chosen lot sizing function.

---

## Multi-Echelon Inventory models;

## Multi-Echelon Inventory Models: An Introduction

Multi-echelon inventory models help manage inventory across different stages in a supply chain. Think of a supply chain as a series of steps from the supplier to the manufacturer, then to the warehouse, and finally to the retailer. Each step is called an echelon.

## Key Concepts

1. **Echelon Stock**: Total inventory at a stage plus all downstream stages.
2. **Echelon Holding Costs**: Costs of holding inventory across multiple stages.

## Basic Models

*1. Serial Multi-Echelon Model*

This model deals with a supply chain where each stage supplies the next one in a series (like a chain).

- **Demand Propagation**: Demand at the end stage (e.g., retailer) moves up the chain.
- **Order-Up-To Levels**: Set optimal inventory levels at each stage.

**Example:** If the retailer needs 100 units, the warehouse needs to have enough to supply the retailer plus its own buffer, and so on up the chain.

### 2. Base Stock Policies

These policies determine how much stock to keep at each stage to meet demand without overstocking.

- **Single-Echelon Base Stock Policy**: Manages stock at one stage.
- **Multi-Echelon Base Stock Policy**: Coordinates stock across multiple stages.

**Example:** A retailer always keeps 50 units in stock. The warehouse ensures it can replenish the retailer while keeping enough for itself.

## Advanced Models

### 1. Installation vs. Echelon Stock Policies

- **Installation Stock Policy**: Each stage manages its inventory independently.
- **Echelon Stock Policy**: Manages total inventory across all stages as a whole.

**Example:** In installation policy, each warehouse, store, etc., decides how much stock to keep. In echelon policy, they coordinate to keep the right amount across the entire chain.

### 2. Multi-Echelon Inventory Optimization

This involves finding the best inventory levels to minimize total costs (holding, ordering, and shortage costs) across the supply chain.

**Objective:** Minimize total costs by balancing inventory levels at each stage.

### 3. Stochastic Multi-Echelon Models

These models consider demand and supply uncertainties.

- **Safety Stock**: Extra stock kept to cover unexpected demand or delays.

**Example:** A retailer keeps extra stock in case more customers buy than expected or deliveries are delayed.

### 4. Inventory Routing Problem (IRP)

Combines inventory management with transportation planning to minimize total costs of holding inventory, ordering, and transporting goods.

**Example:** A company decides not only how much inventory to keep but also the best routes and schedules for delivery trucks to minimize costs.

## Simplified Example

Imagine a simple supply chain with three stages: a supplier, a warehouse, and a retailer.

1. **Demand at Retailer**: The retailer needs 50 units each week.
2. **Warehouse Stock**: The warehouse keeps enough to supply the retailer plus extra in case of higher demand or delivery delays (safety stock).
3. **Supplier Replenishment**: The supplier sends stock to the warehouse based on how much the warehouse uses and how much safety stock it keeps.

## Conclusion

Multi-echelon inventory models help manage inventory efficiently across different stages in a supply chain. They ensure that each stage has the right amount of stock to meet demand, minimize costs, and handle uncertainties. By coordinating inventory across all stages, businesses can reduce total costs and improve service levels.

## OR

Multi-echelon inventory models are designed to manage inventory across multiple stages or levels within a supply chain. These models are crucial for optimizing inventory control in complex supply chains that consist of multiple interconnected echelons, such as suppliers, manufacturers, distribution centers, and retailers.

## Key Concepts and Models in Multi-Echelon Inventory Management

### *1. Echelon Stock and Echelon Holding Costs*

- **Echelon Stock:** The total inventory within a supply chain network, including inventory at the current level and all downstream levels.
- **Echelon Holding Cost:** The holding cost associated with echelon stock, accounting for inventory holding at all levels.

### *2. Base Stock Policies*

- **Single-Echelon Base Stock Policy:** Determines the optimal inventory level at one echelon.
- **Multi-Echelon Base Stock Policy:** Extends the base stock policy to multiple echelons, ensuring coordinated inventory levels across the supply chain.

---

## Multi-Echelon Inventory Models

### *1. Serial Multi-Echelon Model*

This model involves a sequence of stages, where each stage supplies the next one. Inventory decisions at each stage affect the subsequent stages.

- **Demand Propagation:** Demand at the final echelon (e.g., retailer) propagates upstream.
- **Order-Up-To Levels:** Determine optimal inventory levels at each echelon.

**Equation:**

$S_i = R_i + \sum_{j=i+1}^{N} E[D_j]$

where:

- $S_i$ = Order-up-to level at echelon $i$
- $R_i$ = Reorder point at echelon $i$
- $E[D_j]$ = Expected demand at echelon $j$
- $N$ = Total number of echelons

### *2. Installation vs. Echelon Stock Policies*

- **Installation Stock Policy:** Inventory control is managed separately at each echelon.
- **Echelon Stock Policy:** Considers the total inventory across all echelons.

**Equation:**

$C_i = h_i S_i + \sum_{j=i+1}^{N} h_j (S_j - S_{j-1})$

where:

- $C_i$ = Holding cost at echelon $i$
- $h_i$ = Holding cost rate at echelon $i$
- $S_{j-1}$ = Order-up-to level at echelon $j-1$

### *3. Multi-Echelon Inventory Optimization*

Optimization techniques aim to minimize total costs across the supply chain, including holding, ordering, and shortage costs.

- **Objective Function:**
  $\min \sum_{i=1}^{N} (h_i I_i + S_i K_i + p_i \beta_i)$
  where:
- $I_i$ = Inventory level at echelon $i$
- $K_i$ = Ordering cost at echelon $i$
- $p_i$ = Shortage cost at echelon $i$
- $\beta_i$ = Service level at echelon $i$

## Advanced Multi-Echelon Models

### 1. **Stochastic Multi-Echelon Models**

Account for demand and supply variability by incorporating stochastic elements.

- **Inventory Position:** Tracks on-hand inventory, on-order inventory, and backorders.
- **Safety Stock:** Ensures service levels under uncertainty.

**Equation:**
$SS_i = z\sqrt{L_i\sigma_d^2 + \sigma_L^2 D_i^2}$
where:

- $SS_i$ = Safety stock at echelon $i$
- $z$ = Service level factor
- $L_i$ = Lead time at echelon $i$
- $\sigma_d$ = Standard deviation of demand
- $\sigma_L$ = Standard deviation of lead time
- $D_i$ = Average demand at echelon $i$

### 2. **Inventory Routing Problem (IRP)**

Combines inventory management with vehicle routing, optimizing both inventory levels and transportation.

- **Objective:** Minimize total costs of holding inventory, ordering, and transportation.

**Equation:**
$\min \sum_{t=1}^{T} \sum_{i=1}^{N} (h_i I_{it} + K_i O_{it} + \sum_{j \in J} c_{ij} x_{ijt})$
where:

- $I_{it}$ = Inventory level at node $i$ at time $t$
- $O_{it}$ = Order quantity at node $i$ at time $t$
- $c_{ij}$ = Transportation cost between nodes $i$ and $j$
- $x_{ijt}$ = Binary variable (1 if route $i \to j$ is used at time $t$, 0 otherwise)

## Conclusion

Multi-echelon inventory models are vital for optimizing complex supply chains. They help in achieving coordinated and cost-effective inventory management across multiple stages, considering factors like demand variability, lead times, and transportation. Employing these models can significantly enhance supply chain efficiency and service levels.

## Aggregate Inventory System and LIMIT

An aggregate inventory system involves managing the total inventory across various products, locations, or stages in the supply chain as a single entity. This approach helps optimize inventory levels, reduce costs, and improve service levels. LIMIT (Leverage, Integrate, Manage, Innovate, and Transform) is a framework that can be applied to improve aggregate inventory systems.

**Components of Aggregate Inventory System:**

1. **Centralized Control:**
   - Centralize inventory decisions to manage overall stock levels more efficiently.
   - Use a centralized inventory management system for better visibility and control.
2. **Inventory Optimization:**
   - Optimize inventory levels across the entire supply chain to balance cost and service.
   - Use advanced forecasting techniques to predict demand accurately.
3. **Collaboration:**
   - Collaborate with suppliers and partners to ensure synchronized inventory replenishment.
   - Share data and insights to improve demand planning and inventory management.
4. **Technology Integration:**
   - Implement technologies like IoT, RFID, and ERP systems to track and manage inventory in real time.
   - Use data analytics and AI for inventory optimization and decision-making.

**LIMIT Framework in Aggregate Inventory System:**

1. **Leverage:**
   - Leverage existing resources and technologies to improve inventory management.
   - Utilize historical data and advanced analytics for better forecasting and planning.
2. **Integrate:**
   - Integrate inventory data from various sources into a centralized system.
   - Ensure seamless integration with suppliers and partners for better coordination.
3. **Manage:**
   - Implement robust inventory management practices to maintain optimal stock levels.
   - Use KPIs to monitor performance and identify areas for improvement.
4. **Innovate:**
   - Adopt innovative technologies and practices to enhance inventory management.
   - Explore new methods like just-in-time (JIT) inventory and vendor-managed inventory (VMI).

5. **Transform:**
    - Continuously transform inventory practices to adapt to changing market conditions.
    - Invest in employee training and development to build expertise in modern inventory management.

---

## Risk Analysis in Supply Chain

Risk analysis in supply chain involves identifying, assessing, and mitigating risks that could disrupt the supply chain operations. Effective risk analysis helps in maintaining continuity, reducing vulnerabilities, and ensuring smooth operations.

**Key Steps in Risk Analysis:**

1. **Identify Risks:**
    - Identify potential risks from various sources such as suppliers, transportation, natural disasters, market fluctuations, and geopolitical factors.
    - Use techniques like SWOT analysis and scenario planning to identify risks.
2. **Assess Risks:**
    - Assess the likelihood and impact of each identified risk.
    - Use risk assessment tools like Failure Modes and Effects Analysis (FMEA) and risk matrices.
3. **Mitigate Risks:**
    - Develop mitigation strategies for each identified risk.
    - Implement measures like diversifying suppliers, increasing safety stock, and developing contingency plans.
4. **Monitor and Review:**
    - Continuously monitor risks and their impact on the supply chain.
    - Regularly review and update risk management strategies.

## Risk Pooling Strategies

Risk pooling is a strategy used to reduce risk and variability in the supply chain by aggregating demand across multiple locations or products. By pooling risks, companies can achieve more stable and predictable outcomes.

**Common Risk Pooling Strategies:**

1. **Centralized Inventory:**
    - Consolidate inventory in a central location rather than maintaining separate inventories at multiple locations.
    - Benefits include reduced safety stock levels and lower overall inventory costs.
2. **Product Pooling:**
    - Combine similar products into a single inventory pool.
    - This strategy reduces variability and improves service levels.
3. **Component Commonality:**
    - Use common components in multiple products to reduce variability and improve flexibility.

    - Benefits include simplified inventory management and reduced lead times.
4. **Demand Pooling:**
    - Aggregate demand across different regions or markets to smooth out variability.
    - This approach helps in better demand forecasting and inventory planning.

**Benefits of Risk Pooling:**

1. **Reduced Inventory Costs:**
    - Lower safety stock levels due to reduced variability.
    - Economies of scale in inventory management and procurement.
2. **Improved Service Levels:**
    - More consistent and reliable inventory availability.
    - Better ability to meet customer demand.
3. **Enhanced Flexibility:**
    - Greater flexibility to respond to changes in demand or supply conditions.
    - Improved ability to manage disruptions and uncertainties.
4. **Streamlined Operations:**
    - Simplified inventory management processes.
    - Improved efficiency in logistics and distribution.

---