Iterative methods: Bisection Method

Numerical Methods

September 29, 2021

Lecture Notes

Bisection Method

Algorithm

To find a root of a function given as $f(x) = 0$,

1. Choose two points a and b such that $f(a).f(b) < 0$

2. Calculate point c using
$$c = \frac{a+b}{2}$$

3. if $f(a).f(c) < 0$
   update $b = c$
   else
   update $a = c$

4. Repeat steps 2 and 3 until $|a - b| <$ Tolerance Value

5. The root is value of c in last iteration.

Example:

Find the root of the expression $f(x) = x^3 + 4x^2 - 3x - 12$

iter	a	b	$c = \frac{a+b}{2}$	f(a)	f(b)	f(c)
0	1	2	1.5	-10	6	-4.125
1	1.5	2	1.75	-4.125	6	0.359375
2	1.5	1.75	1.625	-4.125	0.359375	-2.021484375
3	1.625	1.75	1.6875	-2.021484375	0.359375	-0.8664550781
4	1.6875	1.75	1.71875	-0.8664550781	0.359375	-0.2624816895
5	1.71875	1.75	1.734375	-0.2624816895	0.359375	0.04619979858
6	1.71875	1.73438	1.726565	-0.2624816895	0.04629929466	-0.1086518402
7	1.72656	1.73438	1.73047	-0.1087506179	0.04629929466	-0.03136618083
8	1.73047	1.73438	1.732425	-0.03136618083	0.04629929466	0.007431404704
9	1.73047	1.73242	1.731445	-0.03136618083	0.007332088488	-0.01202578653
10	1.73144	1.73242	1.73193	-0.01212501263	0.007332088488	-0.00239866998
11	1.73193	1.73242	1.732175	-0.00239866998	0.007332088488	0.002466157233

Table 1: Solution of $f(x) = x^3 + 4x^2 - 3x - 12$

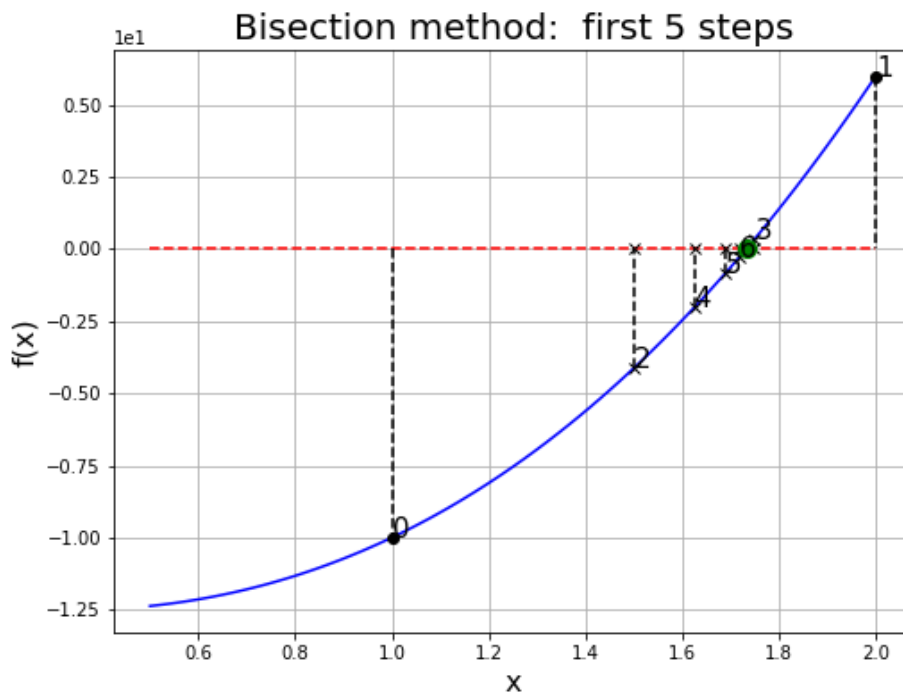From the table, the root of given equation is 1.73193.

Figure 1: Bisection method on $f(x) = x^3 + 4x^x - 3x - 12$

- Bracketing method.

- Linear convergence.

- Certainty of finding root.

Practice Exercises

- Find the root of the following expressions using Bisection method:

  1. $x^4 - 19 = 0$
  2. $f(x) = \sin(x) - 4e^{-2x}$
  3. $4\ln(x) + 0.2x^2$
  4. $x^5 + 6x^3 - 4x + 10$