

Iterative methods: Newton Raphson Method

Numerical Methods

October 6, 2021

Lecture Notes

Newton Raphson Method

Algorithm

To find the root of a function $f(x) = 0$,

1. Calculate the derivative of $f(x)$, $(f'(x))$.
2. Choose an initial point x_0 and evaluate $f(x_0)$ and $f'(x_0)$ at that point.
3. Calculate next approximation point x_1 using

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

4. Set $x_0 = x_1$
5. Repeat steps 2 and 3 until $|x_0 - x_1| < \epsilon_1$ or $|f(x_0)| < \epsilon_2$ where ϵ_1 and ϵ_2 are tolerance values.
6. The root is value of x_n in last iteration.

Example:

Find the root of the expression $f(x) = x^3 + 4x^2 - 3x - 12$

i	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	1	-10	8	2.25
1	2.25	12.890625	30.1875	1.822981366
2	1.822981366	1.882343143	21.55363412	1.735648384
3	1.735648384	0.07155401488	19.92261302	1.732056786
4	1.732056786	0.0001187193532	19.85651643	1.732050808
5	1.732050808	0.0000000003287325967	19.85640646	1.732050808

Table 1: Solution of $f(x) = x^3 + 4x^2 - 3x - 12$

From the table, the root of given equation is 1.73205.

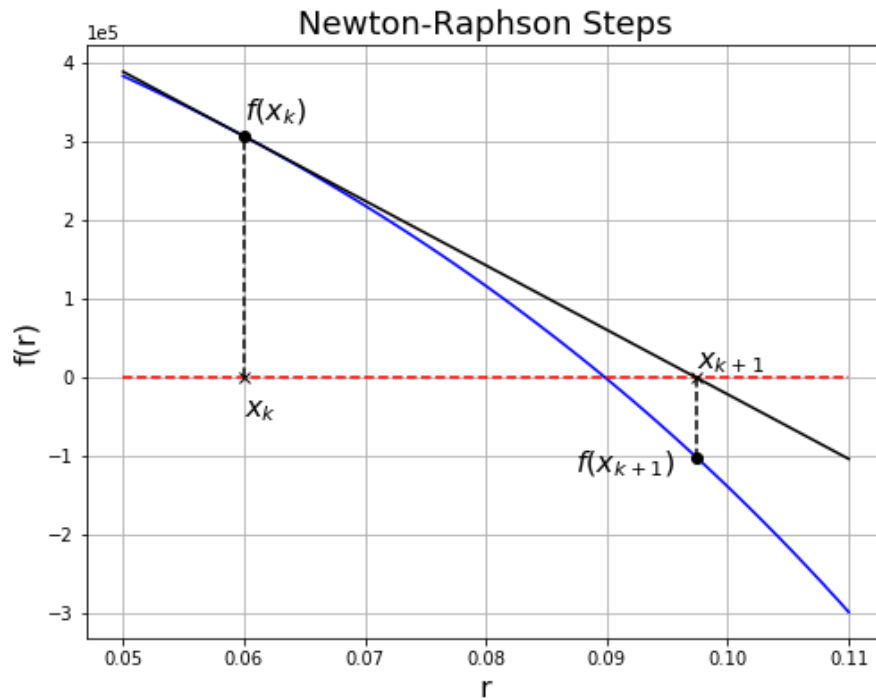


Figure 1: Newton Raphson method on $f(x) = x^3 + 4x^x - 3x - 12$

- Non-Bracketing method.
- Quadratic convergence.
- Might not converge.

Practice Exercises

- Find the root of the following expressions using Newton Raphson method:
 1. $x^4 - 19 = 0$
 2. $f(x) = \sin(x) - 4e^{-2x}$
 3. $4 \ln(x) + 0.2x^2$
 4. $x^5 + 6x^3 - 4x + 10$