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Convert from cylindrical to spherical.

$x = r\cos\phi$, $x = R\sin\phi\cos\phi \rightarrow r = R\sin\phi$

$y = r\sin\phi$, $y = R\sin\phi\sin\phi$

$z = z$, $z = R\cos\phi$

Now ,

$$\frac{\partial U_r}{\partial R} = \frac{\partial U_r}{\partial r}\cdot\frac{\partial r}{\partial R} + \frac{\partial U_r}{\partial \phi}\cdot\frac{\partial \phi}{\partial R} + \frac{\partial U_r}{\partial z}\cdot\frac{\partial z}{\partial R}$$ ($\partial z = 0$)

$$= \frac{\partial}{\partial r}(U_r\cos\phi + U_y\sin\phi)\cdot 2(R\sin\phi)\cdot\frac{\partial \phi}{\partial R} + \frac{\partial}{\partial \phi}(U_r\cos\phi + U_y\sin\phi)$$

$$= \left[\frac{\partial U_r}{\partial r}\cdot\cos\phi + \frac{\partial U_y}{\partial r}\sin\phi\right]\cdot\sin\phi + \left[\frac{\partial U_r}{\partial \phi}\cos\phi + U_r(-\sin\phi) + \frac{\partial U_y}{\partial \phi}\sin\phi + U_y\cos\phi\right]\frac{\partial \phi}{\partial R}$$

$$\Rightarrow \frac{\partial U_r}{\partial R} = \left[\frac{\partial U_r}{\partial r}\cos\phi + \frac{\partial U_y}{\partial r}\sin\phi\right]\sin\phi + \left[\frac{\partial U_r}{\partial \phi}\cos\phi - U_r\sin\phi + \frac{\partial U_y}{\partial \phi} + U_y\cos\phi\right]\frac{\partial \phi}{\partial R}$$

$$\Rightarrow \frac{\partial U_\phi}{\partial \phi} = \frac{\partial U_\phi}{\partial r}\cdot\frac{\partial r}{\partial \phi} + \frac{\partial U_\phi}{\partial \phi}\cdot\frac{\partial \phi}{\partial \phi}\cdot\frac{\partial z}{\partial \phi}$$ ($\partial z = 0$)

$$= \frac{\partial}{\partial r}(-r U_r\sin\theta + r U_y\cos\theta)\cdot\frac{\partial}{\partial \phi}(R\sin\phi) + \frac{\partial}{\partial \phi}(-r U_r\sin\theta + r U_y\cos\theta)\cdot 1$$

$$= \left[-U_r\sin\phi\frac{\partial r}{\partial r} - r\sin\theta\frac{\partial U_r}{\partial r} - r U_r 2\sin\phi + U_y\cos\theta\frac{\partial r}{\partial r} + r\cos\theta\frac{\partial U_y}{\partial r} + r U_y 2\cos\theta\right]$$

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$$\cdot\cdot \sin\phi \frac{\partial R}{\partial \theta} + \left[-u_r \sin\theta \frac{\partial r}{\partial \theta} - r\sin\theta \frac{\partial u_r}{\partial \theta} - r u_{r2} \frac{\sin\theta}{\partial \theta} \right.$$

$$\left. + u_y \cos\theta \frac{\partial r}{\partial \varphi} + r\cos\theta \frac{\partial u_y}{\partial \theta} + r u_{y2} \frac{\cos\theta}{\partial \theta} \right]$$

$$= \left[-u_r \sin\theta - r\sin\theta \frac{\partial u_r}{\partial r} + u_y \cos\theta + r\cos\theta \frac{\partial u_y}{\partial r} \right]$$

$$\sin\theta \frac{\partial R}{\partial \theta} + \left[(-u_r \sin\theta + u_y \cos\theta) \frac{\partial r}{\partial \theta} - \right.$$

$$\left. r\sin\theta \frac{\partial u_r}{\partial \theta} + r\cos\theta \frac{\partial u_y}{\partial \theta} - r u_r \cos\theta - r u_y \sin\theta \right]$$

$$\frac{\partial u_2}{\partial \varphi} = \frac{\partial u_2}{\partial r} \frac{\partial r}{\partial \varphi} + \frac{\partial u_2}{\partial \theta} \cdot \frac{\partial \theta}{\partial \varphi} + \frac{\partial u_2}{\partial \theta} \cdot \frac{\partial^2}{\partial \varphi}$$

$$= \frac{\partial u_2}{\partial r} \cdot \frac{\partial}{\partial \varphi} (R\sin\varphi\sin\theta) + \frac{\partial u_2}{\partial \varphi} \cdot \frac{\partial \theta}{\partial \varphi} + \frac{\partial u_2}{\partial \theta} \cdot \frac{\partial}{\partial \varphi} (R\cos\varphi)$$

$$= \frac{\partial u_2}{\partial r} \left(\frac{\partial R}{\partial \varphi} \sin\varphi + R\cos\varphi \right) + \frac{\partial u_2}{\partial \theta} \cdot \frac{\partial \theta}{\partial \varphi} + \frac{\partial u_2}{\partial \varphi} \cdot \left(\frac{\partial R}{\partial \varphi} \cos\varphi - \sin\varphi \cos\varphi \right)$$

$$\Rightarrow \frac{\partial u_2}{\partial \varphi} = \frac{\partial u_2}{\partial r} \left(\frac{\partial R}{\partial \varphi} \cdot \sin\varphi + R\cos\varphi \right) + \frac{\partial u_2}{\partial \theta} \cdot \frac{\partial \varphi}{\partial \varphi} + \frac{\partial u_2}{\partial \varphi} \left(\frac{\partial R}{\partial \varphi} \cos\varphi - \sin\varphi \cos\varphi \right)$$