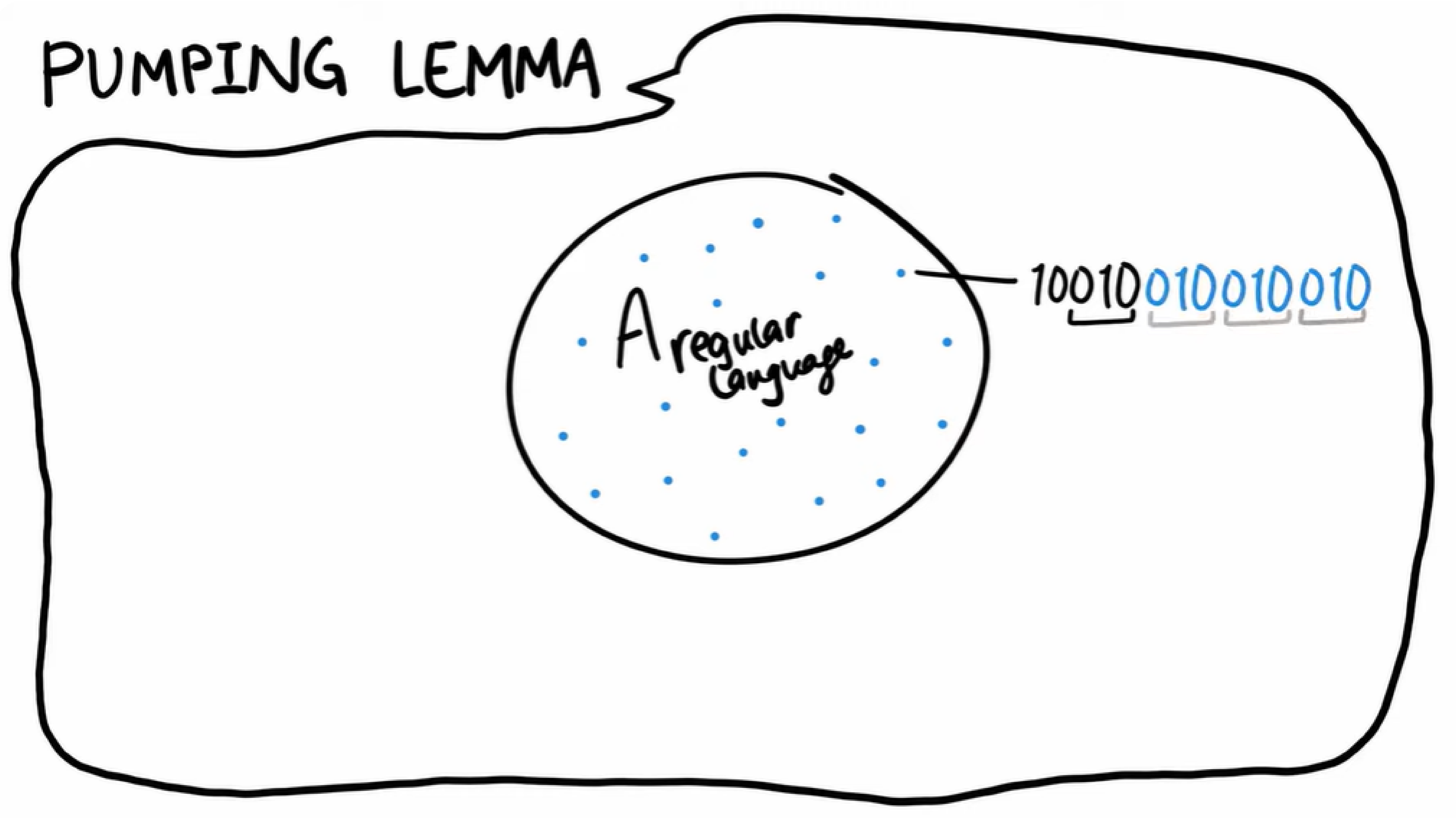
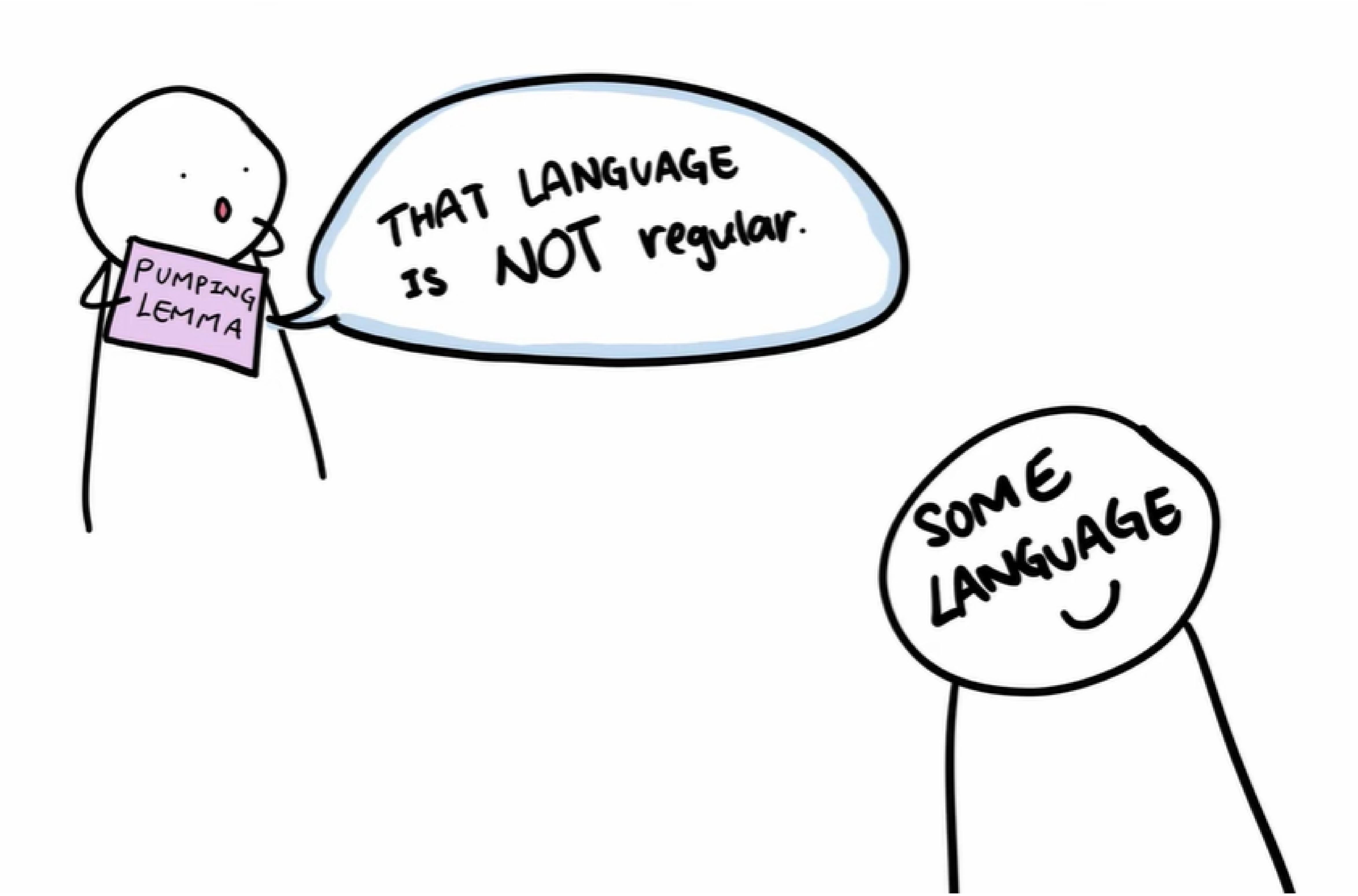
Coretan Nadiyya

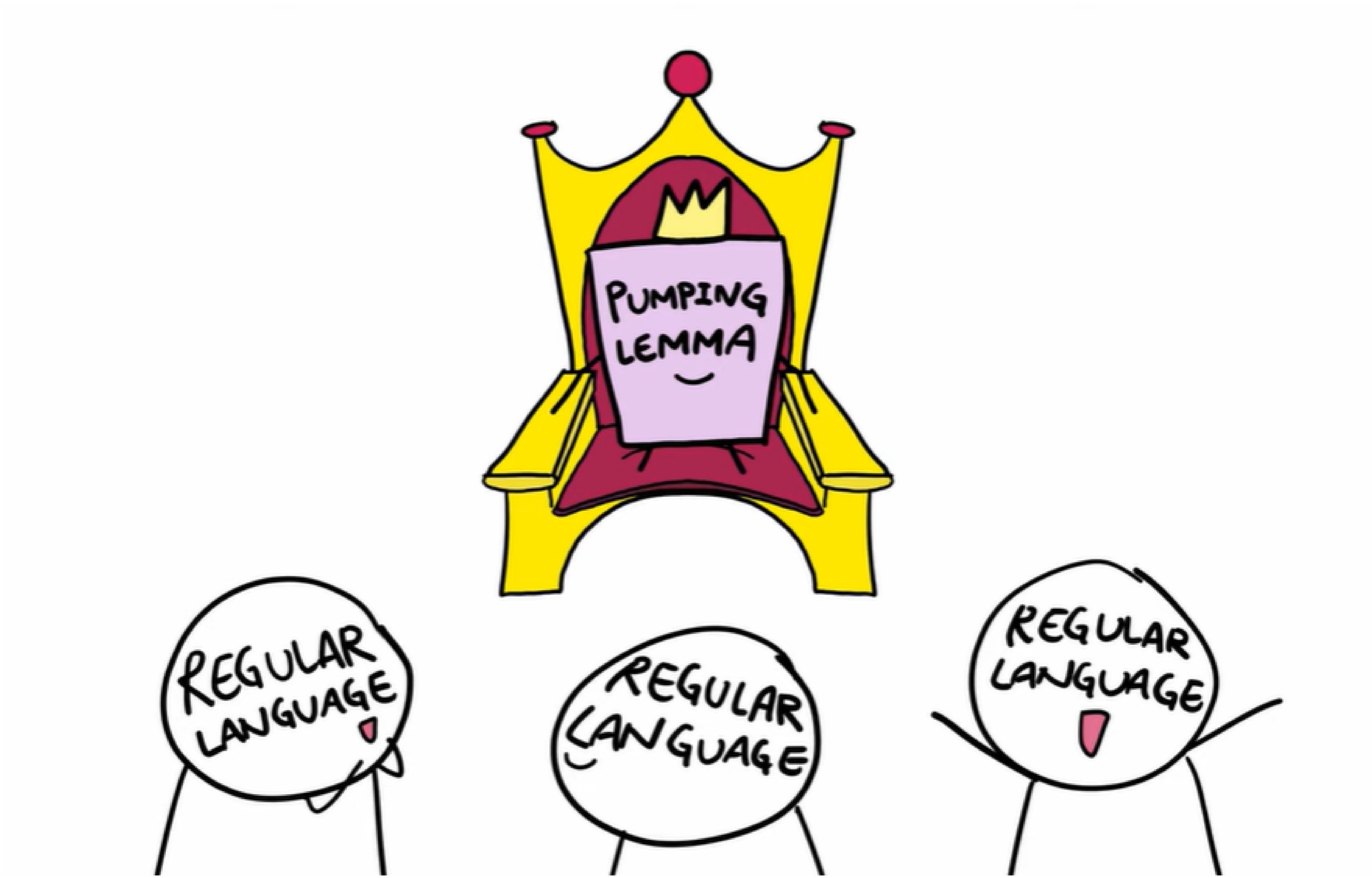
PUMPING LEMMA

Teori Bahasa Otomata

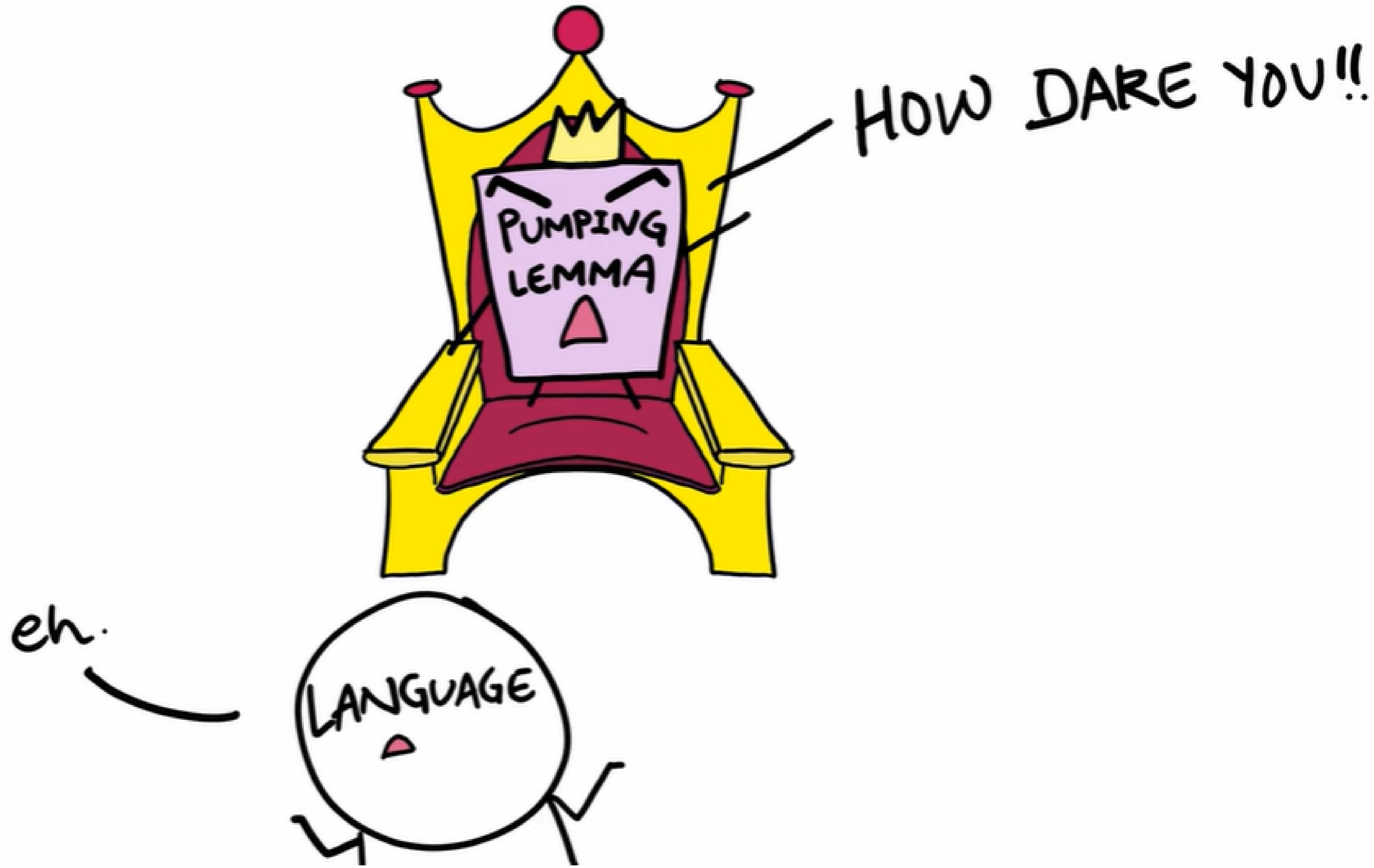


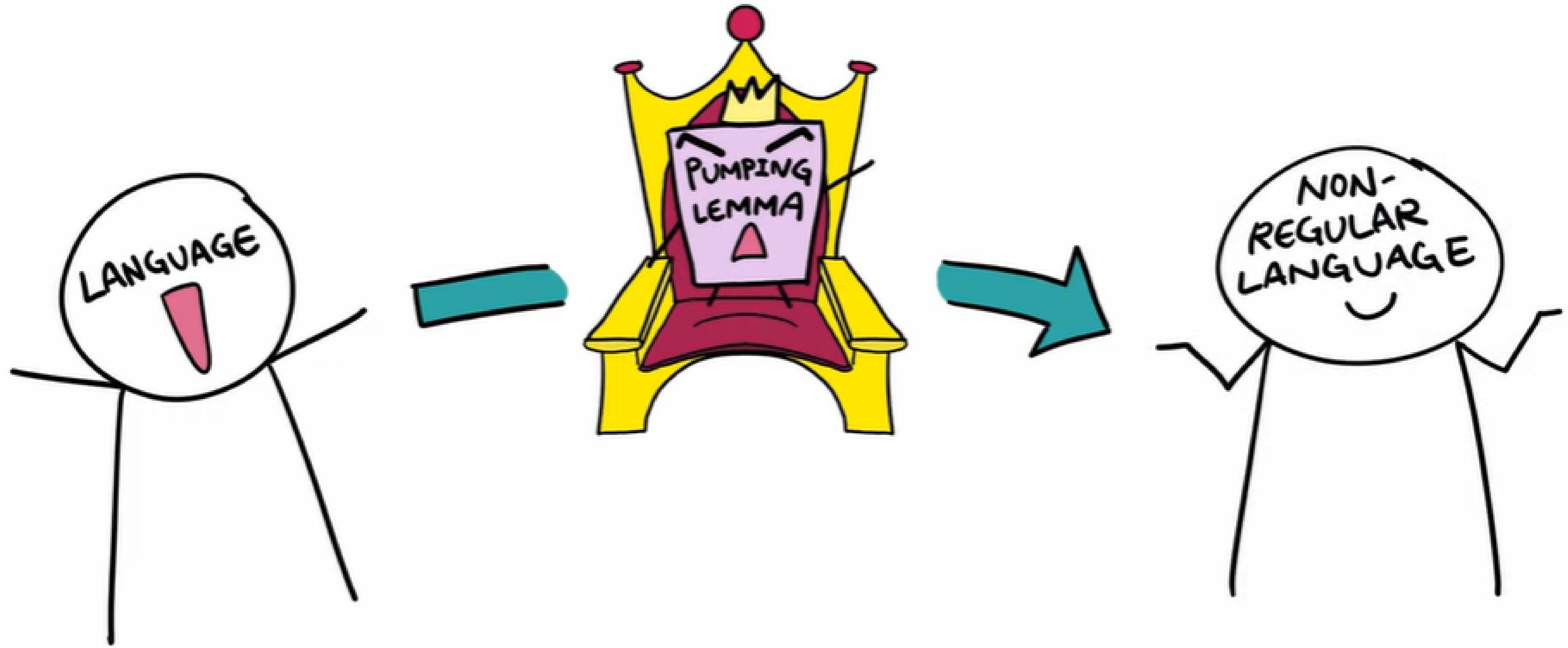
Pumping lemma menyatakan bahwa jika suatu bahasa Regular maka setiap string dalam bahasa tersebut akan memiliki bagian yang dapat diulang atau dipompa beberapa kali dan tetap berada dalam bahasa tersebut


PUMPING LEMMA
THAT LANGUAGE IS NOT regular.
SOME LANGUAGE



Memuaskan/memenuhi Pumping Lemma = Regular Language


HOW DARE YOU!!
PUMPING LEMMA
eh.
LANGUAGE



Tidak memuaskan/memenuhi Pumping Lemma = Non Regular Language

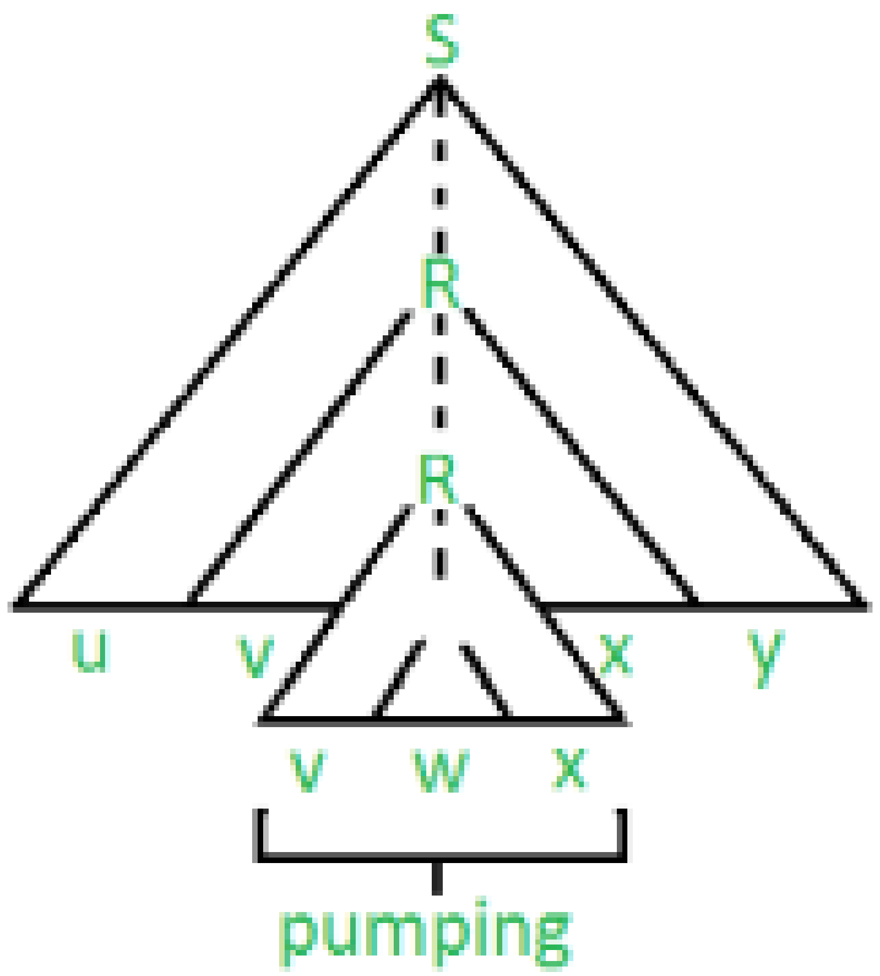
PROOF BY CONTRADICTION

Fungsi Pumping Lemma

Membuktikan suatu bahasa merupakan RL.

Membuktikan suatu bahasa merupakan CFL.

Pumping Lemma for CFL



Pumping Lemma untuk CFL menyatakan bahwa untuk Bahasa Bebas Konteks L apa pun, dimungkinkan untuk menemukan dua substring yang dapat 'dipompa' berapa kali dan tetap berada dalam bahasa yang sama. Untuk bahasa L apa pun, membagi stringnya menjadi lima bagian dan memompa substring kedua dan keempat. Pumping Lemma di sini juga digunakan sebagai alat untuk membuktikan bahwa suatu bahasa bukanlah CFL.

Langkah-langkah menerapkan lemma pemompaan

- Asumsikan bahwa L bebas konteks.
- Panjang pemompaan adalah n.
- Semua string yang lebih panjang dari panjang(n) dapat dipompa-> |w|>=n.
- Sekarang kita harus mencari string 'w' di L sehingga |w|>=n.
- Kami akan membagi string 'w' menjadi uvxyz.
- Sekarang tunjukkan bahwa $uv^ixy^iz \notin L$ untuk beberapa konstanta i.
- Kemudian, kita harus mempertimbangkan cara membagi w menjadi uvxyz.
- Tunjukkan bahwa tidak ada satupun yang dapat memenuhi ketiga kondisi pemompaan secara bersamaan.
- String 'w' tidak dapat dipompa (kontradiksi).

Jika L adalah CFL, terdapat bilangan bulat n, sehingga untuk semua $x \in L$ dengan $|x| \geq n$, terdapat $u, v, w, x, y \in \Sigma^*$, sehingga $x = uvwxy$, dan

- (1) $|vwx| \leq n$
- (2) $|vx| \geq 1$
- (3) untuk semua $i \geq 0$: $uv^iwx^iy \in L$

Pumping Lemma for Context-Free Languages

- **Lemma:**

  Let G = (V, T, P, S) be a CFG in CNF, and let $n = 2^{|V|}$. If z is a string in L(G) and $|z| >= n$, then there exist strings u, v, w, x and y in T* such that z=uvwxy and:

  - $|vx| >= 1$ (i.e., $|v| + |x| >= 1$)
  - $|vwx| <= n$
  - uv^iwx^iy is in L(G), for all $i >= 0$

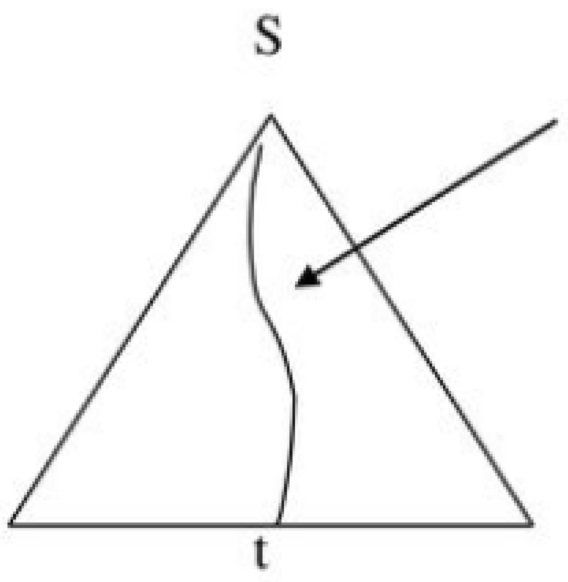
- **Proof:**

  Let G = (V, T, P, S) be a CFG in CNF, let $n = 2^{|V|}$, and let z be a string in L(G) where $|z| >= n$.

  Since $|z| >= n = 2^k$, it follows from the corollary that any derivation tree for z has height at least k+1.

  By definition such a tree contains a path of length at least k+1.

  Consider the longest such path in the tree:



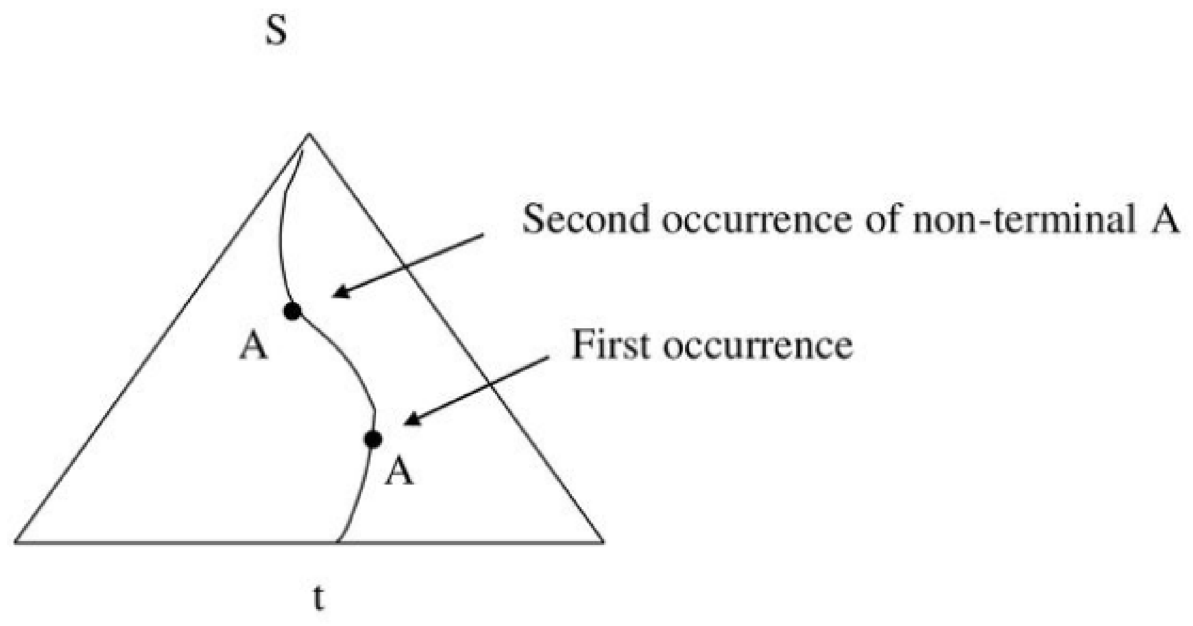
yield of T is z

Such a path has:

- Length >= k+1 (i.e., number of edges in the path is >= k+1)
- At least k+2 nodes
- 1 terminal
- At least k+1 non-terminals

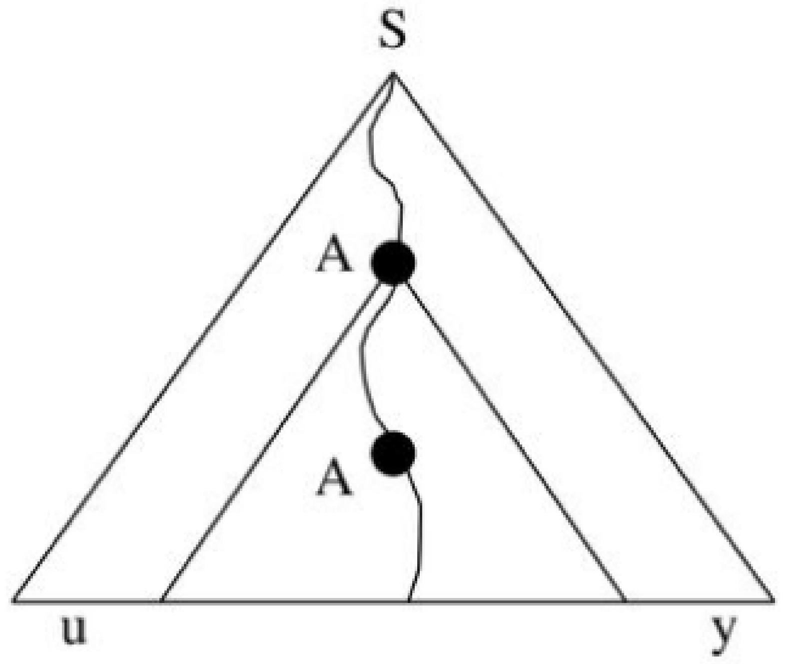
- Since there are only k non-terminals in the grammar, and since k+1 appear on this path, it follows that some non-terminal (perhaps many) appears at least twice on this path.

- Consider the first non-terminal that is repeated, when traversing the path from the leaf to the root.

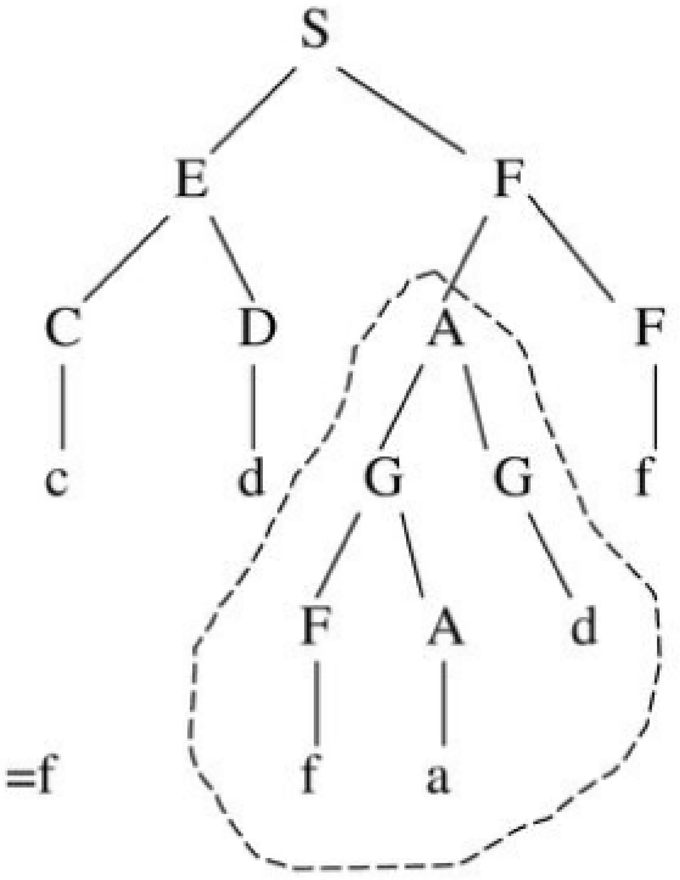


This path, and the non-terminal A will be used to break up the string z.

- **Generic Description:**

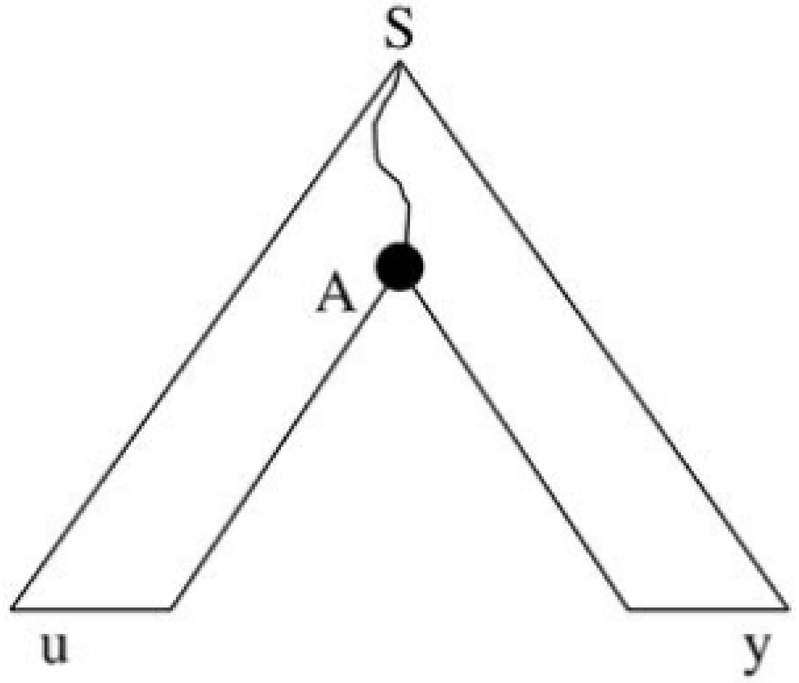


- **Example:**



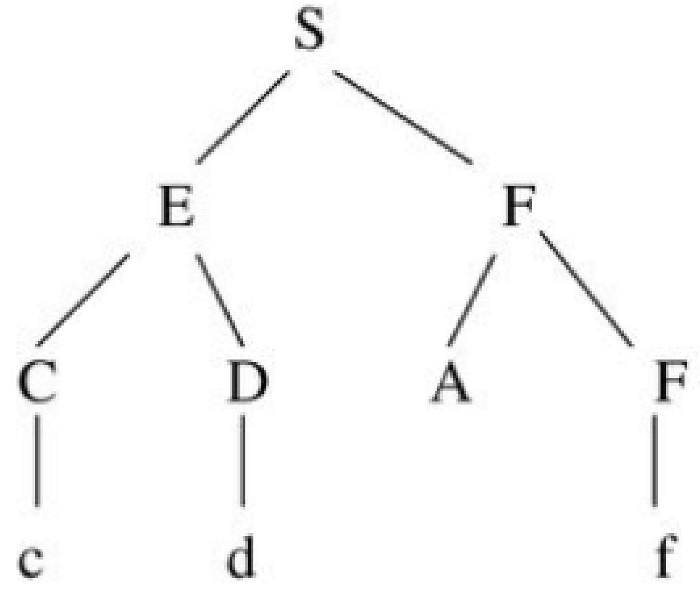
In this case u = cd and y =f

- **Cut out the subtree rooted at A:**



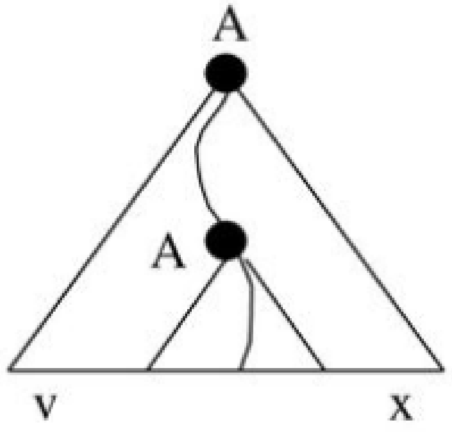
$$S \Rightarrow^* uAy \qquad (1)$$

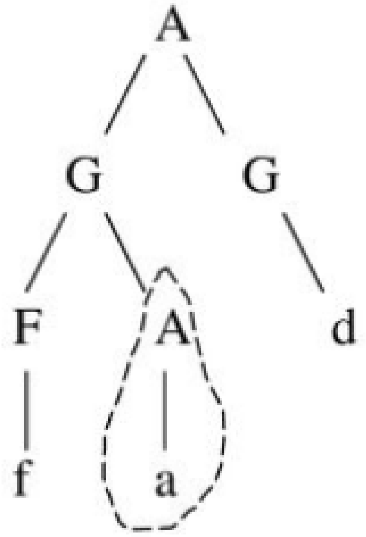
- **Example:**



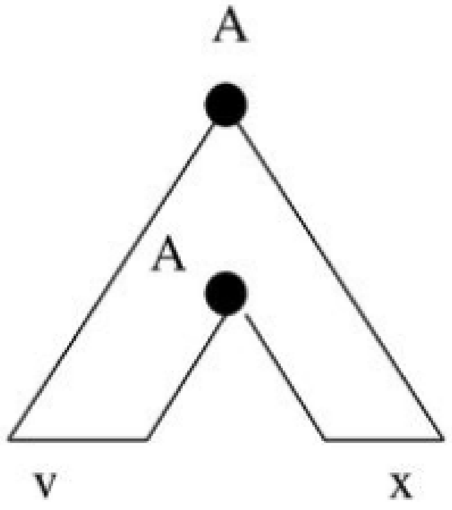
$$S \Rightarrow^* cdAf$$

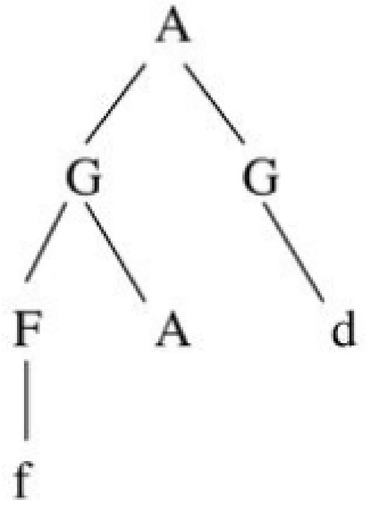
- **Consider the subtree rooted at A:**





- **Cut out the subtree rooted at the first occurrence of A:**

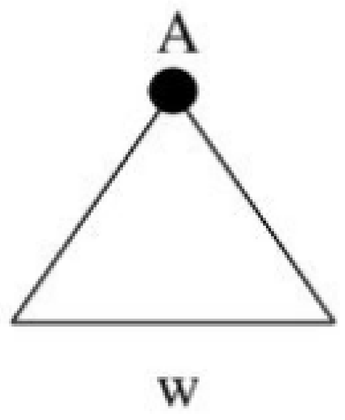


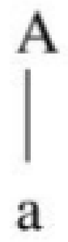


A =>* vAx (2)

A =>* fAd

- **Consider the smallest subtree rooted at A:**





A =>* w (3)

A =>* a

- **Collectively (1), (2) and (3) give us:**

S =>* uAy	(1)
=>* uvAxy	(2)
=>* uvwxy	(3)
=>* z	since z=uvwxy

- **In addition, (2) also tells us:**

$$
\begin{array}{ll}
S \Rightarrow^* uAy & (1) \\
\quad \Rightarrow^* uvAxy & (2) \\
\quad \Rightarrow^* uv^2Ax^2y & (2) \\
\quad \Rightarrow^* uv^2wx^2y & (3)
\end{array}
$$

- **More generally:**

$$S \Rightarrow^* uv^iwx^iy \qquad \text{for all } i>=1$$

- **And also:**

$$
\begin{array}{ll}
S \Rightarrow^* uAy & (1) \\
\quad \Rightarrow^* uwy & (3)
\end{array}
$$

- **Hence:**

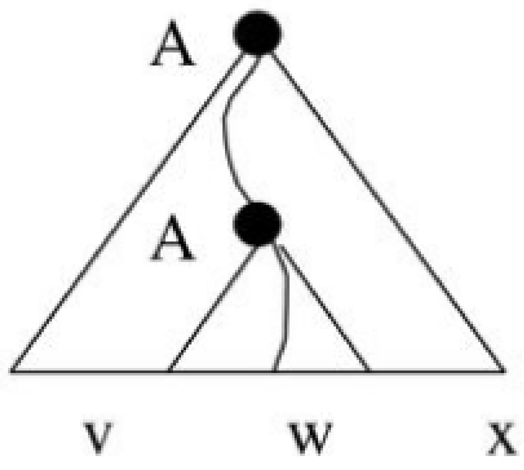
$$S \Rightarrow^* uv^iwx^iy \qquad \text{for all } i>=0$$

- **Consider the statement of the Pumping Lemma:**

–*What is n?*

$n = 2^k$, where k is the number of non-terminals in the grammar.

–*Why is $|v| + |x| >= 1$?*



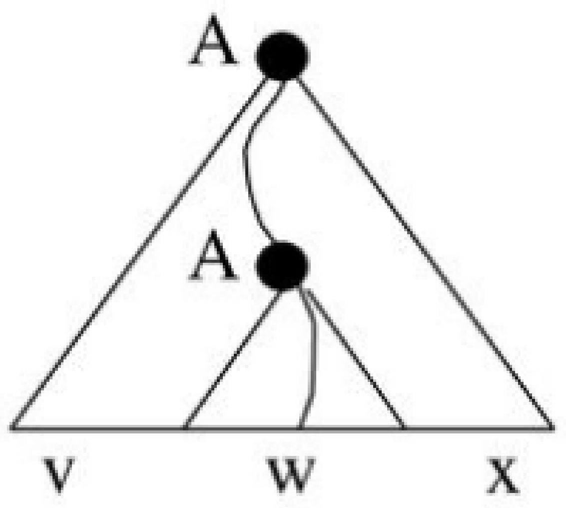
Since the height of this subtree is >=2, the first production is $A \to V_1V_2$. Since no non-terminal derives the empty string (in CNF), either V_1 or V_2 must derive a non-empty v or x. More specifically, if w is generated by V_1, then x contains at least one symbol, and if w is generated by V_2, then v contains at least one symbol.

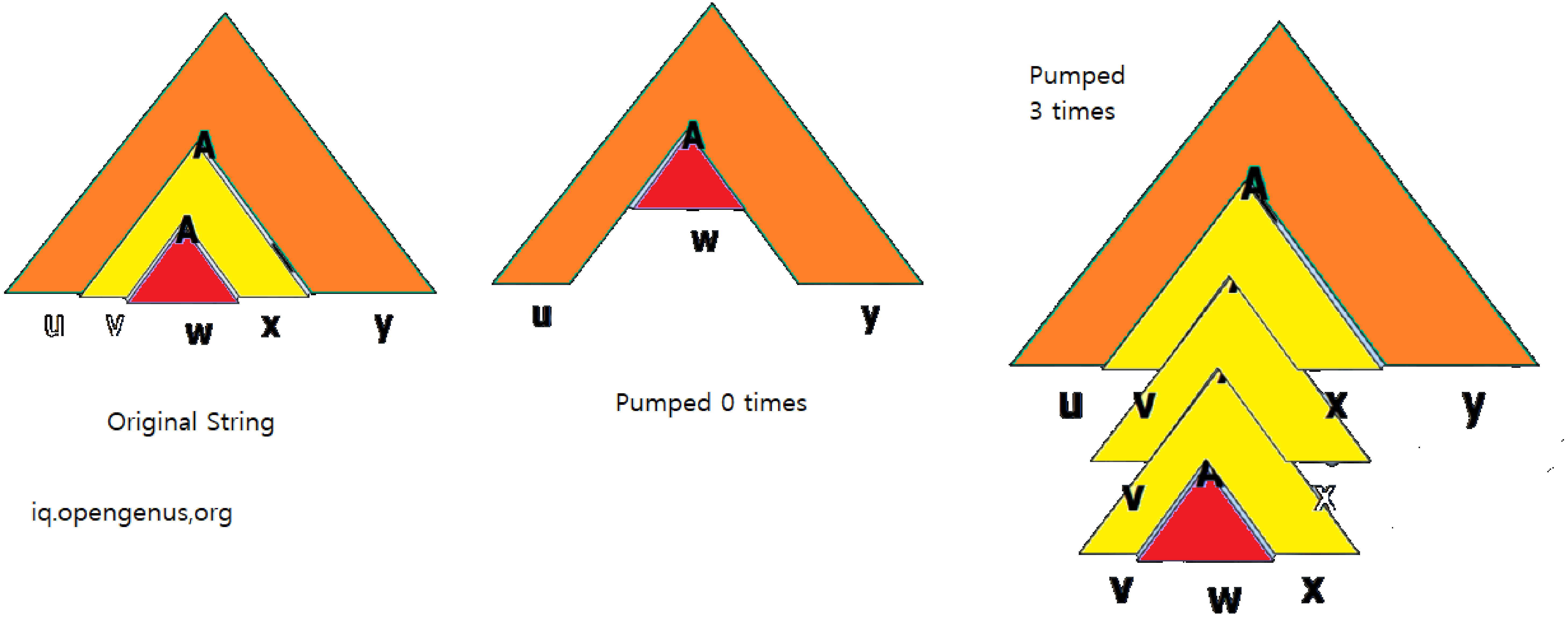
–Why is $|vwx| <= n$*?*

Observations:

- The repeated variable was the <u>first</u> repeated variable on the path from the bottom, and therefore (by the pigeon-hole principle) the path from the leaf to the second occurrence of the non-terminal has length <u>*at most*</u> k+1.

- Since the path was the largest in the entire tree, this path is the longest in the subtree rooted at the second occurrence of the non-terminal. Therefore the subtree has height <=k+1. From the lemma, the yield of the subtree has length $<=2^k=n$.•





Original String

Pumped 0 times

Pumped 3 times

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