# Team Notebook

# $NSU\_NoAC$

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Contents				1.3.5	DSU [MB]	5		1.5.2	Extended GCD [NK]	13
				1.3.6	DSU [NK]	5		1.5.3	Fraction-Functions [SK]	14
1  src		$^2$		1.3.7	LCA [MB]	6		1.5.4	Fraction[MB]	14
1.1				1.3.8	Lazy Segment Tree [MB]	6			Miller-Rabin-for-prime-checking [SK] .	
	1.1.1 C++ Include GNU PBDS [NK]			1.3.9	Lazy Segment Tree [SK]	7			Modular Binary Exponentiation	
	1.1.2 C++ Starter [MB]				Mos Algorithm [MB]				(Power) [NK]	
	1.1.3 C++ Starter [NK] $\dots$				SCC, Condens Graph [NK]			1.5.7	Modular Int [MB]	
	1.1.4 C++ Starter [SK] $\dots$			1.3.12	Segment Tree [SK]	9			Modular inverse [NK]	
	1.1.5 $C++$ Starter debug[MB]			1.3.13	Segment Tree[MB]	10			Prime Phi Sieve [MB]	
	1.1.6 Unordered Map [MB]			1.3.14	$SparseTable[MB] \dots \dots$	10				
1.2				1.3.15	Treap[MB]	11			Prime Sieve [MB]	
	1.2.1 Power Set [NK]	3	1.4	Graph	1	11			nCrp-O(1) [SK]	
1.3	Data Structures	3		1.4.1	Edge Remove CC [MB]	11	1.6	_		
	1.3.1 Articulation Points in $O(N + M)$ [NK]	3		1.4.2	Kruskal's [NK]	12		1.6.1	Hashing [MB]	17
	1.3.2 BIT [MB]	3		1.4.3	Tree Rooting [MB]	12		1.6.2	Hashing [NK]	18
	1.3.3 Bridges Online [NK]	4	1.5	Math		13		1.6.3	Hashing [SK]	19
	1.3.4 Bridges in $O(N + M)$ [NK]	5		1.5.1	Combinatrics [MB]	13		1.6.4	Z-Function [MB]	20

#### src

#### 1.1 -Starters-

### 1.1.1 C++ Include GNU PBDS [NK]

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
namespace pbds = __gnu_pbds;
template <class T>
using ordered_set = pbds::tree<T, pbds::null_type, std::less</pre>
                           pbds::rb tree tag.
                           pbds::
                                tree_order_statistics_node_update.3 C++ Starter [NK]
template <class K, class V>
using hash_map = pbds::gp_hash_table<K, V>;
```

#### 1.1.2 C++ Starter [MB]

```
#if defined LOCAL && !defined ONLINE JUDGE
#include "debug.cpp"
#else
#include <bits/stdc++.h>
using namespace std;
#define dbg(...);
#endif
typedef long long 11;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;
#define mem(x, n) memset(x, n, sizeof(x))
#define all(x) x.begin(), x.end()
#define sz(x) ((int)(x).size())
#define vec vector
inline bool read(auto &...a) { return (((cin >> a) ? true :
    false) && ...): }
inline void print(const auto &...a) { ((cout << a), ...); }</pre>
inline void println(const auto &...a) { print(a..., '\n'): }
void run_case([[maybe_unused]] const int &TC)
```

```
int main()
ios_base::sync_with_stdio(false), cin.tie(0);
int tt = 1;
read(tt):
for (int tc = 1; tc <= tt; tc++)</pre>
 run case(tc):
return 0;
```

```
#include <bits/stdc++.h>
using namespace std;
constexpr double eps = 1e-9;
constexpr int inf = 1 << 30;</pre>
constexpr int mod = 1e9 + 7;
constexpr int nmax = 1e6;
void runcase(int casen) {
   // cout << "Case " << casen << ": " << '\n':
int main() {
   ios_base::sync_with_stdio(false);
   cin.tie(nullptr);
   int ncases = 1:
   cin >> ncases: // Comment out for single-case tests
   for (int casen = 1; casen <= ncases; ++casen) {</pre>
       runcase(casen):
   return 0:
```

### 1.1.4 C++ Starter [SK]

```
#include<bits/stdc++.h>
using namespace std;
```

```
typedef long long 11;
typedef unsigned long long ull;
#define endl "\n"
#define pi 3.142
const double eps = 1e-10;
int dx[] = \{1.0, -1.0\}:
int dv[] = \{0,1,0,-1\};
const 11 M = (11)(1e9) + 7;
const ll inf = (ll)1e17:
const int N = (11)(1e6 + 10):
int main()
   cin.tie(0):
   cout.tie(0):
   ios_base::sync_with_stdio(false);
   //freopen("two.in", "r", stdin);
   //freopen("out.txt", "w", stdout);
```

### 1.1.5 C++ Starter debug[MB]

```
#include <bits/stdc++.h>
using namespace std:
template <typename T, typename C = typename T::value_type>
typename enable_if<!is_same<T, string>::value, ostream &>::
    type operator << (ostream &out, const T &c)
for (auto it = c.begin(); it != c.end(); it++)
out << (it == c.begin() ? "{" : ",") << *it;
return out << (c.emptv() ? "{" : "") << "}";</pre>
template <typename T, typename S>
ostream &operator << (ostream &out, const pair <T, S> &p)
```

```
{
  return out << "{" << p.first << ", " << p.second << "}";
}

#define dbg(...) _dbg_print(#__VA_ARGS__, __VA_ARGS__);

template <typename Arg1>
void _dbg_print(const char *name, Arg1 &&arg1)
{
  if (name[0] == ' ')
    name++;
  cout << "[" << name << ": " << arg1 << "]"
    << "\n";
}

template <typename Arg1, typename... Args>
void _dbg_print(const char *names, Arg1 &&arg1, Args &&...
    args)
{
  const char *comma = strchr(names + 1, ',');
  cout << "[";
  cout.write(names, comma - names) << ": " << arg1 << "] ";
  _dbg_print(comma + 1, args...);
}</pre>
```

### 1.1.6 Unordered Map [MB]

```
#include <bits/stdc++.h>

// For gp_hash_table
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;

struct custom_hash
{
    static uint64_t splitmix64(uint64_t x)
    {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
}
size_t operator()(uint64_t x) const
{
```

#### 1.2 Brute-force

#### 1.2.1 Power Set [NK]

```
template <class T>
vector<vector<T>> power_set(const vector<T>& vec) {
    vector<vector<T>> res;
    list<T> buf;
    function<void(int)> recurse = [&](int i) -> void {
        if (i == vec.size()) {
            res.emplace_back(buf.begin(), buf.end());
            return;
        }
        recurse(i + 1);
        buf.push_back(vec[i]), recurse(i + 1), buf.pop_back()
        ;
    };
    recurse(0);
    return res;
}
```

#### 1.3 Data Structures

### 1.3.1 Articulation Points in O(N + M) [NK]

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited;
vector<int> tin, low;
int timer;

void dfs(int v, int p = -1) {
   visited[v] = true;
   tin[v] = low[v] = timer++;
```

```
int children=0:
   for (int to : adj[v]) {
       if (to == p) continue;
       if (visited[to]) {
          low[v] = min(low[v], tin[to]);
      } else {
          dfs(to, v);
          low[v] = min(low[v], low[to]);
          if (low[to] >= tin[v] && p!=-1)
              IS_CUTPOINT(v);
           ++children:
   if(p == -1 && children > 1)
       IS CUTPOINT(v):
void find_cutpoints() {
   timer = 0:
   visited.assign(n, false);
   tin.assign(n, -1);
   low.assign(n, -1);
   for (int i = 0; i < n; ++i) {</pre>
       if (!visited[i])
          dfs (i);
```

#### 1.3.2 BIT [MB]

```
struct BIT
{
private:
    std::vector<long long> mArray;

public:
    BIT(int sz) // Max size of the array
{
    mArray.resize(sz + 1, 0);
}

void build(const std::vector<long long> &list)
{
    for (int i = 1; i <= list.size(); i++)
    {
        mArray[i] = list[i];
    }

for (int ind = 1; ind <= mArray.size(); ind++)</pre>
```

```
int ind2 = ind + (ind & -ind);
  if (ind2 <= mArray.size())</pre>
   mArray[ind2] += mArray[ind];
}
 long long prefix_query(int ind)
 int res = 0:
 for (; ind > 0; ind -= (ind & -ind))
  res += mArray[ind];
 return res:
 long long range_query(int from, int to)
 return prefix_query(to) - prefix_query(from - 1);
 void add(int ind, long long add)
 for (; ind < mArray.size(); ind += (ind & -ind))</pre>
  mArray[ind] += add;
}
};
```

### 1.3.3 Bridges Online [NK]

```
vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges;
int lca_iteration;
vector<int> last_visit;

void init(int n) {
   par.resize(n);
   dsu_2ecc.resize(n);
   dsu_cc_resize(n);
   dsu_cc_size.resize(n);
   lca_iteration = 0;
   last_visit.assign(n, 0);
   for (int i=0; i<n; ++i) {
      dsu_2ecc[i] = i;
   }
}</pre>
```

```
dsu cc[i] = i:
      dsu_cc_size[i] = 1;
      par[i] = -1;
   bridges = 0;
int find_2ecc(int v) {
   if (v == -1)
      return -1;
   return dsu 2ecc[v] == v ? v : dsu 2ecc[v] = find 2ecc(
        dsu 2ecc[v]):
int find cc(int v) {
   v = find_2ecc(v);
   return dsu cc[v] == v ? v : dsu cc[v] = find cc(dsu cc[v
       1):
void make_root(int v) {
   v = find 2ecc(v):
   int root = v;
   int child = -1:
   while (v != -1) {
      int p = find_2ecc(par[v]);
      par[v] = child;
      dsu cc[v] = root:
      child = v;
      v = p;
   dsu_cc_size[root] = dsu_cc_size[child];
void merge_path (int a, int b) {
   ++lca iteration:
   vector<int> path_a, path_b;
   int lca = -1;
   while (lca == -1) {
      if (a != -1) {
          a = find 2ecc(a):
          path_a.push_back(a);
          if (last visit[a] == lca iteration){
             lca = a;
              break;
          last_visit[a] = lca_iteration;
          a = par[a];
      if (b != -1) {
```

```
b = find 2ecc(b):
          path_b.push_back(b);
          if (last_visit[b] == lca_iteration){
              lca = b:
              break;
          last_visit[b] = lca_iteration;
          b = par[b];
   for (int v : path_a) {
       dsu_2ecc[v] = 1ca;
       if (v == 1ca)
          break;
       --bridges;
   for (int v : path_b) {
       dsu 2ecc[v] = 1ca:
       if (v == lca)
          break:
       --bridges;
void add_edge(int a, int b) {
   a = find 2ecc(a):
   b = find_2ecc(b);
   if (a == b)
       return;
   int ca = find cc(a):
   int cb = find_cc(b);
   if (ca != cb) {
       ++bridges:
       if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
          swap(a, b);
          swap(ca, cb);
       make_root(a);
       par[a] = dsu_cc[a] = b;
       dsu_cc_size[cb] += dsu_cc_size[a];
   } else {
       merge_path(a, b);
```

### 1.3.4 Bridges in O(N + M) [NK]

```
int n; // number of nodes
vector<vector<int>> adj; // adjacency list of graph
vector<bool> visited:
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
   visited[v] = true;
   tin[v] = low[v] = timer++:
   for (int to : adj[v]) {
       if (to == p) continue;
       if (visited[to]) {
          low[v] = min(low[v], tin[to]);
       } else {
          dfs(to, v);
          low[v] = min(low[v], low[to]);
          if (low[to] > tin[v])
              IS_BRIDGE(v, to);
       }
void find_bridges() {
   timer = 0;
   visited.assign(n, false);
   tin.assign(n, -1);
   low.assign(n, -1);
   for (int i = 0; i < n; ++i) {</pre>
       if (!visited[i])
          dfs(i);
}
```

### 1.3.5 DSU [MB]

```
#include <bits/stdc++.h>
// O based
class DSU
{
   std::vector<int> p, csz;

public:
   DSU() {}
   // Max size
   DSU(int dsz)
```

```
//Default empty
p.resize(dsz + 5, 0), csz.resize(dsz + 5, 0);
init(dsz);
void init(int n)
// n = size
for (int i = 0: i <= n: i++)
 p[i] = i, csz[i] = 1;
//Return parent Recursively
int get(int x)
if (p[x] != x)
 p[x] = get(p[x]);
return p[x];
// Return Size
int get_comp_size(int component) { return csz[get(component
// Return if Union created Successfully or false if they
    are already in Union
bool merge(int x, int y)
x = get(x), y = get(y);
if (x == y)
 return false;
 if (csz[x] > csz[y])
 std::swap(x, y);
 p[x] = y;
 csz[v] += csz[x]:
return true:
```

### 1.3.6 DSU [NK]

```
struct DSU {
```

```
int n nodes = 0:
   int n_components = 0;
   vector<int> component_size;
   vector<int> component_root;
   DSU(int n nodes, bool make all nodes = false)
       : n_nodes(n_nodes),
         component_root(n_nodes, -1),
         component_size(n_nodes, 0) {
       if (make_all_nodes) {
          for (int i = 0: i < n nodes: ++i) {</pre>
              make node(i):
          }
       }
   }
   void make node(int v) {
       if (component_root[v] == -1) {
          component root[v] = v:
          component_size[v] = 1;
           ++n_components;
       }
   }
   int root(int v) {
       auto res = v:
       while (component_root[res] != res) {
          res = component_root[res];
       }
       while (v != res) {
           auto u = component_root[v];
          component_root[v] = res;
          v = u:
       }
       return res;
   int connect(int u, int v) {
       u = root(u), v = root(v);
       if (u == v) return u:
       if (component_size[u] < component_size[v]) {</pre>
          swap(u, v);
       }
       component_root[v] = u;
       component_size[u] += component_size[v];
       --n_components;
};
```

5

#### 1.3.7 LCA [MB]

```
struct LCA
private:
int n, lg;
std::vector<int> depth;
std::vector<std::vector<int>> up;
std::vector<std::vector<int>> g;
public:
LCA(): n(0), lg(0) {}
LCA(int n)
 this \rightarrow n = n:
 lg = (int)log2(n) + 2;
 depth.resize(n + 5, 0);
 up.resize(n + 5, std::vector<int>(lg, 0));
 g.resize(n + 1);
LCA(std::vector<std::vector<int>> &graph) : LCA((int)graph.
     size())
 for (int i = 0; i < (int)graph.size(); i++)</pre>
  g[i] = graph[i];
 dfs(1, 0);
void dfs(int curr, int p)
 up[curr][0] = p;
 for (int next : g[curr])
  if (next == p)
   continue:
  depth[next] = depth[curr] + 1;
  up[next][0] = curr;
  for (int j = 1; j < lg; j++)</pre>
   up[next][j] = up[up[next][j - 1]][j - 1];
  dfs(next, curr):
void clear_v(int a)
 g[a].clear();
```

```
void clear(int n_{-} = -1)
if (n == -1)
 n_{-} = ((int)(g.size())) - 1;
 for (int i = 0; i <= n_; i++)</pre>
 g[i].clear();
void add(int a, int b)
g[a].push_back(b);
int par(int a)
return up[a][0];
int get_lca(int a, int b)
if (depth[a] < depth[b])</pre>
 std::swap(a, b);
 int k = depth[a] - depth[b];
 for (int j = lg - 1; j >= 0; j--)
 if (k & (1 << j))
  a = up[a][j];
if (a == b)
 return a:
 for (int j = lg - 1; j >= 0; j--)
 if (up[a][i] != up[b][i])
  a = up[a][j];
  b = up[b][j];
return up[a][0];
int get_dist(int a, int b)
return depth[a] + depth[b] - 2 * depth[get_lca(a, b)];
```

```
1.3.8 Lazy Segment Tree [MB]
```

**}**;

```
template <typename T, typename F, T(*op)(T, T), F(*
    lazy_to_lazy)(F, F), T(*lazy_to_seg)(T, F, int, int)>
struct LazySegTree
private:
std::vector<T> segt;
std::vector<F> lazy;
int n:
T neutral;
F lazvE:
int left(int si) { return si * 2; }
int right(int si) { return si * 2 + 1; }
int midpoint(int ss, int se) { return (ss + (se - ss) / 2);
T query(int ss, int se, int si, int qs, int qe)
 // **** //
 if (lazy[si] != lazyE)
 F curr = lazv[si];
  lazy[si] = lazyE;
  segt[si] = lazy_to_seg(segt[si], curr, ss, se);
  if (ss != se)
   lazy[left(si)] = lazy_to_lazy(lazy[left(si)], curr);
   lazy[right(si)] = lazy_to_lazy(lazy[right(si)], curr);
 if (se < qs || qe < ss)</pre>
 return neutral;
 if (qs <= ss && qe >= se)
 return segt[si]:
 int mid = midpoint(ss, se);
 return op(query(ss, mid, left(si), qs, qe), query(mid + 1,
       se, right(si), qs, qe));
void update(int ss, int se, int si, int qs, int qe, F val)
 // **** //
 if (lazv[si] != lazvE)
  F curr = lazy[si];
  lazv[si] = lazvE;
```

```
segt[si] = lazy_to_seg(segt[si], curr, ss, se);
  if (ss != se)
  {
   lazy[left(si)] = lazy_to_lazy(lazy[left(si)], curr);
   lazy[right(si)] = lazy_to_lazy(lazy[right(si)], curr);
 if (se < qs || qe < ss)
  return:
 if (qs <= ss && qe >= se)
  // **** //
  segt[si] = lazy_to_seg(segt[si], val, ss, se);
  if (ss != se)
   lazy[left(si)] = lazy_to_lazy(lazy[left(si)], val);
   lazy[right(si)] = lazy_to_lazy[right(si)], val);
  }
  return:
 int mid = midpoint(ss, se);
 update(mid + 1, se, si * 2 + 1, qs, qe, val);
 update(ss, mid, left(si), qs, qe, val);
 segt[si] = op(segt[left(si)], segt[right(si)]);
void build(const std::vector<T> &a. int si. int ss. int se)
 if (ss == se)
  segt[si] = a[ss];
  return:
 int mid = midpoint(ss, se);
 build(a, left(si), ss, mid);
 build(a, right(si), mid + 1, se);
 segt[si] = op(segt[left(si)], segt[right(si)]);
}
public:
LazvSegTree() : n(0) {}
LazySegTree(int sz, T ini, T _neutral, F _lazyE)
 this \rightarrow n = sz + 1:
 this->neutral = _neutral;
 this->lazyE = _lazyE;
 segt.resize(n * 4 + 5, ini);
 lazy.resize(n * 4 + 5, _lazyE);
```

### 1.3.9 Lazy Segment Tree [SK]

```
11 v[4*N];
11 add[4*N];
int arr[N];

void push(int cur)
{
   add[cur*2] += add[cur];
   add[cur*2 + 1] += add[cur];
   add[cur] = 0;
}

/*
void build(int cur,int l,int r)
{
   if(l==r)
   {
     v[cur] = arr[l];
}
```

```
return:
   int mid = 1 + (r-1)/2:
   build(cur*2.1.mid):
   build(cur*2 + 1,mid+1,r);
   v[cur] = v[cur*2] + v[cur*2 + 1]:
   return:
*/
11 query(int cur,int l,int r,int x,int y)
   if(x>r || y<1)</pre>
       return 0:
   if(1==r)
       return v[cur] + add[cur];
   if(l==x && r==v)
       return v[cur] + add[cur]*(r-l+1):
   int mid = 1 + (r-1)/2:
   v[cur] += add[cur]*(r-l+1):
   push(cur);
   11 left = query(cur*2,1,mid,x,min(mid,y));
   11 right = query(cur*2 + 1,mid+1,r,max(mid+1,x),y);
   11 \text{ res} = 0:
   res = left + right :
   return res:
void update(int cur,int l,int r,int s,int e,int val)
```

sort(queries.begin(), queries.end());

```
if(l==s && r==e)
{
    add[cur] += val;
    return;
}

if(s>r || e<1)
{
    return;
}

int mid = 1 + (r-1)/2;

push(cur);

update(cur*2,1,mid,s,min(e,mid),val);
 update(cur*2 + 1,mid+1,r,max(s,mid+1),e,val);

v[cur] = (v[cur*2] + add[cur*2]*(mid-1+1)) + (v[cur*2 + 1] + add[cur*2 + 1]*(r-mid));

return;</pre>
```

### 1.3.10 Mos Algorithm [MB]

```
#include <bits/stdc++.h>
using namespace std;
const int N = 3e4 + 5:
const int blck = sqrt(N) + 1;
struct Query
 int 1, r, i;
 bool operator<(const Query q) const</pre>
 if (this->1 / blck == q.1 / blck)
  return this->r < q.r;
 return this->1 / blck < q.1 / blck;</pre>
 }
};
vector<int> mos_alogorithm(vector<Query> &queries, vector<</pre>
     int> &a)
{
 vector<int> answers(queries.size());
```

```
int sza = 1e6 + 5:
vector<int> freq(sza);
int cnt = 0:
auto add = [&](int x) -> void
freq[x]++;
if (freq[x] == 1)
 cnt++:
};
auto remove = \lceil k \rceil (int x) -> void
freq[x]--;
if (freq[x] == 0)
 cnt--;
};
int 1 = 0:
int r = -1;
for (Query q : queries)
while (1 > q.1)
 1--:
 add(a[1]);
while (r < q.r)
 r++:
 add(a[r]);
while (1 < q.1)
 remove(a[1]);
 1++;
while (r > q.r)
 remove(a[r]);
answers[q.i] = cnt;
return answers;
```

```
int main()
int n;
cin >> n:
vector<int> a(n):
for (int i = 0; i < n; i++)</pre>
 cin >> a[i];
int q;
cin >> q;
vector<Query> qr(q);
for (int i = 0; i < q; i++)
 int 1, r;
 cin >> 1 >> r;
 1--. r--:
 qr[i].1 = 1, qr[i].r = r, qr[i].i = i;
vector<int> res = mos_alogorithm(qr, a);
for (int i = 0; i < q; i++)</pre>
 cout << res[i] << endl:</pre>
return 0;
```

### $1.3.11 \quad SCC, \ Condens \ Graph \ [NK]$

```
#include <bits/stdc++.h>
using namespace std;

vector<vector<int>> adj, adj_rev;
vector<bool> used;
vector<int> order, component;

void dfs1(int v) {
    used[v] = true;

    for (auto u : adj[v])
        if (!used[u])
            dfs1(u);

    order.push_back(v);
}
```

```
void dfs2(int v) {
    used[v] = true;
    component.push_back(v);
    for (auto u : adj_rev[v])
       if (!used[u])
           dfs2(u);
}
int main() {
    int n:
    // ... read n ...
    for (::) {
       int a, b;
       // ... read next directed edge (a,b) ...
       adj[a].push_back(b);
       adj_rev[b].push_back(a);
    used.assign(n, false);
    for (int i = 0; i < n; i++)</pre>
       if (!used[i])
           dfs1(i);
    used.assign(n. false):
    reverse(order.begin(), order.end());
    for (auto v : order)
       if (!used[v]) {
           dfs2(v);
           // ... processing next component ...
           component.clear();
       }
    vector<int> roots(n, 0):
    vector<int> root nodes:
    vector<vector<int>> adj_scc(n);
    for (auto v : order)
       if (!used[v]) {
           dfs2(v):
           int root = component.front();
           for (auto u : component) roots[u] = root;
           root_nodes.push_back(root);
```

```
component.clear();
}

for (int v = 0; v < n; v++)
    for (auto u : adj[v]) {
        int root_v = roots[v],
            root_u = roots[u];

        if (root_u != root_v)
            adj_scc[root_v].push_back(root_u);
     }
}</pre>
```

### 1.3.12 Segment Tree [SK]

```
pair<int,int>v[4*N];
int arr[N]:
void build(int cur,int l,int r)
   if(l==r)
       pair<int,int> tmp = {0,0};
       if(arr[1]==0)
          tmp.second++;
       else if(arr[1]<0)</pre>
          tmp.first++;
       v[cur] = tmp;
       return:
   int mid = 1 + (r-1)/2:
   build(cur*2.1.mid):
   build(cur*2 + 1,mid+1,r);
   v[cur].first = v[cur*2].first + v[cur*2 + 1].first:
   v[cur].second = v[cur*2].second + v[cur*2 + 1].second;
   return;
pair<int,int>query(int cur,int l,int r,int x,int y)
   if(l==x && r==y)
```

```
return v[cur];
   if(x>r || y<1)
       return {-1,-1};
   int mid = 1 + (r-1)/2:
   pair<int,int> left = query(cur*2,1,mid,x,min(mid,y));
   pair<int,int> right = query(cur*2 + 1,mid+1,r,max(mid+1,x)
   pair<int, int> res = {0,0};
   res.first = ((left.first!=-1)?left.first:0) + ((right.
        first!=-1)?right.first:0);
   res.second = ((left.second!=-1)?left.second:0) + ((right.
        second!=-1)?right.second:0);
   return res:
void update(int cur,int l,int r,int pos,int val)
   if(l==r)
       arr[1] = val:
       pair<int, int> tmp = {0,0};
       if(arr[1]==0)
           tmp.second++;
       else if(arr[1]<0)</pre>
           tmp.first++;
       v[cur] = tmp;
       return;
   int mid = 1 + (r-1)/2;
   if(pos<=mid)</pre>
       update(cur*2,1,mid,pos,val);
   }
   else
   {
       update(cur*2 + 1,mid+1,r,pos,val);
```

10

```
v[cur].first = v[cur*2].first + v[cur*2 + 1].first;
v[cur].second = v[cur*2].second + v[cur*2 + 1].second;
return;
}
```

### 1.3.13 Segment Tree[MB]

NSU

```
template <typename T, T(*op)(T, T)>
struct SegTree
private:
std::vector<T> segt;
int n;
Te;
int left(int si) { return si * 2: }
int right(int si) { return si * 2 + 1; }
int midpoint(int ss, int se) { return (ss + (se - ss) / 2);
T query(int ss, int se, int qs, int qe, int si)
 if (se < qs || qe < ss)
 return e;
 if (qs <= ss && qe >= se)
 return segt[si];
 int mid = midpoint(ss, se);
 return op(query(ss, mid, qs, qe, left(si)), query(mid + 1,
       se, qs, qe, right(si)));
void update(int ss. int se. int kev. int si. T val)
 if (ss == se)
  segt[si] = val;
  return:
 int mid = midpoint(ss. se);
 if (kev > mid)
  update(mid + 1, se, key, right(si), val);
  update(ss, mid, key, left(si), val);
 segt[si] = op(segt[left(si)], segt[right(si)]);
void build(const std::vector<T> &a, int si, int ss, int se)
 if (ss == se)
```

```
segt[si] = a[ss]:
  return;
 int mid = midpoint(ss, se);
 build(a, left(si), ss, mid);
 build(a, right(si), mid + 1, se);
 segt[si] = op(segt[left(si)], segt[right(si)]);
public:
SegTree() : n(0) {}
SegTree(int sz. T e)
 this->e = _e;
 this \rightarrow n = sz:
 segt.resize(n * 4 + 5, _e);
SegTree(const std::vector<T> &arr, T _e) : SegTree((int)arr
     .size(), _e) { init(arr); }
void init(const std::vector<T> &arr) { this->n = (int)(arr.
     size()):build(arr, 1, 0, n - 1): }
T get(int qs, int qe) { return query(0, n - 1, qs, qe, 1);
void set(int key, T val) { update(0, n - 1, key, 1, val); }
int op(int a, int b)
return min(a, b):
```

### 1.3.14 SparseTable[MB]

```
template <typename T, T (*op)(T, T)>
struct SparseTable
{
    private:
    std::vector<std::vector<T>> st;
    int n, lg;
    std::vector<int> logs;
    T e;

public:
    SparseTable() : n(0) {}

    SparseTable(int _n)
    {
        this->n = _n;
        int bit = 0;
        while ((1 << bit) <= n)</pre>
```

```
bit++:
 this->lg = bit;
 st.resize(n, std::vector<T>(lg));
 logs.resize(n + 1, 0);
 logs[1] = 0:
 for (int i = 2; i <= n; i++)
 logs[i] = logs[i / 2] + 1;
SparseTable(const std::vector<T> &a) : SparseTable((int)a.
     size())
 init(a);
 void init(const std::vector<T> &a)
 this->n = (int)a.size();
 for (int i = 0; i < n; i++)</pre>
 st[i][0] = a[i];
 for (int j = 1; j <= lg; j++)
  for (int i = 0; i + (1 << i) <= n; i++)
   st[i][j] = op(st[i][j-1], st[std::min(i + (1 << (j-1)
       ), n - 1)][j - 1]);
T get(int 1, int r)
 int j = logs[r - l + 1];
 return op(st[1][j], st[r - (1 << j) + 1][j]);</pre>
}:
int min(int a, int b)
return std::min(a, b);
```

#### 1.3.15 Treap[MB]

```
#include <bits/stdc++.h>
#define mem(x, n) memset(x, n, sizeof(x))
#define all(x) x.begin(), x.end()
#define endl "\n"
#include <ext/pb_ds/assoc_container.hpp> // Common file
// using namespace __gnu_pbds;
// https://codeforces.com/blog/entry/11080
//cout<<*X.find_by_order(4)<<endl; // 16
// cout<<(end(X)==X.find_by_order(6))<<endl; // true</pre>
// cout<<X.order_of_key(-5)<<endl; // 0
template <typename T, typename order = std::less<T>>
using ordered_set = __gnu_pbds::tree<T, __gnu_pbds::</pre>
    null_type, order, __gnu_pbds::rb_tree_tag, __gnu_pbds::
    tree_order_statistics_node_update>;
int main()
ł
 ordered set<int> X:
 std::cout << *X.find_by_order(4) << endl;</pre>
                                                     // 16
 std::cout << (std::end(X) == X.find_by_order(6)) << endl;</pre>
     // true
 std::cout << X.order_of_key(-5) << endl;</pre>
                                                    // 0
 return 0;
```

## 1.4 Graph

### 1.4.1 Edge Remove CC [MB]

```
#define endl "\n"
class DSU
std::vector<int> p, csz;
public:
DSU() {}
DSU(int dsz) // Max size
 //Default empty
 p.resize(dsz + 5, 0), csz.resize(dsz + 5, 0);
 init(dsz):
void init(int n)
 // n = size
 for (int i = 0; i <= n; i++)</pre>
 p[i] = i, csz[i] = 1;
//Return parent Recursively
int get(int x)
 if (p[x] != x)
 p[x] = get(p[x]);
 return p[x];
// Return Size
int getSize(int x) { return csz[get(x)]; }
// Return if Union created Successfully or false if they
     are already in Union
bool merge(int x, int y)
 x = get(x), y = get(y);
 if (x == v)
  return false;
 if (csz[x] > csz[y])
  std::swap(x, y);
 p[x] = y;
 csz[y] += csz[x];
```

```
return true:
}:
void runCase([[maybe unused]] const int &TC)
int n, m;
cin >> n >> m;
auto g = vec(n + 1, set < int > ());
auto dsu = DSU(n + 1);
for (int i = 0: i < m: i++)
 int u, v;
 cin >> u >> v;
 g[u].insert(v);
 g[v].insert(u);
set<int> elligible;
for (int i = 1; i <= n; i++)
 elligible.insert(i);
int i = 1;
int cnt = 0;
while (sz(elligible))
 cnt++:
 queue<int> q;
 q.push(*elligible.begin());
 elligible.erase(elligible.begin());
 while (sz(a))
 int fr = q.front();
  q.pop();
  auto v = elligible.begin();
  while (v != elligible.end())
   if (g[fr].find(*v) == g[fr].end())
```

```
{
    q.push(*v);
    v = elligible.erase(v);
}
else
{
    v++;
}
}
}

cout << cnt - 1 << endl;
}

int main()
{
    ios_base::sync_with_stdio(false), cin.tie(0);
    int t = 1;
//cin >> t;

for (int tc = 1; tc <= t; tc++)
    runCase(tc);

return 0;
}</pre>
```

### 1.4.2 Kruskal's [NK]

```
DEF_EDGE_OP(<=)
   DEF EDGE OP(>)
   DEF EDGE OP(>=)
constexpr Edge::weight_type Edge::bad_w = numeric_limits
    Edge::weight_type>::max();
template <class EdgeCompare = less<Edge>>
constexpr vector<Edge> kruskal(const int n, vector<Edge>
    edges, EdgeCompare compare = EdgeCompare()) {
   // define dsu part and initlaize forests
   vector<int> parent(n);
   iota(parent.begin(), parent.end(), 0);
   vector<int> size(n, 1);
   auto root = [&](int x) {
      int r = x:
      while (parent[r] != r) {
          r = parent[r];
      while (x != r) {
          int tmp_id = parent[x];
          parent[x] = r;
          x = tmp_id;
      return r;
   auto connect = [&](int u, int v) {
      u = root(u):
      v = root(v):
      if (size[u] > size[v]) {
          swap(u, v);
       parent[v] = u:
      size[u] += size[v]:
       size[v] = 0:
   };
   // connect components (trees) with edges in order from
        the sorted list
   sort(edges.begin(), edges.end(), compare);
   vector<Edge> edges_mst;
   int remaining = n - 1;
   for (const Edge& e : edges) {
      if (!remaining) break;
      const int u = root(e.u);
       const int v = root(e.v);
       if (u == v) continue;
```

```
--remaining;
edges_mst.push_back(e);
connect(u, v);
}

return edges_mst;
}
```

#### 1.4.3 Tree Rooting [MB]

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11:
const int N = 2e5 + 5;
vector<int> g[N];
11 sz[N], dist[N], sum[N];
void dfs(int s, int p)
sz[s] = 1:
dist[s] = 0;
for (int nxt : g[s])
 if (nxt == p)
 continue:
 dfs(nxt, s);
 sz[s] += sz[nxt]:
 dist[s] += (dist[nxt] + sz[nxt]):
void dfs1(int s, int p)
if (p != 0)
 11 my_size = sz[s];
 11 my_contrib = (dist[s] + sz[s]);
 sum[s] = sum[p] - my\_contrib + sz[1] - sz[s] + dist[s];
for (int nxt : g[s])
 if (nxt == p)
  continue;
```

```
dfs1(nxt, s);
}
// problem link: https://cses.fi/problemset/task/1133
int main()
{
 int n;
 cin >> n;
 for (int i = 1, u, v: i < n: i++)
 cin >> u >> v, g[u].push_back(v), g[v].push_back(u);
 dfs(1, 0):
 sum[1] = dist[1]:
 dfs1(1, 0):
 for (int i = 1; i <= n; i++)</pre>
 cout << sum[i] << " ";
 cout << endl;</pre>
 return 0;
```

#### 1.5 Math

### 1.5.1 Combinatrics [MB]

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;

struct Combinatrics
{
   vector<ll> fact, fact_inv, inv;
   ll mod, nl;

Combinatrics() {}

Combinatrics(ll n, ll _mod)
   {
    this->nl = n;
    this->mod = _mod;
}
```

```
fact.resize(n + 1, 1), fact inv.resize(n + 1, 1), inv.
     resize(n + 1, 1):
init();
void init()
fact[0] = 1;
for (int i = 1; i <= nl; i++)</pre>
 fact[i] = (fact[i - 1] * i) % mod:
inv[0] = inv[1] = 1:
for (int i = 2; i <= nl; i++)</pre>
 inv[i] = inv[mod % i] * (mod - mod / i) % mod:
fact inv[0] = fact <math>inv[1] = 1:
for (int i = 2; i <= nl; i++)
fact inv[i] = (inv[i] * fact inv[i - 1]) % mod:
11 ncr(11 n. 11 r)
if(n < r){
return 0:
if (n > n1)
 return ncr(n, r, mod);
return (((fact[n] * 1LL * fact inv[r]) % mod) * 1LL *
     fact_inv[n - r]) % mod;
ll npr(ll n, ll r)
if(n < r){
return 0:
if (n > n1)
return npr(n, r, mod);
return (fact[n] * 1LL * fact_inv[n - r]) % mod;
ll big_mod(ll a, ll p, ll m = -1)
m = (m == -1 ? mod : m);
```

```
ll res = 1 \% m, x = a \% m:
 while (p > 0)
  res = ((p \& 1) ? ((res * x) % m) : res), x = ((x * x) % m)
      ), p >>= 1;
 return res;
ll mod_inv(ll a, ll p)
 return big_mod(a, p - 2, p);
11 ncr(ll n, ll r, ll p)
 if (n < r)
 return 0;
 if (r == 0)
 return 1;
 return (((fact[n] * mod_inv(fact[r], p)) % p) * mod_inv(
      fact[n - r], p)) % p;
ll npr(ll n, ll r, ll p)
if (n < r)
 return 0;
 if (r == 0)
 return 1:
 return (fact[n] * mod_inv(fact[n - r], p)) % p;
};
const int N = 1e6, MOD = 998244353:
Combinatrics comb(N. MOD):
```

13

### 1.5.2 Extended GCD [NK]

```
template <class Z>
constexpr Z extended_gcd(Z a, Z b, Z& x_ref, Z& y_ref) {
    x_ref = 1, y_ref = 0;
    Z x1 = 0, y1 = 1, tmp = 0, q = 0;
    while (b > 0) {
        q = a / b;
        tmp = a, a = b, b = tmp - (q * b);
        tmp = x_ref, x_ref = x1, x1 = tmp - (q * x1);
        tmp = y_ref, y_ref = y1, y1 = tmp - (q * y1);
    }
    return a;
```

}

### 1.5.3 Fraction-Functions [SK]

```
pair<11,11> frac_add(pair<11,11> a,pair<11,11> b)
   11 g = a.second*b.second;
   pair<ll,ll> x;
   x.second = g:
   x.first = a.first * (b.second) + b.first * (a.second);
   11 y = __gcd(x.first,x.second);
   x.first/=y;
   x.second/=y;
   return x;
pair<11,11> frac_mult(pair<11,11> a,pair<11,11> b)
   pair<11,11> x;
   x.first = a.first * b.first;
   x.second = a.second * b.second:
   11 y = __gcd(x.first,x.second);
   x.first/=y;
   x.second/=y;
   return x;
}
```

### 1.5.4 Fraction[MB]

```
struct Fraction {
  int p, q;

Fraction (int _p, int _q) : p(_p), q(_q) {
  }

std::strong_ordering operator<=> (const Fraction &oth)
        const {
    return p * oth.q <=> q * oth.p;
  }
};
```

### 1.5.5 Miller-Rabin-for-prime-checking [SK]

```
typedef long long 11;
```

```
11 mulmod(l1 a, 11 b, 11 c) {
 11 x = 0, y = a % c;
 while (b) {
   if (b & 1) x = (x + y) \% c;
   v = (v << 1) \% c:
 return x % c;
11 fastPow(11 x, 11 n, 11 MOD) {
 ll ret = 1;
 while (n) {
   if (n & 1) ret = mulmod(ret, x, MOD);
   x = mulmod(x, x, MOD);
   n >>= 1:
 return ret:
bool isPrime(ll n) {
 11 d = n - 1;
 int s = 0:
 while (d % 2 == 0) {
   s++:
   d >>= 1:
 }
 // It's guranteed that these values will work for any
      number smaller than 3*10**18 (3 and 18 zeros)
 int a[9] = { 2, 3, 5, 7, 11, 13, 17, 19, 23 };
 for(int i = 0: i < 9: i++) {</pre>
   bool comp = fastPow(a[i], d, n) != 1;
   if(comp) for(int j = 0; j < s; j++) {</pre>
     ll fp = fastPow(a[i], (1LL << (ll)j)*d, n);
     if (fp == n - 1) {
      comp = false;
       break;
   if(comp) return false;
 return true;
```

# $\begin{array}{ccc} \textbf{1.5.6} & \textbf{Modular} & \textbf{Binary} & \textbf{Exponentiation} & \textbf{(Power)} \\ & & [\textbf{NK}] \end{array}$

```
template <class B, class E, class M>
constexpr B power(B base, E expo, M mod = 0) {
   assert(expo >= 0):
   if (mod == 1) return 0;
   if (base == 0 || base == 1) return base:
   B res = 1:
   if (!mod) {
       while (expo) {
          if (expo & 1) res *= base;
          base *= base;
          expo >>= 1:
       }
   } else {
       assert(mod > 0):
       base %= mod;
       if (base <= 1) return base:</pre>
       while (expo) {
          if (expo & 1) res = (res * base) % mod;
          base = (base * base) % mod;
           expo >>= 1;
      }
   }
   return res;
```

### 1.5.7 Modular Int [MB]

```
#include <bits/stdc++.h>
// Tested By Ac
// submission : https://atcoder.jp/contests/abc238/
    submissions/29247261
// problem : https://atcoder.jp/contests/abc238/tasks/
    abc238 c
template <const int MOD>
struct ModInt
int val:
ModInt() { val = 0: }
ModInt(long long v) { v += (v < 0 ? MOD : 0), val = (int)(v) }
      % MOD): }
ModInt &operator+=(const ModInt &rhs)
 val += rhs.val, val -= (val >= MOD ? MOD : 0):
 return *this;
ModInt &operator -= (const ModInt &rhs)
```

```
val -= rhs.val, val += (val < 0 ? MOD : 0):</pre>
 return *this:
 ModInt &operator*=(const ModInt &rhs)
 val = (int)((val * 1ULL * rhs.val) % MOD);
 return *this;
 ModInt pow(long long n) const
 ModInt x = *this, r = 1:
 while (n)
  r = ((n \& 1) ? r * x : r), x = (x * x), n >>= 1;
 return r:
 ModInt inv() const { return this->pow(MOD - 2); }
 ModInt &operator/=(const ModInt &rhs) { return *this = *
      this * rhs.inv(): }
 friend ModInt operator+(const ModInt &lhs. const ModInt &
     rhs) { return ModInt(lhs) += rhs: }
 friend ModInt operator-(const ModInt &lhs, const ModInt &
     rhs) { return ModInt(lhs) -= rhs: }
 friend ModInt operator*(const ModInt &lhs, const ModInt &
     rhs) { return ModInt(lhs) *= rhs; }
 friend ModInt operator/(const ModInt &lhs, const ModInt &
     rhs) { return ModInt(lhs) /= rhs; }
 friend bool operator==(const ModInt &lhs, const ModInt &rhs
     ) { return lhs.val == rhs.val: }
 friend bool operator!=(const ModInt &lhs, const ModInt &rhs
     ) { return lhs.val != rhs.val: }
 friend std::ostream &operator<<(std::ostream &out, const</pre>
     ModInt &m) { return out << m.val; }</pre>
 friend std::istream &operator>>(std::istream &in, ModInt &m
     ) { return in >> m.val; }
 operator int() const { return val: }
}:
const int MOD = 1e9 + 7;
using mint = ModInt<MOD>;
```

### 1.5.8 Modular inverse [NK]

```
template <class Z>
constexpr Z inverse(Z num, Z mod) {
   assert(mod > 1);
   if (!(0 <= num && num < mod)) {
      num %= mod;
      if (num < 0) num += mod;
   }</pre>
```

```
Z res = 1, tmp = 0;
assert(extended_gcd(num, mod, res, tmp) == 1);
if (res < 0) res += mod;
return res;
}</pre>
```

#### 1.5.9 Prime Phi Sieve [MB]

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef pair<int, int> pii;
typedef pair<11, 11> pll:
struct PrimePhiSieve
private:
vector<ll> primes, phi;
vector<bool> is_prime;
PrimePhiSieve() {}
 PrimePhiSieve(11 n)
 this->n = n, is_prime.resize(n + 5, true), phi.resize(n +
      5, 1);
 phi sieve():
 void phi_sieve()
 is_prime[0] = is_prime[1] = false;
  for (11 i = 1: i <= n: i++)</pre>
  phi[i] = i;
 for (ll i = 1: i <= n: i++)</pre>
  if (is_prime[i])
   primes.push_back(i);
   phi[i] *= (i - 1), phi[i] /= i;
   for (ll j = i + i; j <= n; j += i)
    is_prime[j] = false, phi[j] /= i, phi[j] *= (i - 1);
```

```
11 get_divisors_count(int number, int divisor)
return phi[number / divisor];
vector<pll> factorize(ll num)
vector<pll> a;
 for (int i = 0; i < (int)primes.size() && primes[i] * 1LL</pre>
      * primes[i] <= num: i++)
  if (num % primes[i] == 0)
 ſ
  int cnt = 0:
  while (num % primes[i] == 0)
   cnt++, num /= primes[i];
  a.push_back({primes[i], cnt});
if (num != 1)
 a.push_back({num, 1});
return a:
11 get_phi(int n)
return phi[n];
// (n/p) * (p-1) \Rightarrow n- (n/p);
void segmented phi sieve(ll l. ll r)
 vector<ll> current_phi(r - 1 + 1);
vector<ll> left_over_prime(r - 1 + 1);
 for (ll i = l: i <= r: i++)
 current phi[i - 1] = i. left over prime[i - 1] = i:
 for (ll p : primes)
 11 \text{ to } = ((1 + p - 1) / p) * p;
 if (to == p)
  to += p:
  for (ll i = to; i <= r; i += p)</pre>
  while (left_over_prime[i - 1] % p == 0)
   left_over_prime[i - 1] /= p;
  current_phi[i - 1] -= current_phi[i - 1] / p;
```

15

```
}
 for (11 i = 1; i <= r; i++)</pre>
   if (left_over_prime[i - 1] > 1)
   current_phi[i - 1] -= current_phi[i - 1] /
        left_over_prime[i - 1];
   cout << current_phi[i - 1] << endl;</pre>
 }
 11 phi_sqrt(ll n)
 11 \text{ res} = n;
 for (ll i = 1; i * i <= n; i++)
  if (n % i == 0)
   res /= i:
   res *= (i - 1);
   while (n \% i == 0)
    n /= i:
  if (n > 1)
  res /= n, res *= (n - 1);
 return res:
}
};
```

### 1.5.10 Prime Sieve [MB]

```
#include <bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef pair<int, int> pii;
typedef pair<ll, ll> pll;

struct PrimeSieve
{
public:
    vector<int> primes;
    vector<bool> isprime;
    int n;
```

```
PrimeSieve() {}
PrimeSieve(int n)
this->n = n. isprime.resize(n + 5, true), primes.clear()
sieve();
void sieve()
isprime[0] = isprime[1] = false;
primes.push_back(2);
for (int i = 4; i <= n; i += 2)</pre>
 isprime[i] = false:
 for (int i = 3: 1LL * i * i <= n: i += 2)
 if (isprime[i])
  for (int j = i * i; j <= n; j += 2 * i)
   isprime[j] = false;
for (int i = 3; i <= n; i += 2)</pre>
 if (isprime[i])
  primes.push_back(i);
vector<pll> factorize(ll num)
vector<pll> a;
for (int i = 0; i < (int)primes.size() && primes[i] * 1LL</pre>
     * primes[i] <= num; i++)
 if (num % primes[i] == 0)
  int cnt = 0:
  while (num % primes[i] == 0)
   cnt++, num /= primes[i];
  a.push_back({primes[i], cnt});
if (num != 1)
 a.push_back({num, 1});
return a;
vector<ll> segemented_sieve(ll l, ll r)
vector<11> seg_primes;
vector<bool> current_primes(r - 1 + 1, true);
```

```
for (ll p : primes)
{
    ll to = (l / p) * p;
    if (to < l)
        to += p;
    if (to == p)
        to += p;
    for (ll i = to; i <= r; i += p)
    {
        current_primes[i - l] = false;
    }
}

for (ll i = l; i <= r; i++)
{
    if (i < 2)
        continue;
    if (current_primes[i - l])
    {
        seg_primes.push_back(i);
    }
}
    return seg_primes;
}
</pre>
```

### 1.5.11 nCrp-O(1) [SK]

```
factorialNumInverse[0] = factorialNumInverse[1] = 1:
    // precompute inverse of natural numbers
    for (int i = 2: i <= N: i++)</pre>
       factorialNumInverse[i] = (naturalNumInverse[i] *
            factorialNumInverse[i - 1]) % p:
}
// Function to calculate factorial of 1 to N
void factorial(ll p)
   fact[0] = 1:
    // precompute factorials
    for (int i = 1; i <= N; i++) {</pre>
       fact[i] = (fact[i - 1] * i) % p;
}
// Function to return nCr % p in O(1) time
11 Binomial(11 N, 11 R, 11 p)
    // n C r = n!*inverse(r!)*inverse((n-r)!)
    11 ans = ((fact[N] * factorialNumInverse[R])
            % p * factorialNumInverse[N - R])
            % p;
    return ans:
```

### 1.6 String

### 1.6.1 Hashing [MB]

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;

const int PRIMES[] = {2147462393, 2147462419, 2147462587, 2147462633, 2147462747, 2147463167, 2147463203, 2147463569, 2147463727, 2147463863, 2147464211, 214746549, 2147464751, 2147465153, 214746563, 2147465599, 2147465743, 2147465953, 214746657, 2147466463, 2147466521, 2147466721, 2147467009, 2147467057, 2147467067, 2147467261, 2147467379, 214746743, 214746891, 2147468591, 214746851, 2147468779, 2147468801, 21474689017, 2147469041, 2147469173,
```

```
2147469229, 2147469593, 2147469881, 2147469983,
    2147470027, 2147470081, 2147470177, 2147470673,
    2147470823, 2147471057, 2147471327, 2147471581,
    2147472137, 2147472161, 2147472689, 2147472697,
    2147472863, 2147473151, 2147473369, 2147473733,
    2147473891, 2147473963, 2147474279, 2147474921,
    2147474929, 2147475107, 2147475221, 2147475347,
    2147475397, 2147475971, 2147476739, 2147476769,
    2147476789, 2147476927, 2147477063, 2147477107,
    2147477249, 2147477807, 2147477933, 2147478017,
    2147478521}:
// 11 base_pow,base_pow_1;
ll base1 = 43, base2 = 47, mod1 = 1e9 + 7, mod2 = 1e9 + 9;
// **** Enable this function for codeforces
void generateRandomBM()
unsigned int seed = chrono::svstem clock::now().
     time since epoch().count():
srand(seed); /// to avoid getting hacked in CF, comment
     this line for easier debugging
int q_len = (sizeof(PRIMES) / sizeof(PRIMES[0])) / 4;
base1 = PRIMES[rand() % q_len];
mod1 = PRIMES[rand() % q_len + q_len];
base2 = PRIMES[rand() % q_len + 2 * q_len];
mod2 = PRIMES[rand() % g len + 3 * g len]:
struct Hash
vector<int> base_pow, f_hash, r_hash;
11 base, mod:
Hash() {}
// Update it make it more dynamic like segTree class and
Hash(int mxSize, 11 base, 11 mod) // Max size
 this->base = base;
 this->mod = mod:
 base_pow.resize(mxSize + 2, 1), f_hash.resize(mxSize + 2,
      0), r_hash.resize(mxSize + 2, 0);
 for (int i = 1; i <= mxSize; i++)</pre>
  base_pow[i] = base_pow[i - 1] * base % mod;
```

```
void init(string s)
 int n = s.size();
 for (int i = 1; i <= n; i++)</pre>
 f_{hash}[i] = (f_{hash}[i-1] * base + int(s[i-1])) % mod;
 for (int i = n; i >= 1; i--)
  r hash[i] = (r hash[i + 1] * base + int(s[i - 1])) % mod:
int forward_hash(int 1, int r)
 int h = f_hash[r + 1] - (1LL * base_pow[r - 1 + 1] *
      f_hash[1]) % mod;
 return h < 0? mod + h: h:
int reverse_hash(int 1, int r)
 int h = r_hash[1 + 1] - (1LL * base_pow[r - 1 + 1] *
      r hash[r + 2]) \% mod:
 return h < 0? mod + h: h;
};
class DHash
public:
Hash sh1, sh2:
DHash() {}
DHash(int mx_size)
 sh1 = Hash(mx size, base1, mod1):
 sh2 = Hash(mx_size, base2, mod2);
void init(string s)
 sh1.init(s);
 sh2.init(s):
```

#### 1.6.2 Hashing [NK]

```
// Primes suitable for use as the constant base in a
    polynomial rolling hash function.
constexpr std::array<int, 10>
   prime_bases = {257, 263, 269, 271, 277, 281, 283, 293,
        307, 311};
// Primes suitable for use as modulus.
constexpr std::array<int, 10>
   prime_moduli = {1000000007, 1000000009, 1000000021,
        1000000033, 1000000087,
                 1000000093, 1000000097, 1000000103,
                      1000000123, 1000000181}:
/**
* Obrief A data structure for computing polynomial hashes
     of sequence keys.
* For a given key defined as an integral sequence of n
     elements S[0], S[1], ....
* S[n - 1], this structure builds and stores for each
     prefix S[0...i] the hash value
* H(i) = S[0] * B^i + S[1] * B^i - 1) + ... + S[i] * B^0,
     modulo M.
* Otparam Base The base B. Should be a prime to reduce
     chances of collision.
* Otparam Modulus The modulus M. Should be a prime to
     reduce chances of collision.
template <std::uint64_t Base, std::uint64_t Modulus>
class Polynomial_hasher {
public:
   using int type = std::uint64 t:
   using value_type = int_type;
   using size_type = std::size_t;
   static constexpr int_type B = Base;
```

```
static constexpr int type M = Modulus:
protected:
   // Base power
   static std::vector<int_type> bpow_;
   // Prefix hash
   std::vector<int_type> pref_hash_;
   // Suffix hash
   std::vector<int_type> suff_hash_;
   // Flag for hashing bidirectionally
   bool bidir = false:
public:
    * @brief Default constructor
   Polvnomial hasher() {}
    * @brief Constructors and builds the hash from a range (
    * Otparam InputIter Type of the iterator of the range
    * Cparam from Iterator pointing to the start of the
    * Oparam until Iterator pointing to the end (one past
         the last element) of the range
    * Oparam bidir Flag for hashing bidirectionally
   template <class InputIter>
   Polynomial hasher(InputIter from, InputIter until, bool
        bidir = false) {
       build_hash(from, until, bidir);
   }
    * Obrief Builds the hash from a range (a "kev").
    * @tparam InputIter Type of the iterator of the range
    * Oparam from Iterator pointing to the start of the
    * Cparam until Iterator pointing to the end (one past
         the last element) of
    * the range
    * Oparam bidir Flag for hashing bidirectionally
   template <class InputIter>
   void build_hash(InputIter from, InputIter until, bool
        bidir = false) {
       const auto n = std::distance(from, until);
       while (bpow_.size() < n) {</pre>
          bpow_.push_back((bpow_.back() * B) % M);
```

```
}
   // Build forward hash
       pref_hash_.resize(n + 1);
       pref_hash_[0] = 0;
       auto it = from:
       for (size_type i = 0; i < n; ++i) {</pre>
          pref_hash_[i + 1] =
              (((pref_hash_[i] * B) % M) + static_cast<</pre>
                   int_type>(*it)) % M;
          ++it:
   }
   // Set and test flag, and build reverse hash
   bidir = bidir:
   if (bidir_) {
       suff_hash_.resize(n + 1);
       suff_hash_[n] = 0;
       auto it = prev(until):
       for (size_type i = n; i; --i) {
          suff_hash_[i - 1] =
              (((suff_hash_[i] * B) % M) + static_cast<</pre>
                   int_type>(*it)) % M;
          --it;
* @brief Returns the polynomial hash value of the
     subsegment S[i], S[i + 1], ...,
* S[i + n - 1], which is the value S[i] * B^n(n - 1) + S[
     i + 1] * B^{(n - 2)} +
* ... + S[i + n - 1] * B^0, modulo M.
* Oparam i Starting index/position of the subsegment
* Cparam n Length of the subsegment
value_type get(size_type i = 0,
             size_type n = std::numeric_limits<size_type</pre>
                  >::max()) const {
   assert(i < pref hash .size());</pre>
   n = std::min(n, pref_hash_.size() - 1 - i);
   return (pref_hash_[i + n] - ((pref_hash_[i] * bpow_[n
        1) % M) + M) % M:
* @brief Returns the polynomial hash value of the
     subsegment S[i], S[i + 1], ....
```

```
* B^i + S[i + 1] *
    * B^{(i+1)} + ... + S[i + n - 1] * B^{(i+n-1)}, modulo
    * Oparam i Starting index/position of the subsegment
    * Cparam n Length of the subsegment
   value_type get_rev(size_type i = 0,
                    size_type n = std::numeric_limits<</pre>
                         size_type>::max()) const {
       assert(bidir ):
       assert(i < suff hash .size()):</pre>
      n = std::min(n, suff_hash_.size() - 1 - i);
       return (suff_hash_[i] - ((suff_hash_[i + n] * bpow_[n
           1) % M) + M) % M:
   /**
    * Obrief Erases hash values of all prefixes (and
         suffixes if hashed
    * bidirectionally) calling 'clear()' on the internal
         vector(s). Resets
    * bidirectional flag.
   void clear() {
       pref_hash_.clear();
       suff hash .clear():
       bidir = false:
    * Obrief Number of elements in the hashed key.
   size_type size() const { return pref_hash_.size() ?
        pref hash .size() - 1 : 0: }
    * Obrief Returns true if no hash values are stored.
   bool empty() const { return pref_hash_.empty(); }
    * Obrief Returns true if the stored hash value is
         bidirectional (i.e., both
    * 'hash' and 'hash_rev' can be called).
   bool bidirectional() const { return bidir_; }
template <std::uint64_t Base, std::uint64_t Modulus>
```

};

```
* S[i + n - 1] in reverse order, which is the value S[i] std::vector<std::uint64 t> Polynomial hasher<Base, Modulus
                                                              >::bpow = {1ULL}:
                                                          using Hasher0 = Polynomial hasher<prime bases[0].
                                                              prime_moduli[0]>;
                                                          using Hasher1 = Polynomial hasher<prime bases[1].
                                                              prime_moduli[1]>;
```

#### 1.6.3 Hashing [SK]

```
int powhash1[ 1000000+ 10]= {};
int powhash2[ 1000000+ 10]= {};
int f_prefhash1[1000000 + 10];
int f_prefhash2[1000000 + 10];
int r prefhash1[1000000 + 10]:
int r_prefhash2[1000000 + 10];
int add(ll x.ll v.ll mod)
   return (x+y>=mod)?(x+y-mod):(x+y);
int subtract(ll x,ll y,ll mod)
   return (x-y<0)?(x-y+mod):(x-y);</pre>
int multp(ll x,ll y,ll mod)
   return (x*y)%mod;
const int BASE1 = 125;
const int MOD1 = 1e9 + 9:
const int BASE2 = 250;
const int MOD2 = 1e9 + 7:
void f prefhashcalc(string& s.int base.int mod.int*prefhash)
   11 \text{ sum} = 0:
   int ns = s.size():
   for(int i=0; i<ns; i++)</pre>
       sum = add(((11)sum*base)%mod,s[i],mod);
       prefhash[i]=sum:
```

```
void r_prefhashcalc(string& s,ll base,ll mod,int*prefhash)
   11 \text{ sum} = 0:
   int ns = s.size();
   prefhash[ns]=0:
   for(int i=ns-1: i>=0: i--)
       sum = add((sum*base)%mod.s[i].mod):
       prefhash[i]=sum;
int f_strhash(string& s,int base,int mod)
   11 \text{ sum} = 0:
   int ns = s.size();
   for(int i=0; i<ns; i++)</pre>
       sum = add(((11)sum*base)%mod,s[i],mod);
   return sum:
int r strhash(string& s.ll base.ll mod)
   11 \text{ sum} = 0:
   int ns = s.size():
   for(int i=ns-1: i>=0: i--)
       sum = add((sum*base)%mod.s[i].mod):
   return sum;
void powhashfill(int base,int mod,int*powhash)
   for(int i=0: i<1000000 + 10: i++)
       if(i==0)
       {
           powhash[0]=1;
```

```
continue;
       powhash[i] = multp(powhash[i-1],base,mod);
}
int f_substrhash(int l,int r,ll mod,int*prefhash,int*powhash
{
    ll x = subtract( prefhash[r], multp(prefhash[l-1],powhash #include<bits/stdc++.h>
        [r-l+1],mod), mod);
    return x;
}
int r_substrhash(int l,int r,ll mod,int*prefhash,int*powhash
```

```
11 x = subtract( prefhash[1], multp(prefhash[r+1],powhash
     [r-l+1], mod), mod);
return x:
```

### 1.6.4 Z-Function [MB]

```
tested by ac
submission: https://codeforces.com/contest/432/submission
    /145953901
problem: https://codeforces.com/contest/432/problem/D
```

```
std::vector<int> z_function(const std::string &s)
int n = (int)s.size();
std::vector<int> z(n, 0);
for (int i = 1, l = 0, r = 0; i < n; i++)
 if (i <= r)</pre>
  z[i] = std::min(r - i + 1, z[i - 1]);
 while (i + z[i] < n && s[z[i]] == s[i + z[i]])
  z[i]++;
 if (i + z[i] - 1 > r)
 1 = i, r = i + z[i] - 1;
return z;
```