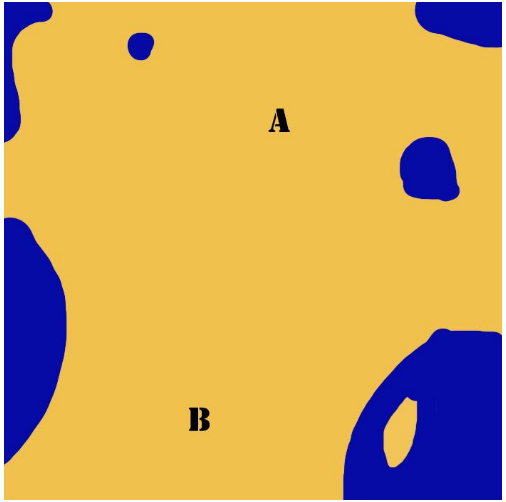
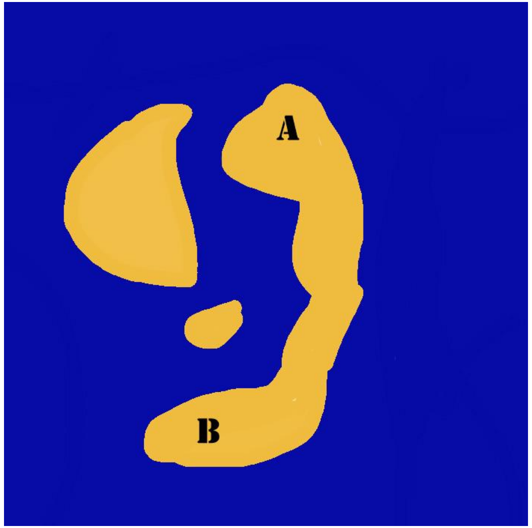
Introduction :

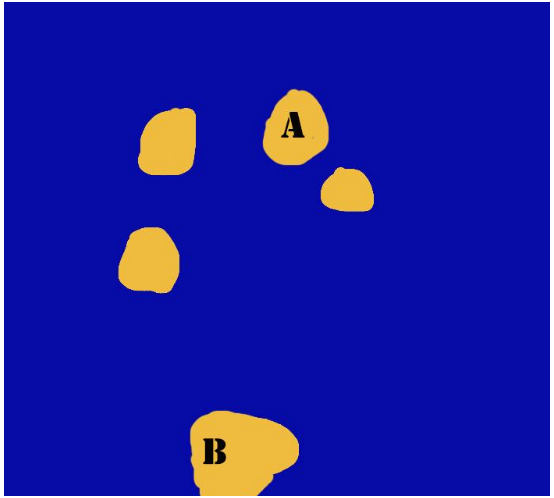
- We consider two points A and B that are located in a continent



- Due to the sea level rise , the areas submerged by water are getting larger and some are merging to form an archipelago of islands.



- At some point, it will no longer be impossible to find a path from point A to point B.



Problematic :

Hypothesis: Any point on a continent can be randomly submerged with a given probability "p"

To what proportion of submerged points is it no longer possible for a living species to move from a point to another?

PLAN :

- **Généralités sur la théorie des graphes**
  - Définitions
  - Réseau carré
- **Percolation sur $\mathbb{L}^2$:**
  - Espace de probabilité sur $\mathbb{L}^2$
  - Cluster sur $\mathbb{L}^2$
  - La probabilité de percolation
- **Simulations**

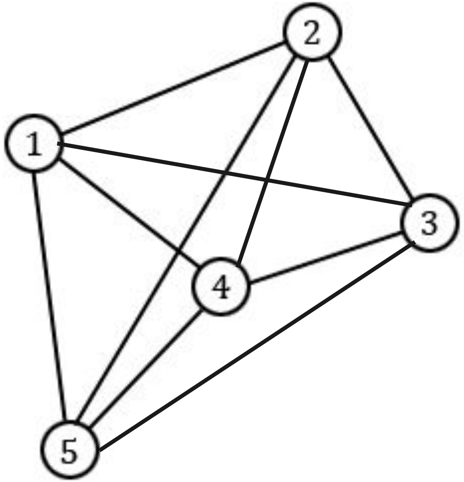
Graph Theory

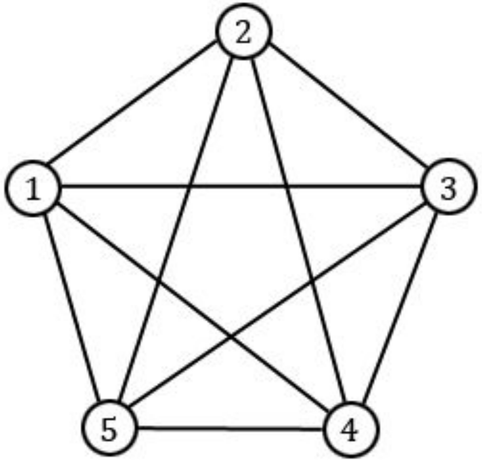
Définitions :

- A simple undirected graph **G** is a an ordered pair **(V,E)**, comprising **V** is a set and **E** is a set of pairs of **V** elements.
- The elements of **V** are called **vertices** or **nodes**, and the elements of **E** are called **edges.**
- A path is is a sequence of distincts nodes **x0, x1, ...** which verify :

$\forall i \geq 0$, $\{x_i , x_{i+1}\}$ is an edge.

- A graphs is said to be connected if every pair of vertices in the graph is connected. This means that there is a path between every pair of vertices





V={1,2,3,4,5}

, **E={(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)}**

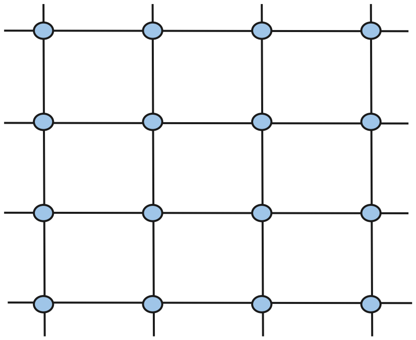
Square Lattice:

Distance between x and y : $\delta(x,y) = \sum_{i=1}^{2} |x_i - y_i|, \forall x, y \in \mathbb{Z}^2.$

- **We consider the graph $\mathbb{L}^2$ obtained from $\mathbb{Z}^2$ as it is shown in the figure**

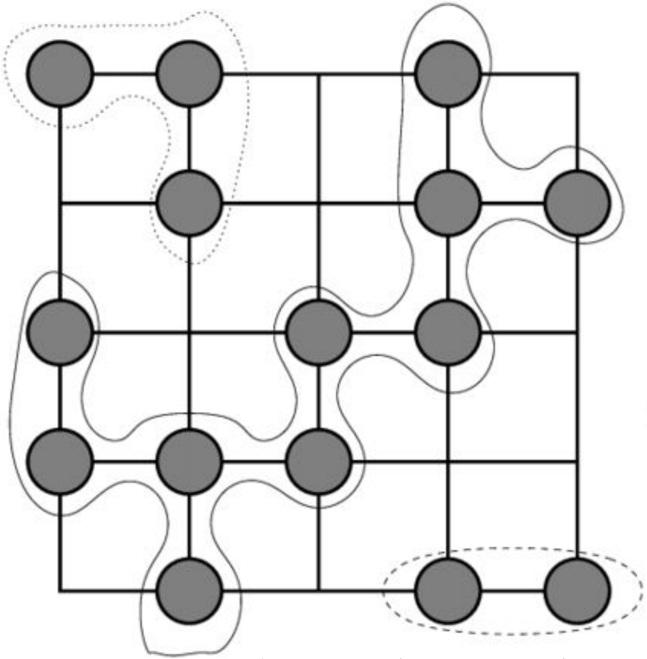
- **$\mathbb{E}^2$ is the set of those vertices**

- **The distance between two vertices that are neighbours is unchanged**



$$\mathbb{L}^2 = (\mathbb{Z}^2, \mathbb{E}^2)$$

Percolation in $\mathbb{L}^2$:



Site percolation (vertices)

1. Probability space in $\mathbb{L}^2$:

- The vertices are independant :

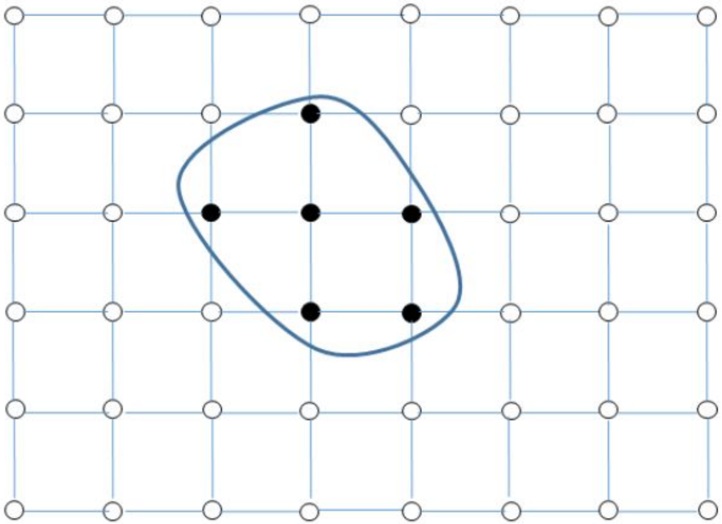
- Considering $\Omega = \{0,1\}^{\mathbb{E}^2}$, we say :
  - an element $\omega \in \Omega$ is a configuration in $\mathbb{L}^2$
  - a site **e** of $\mathbb{L}^2$ is open if $\omega(e) = 0$ and closed if $\omega(e) = 1$
- Each site is open (resp. closed) with a probability p (resp. 1-p) where p is a real number in [0,1]

- We provide Ω with a σ -algebra $\mathcal{F}$ and a probability $\mathbb{P}_p$ of a Bernoulli distribution having p as a parameter

2. **Cluster in $\mathbb{L}^2$:**

- For a configuration ω , we define $K(\omega)$ as the set obtained from $\mathbb{Z}^2$ by removing the unavailable sites
- The connected elements of the graph $G = (K(\omega),\ E)$ are called open clusters.

- A path is said to be open if all its vertices are open.
- For a node **x** , C(x) is the sub-set of the vertices that are linked to x by an open path.

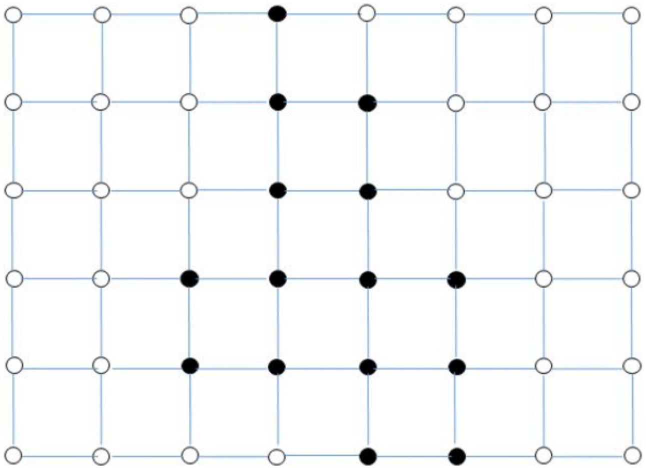


\- Cluster example -

3. Percolation probability :

- In theory , and in an infinite graph, the cluster must be infinite to have percolation

$$|C| = \infty$$



- Percolation example -

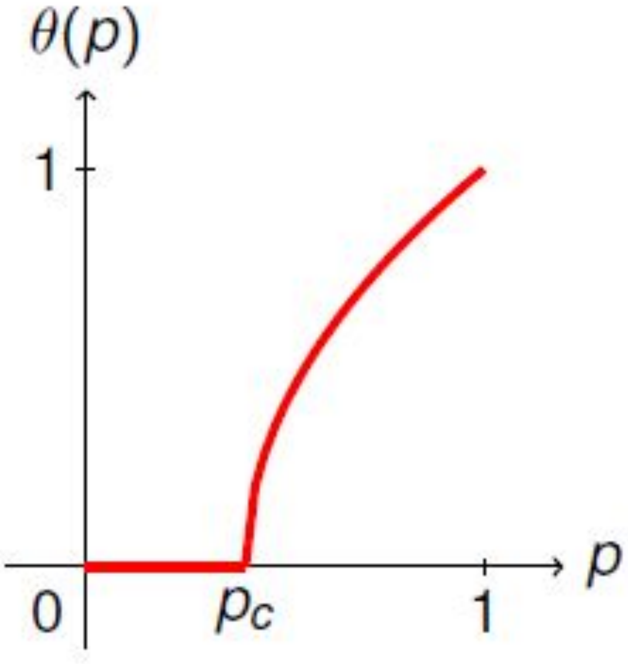
- We introduce the function θ as a percolation probability ;

$$\theta(p) : [0,1] \to [0,1]$$

$$p \to \theta(p) = \mathbb{P}_p(|C| = \infty)$$

- θ is a monotonically increasing function, that takes the value 0 when p=0 and 1 when p=1 .

- **Percolation threshold :** $p_c := sup\{p|\theta(p) = 0\}$



➔ Threshold theoretical value for site percolation in $\mathbb{L}^2$: $\boxed{p_c = 0.592746}$