QMM Assignment-3

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The Transportation Model

Decision variables

Xji is the total number of AEDs shipped from plant j to warehouse i

Here, (j = A,B) and (i = 1,2,3)

Answer 1 - Formulating the transportation model

Z = 622(XPA1) +614(XPA2) +630(XPA3) +641(XPB1) +645(XPB2) +649(XPB3)

Constraints XPA1+XPA2+XPA3 <= 100 XPB1+XPB2+XB3 <= 120 XPA1+XPB1 = 80 XPA2+XPB2 = 60 XPA3+XPB3= 70 and Xji >= 0

Install and use lpSolveAPI

```
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.1.3

library(lpSolveAPI)

## Warning: package 'lpSolveAPI' was built under R version 4.1.3
```

We have 5 constraints, 6 decision variables in this problem

```
lprec <- make.lp(5,6)

# Objective function
set.objfn(lprec, c(622,614,630,641,645,649))

# Finding the direction towards minimum
lp.control(lprec, sense = "min")

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
```

```
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"       "dynamic"      "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##       epsb       epsd      epsel     epsint epsperturb   epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##    1e-11    1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"    "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"   "equilibrate" "integers"
```

```
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual"   "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Add all the constraints

```
# Using the constraint values to both the plants A and B

# Production capacity constraints
set.row(lprec, 1, c(1,1,1), indices = c(1,2,3))
set.row(lprec, 2, c(1,1,1), indices = c(4,5,6))

# Warehouse demand constraints
set.row(lprec, 3, c(1,1), indices = c(1,4))
set.row(lprec, 4, c(1,1), indices = c(2,5))
set.row(lprec, 5, c(1,1), indices = c(3,6))

# Formulate the right hand side values
rhs <- c(100,120,80,60,70)
set.rhs(lprec, rhs)

# Finding the constraint type
set.constr.type(lprec, c("<=","<=","=","=","="))
```

Here all the values are greater than 0

```
# Add the boundary conditions to the decision variables
set.bounds(lprec, lower = rep(0, 6))

# Name all the rows and columns for the problem
lp.rownames <- c("Plant A Capacity", "Plant B Capacity", "Warehouse 1
Demand", "Warehouse 2 Demand", "Warehouse 3 Demand")
lp.colnames <- c("PlantA to Warehouse 1", "PlantA to Warehouse 2", "PlantA to
Warehouse 3", "PlantB to Warehouse 1", "PlantB to Warehouse 2", "PlantB to
Warehouse 3")

dimnames(lprec) <- list(lp.rownames, lp.colnames)

# Re-check all the values
lprec
```

```
## Model name:
##                       PlantA to Warehouse 1  PlantA to Warehouse 2  PlantA
to Warehouse 3  PlantB to Warehouse 1  PlantB to Warehouse 2  PlantB to
Warehouse 3
## Minimize                                 622                    614
630                    641                    645                    649
## Plant A Capacity                           1                      1
1                      0                      0                      0  <=
100
## Plant B Capacity                           0                      0
0                      1                      1                      1  <=
120
## Warehouse 1 Demand                         1                      0
0                      1                      0                      0   =
80
## Warehouse 2 Demand                         0                      1
0                      0                      1                      0   =
60
## Warehouse 3 Demand                         0                      0
1                      0                      0                      1   =
70
## Kind                                     Std                    Std
Std                    Std                    Std                    Std
## Type                                    Real                   Real
Real                   Real                   Real                   Real
## Upper                                    Inf                    Inf
Inf                    Inf                    Inf                    Inf
## Lower                                      0                      0
0                      0                      0                      0
```

Formulating the linear programming problem to find the optimal solution. Say if the result says 0, then it the optimal solution.

```
# Solving the linear program
solve(lprec)

## [1] 0
```

The model returned a 0, so there is an optimal solution

Fix a minimum value to the objective function

```
# The value of the objective function is
get.objective(lprec)

## [1] 132790
```

The minimum shipping and production costs is $132,790

Adding the decision variables to find the production and units shipped

```r
# Optimum decision variable values
get.variables(lprec)

## [1]  0 60 40 80  0 30
```

Results

Plant A ships 0 units to Warehouse 1, Plant A ships 60 units to Warehouse 2, Plant A ships 40 units to Warehouse 3, Plant B ships 80 units to Warehouse 1, Plant B ships 0 units to Warehouse 2, Plant B ships 30 units to Warehouse 3.

The distribution minimizes the cost and maximize production of all the 210 units out of both the plants.

Answer 2 - Dual for the transportation model

VA = Pi^j - Pi^0 Max VA = (80p1^d +60p2^{d+70p3}d)-(100p1$^{0-120p2}$0) Plant A p1^d -p1^0 >=22 p2^d -p1^0 >=14 p3^d -p1^0 >=30

Plant B p1^d -p2^o >=16 p2^d -p2^0 >=20 p3^d -p2^0 >=24

Here all non-negative variables we need pi^j >=0

Answer 3 - Concluding the economic interpretation

```r
# Switch the matrix to calculate the dual
costs <- matrix(c(622,614,630,0,
 641,645,649,0) , ncol=4 , byrow=TRUE)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)


lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
lptrans$duals

##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

Since we are taking the min of this specific function seeing the number go down by 19 means the shadow price is 19, that was found from the primal and adding 1 to each of the

Plants. However with Plant B does not have a shadow price. We also found that the dual variable where Marginal Revenue (MR) <= Marginal Cost (MC).

Conclusion from the primal

60x12 which is 60 Units from Plant A to Warehouse 2. 40x13 which is 40 Units from Plant A to Warehouse 3. 80x21 which is 60 Units from Plant B to Warehouse 1. 30x23 which is 60 Units from Plant B to Warehouse 3. from the dual So, MR=MC. Five of the six MR<=MC. The only equation that does not satisfy this requirement is Plant B to Warehouse 2. The primal that we will not be shipping any AED device there.