An Introduction To Machine Learning¹

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"Computers are able to see, hear and learn.
Welcome to the future"

Dave Waters

"This is nothing. In a few years, that bot will move so fast you'll need a strobe light to see it. Sweet dreams..."

Flon Musk

"Machine intelligence is the last invention that humanity will ever need to make."

Nick Bostrom



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Roadmap

- An overview
- Supervised Learning
- Unsupervised Learning
- Artificial Neural Network
- ML Landscape through Quizzes





- An overview
- Supervised Learning
- Unsupervised Learning
- 4 Artificial Neural Network
- ML Landscape through Quizzes



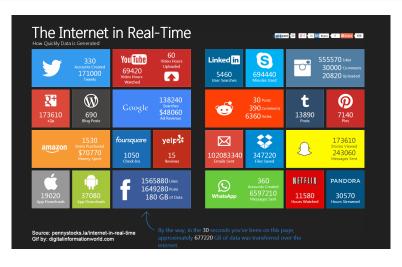


"Numbers represent search interest relative to the highest point on the chart for the given region and time

- A value of 100 is the peak popularity for the term;
- A value of 50 means that the term is half as popular;
- A score of 0 means there was not enough data for this term."



Global Data Traffic



Update on the internet in real time is available here.



5/78 A. MHAMDI An Introduction To ML

Top Uses



























Literature Review (1/3)

"The field of machine learning is concerned with the question of how to construct computer programs that automatically improve with experience."

Mitchell, T. (1997) Machine Learning. McGraw-Hill International Editions. McGraw-Hill.



Literature Review (2/3)

"Machine learning (ML) is a scientific discipline that concerns developing learning capabilities in computer systems. Machine learning is one of central areas of Artificial Intelligence (AI). It is an interdisciplinary area that combines results from statistics, logic, robotics, computer science, computational intelligence, pattern recognition, data mining, cognitive science, and more."

Wojtusiak, J. (2012) Machine learning. In Encyclopedia of the Sciences of Learning, pages 2082–2083. Springer US.



"Machine learning is an evolving branch of computational algorithms that are designed to emulate human intelligence by learning from the surrounding environment. They are considered the working horse in the new era of the so-called big data. Techniques based on machine learning have been applied successfully in diverse fields ranging from pattern recognition, computer vision, spacecraft engineering, finance, entertainment, and computational biology to biomedical and medical applications. [...] The ability of machine learning algorithms to learn from current context and generalize into unseen tasks would allow improvements in both the safety and efficacy of radiotherapy practice leading to better outcomes."

El Naqa, I. and Murphy, M. J. (2015) What Is Machine Learning?, pages 3–11. Springer International Publishing.



9/78 A. MHAMDI An Introduction To ML

Arthur Samuel (1959)

<u>Machine Learning:</u> Field of study that gives computers the ability to learn without being explicitly programmed.

Tom Mitchell (1998)

<u>Well-posed Learning Problem:</u> A computer is said to learn from experience $\mathcal E$ with respect to some task $\mathcal T$ and some performance measure $\mathcal P$, if its performance on $\mathcal T$, as measured by $\mathcal P$, improves with experience $\mathcal E$.



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Task #1

Suppose your email program watches which emails you do or do not mark as spam, and based on that learns how to better filter spam. What is the task \mathcal{T} in this setting?

- Classifying emails as spam or not spam;
- Watching you label emails as spam or not spam;
- The number (or fraction) of emails correctly classified as spam/not spam;
- None of the above-this not a machine learning problem.



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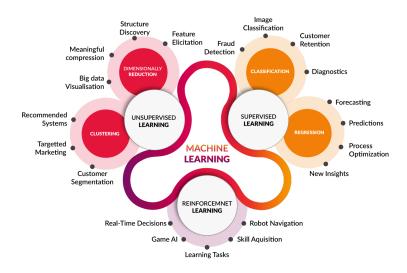


An Introduction To ML

Overall Methodology

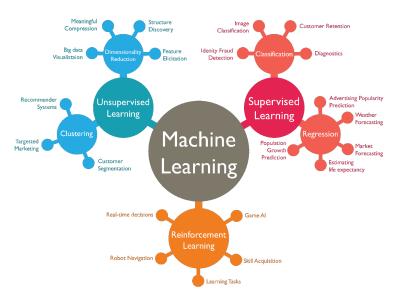
- Define the problem;
- Gather dataset;
- Ohoose measure of success;
- Decide evaluation protocol;
- Prepare the data;
- Oevelop a model;
- Iterate models.





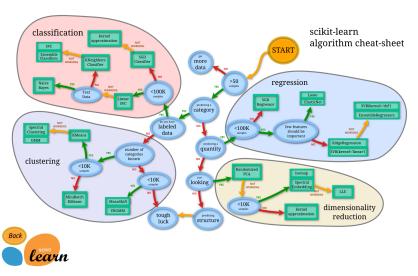
https://www.cognub.com/index.php/cognitive-platform/





https://vitalflux.com/great-mind-maps-for-learning-machine-learning/

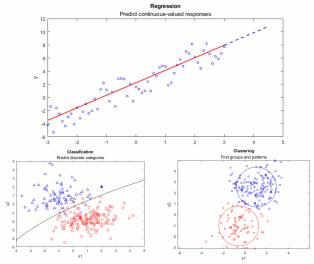




https://scikit-learn.org/stable/tutorial/machine_learning_map/index.html



Regression | Classification | Clustering



https://github.com/MathWorks-Teaching-Resources/Machine-Learning-for-Regression



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Programming Language







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Development Environments







- \$ docker compose up
- \$ docker compose down















Required Packages

Valid only for...

python

A full list is available @ https://pypi.org/

(Numpy)

(Matplotlib

Pandas

Scikit – learn

Keras



matpl stlib







- \$ pip install virtualenv
- \$ virtualenv -version
- \$ virtualenv «virtualenv_name»
- \$ source «virtualenv_name»/bin/activate # ACTIVATE
- \$ deactivate # DEACTIVATE

- > pip install virtualenv
- > virtualenv -version
- > virtualenv «virtualenv_name»
- > «virtualenv_name»\Scripts\activate %= ACTIVATE = %
- > deactivate %= DEACTIVATE =%

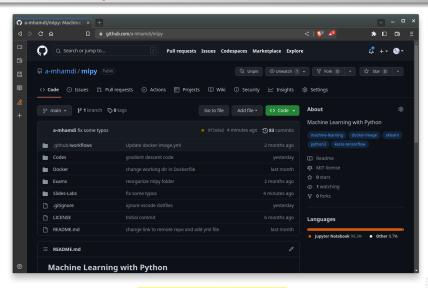






Source Control Management (SCM)



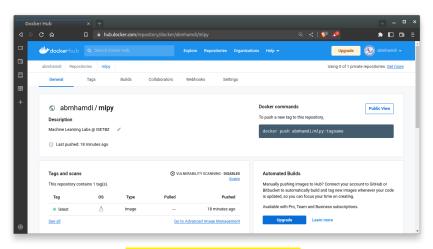


https://github.com/a-mhamdi/mlpy



Continuous Integration (CI)





https://hub.docker.com/r/abmhamdi/mlpy





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Workflow in Machine Learning





Data Preprocessing

How?

Cleaning Identifying and correcting or removing inaccuracies and inconsistencies in the data.

Transformation Converting data from one format or structure to another.

Normalization Scaling the data so that it fits within a specific range. This is often done to make the data more amenable to certain operations or algorithms.



Data Preprocessing

Why?

- Raw data is often messy and may need to be cleaned and formatted before it can be used for machine learning.
 - (This may involve removing missing or invalid data, handling outliers, and encoding categorical variables.)
- Normalizing the data can help to scale the features so that they are on the same scale.
 (This can be important for algorithms that use distance measures, as features on different scales can dominate the distance measure.)
- Preprocessing techniques such as feature selection and feature extraction can help to reduce the dimensionality of the data.
 - (This may improve the performance of the model and reduce the risk of overfitting.)
- Preprocessing techniques such as feature selection can help to identify the most important features in the data
 - (This can make the model more interpretable and easier to understand.)



Data Preprocessing

Feature Scaling

Normalization (MinMaxScaler)	Standardization (StandardScaler)		
$X \triangleq \frac{X - X.\min()}{X.\max() - X.\min()}$	$X \triangleq \frac{X - \mu}{\sigma}$		
▲ No assumption on data distribution	▲ More recommended when following normal distribution		



Data Preprocessing Template

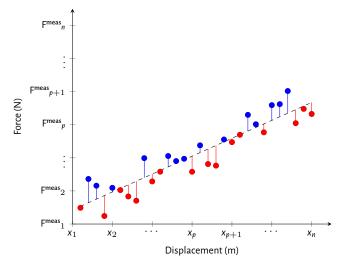
```
from sklearn.preprocessing import StandardScaler
sc = StandardScaler()
X_train = sc.fit_transform(X_train)
X_test = sc.transform(X_test)
```







This kind of supervised learning deals with labelled data. A subset of this data is used later to predict in continuous form. Regression problems involve tasks where the outputs form generally a set of real numbers. They oftenly follow linear formats.





Consider the example of a spring. Our main goal is to determine the stiffness k of this spring, given some experimental data. The mathematical model (Hooke's law):

$$F = kx (1)$$

Restoring force is proportional to displacement.

Table: Measurements of couple (x_i, F^{meas_i})

ĺ	Xi	<i>x</i> ₁	 Χp	 Xn
	F ^{meas} ;	F ^{meas} 1	 F ^{meas} _p	 F ^{meas} n

$$F^{\text{meas}}_{i} = F_{i} + \varepsilon_{i}$$

= $kx_{i} + \varepsilon_{i}$, (2)

where F_i denotes the unknown real value of the force applied to the spring. In order to estimate the stiffness value k, we can consider the quadratic criterion:

$$\mathcal{J} = \sum_{i=1}^{n} \varepsilon_{i}^{2}$$
$$= \sum_{i=1}^{n} (F^{\text{meas}}_{i} - kx_{i})^{2}$$



$$\frac{\partial \mathcal{J}}{\partial k} = 0 \tag{3}$$

$$2\sum_{i=1}^{n} (F^{\text{meas}}_{i} - kx_{i}) \sum_{i=1}^{n} \frac{\partial (F^{\text{meas}}_{i} - kx_{i})}{\partial k} = 0$$
$$\sum_{i=1}^{n} (F^{\text{meas}}_{i} - kx_{i}) \sum_{i=1}^{n} x_{i} = 0$$

$$\sum_{i=1}^{n} F^{\text{meas}}_{i} x_{i} = k \sum_{i=1}^{n} x_{i}^{2} \iff \hat{k} = \frac{\sum_{i=1}^{n} F^{\text{meas}}_{i} x_{i}}{\sum_{i=1}^{n} x_{i}^{2}}$$

$$\hat{k} = \frac{\sum_{i=1}^{n} \mathsf{F}^{\mathsf{meas}_{i}} x_{i}}{\sum_{i=1}^{n} x_{i}^{2}}$$



Simple Linear Regression

Training the Simple Linear Regression model on the Training set

```
from sklearn.linear_model import LinearRegression
```

```
lr = LinearRegression()
lr.fit(X_train, y_train)
```

LinearRegression()

Predicting the Test set results

```
y_pred = lr.predict(X_test)
```





The notebook is available at https://github.com/a-mhamdi/mlpy/ \rightarrow Codes \rightarrow Python \rightarrow simple-linear-regression.ipynb



Training the multiple linear regression model on the training set

```
[]: from sklearn.linear_model import LinearRegression
```

```
[]: lr = LinearRegression()
lr.fit(X_train, y_train)
```

[]: LinearRegression()

Making predictions using the X test set and comparison

```
[ ]: y_pred = lr.predict(X_test)
```





The notebook is available at https://github.com/a-mhamdi/mlpy/ \rightarrow Codes \rightarrow Python \rightarrow multiple-linear-regression.ipynb



Consider the following multivariable equation:

$$y = \theta_1 x_{(1)} + \theta_2 x_{(2)} + \dots + \theta_m x_{(m)}$$
 (4)

For a particular single measurement, eq. (4) can be updated as

$$y_k = \theta_1 x_{(1,k)} + \theta_2 x_{(2,k)} + \dots + \theta_m x_{(m,k)} + \varepsilon_k$$
 (5)

We denote hereafter by θ the vector $\begin{vmatrix} v_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{vmatrix}$. The function y_k becomes:

$$y_k = \underbrace{\left[x_{(1,k)}, x_{(2,k)}, \cdots, x_{(m,k)}\right]}_{X_L^T} \theta + \varepsilon_k \tag{6}$$

We assume that we have n measurements for y. Then we can transform the previous equation into

$$y = X\theta + \varepsilon, \tag{7}$$

where
$$\mathbf{y}^T = [y_1, y_2, \cdots, y_n], X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix}$$
 and $\boldsymbol{\varepsilon}^T = [\boldsymbol{\varepsilon}_1, \, \boldsymbol{\varepsilon}_2, \, \cdots, \, \boldsymbol{\varepsilon}_n].$



An estimate of $\hat{\theta}$ is given by

$$\hat{\theta} = (X^T X)^{-1} X^T y$$



 X^TX is not invertible (singular/degenerate)

▼ Redundant Features

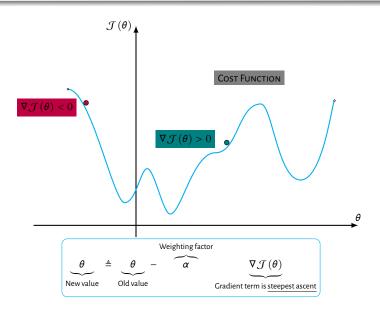
Some features are linearly dependent, i.e, \exists some $x_p \propto \text{some } x_l$.

▼ Too many features

Fewer observations compared to the number of features, i.e, $m \ge n$.

- Delete some features
- ▲ Add extra observations

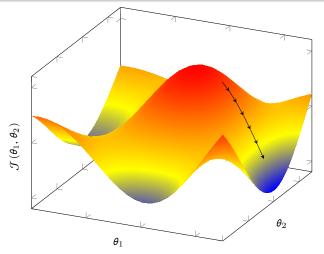






32/78 A. MHAMDI An Introduction To ML

Gradient Descent (2/3)



- ① Start with some random values of θ_1 and θ_2
- ② Keep changing $heta_1$ and $heta_2$ to reduce $\mathcal{J}(heta_1,\, heta_2)$ until we hopefully end up at minimum



Gradient Descent (3/3)

The linear regression is given by:

$$\theta_0 \triangleq \theta_0 + \alpha \frac{1}{n} \sum_{k=1}^{n} (y_k - h_\theta(x_k)) x_{(0,k)}$$

$$\theta_{1} \triangleq \theta_{1} + \alpha \frac{1}{n} \sum_{k=1}^{n} (y_{k} - h_{\theta}(x_{k})) x_{(1,k)}$$

$$\theta_{m} \triangleq \theta_{m} + \alpha \frac{1}{n} \sum_{k=1}^{n} (y_{k} - h_{\theta}(x_{k})) x_{(m,k)}$$

The hyperparameter α is the learning rate.

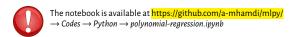


(8)

An Introduction To ML 34/78 Δ ΜΗΔΜΟΙ

```
poly_reg = PolynomialFeatures(degree=4)
X_poly = poly_reg.fit_transform(X)
print(X_poly[:5])
lr_2 = LinearRegression()
lr_2.fit(X_poly, y)
```





from sklearn.preprocessing import PolynomialFeatures



Task #2

The yield y of a chemical process is a random variable whose value is considered to be a linear function of the temperature x. The following data of corresponding values of x and yis found:

Temperature in °C (x)	0	25	50	75	100
Yield in grams (y)	14	38	54	76	95

The linear regression model $y = \theta_0 + \theta_1 x$ is used. Determine the values of θ_0 , θ_1 using normal equation.

$$y = \begin{bmatrix} 14\\38\\54\\76\\95 \end{bmatrix} \text{ and } X = \begin{bmatrix} 1&0\\1&25\\1&50\\1&75\\1&100 \end{bmatrix} \implies X^{T}X = \begin{bmatrix} 5&250\\250&18750 \end{bmatrix}$$
$$\hat{\theta} = \begin{bmatrix} \hat{\theta}_{0}\\\hat{\theta}_{1} \end{bmatrix} = \begin{bmatrix} 15.4\\0.8 \end{bmatrix}$$



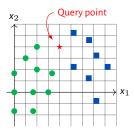
36/78 Δ ΜΗΔΜΟΙ An Introduction To MI

Code implementation



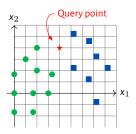
```
1
    import numpy as np
    import matplotlib.pyplot as plt
2
3
    X = np.array([[1, 0], [1, 25], [1, 50], [1, 75], [1, 100]])
    y = np.array([14, 38, 54, 76, 95])
5
6
    # NORMAL EQUATION
    theta_ne = np.linalg.inv(X.T @ X) @ X.T @ y
8
9
    # GRADIENT DESCENT
10
    theta_gd = np.zeros(shape=(2, 1001))
11
    theta_gd[:, 0] = np.array([10, .5])
12
    cost = \Pi
13
    for k in range(1000):
14
        eps = y-(X @ theta_gd[:, k])
15
        cost.append(1/10*(eps @ eps))
16
        theta_gd[:, k+1] = theta_gd[:, k] + .003/5*(eps @ X)
17
18
    plt.plot(theta_gd[0, :], label=r'$\hat{\theta}_0$')
19
    plt.plot(theta_gd[1, :], label=r'$\hat{\theta}_1$')
20
    plt.legend(); plt.grid(); plt.show()
21
22
    plt.plot(cost); plt.grid(); plt.show()
23
```

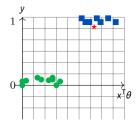
Classification is a type of supervised machine learning algorithm. A model is trained on a set of labeled data, where each data point is associated with a known class or category. The goal of the algorithm is to learn the relationship between the input features x and the corresponding output classes y, so that it can accurately predict the class of new, unseen query points.





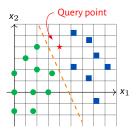
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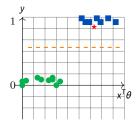






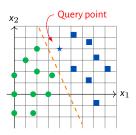
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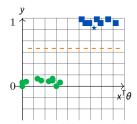






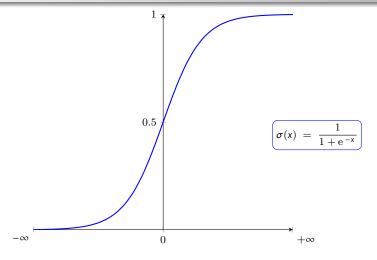
Classification is a type of supervised machine learning algorithm. A model is trained on a set of *labeled data*, where each data point is associated with a known <u>class</u> or category. The goal of the algorithm is to learn the relationship between the *input features x* and the corresponding *output classes y*, so that it can accurately predict the class of new, unseen query points.







Logistic or S-shaped function σ



- $\boldsymbol{\Delta}$ $\boldsymbol{\sigma}$ squashes range of distance from $]-\infty, +\infty[$ to [0, 1]
- $oldsymbol{\sigma}$ is differentiable and easy to compute: $\left(\dot{\sigma} = \sigma imes (1-\sigma)
 ight)$



$$y = \sigma \left(\theta_1 x_{(1)} + \theta_2 x_{(2)} + \dots + \theta_m x_{(m)}\right)$$
$$y = \frac{1}{1 + e^{-x^T \theta}}$$

Hypothesis

Considering
$$h_{\theta}\left(x\right) = \frac{1}{1 + \mathrm{e}^{-x^{T}\theta}}$$
 yields $h_{\theta}\left(x_{k}\right) = \frac{1}{1 + \mathrm{e}^{-x_{k}^{T}\theta}}$

$$\vdots$$

$$\theta_{m} \triangleq \theta_{m} + \alpha \frac{1}{n} \sum_{k=1}^{n} \left(y_{k} - h_{\theta}\left(x_{k}\right)\right) x_{(m,k)}$$



Training the logistic regressor

```
[]: from sklearn.linear_model import LogisticRegression
```

```
clf = LogisticRegression(random_state=123)
clf.fit(X_train, y_train)
```

[]: LogisticRegression(random_state=123)

Predicting the test set results

```
[]: y_pred = clf.predict(X_test)
```





The notebook is available at https://github.com/a-mhamdi/mlpy/ \rightarrow Codes \rightarrow Python \rightarrow logistic-regression.ipynb



Evaluation metrics

Accuracy

Precision

Recall

f1-score

Accuracy denotes the ratio of how much we got right over all cases.

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN}$$

Precision designates how much positives do we get right over all positive predictions:

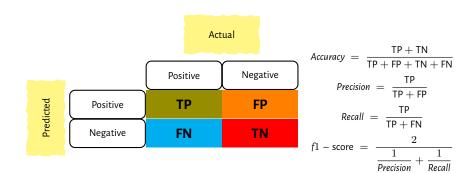
$$Precision = \frac{TP}{TP + FP}$$

Recall is the ratio of how much positives we got right over all actual positive cases:

$$Recall = \frac{TP}{TP + FN}$$

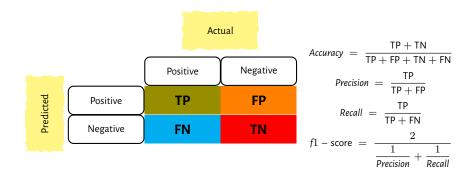
f1 - score denotes the Harmonic Mean of Precision & Recall







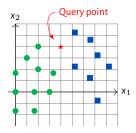
Confusion Matrix







k-Nearest Neighbors (1/5)



► Evelyn Fix and Joseph Hodges, 1951 Thomas Cover, 1966

Algorithm Summary Construction

1: **procedure** How DOES k-NN WORK? (Finding Nearest Neighbors) Input: A query point;

Output: Assign a class label to that point.

- Define how many neighbors will be checked to classify the specific query point; 2:
- Compute the distance d(x; y) of the query point to other data points; 3.
- Count the number of the data points in each category;
- Assign the query point to the class with most frequent neighbors.
- 6: end procedure



$$d(x; y) = \left(\sum_{i=1}^{n} |y_i - x_i|^p\right)^{1/p}$$

(p=1)

$$d(x; y) = \sum_{i=1}^{n} |y_i - x_i|$$

$$d(x; y) = \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2}$$

Task #3

Let be the following coordinate points:

$$A(1, 6); B(2, 6); C(3, 1); D(4, 2); E(6, 0); F(7, 5); C(7, 3); H(10, 3); I(-4, -1)$$

Using the Euclidean distance, what are the two closest neighbors of point P(5, 5)?

$$d(A; P) = \sqrt{17} \approx 4.12$$
 $d(B; P) = \sqrt{10} \approx 3.16$ $d(C; P) = \sqrt{20} \approx 4.47$

$$d(D; P) = \sqrt{10} \approx 3.16$$
 $d(E; P) = \sqrt{26} \approx 5.1$ $d(F; P) = \sqrt{4} = 2$

$$\textit{d(G; P)} = \sqrt{8} \approx 2.83 \qquad \textit{d(H; P)} = \sqrt{29} \approx 5.38 \qquad \textit{d(I; P)} = \sqrt{117} \approx 10.82$$



45/78

k-Nearest Neighbors (3/5)

```
from math import sqrt
def dds(a, b): # `a` and `b` are coordinates of some point
    d_squared = (a-5)**2+(b-5)**2
    return (d_squared, sqrt(d_squared))

dds(1, 6) # Point `A`
dds(2, 6) # Point `B`
```



k-Nearest Neighbors (4/5)

Task #4ª

From Prof Winston's hook

We try to predict the color of a fruit according to its width (w) and height (h). The following training data is available:

Fruit	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F ₈
w	2	5	2	6	1	4	2	6
h	6	6	5	5	2	2	1	1
Color	Red	Yellow	Orange	Purple	Red	Blue	Violet	Green

The goal here is to study the influence of neighbors on the color property of a fruit. Let ${\it U}$ be the new fruit of width ${\it w}=1$ and height ${\it h}=4$

- What is its color if we consider 1 neighbor?
- What is its color if we consider 3 neighbors?
- **Q** Rather than majority voting, we would like to consider the vote of neighbors weighted by the distance. Each neighbor votes according to a weight inversely proportional to the square of its distance: $\frac{1}{d^2}$. We take 3 neighbors, what is the color of *U*? Compare your results to those in question 2.



k-Nearest Neighbors (5/5)

$$d(U; F_1) = \sqrt{5} \approx 2.24 \qquad d(U; F_2) = \sqrt{20} \approx 4.47 \qquad d(U; F_3) = \sqrt{2} \approx 1.41$$

$$d(U; F_4) = \sqrt{26} \approx 5.1 \qquad d(U; F_5) = \sqrt{4} = 2 \qquad d(U; F_6) = \sqrt{13} \approx 3.6$$

$$d(U; F_7) = \sqrt{10} \approx 3.16 \qquad d(U; F_8) = \sqrt{34} \approx 5.83$$

- Color of *U* is Orange because $d(U; F_3)$ is the smallest.
- ② Color of U is Red: F_1 and F_5 (+2 to Red class), F_3 (+1 to Orange class)
- Color of U is Orange

$$S(\text{Red}) = \frac{1}{d^2(\textit{U}; \, \textit{F}_1)} + \frac{1}{d^2(\textit{U}; \, \textit{F}_5)} = 0.45 \\ S(\text{Orange}) = \frac{1}{d^2(\textit{U}; \, \textit{F}_3)} = 0.5$$

```
from math import sqrt
def dds(w, h): # `w` and `h` are width and height of some fruit
    d_squared = (w-1)**2+(h-4)**2
    return (d_squared, sqrt(d_squared))

dds(2, 6) # Fruit `F_1`
dds(5, 6) # Fruit `F 2`
```



48/78 A. MHAMDI An Introduction To ML

Importing the classifier

```
[]: from sklearn.neighbors import KNeighborsClassifier
```

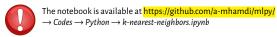
Training the K-NN model on the training set

```
clf = KNeighborsClassifier(n_neighbors, metric, p)
clf.fit(X_train, y_train)
```

Predicting the test set results

```
[]: y_pred = clf.predict(X_test)
```







Rule of Thumb to Choose k

k is even if the number of classes is odd

k is odd if the number of classes is even

k is an important hyperparameter that can affect the performance of the model.

- Larger values of k will result in a smoother decision boundary, which can lead to a more generalized model.
- Smaller values of k will result in a more complex decision boundary, which can lead to a model that is more prone to overfitting.
- 1 The optimal value of K may depend on the specific dataset and the characteristics of the data.



50/78 A. MHAMDI An Introduction To ML

Outroduction

Method		Pros		Cons
Logistic Regression	A	Probabilistic	▼	Almost linearly separable data
K-NN	A	Simple Fast Efficient	*	Number of neighbors <i>k</i> Detecting outliers ²





- An overview
- Supervised Learning
- Unsupervised Learning
- Artificial Neural Network
- ML Landscape through Quizzes



K-Means Clustering (1/3)

The algorithm K-Means allows to display regularities or patterns in unlabeled data.

- ▶ The term 'means' refers to averaging the data when computing each centroid;
- A centroid is the arithmetic mean of all the data points belonging to a particular cluster.

This technique identifies a certain number of centroids within a data set. The algorithm then allocates every data point to the nearest cluster as it attempts to keep the clusters as small as possible. At the same time, K-Means attempts to keep the other clusters as different as possible.



53/78 A. MHAMDI An Introduction To ML

K-Means Clustering (2/3)





K-Means Clustering (3/3)

Algorithm Summary Construction

- procedure How DOES K-MEANS WORK? (Discovering similarities)
 Input: Unlabeled data sets;
 - Output: Grouping into clusters.
- 2: Define how many clusters will be used to group the data sets;
- 3: Initialize all the coordinates of the k cluster centers
- 4: repeat
- 5: Assign each point to its nearest cluster;
- 6: Update the centroids coordinates;
- 7: **until** No changes to the centers of the clusters
- 8: Assign new cases to one of the clusters
- 9: end procedure



Task #5ª

"From 'Machine Learning' course on 'Coursera

Of the following examples, which would you address using an <u>unsupervised learning</u> algorithms? (Check all that apply.)

- Given email labeled as spam/not spam, learn a spam filter
- Given a set of news articles found on the web, group them into set of articles about the same story
- Given a database of customer data, automatically discover market segments and group customers into different market segments
- Given a dataset of patients diagnosed as either having diabetes or not, learn to classify new patients as having diabetes or not.



56/78 A. MHAMDI An Introduction To ML

Task #5ª

From 'Machine Learning' course on 'Coursera

Of the following examples, which would you address using an <u>unsupervised learning</u> algorithms? (Check all that apply.)

- Given email labeled as spam/not spam, learn a spam filter
- Given a set of news articles found on the web, group them into set of articles about the same story
- Given a database of customer data, automatically discover market segments and group customers into different market segments
- Given a dataset of patients diagnosed as either having diabetes or not, learn to classify new patients as having diabetes or not.



56/78 A. MHAMDI An Introduction To ML

"Credit: Shokoufeh Mirzaei, PhD

$$A(2, 10); B(2, 5); C(8, 4); D(5, 8); E(7, 5); F(6, 4); G(1, 2)$$
 and $H(4, 9).$

- Initial cluster centers are: $\alpha(2, 10)$; $\beta(5, 8)$ and $\gamma(1, 2)$
- ► The distance between two points: $M(x_m, y_m)$ and $N(x_n, y_n)$ is defined as

$$d(M; N) = |x_m - x_n| + |y_m - y_n|$$

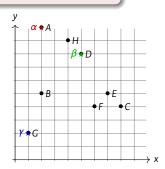


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Task #6^a

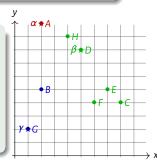
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$$d(M; N) = |x_m - x_n| + |y_m - y_n|$$

Point	$\alpha(2, 10)$	$\beta(5, 8)$	$\gamma(1, 2)$	#
A(2, 10)	0	5	9	1
B(2, 5)	5	6	4	3
C(8, 4)	12	7	9	2
D(5, 8)	5	0	10	2
E(7, 5)	10	5	9	2
F(6, 4)	10	5	7	2
G(1, 2)	9	10	0	3
H(4, 9)	3	2	10	2





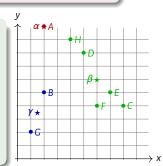
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D(5, 8)	5	0	10	2
E(7, 5)	10	5	9	2
F(6, 4)	10	5	7	2
G(1, 2)	9	10	0	3
H(4, 9)	3	2	10	2
$\alpha(2,$	10) \(\beta\)	(6, 6)	γ(1.5, 3.5	((i



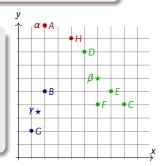


$$\textit{A}(2,\,10);\,\textit{B}(2,\,5);\textit{C}(8,\,4);\textit{D}(5,\,8);\textit{E}(7,\,5);\textit{F}(6,\,4);\textit{G}(1,\,2)\,\,\text{and}\,\,\textit{H}(4,\,9).$$

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C(8, 4)	12	4	7	2
D(5, 8)	5	3	8	2
E(7, 5)	10	2	7	2
F(6, 4)	10	2	5	2
G(1, 2)	9	9	2	3
H(4, 9)	3	5	8	1





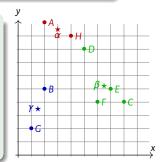
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E(7, 5)	10	2	7	2
F(6, 4)	10	2	5	2
G(1, 2)	9	9	2	3
H(4, 9)	3	5	8	1
$\alpha(3.9)$	β	5 5 25)	v(1.5.3.5)	





Task #6ª

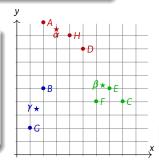
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$$d(M; N) = |x_m - x_n| + |y_m - y_n|$$

Point	$\alpha(3, 9.5)$	$\beta(6.5, 5.25)$	$\gamma(1.5, 3.5)$	#
A(2, 10)	1.5	9.25	7	1
B(2, 5)	5.5	4.75	2	3
C(8, 4)	10.5	2.75	7	2
D(5, 8)	3.5	4.25	8	1
E(7, 5)	8.5	0.75	7	2
F(6, 4)	8.5	1.75	5	2
G(1, 2)	9.5	8.75	2	3
H(4, 9)	1.5	6.25	8	1





Task #6ª

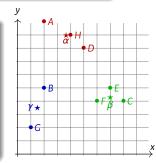
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E(7, 5)	8.5	0.75	7	2
F(6, 4)	8.5	1.75	5	2
G(1, 2)	9.5	8.75	2	3
H(4, 9)	1.5	6.25	8	1
α (3	.67, 9)	$\beta(7, 4.3)$	γ(1.5, 3.5)	





Task #6^a

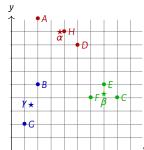
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$$d(M; N) = |x_m - x_n| + |y_m - y_n|$$

Point	$\alpha(3.67, 9)$	$\beta(7, 4.3)$	$\gamma(1.5, 3.5)$	#
A(2, 10)	2.67	10.7	7	1
B(2, 5)	5.67	5.7	2	3
C(8, 4)	9.33	1.3	7	2
D(5, 8)	2.33	5.7	8	1
E(7, 5)	7.33	0.7	7	2
F(6, 4)	7.33	1.3	5	2
G(1, 2)	9.67	8.3	2	3
H(4, 9)	0.33	7.7	8	1





Task #6ª

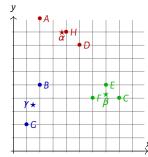
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G(1, 2)	9.67	8.3	2	3
H(4, 9)	0.33	7.7	8	1
$\alpha(3.$	67, 9) β	(7, 4.3)	$\gamma(1.5, 3.5)$	





Import KMeans class

[]: from sklearn.cluster import KMeans

Training the K-Means model on the dataset

kmeans = KMeans(n_clusters, init, random_state)
y_pred = kmeans.fit_predict(X)







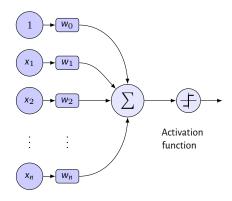


- An overview
- Supervised Learning
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- ML Landscape through Quizzes



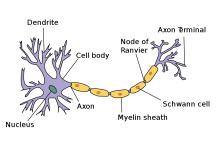
Fundamental unit of a neural network (1/3)

Artificial neuron



Inputs Weights

Biological neuron



https://id.wikipedia.org/wiki/Sel_saraf



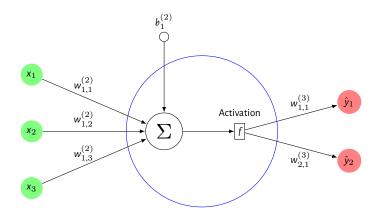
Fundamental unit of a neural network (2/3)

Task #7 Compute the output of the following neuron.

$$y = sign(1 \times -1 + 2.6 \times 0.7 + 0.7 \times 2.5 - 2 \times 0.15 + 0.8 \times 1.3) = 1$$

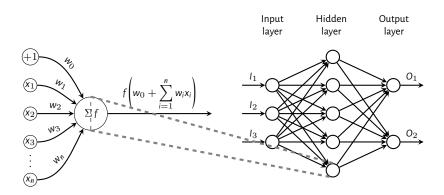


Fundamental unit of a neural network (3/3)





Multilayer Perceptron (MLP)



Task #8

For the above structure, determine how many parameters are to be adjusted.

Params # =
$$5 \times 3 + 5 + 2 \times 5 + 2 = 32$$



- ✓ Design a structure
- ✓ Specify a loss function to minimize
- ✓ Optimize using gradient descent
 - ① Feedforward propagation (matrix multiplication and point-wise activation)
 - ② Back propagation (multivariate chain rule)
 - ③ Update the weights accordingly

 Input

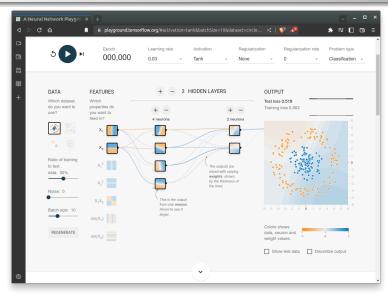
 Synaptic Weights

 Compare

UPDATE WEIGHTS



Tinker with a neural network





https://playground.tensorflow.org/

```
[]: from keras.models import Sequential
    from keras.layers import Dense

[]: clf = Sequential()
    ndim = X_train.shape[1]
    clf.add(Dense(units=8, activation="relu", input_dim=ndim))
    clf.add(Dense(units=4, activation="relu"))
    clf.add(Dense(units=4, activation="relu"))
    clf.add(Dense(units=1, activation="relu"))
```

```
[]: clf.compile(optimizer="adam", loss="binary_crossentropy", metrics=["accuracy"])
```

```
[ ]: clf.fit(X_train, y_train, batch_size=16, epochs=32);
```





The notebook is available at https://github.com/a-mhamdi/mlpy/ \rightarrow Codes \rightarrow Python \rightarrow artificial-neural-network.ipynb

List of available optimizers

Here is a list of some common optimizers for artificial neural networks:

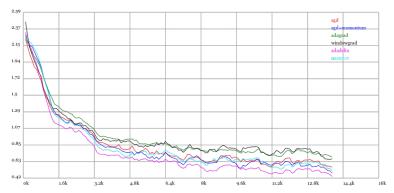
$$\boxed{ \Delta \hat{\mathcal{W}} \triangleq \mathcal{F} \left(\nabla \underbrace{\mathcal{J} \left(\hat{\mathcal{W}} \right)}_{\text{Loss Function}} \right) \equiv \underbrace{ \left(\hat{\mathcal{W}} \triangleq \hat{\mathcal{W}} + \mathcal{F} \left(\nabla \mathcal{J} \left(\hat{\mathcal{W}} \right) \right) \right) }_{\text{Loss Function}} \nabla \mathcal{J} \left(\hat{\mathcal{W}} \right) = \begin{bmatrix} \frac{\partial \mathcal{J}}{\partial \hat{\mathcal{W}}_0} \\ \vdots \\ \frac{\partial \mathcal{J}}{\partial \hat{\mathcal{W}}_n} \end{bmatrix}$$

SGD+MOMENTUM ADAGRAD

WINDOWGRAD ADADELTA NESTEROV



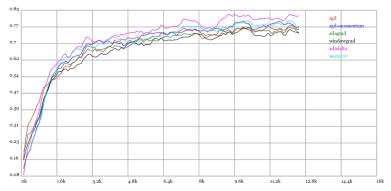
Effect of optimizer on loss values



https://cs.stanford.edu/people/karpathy/convnetjs/demo/trainers.html



Effect of optimizer on testing accuracy values



https://cs.stanford.edu/people/karpathy/convnetjs/demo/trainers.html





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MCQ (1/8)

What is Machine Learning (ML)?

- × The selective acquisition of knowledge through the use of computer programs
- × The selective acquisition of knowledge through the use of manual programs
- ✓ The autonomous acquisition of knowledge through the use of computer programs
 - imes The autonomous acquisition of knowledge through the use of manual programs

Successful applications of ML

- × Learning to recognize spoken words
- × Learning to drive an autonomous vehicle
- × Learning to classify new astronomical structures
- × Learning to play world-class backgammon
- √ All of the above

Features of Machine Learning are ...

- √ Automation
- √ Improved customer experience
- √ Business intelligence

... is the machine learning algorithm that can be used with labeled data.

- √ Regression algorithm
- × Clustering algorithm
- × Association algorithm



MCQ (2/8)

- Replace missing values with mean/median/mode helps to handle missing or corrupted data in a dataset. True/False?
 - ✓ True
 - × False
- Which among the following algorithms are used in Machine learning?
 - √ Naive Bayes
 - √ Support Vector Machines
 - √ k-Nearest Neighbors
- The term machine learning was coined by ...
 - × James Gosling
 - ✓ Arthur Samuel
 - × Guido van Rossum
 - × None of the above
- The Real-world machine learning use cases are
 - √ Digital assistants
 - √ Chatbots
 - √ Fraud detection



MCQ (3/8)

- Machine learning approaches can be traditionally categorized into ... categories.
 - √ 3
 - \times 4
 - \times 7
 - \times 9
- ... is a part of machine learning that works with neural networks.
 - × Artificial intelligence
 - ✓ Deep learning
 - × All of the above
 - × None of the above
- The supervised learning problems can be grouped as ...
 - × Regression problems
 - × Classification problems
 - √ All of the above
 - × None of the above
- The unsupervised learning problems can be grouped as ...
 - × Clustering
 - × Association
 - √ All of the above
 - × None of the above



MCQ (4/8)

- Overfitting is a type of modelling error which results in the failure to predict future observations effectively or fit additional data in the existing model. Yes/No?
 - × Probably
 - √ Yes
 - × No
 - × Can not say
- ... is the scenario when the model fails to decipher the underlying trend in the input data.
 - √ Underfitting
 - × Overfitting
 - × All of the above
 - × None of the above
- The categories in which Machine learning approaches can be traditionally categorized are ...
 - × Supervised learning
 - × Unsupervised learning
 - \times Reinforcement learning
 - √ All of the above
- In general, to have a well-defined learning problem, we must identity which of the following
 - × The class of tasks
 - × The measure of performance to be improved
 - × The source of experience
 - √ All of the above



MCQ (5/8)

- The average positive difference between computed and desired outcome values
 - × Root Mean Squared Error
 - × Mean Squared Error
 - × Mean Absolute Error
 - √ Mean Positive Error
- ... is used as an input to the machine learning model for training and prediction purposes.
 - × Target variable
 - √ Feature vector
 - × All of the above
 - × None of the above
- Simple regression assumes a ... relationship between the input attribute and output attribute.
 - √ linear
 - × quadratic
 - × reciprocal
 - × inverse
- The correlation between the number of years an emplyee has worked for a company and the salary of the employee is 0.75. What can be said about employee salary and years worked?
 - × There is no relationship between salary and years worked.
 - ✓ Individuals that have worked for the company the longest have higher salaries.
 - × Individuals that have worked for the company the longest have lower salaries.
 - × The majority of employees have been with the company a long time.



MCQ (6/8)

- Which machine learning models are trained to make a series of decisions based on the rewards and feedback they receive for their actions?
 - × Supervised learning
 - × Unsupervised learning
 - √ Reinforcement learning
 - × All of the above
- Which of the following is not a type of supervised learning?
 - × Classification
 - × Regression
 - √ Clustering
 - × None of the above
- As the amount of training data increases
 - × Training error usually increases and generalization error usually increases
 - √ Training error usually increases and generalization error usually decreases
 - × Training error usually decreases and generalization error usually decreases
 - × Training error usually decreases and generalization error usually increases
- Which of the following are not classification tasks?
 - × Find the gender of a person by analyzing his writing style
 - × Detect Pneumonia from Chest X-ray images
 - ✓ Predict the price of a house based on floor area, number of rooms, etc.
 - × Predict whether there will be abnormally heavy rainfall next year



MCQ (7/8)

- Which of the following is a categorical feature?
 - × Height of a person
 - × Price of petroleum
 - × Amount of rainfall in a day
 - √ Mother tongue of a person
- What is the use of validation dataset in Machine Learning?
 - × To train the machine learning model.
 - ✓ To tune the hyperparameters of the machine learning model
 - × To evaluate the performance of the machine learning model
 - × None of the above
- Which of the following criteria is typically used for optimizing in linear regression.
 - × Maximize the number of points it touches.
 - × Minimize the number of points it touches.
 - ✓ Minimize the squared distance from the points.
 - × Minimize the maximum distance of a point from a line.
- For two runs of K-Mean clustering, is it expected to get same clustering results?
 - × Yes
 - ✓ No



MCQ (8/8)

- Logistic Regression is used for ...
 - × regression purposes
 - √ classification purposes
 - × all of the above
 - × none of the above
- Which of the following methods do we use to best fit the data in Logistic Regression?
 - × Least Squared Error
 - √ Maximum Likelihood
 - × laccard distance
- When there is noise in data, which of the following options would improve the performance of the k-NN algorithm?
 - ✓ Increase the value of k
 - \times Decrease the value of k
 - \times Changing value of k will not change the effect of the noise
 - × None of these



Link Bundle

https://karpathy.ai/ https://colah.github.io/posts/2014-03-NN-Manifolds-Topology/
https://yann.lecun.com/ https://www.ibm.com/downloads/cas/GB8ZMQZ3
https://www.hackingnote.com/ https://stanford.edu/shervine/teaching/
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