1. Introduction

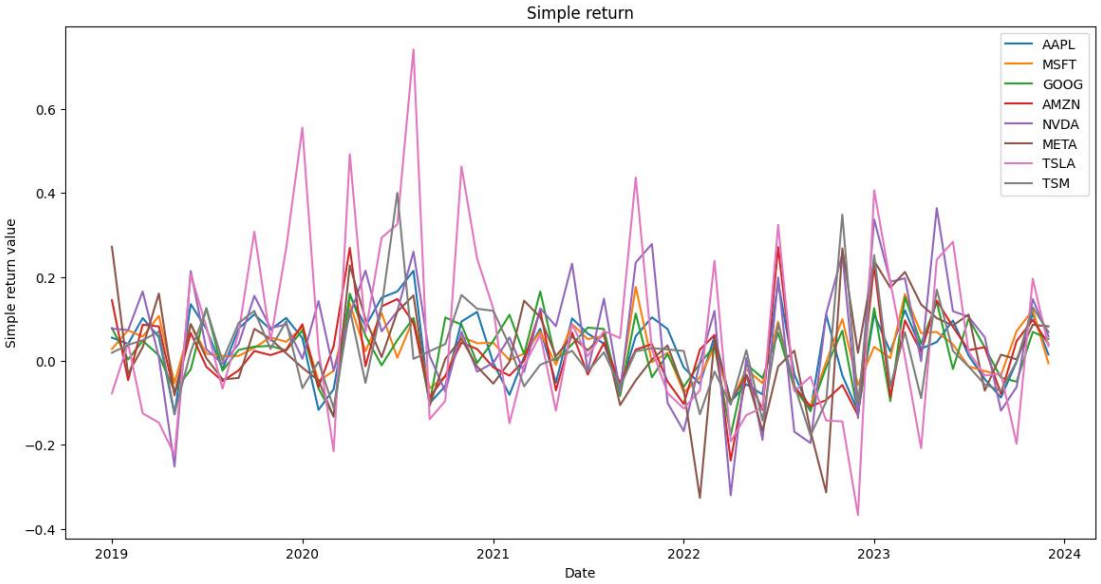
This report focuses on eight specific technology companies—Apple (AAPL), Microsoft (MSFT), Alphabet (GOOG), Amazon (AMZN), Nvidia (NVDA), Meta (META), Tesla (TSLA), and Taiwan Semiconductor Manufacturing Company (TSMC). Under the condition that short selling is prohibited, it establishes two benchmarks: the tangency portfolio and the market portfolio, to evaluate the active portfolio. The report also estimates the risk associated with each portfolio and compares the differences between them.

The date of sample data is forming December 2018 to December 2023, with monthly intervals. These data are only used as training model data and do not perform model verification operations. In addition, the methodology includes Initial processing, Tangency portfolio, Black-Litterman Model and Value at Risk (VaR) and Conditional Value at Risk (CVaR) to analyze the performance of each portfolio. The results show that the active portfolio can achieve lower risks than the tangency portfolio and higher returns than the market portfolio.

This report is composed of four parts. The first paragraph of the introduction outlines the purpose and background. The first part lists the input information required for the report. The second part explains the steps and purposes of each method. The third part describes the performance of models under these data sets. The final part summarizes all the data and lists areas that need improvement.

2. Original data

This report uses the eight company's stock price start with 2018/12/01 end in 2023/12/01 to complete calculations. Moreover, it uses the adjust close stock price rather than simple close stock price to avoid dividends, stock splits and right offerings. Then we use the adjust close stock price to generate the simple return. Graphs 1 shows the trend of the simple returns.



GRAPH 1. Simple Return

Graph 1 displays the trend of simple returns for stocks from eight tech companies from the end of 2018 to the end of 2023. All companies exhibit noticeable triannual cyclical fluctuations in their stock returns, indicating similar market influences. These patterns are crucial for understanding the periodic performance of each company's stock and for devising portfolio strategies.

Simple return								
	AAPL	MSFT	GOOG	AMZN	NVDA	META	TSLA	TSM
Maximum	21.44%	17.63%	16.51%	27.06%	36.34%	27.16%	74.15%	40.00%
Minimum	-10.74%	-17.67%	-23.75%	-32.02%	-32.63%	-36.73%	-17.74%	0.00%
Median	4.93%	2.54%	2.89%	2.45%	6.97%	1.84%	2.72%	2.28%
Mean	3.11%	2.49%	1.98%	1.62%	5.61%	2.39%	6.24%	2.48%
STD	8.64%	6.35%	7.80%	9.54%	14.25%	11.90%	22.15%	10.79%

Table 1. Simple returns analysis

According Table1, TSLA leads the way with the highest maximum return of 74.15% and standard deviation STD of 22.15%. NVIDIA NVDA performed solidly, with the highest median return of 6.97% and a respectable average return of 5.61%. AMZN and META experienced significant lows with minimum returns of -32.02% and -36.73% respectively. TSM performed outstandingly, did not suffer losses, and maintained a minimum return rate of 0.00%. Other stocks such as AAPL, MSFT, and GOOG have lower standard deviations ranging from 6.35% to 8.64%, indicating greater stability.

3.Methodology

3.1 Initial processing (Mean return, Covariance matrix)

This First step begins by converting the adjusted price data from the end of 2018 to the end of 2023 into simple returns. Subsequently, we calculate the average return using these simple returns. Next, use the simple returns and the average returns to form the covariance matrix (Benninga, 2014). Within this matrix, the elements on the diagonal represent the variance of each stock itself, while the off-diagonal elements represent the covariance between different stocks. Furthermore, the covariance matrix aids in constructing a diversified investment portfolio to mitigate risk.

3.2 Tangency Portfolio

According to, the covariance matrix provided in section 3.1 and the given annual risk-free interest rate of 5.386% that generate the monthly risk-free rate is 0.438%. Using this information, the Tangency Portfolio is derived according to the following formula:

$$P = \frac{V^{-1}(E(R) - R_F)}{1^T V^{-1}(E(R) - R_F)}$$

P: portfolio weights vector.

V: covariance matrix of asset returns.

$E(R)$: expected returns vector.

R_F: Risk free rate.

1^T: refers to the transpose of a vector of ones.

3.3 Black-Litterman Model

The Black-Litterman model is a portfolio optimization framework that extends the foundational concepts of Modern Portfolio Theory (MPT) and the Capital Asset Pricing Model (CAPM). The Black-Litterman model offers the adaptability to integrate the market's equilibrium state with the investor's supplementary views of the market (He & Litterman, 2002).

Within this context, the report incorporates λ as a modifier to adjust the impact of returns on the portfolio optimization process. In fact, it utilizes market information to

derive the expected excess return per unit of risk and the formula is as follows:

$$\lambda = \frac{E(R_P) - R_F}{\sigma_P^2}$$

σ_P^2 ： variance of the portfolio's returns.

$E(R_P)$: expected return of the portfolio.

R_F : $risk\ free\ rate.$

According to the requirement we expect the performance of AAPL and META to decline by 0.1% and 0.2% per month respectively. These views are fed into δ , where analyst return expectations are based on previously calculated market-implied expected returns. However, this action will affect the expected returns of other assets. Therefore, we adopt the following approach to calculate adjusted expected returns, considering two different perspectives:

$$E(R)_{Adjusted} = E(R)_{implied} + \theta\delta$$

θ: Black − Litterman Tracking Factor Matrix

$E(R)_{Adjusted}$: adjusted expected return.

$E(R)_{implied}$: original market − implied expected.

It should be noted that we apply Solver to modify θ to align with the anticipated forecasts for AAPL and META. The objective of this report is to minimize the absolute difference between the adjusted expected returns and the analyst-predicted expected returns.

3.4 Value at Risk & Conditional Value at Risk (VaR & CVaR)

VaR is calculated as the maximum potential loss allowed under a specific confidence level (Allen & Powell, 2007), and its formula is as follows (Hull, 2015, p.264):

$$VaR = -(q_\alpha^S \sigma + \mu)$$

$$T\ day\ VaR = 1\ day\ VaR \times \sqrt{T}$$

q_α^S: $\alpha\ percent\ quantile\ of\ the\ standard\ normal\ distribution.$

σ: $standand\ error.$

$\mu: mean.$

CVaR is calculated as the loss obtained beyond the VaR at a specific confidence level (Allen & Powell, 2007), and its formula is as follows (Hull, 2015, p.264):

$$CVaR = \frac{e^{\frac{-(q_\alpha^S)^2}{2}}}{\alpha(2\pi)^{\frac{1}{2}}}\sigma$$

$$T\ day\ CVaR = 1\ day\ CVaR \times \sqrt{T}$$

$q_\alpha^S: \alpha\ percent\ quantile\ of\ the\ standard\ normal\ distribution$

$\sigma: standand\ error.$

$\mu: mean.$

Based on the formulas mentioned above and standard deviation of the investment returns for the tangency portfolio, market portfolio, and adjusted portfolio respectively, with a mean of 0, we can compute the VaR and CVaR at a 95% confidence level with variance-covariance method that.

4. Results

The following section shows information about the requirements and is divided into 4 sections to show the performance of 3 portfolios using average return, standard deviation, Sharpe ratio, Var and CVaR indicators for comparison.

4.1 Tangency portfolio

This report has calculated the weights of the tangency portfolio for eight companies, as described in Section 3. These calculations are determined by the expected returns, which in turn are derived from the average of historical returns. After incorporating the constraint that short selling is not allowed, we obtain the Tangency portfolio weight. According to table 2, it can be observed that the portfolio is primarily concentrated in two stocks, MSFT and NVDA, which together account for nearly 83% of the weights. This indicates that under these conditions, they have a distinct advantage in terms of expected risk and return trade-off.

	Tangency portfolio	
	Weight	**ER**
AAPL	12.56%	3.11%
MSFT	43.04%	2.49%
GOOG	0.00%	1.98%
AMZN	0.00%	1.62%
NVDA	39.92%	5.61%
META	0.00%	2.39%
TSLA	4.48%	6.24%
TSM	0.00%	2.48%

Table 2. Tangency portfolio (Weight& Expect return)

4.2 Black-Litterman model

Based on Section 3 mentioned above, we divide the Black-Litterman model into two parts: we first explore the passive component before incorporating the active component.

4.2.1 Passive Portfolio (Market portfolio)

By the information given in the requirement, we have calculated the market portfolio weights as shown in the Table 3, which greatly differ from those of the tangency portfolio. The market weights exhibit more diversity, unlike the tangency portfolio. Following this, we use the weights, the risk-free rate, lambda, and the covariance matrix to derive the implied expected returns (Thomas Idzorek, 2007). Furthermore, the implied expected returns of the passive portfolio are very close to the actual market returns, approximately at 1.02%.

	Black-Litterman			
	Market portfolio		**Active portfolio**	
	weight	**ER**	**weight**	**ER**
AAPL	22.82%	1.00%	15.35%	0.90%
MSFT	22.17%	0.84%	26.06%	0.77%
GOOG	14.07%	0.89%	16.76%	0.82%
AMZN	12.43%	1.04%	15.60%	0.94%
NVDA	10.53%	1.30%	14.54%	1.18%
META	7.89%	1.01%	0.00%	0.87%
TSLA	5.91%	1.67%	7.06%	1.50%
TSM	4.19%	0.95%	4.63%	0.86%

Table 3. Black-Litternman (Weight& Expect return)

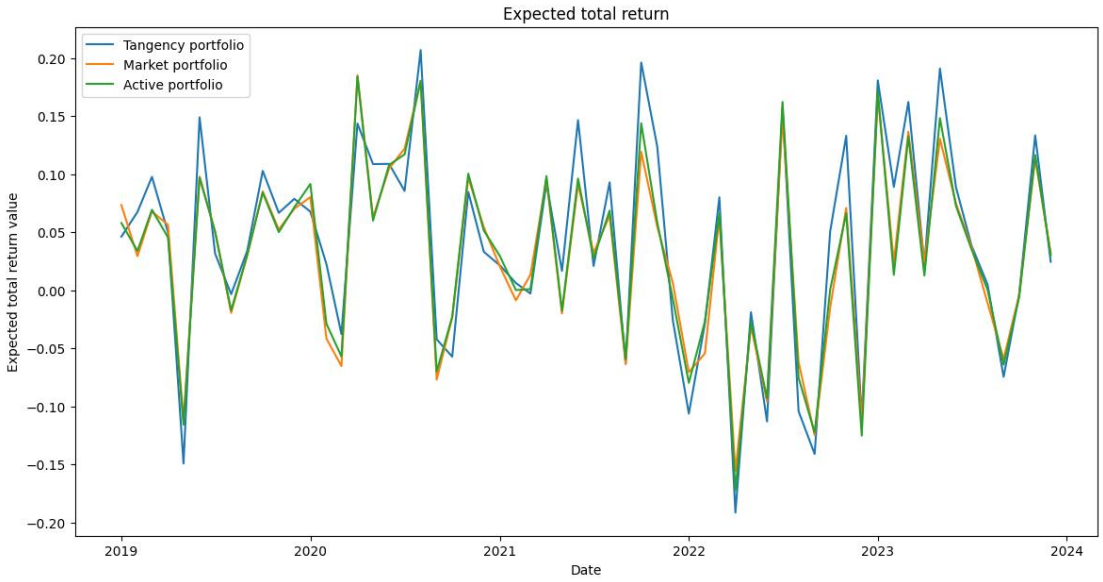
4.2.2 Active Portfolio

The active portfolio integrates the passive portfolio returns from section 4.2.1 with lambda and the earlier covariance matrix, alongside the investor's market perspectives, to generate adjusted expected returns and the active portfolio weights among individual stocks. It can be found that the AAPL weight of the active portfolio is lower than the market portfolio, falling from the original 22.82% to 15.35%. In addition, the

META has also dropped significantly from the original 7.89% to about 0.00%. The final weight optimization results are similar to investors' initial predictions, and AAPL and META will underperform.

4.3 Comparison

The Graph 2 below reveals that all three investment portfolios exhibit a certain degree of volatility in their expected total returns. The market portfolio represents the average performance of the entire market, while the tangency portfolio generally achieves higher profitability, particularly reaching its highest returns towards the end of 2020. However, it also leads to the largest negative expected total return. As for the active portfolio, its performance most of the time falls between the tangency portfolio and the market portfolio.



Graph 2. Expected Total Return

Following the data presented in the Table 4, it can be observed that the Tangency portfolio offers the highest average return, approximately 3.98%, with a standard deviation of 9.19%, indicating that the return volatility is also higher relative to the other two groups. The Sharpe ratio is about 113.43%, suggesting that it yields the highest excess return per unit of total risk. Overall, while the Tangency portfolio

provides the highest returns and the highest risk-adjusted returns, it also has the highest volatility. Moreover, the data indicate that the Market portfolio offers a more conservative and relatively stable option. The Active portfolio, on the other hand, strikes a balance between these two, demonstrating a Sharpe ratio close to that of the market.

	Tangency portfolio	**Market portfolio**	**Active portfolio**
Average return	3.98%	2.99%	3.08%
Standard Deviation	9.19%	7.92%	8.11%
Sharpe Ratio (annualized)	133.43%	111.75%	112.93%

Table 4. Comparison different index

4.4 VaR &CVaR

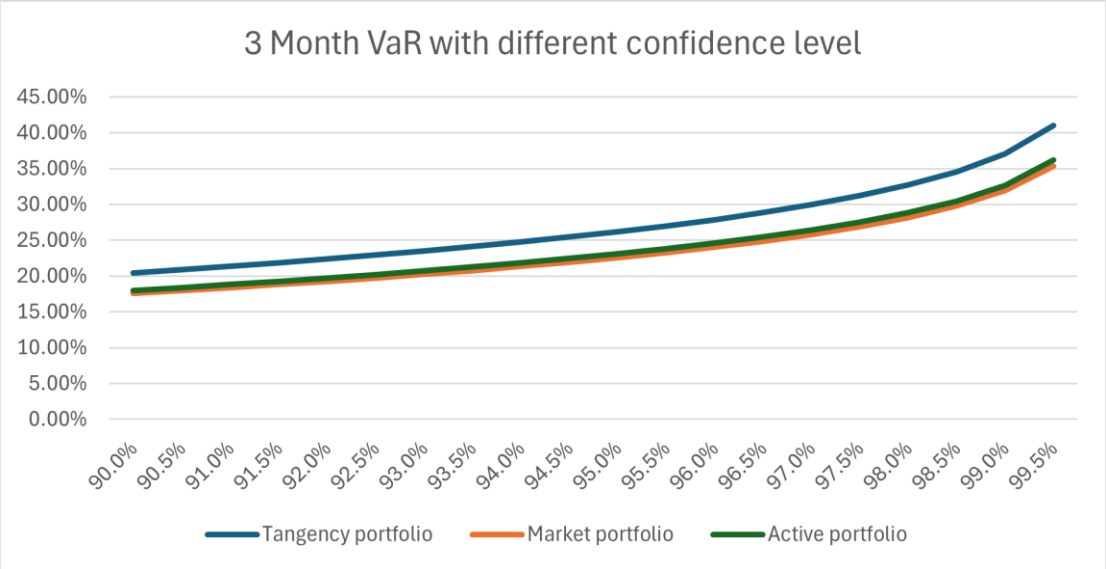
Referring to Table 5, it is observed that at a 95% confidence level over a 3-month period, the Tangency portfolio has the largest loss at approximately 26.18%, followed by the Black-Litterman's Active portfolio at around 23.09%, with the Market portfolio having the smallest at 22.56%, indicating it bears the least risk.

As for the CVaR, the order is the same with the Tangency portfolio being the highest at approximately 32.84%, followed by the Active portfolio at about 28.96% and the Market portfolio at around 28.29%, implying that in the worst 5% of cases, the Market portfolio also sustains the least risk.

	Tangency portfolio	Market portfolio	Active portfolio
3 MONTH VaR (zero mean)	26.18%	22.56%	23.09%
3-MONTH CvaR (zero mean)	32.84%	28.29%	28.96%

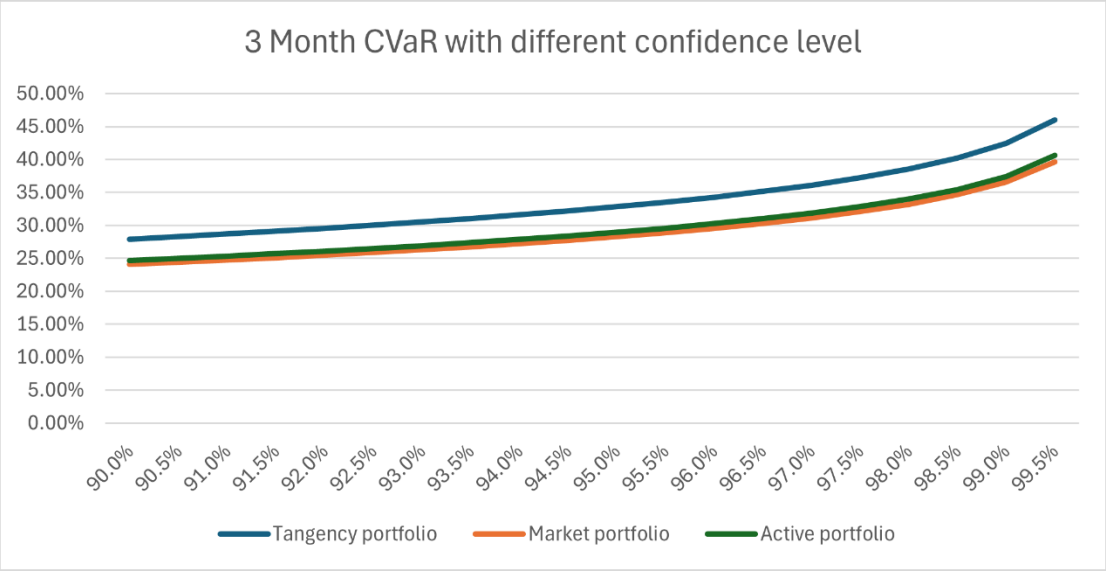
Table 5. VaR & CvaR

The Graph 3 below shows the VaR in difference confidence level between 90% and 99.5% with the index 0.5%. Obviously, all of them show the similar trends. Moreover, Tangency portfolio has the highest VaR value in all confidence level. Subsequently, Market portfolio displays the lowest VaR value in every confidence level. As for Active portfolio, its VaR values are between other two in each confidence level.



Graph 3. VaR Distribution

The Graph 4 displays the CVaR in difference confidence level between 90% and 99.5% with the index 0.5%. Obviously, they all have similar trends. However, Tangency portfolio has the highest CVaR value in all confidence level and it between 25% and 50%. Following next, Market portfolio displays the lowest CVaR value in every confidence level it about 24% to 34%. As for Active portfolio, its CVaR values are between other two in each confidence level.



Graph 4. CVaR Distribution

5.Conclusion

The primary purpose of this report is to construct a "Tangent Portfolio" and a "Black-Litterman Model" using selected information from eight chosen stocks. It compares tangency portfolios, passive and active portfolios to show their difference. With the addition of a restriction no short selling, the Tangency Portfolio becomes more consistent with real-world conditions. According to the results in Section 4.1, the weights of the Tangency Portfolio are MSFT 43.0%, NVDA 39.9%, AAPL just for 12.56% and TSLA 4.48%, while the rest are zero. Sections 4.3 and 4.4 indicate that although the Tangent Portfolio has higher returns, it also exhibits greater volatility and a higher risk over the next three months at a 95% confidence level, compared to the other two portfolios. Its VaR is 26.18% and CVaR is 32.84% show that has highest risk.

In contrast, the Black-Litterman model results, though offering lower returns and a lower Sharpe ratio, provide a more diversified portfolio with lower volatility. Its indicators fall between those of the Tangency and passive portfolios. The passive portfolio yields a return of 2.99% with a standard deviation of 7.92% and a Sharpe ratio of 111.75%, and it bears the lowest risk over the next three months at a 95% confidence level. Its VaR is 22.56% and CVaR is about 28.29%, indicating that while its returns and efficiency are not as high as those of the Tangency Portfolio, its risks are comparatively lower.

Following this, the Active Portfolio, which sits between the Tangent and passive portfolios in terms of data, shows a return of 3.08% with a standard deviation of 8.11% and a Sharpe ratio of 112.93%. Its VaR and CVaR are 23.09% and 28.96%, respectively. Moreover, its weight distribution is more diversified than that of the Tangent Portfolio, making it suitable for investors who seek returns slightly above the market without taking on extra risk. This indicates that the model can reduce portfolio risk exposure when calculating expected returns using weights, the risk-free rate, lambda, and the covariance matrix.

In conclusion, the active portfolio can achieve better profits than the market while considering market information to reduce risk. However, it didn't have the evaluation step there may be an overfitting issue, leading to performance that may not meet expectations in real-world. Moreover, since the selected samples have high industry redundancy, it might be necessary to include different industries to diversify risks, or to incorporate more potential market-influencing factors and additional tests to persuade investors to invest.

6. Reference

Benninga, S. (2014) Financial modeling. Cambridge, MA: The MIT Press, Chapter 9-12.

He, G. and Litterman, R. (2002) The intuition behind Black-Litterman model portfolios, SSRN.

Hull, J. (2015) Risk management and financial institutions: WIlEY finance. Hoboken, NJ: John wiley & Sons.

Allen, D.E. and Powell, R.J. (2007) Thoughts on VaR and CVaR, Research Online at Edith Cowan University. Available at: https://www.researchgate.net/publication/49284280_Thoughts_on_VaR_and_CVaR (Accessed: 18 April 2024).

Idzorek, T.M. (2007) 'A step-by-step guide to the Black-Litterman model,' in Elsevier eBooks, pp. 17–38.