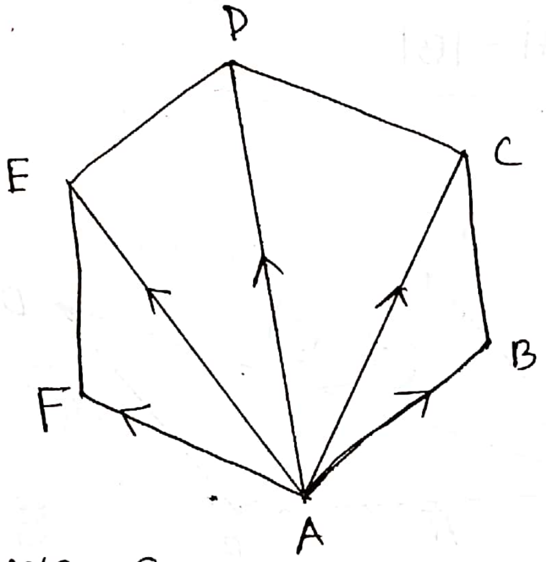
Q-37 | if ABCD are the vectors of a regular hexagon, find the resultant of the forces represented by the vectors AB, AC, AD, AE & AF.

Solnº




From fig. we see —

$$\overline{BC} = \overline{EF}, \quad \overline{AF} = \overline{CD}, \quad \overline{ED} = \overline{AB}$$

$\Delta ACD$ gives $\rightarrow$ $\overline{AC} + \overline{CD} = \overline{AD}$ —— (i)

$\Delta AED$ gives $\rightarrow$ $\overrightarrow{AE} + \overline{ED} = \overline{AD}$ —— (ii)

(i) + (ii)

$$2\overline{AD} = \overline{AC} + \overline{CD} + \overline{AE} + \overline{ED}$$

$$= \overline{AC} + \overline{AF} + \overline{AE} + \overline{AB}$$

Resultant, $\overline{R} = \overline{AC} + \overline{AB} + \overline{AD} + \overline{AE} + \overline{AF}$

$$= 2\overline{AD} + \overline{AD}$$

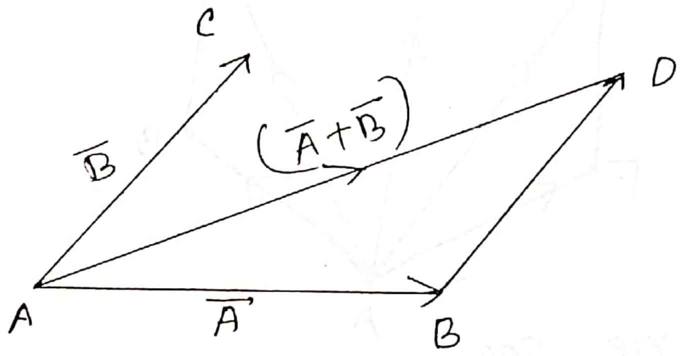
$$= 3\overline{AD}$$

Q-38|| If A & B are given vectors, show that

(a) $|A+B| \leq |A| + |B|$

(b) $|A-B| \geqslant |A| - |B|$

Sol$^n$:




In fig we see, $\overrightarrow{BD} \parallel \overrightarrow{AC}$

let, $\bar{AB} = \bar{A}$ & $\bar{AC} = \bar{B}$

$\bar{AB}$ has a magnitude $|\bar{A}|$ & direction along $\bar{A}$

Similarly for $\bar{AC}$

As $\bar{AC} \parallel \bar{BD}$, So we conclude $\bar{AC} = \bar{BD} = \bar{B}$

Now fm $\triangle ABD \longrightarrow |\bar{AB}| + |\bar{BD}| > |\bar{AD}|$

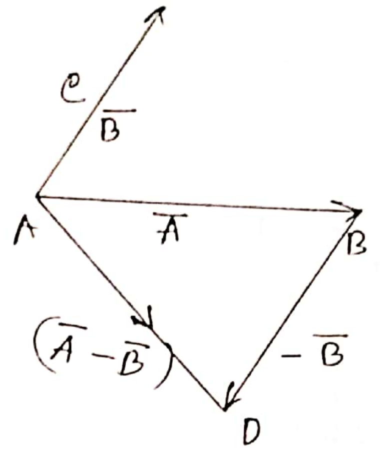
$\bar{AD} = \bar{AB} + \bar{BD}$ $\Rightarrow$ $|\bar{A}| + |\bar{B}| > |\bar{A}+\bar{B}|$

$= \bar{A} + \bar{B}$

If the components of $\bar{A}$ & B Both 'zero' then we can see $|\bar{A}+\bar{B}| = |\bar{A}| + |\bar{B}|$

$\therefore$ $|\bar{A}+\bar{B}| \leq |\bar{A}| + |\bar{B}|$

(b)




From fig, we get $\triangle ABD$ which gives

$$|\bar{B}| + |\bar{A}-\bar{B}| > |A|$$

$$\Rightarrow \quad |\bar{A}-\bar{B}| > |\bar{A}| - |\bar{B}|$$

when the component of $\bar{A}$ & $\bar{B}$ are 'zero'

then $\quad |\bar{A}-\bar{B}| = |\bar{A}| - |\bar{B}|$

$$\therefore \quad |\bar{A}-\bar{B}| \geqslant |\bar{A}| - |\bar{B}|$$

Q.39|| Show that, $|A+B+C| \leq |A| + |B| + |C|$

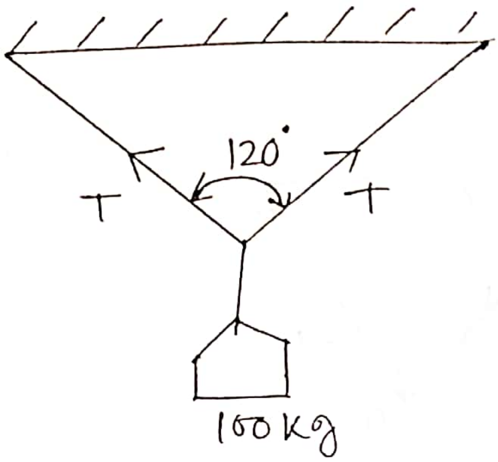
Soln: We know that, $|A+B| \leq |A| + |B|$

$$\therefore \quad |A+(B+C)| \leqslant |A| + |B+C|$$

$$\Rightarrow |A+(B+C)| \leqslant |A| + |B| + |C|$$

Since, $|B+C| \leq |B| + |C|$

Q. No. 42




A 100 kg weight is suspended from the centre of a rope as shown in the fig. Determine Tension T.

Solⁿ: Clearly All forces are in equilibrium. So the Resultant of the Two forces of T & T is 100 Kg.

$\therefore \quad R = 100 = \sqrt{T^2 + T^2 + 2 \cdot T \cdot T \cos 120}$

$\Rightarrow 100 = \sqrt{2T^2(1 + \cos 120)}$

$\Rightarrow T = 100$

Ans. T = 100 kg

Q.44|| If a & b are non-collinear vectors & $A = (x+4y)a + (2x+y+1)b$ & $B = (y-2x+2)a + (2x-3y-1)b$ find, x & y such that $3A = 2B$.

Solⁿ: Given,

$3A = 2B$

$\Rightarrow 3(ax+4ay) + 3(2bx+by+b) = 2(ay-2ax+2a) + 2(2bx-3by-b)$

$\Rightarrow 3ax + 12ay + 6bx + 3by + 3b = 2ay - 4ax + 4a + 4bx - 6by - 2b$

$\Rightarrow 7ax + 10ay + 2bx + 9by + 5b - 4a = 0$

$\Rightarrow (7x + 10y - 4)a + (2x + 9y + 5)b = 0$

From example-12 →

If a & b are non-collinear, the $xa + yb = 0$

implies $x = y = 0$

$\therefore 7x + 10y - 4 = 0$

$2x + 9y + 5 = 0$

Solving x & y give →

$x = 2$

$y = -1$

Ans

Q-46| If a, b, c are non-coplanar vectors determine whether the vectors $r_1 = 2a - 3b + c$, $r_2 = 3a - 5b + 2c$ & $r_3 = 4a - 5b + c$ are linearly dependent or independent.

Solⁿ: 3 vectors are said to be linearly independent if $K_1 r_1 + K_2 r_2 + K_3 r_3 = 0$ where, $K_1 = K_2 = K_3 = 0$

we can written it as this form,

$$\begin{bmatrix} 2 & 3 & 4 \\ -3 & -5 & -5 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = 0$$

we will do, row reducing for echelon form.

$$\begin{bmatrix} 1 & 2 & 1 \\ \cancel{-3} & \cancel{-5} & \cancel{-5} \\ 2 & 3 & 4 \\ -3 & -5 & -5 \end{bmatrix}$$ Swaping the $r_3$ to $r_1$

$r_2' = r_2 - 2r_1$, $r_3' = 3r_1 + r_3 \longrightarrow$ $$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 1 & -2 \end{bmatrix}$$

$r_2' = -r_2 \longrightarrow$ $$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix}$$

$r_3' = r_3 - r_2 \longrightarrow$ $$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

As we can see,

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = 0$$

$\Rightarrow$ 1st row $\rightarrow$ $k_1 + 2k_2 + k_3 = 0$ — (I)

2nd row $\rightarrow$ $k_2 - 2k_3 = 0$ — (II)

3rd row $\rightarrow$ $0 = 0$

$k_2 = 2k_3 \rightarrow$ in eqn – (I)

$k_1 + 4k_3 + k_3 = 0$

$k_1 = -5k_3$

So we see $k_1 \neq k_2 \neq 0 \neq k_3$

As a result vectors are linearly dependent

That gives, $k_1 r_1 + k_2 r_2 + k_3 r_3 = 0$

$\Rightarrow -5r_1 k_3 + 2k_3 r_2 + k_3 r_3 = 0$

$\Rightarrow r_3 = 5r_1 - 2r_2$

Ans

** For better explanation see prob. 61 (b) page (12-13)

Q.NO.56 || A quadrillateral ABCD has masses of 1, 2, 3, 4 units respectively at it's vertices A(-1, -2, 2) B(3, 2, -1), C(1, -2, 4) & D(3, 1, 2). Find the Co-ordinates of centroid.

Solⁿ: From Q-35 → position vector of centroid for masses $m_1, m_2, m_3, \ldots m_n$ & position vector $r_1, r_2, r_3, \ldots r_n$ is given by

$$r = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3 + \cdots + m_n r_n}{m_1 + m_2 + m_3 + \cdots m_n}$$

Now for our question →

$$x = \frac{1\times -1 + 2\times 3 + 3\times 1 + 4\times 3}{1+2+3+4} = 2$$

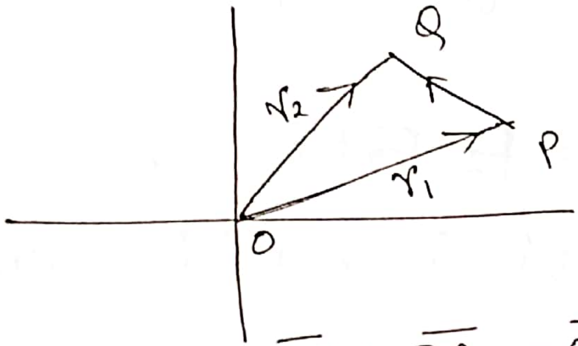
$$y = \frac{1\times -2 + 2\times 2 + 3\times -2 + 4\times 1}{1+2+3+4} = 0$$

$$z = \frac{1\times 2 + 2\times -1 + 3\times 4 + 4\times 2}{1+2+3+4} = 2$$

$\therefore (x, y, z) = (2, 0, 2)$

$\bar{r} = 2\hat{i} + 2\hat{k}$

**Q.58** The position vectors of point $P$ & $Q$ are given by $r_1 = 2\hat{i}+3\hat{j}-\hat{k}$, $r_2 = 4\hat{i}-3\hat{j}+2\hat{k}$. Determine $PQ$ in term of $\hat{i}, \hat{j}, \hat{k}$. And find it's magnitude.




$$\overline{OP} + \overline{PQ} = \overline{OQ}$$

$$\Rightarrow \overline{PQ} = \overline{OQ} - \overline{OP}$$

$$= \bar{r}_2 - \bar{r}_1$$

$$= 4\hat{i} - 3\hat{j} + 2\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$

$$= 2\hat{i} - 6\hat{j} + 3\hat{k}$$

$$|\overline{PQ}| = \sqrt{2^2 + 6^2 + 3^2} = 7$$

Ans.

Q.NO. 60|| The following forces act on a particle p:

$F_1 = 2\hat{i} + 3\hat{j} - 5\hat{k}$ , $F_2 = -5\hat{i} + \hat{j} + 3\hat{k}$ , $F_3 = \hat{i} - 2\hat{j} + 4\hat{k}$

$F_4 = 4\hat{i} - 3\hat{j} - 2\hat{k}$ measured in Newtons. Find

(a) resultant (b) magnitude of resultant.

Solⁿ:

Resultant $= F_1 + F_2 + F_3 + F_4$

$= (2\hat{i} + 3\hat{j} - 5\hat{k}) + (-5\hat{i} + \hat{j} + 3\hat{k}) + (\hat{i} - 2\hat{j} + 4\hat{k}) + (4\hat{i} - 3\hat{j} - 2\hat{k})$

$= 2\hat{i} - \hat{j}$

Magnitude $= \sqrt{2^2 + 1^2} = \sqrt{5}$

Ans

**Q.61** In each case determine whether the vectors are linearly independent or dependent.

(a) $\bar{A} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\bar{B} = \hat{i} - 4\hat{k}$, $\bar{C} = 4\hat{i} + 3\hat{j} - \hat{k}$

**Solⁿ:** 3 vectors are said to linearly independent if $k_1\bar{A} + k_2\bar{B} + k_3\bar{C} = 0$ where $k_1 = k_2 = k_3 = 0$. Which can be

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \\ -3 & -4 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

We will do the matrix in the echolon form.

$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ -3 & -4 & -1 \end{bmatrix} \quad \text{swaping } r_2 \text{ in } r_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & -4 & 8 \end{bmatrix} \quad \begin{aligned} r_2' &= r_2 - 2r_1 \\ r_3' &= r_3 + 3r_1 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix} \quad r_3' = r_3/-4$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad r_3' = r_3 - r_2 \Rightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

which gives us $\longrightarrow$ row-1 $\longrightarrow$ $k_1 + 3k_3 = 0$

$k_2 - 2k_3 = 0$

Where $k_1 \neq 0$ $k_2 \neq 0$, $k_3 \neq 0$ So linearly dependent.

(b) 3 vectors are said to be linearly independent if the vectors can be expressed as follow.

$$K_1 A + K_2 B + K_3 C = 0$$

$$K_1(\hat{i} - 3\hat{j} + 2\hat{k}) + K_2(2\hat{i} - 4\hat{j} - \hat{k}) + K_3(3\hat{i} + 2\hat{j} - \hat{k}) = 0$$

$$\Rightarrow K_1\hat{i} - 3K_1\hat{j} + 2K_1\hat{k} + 2K_2\hat{i} - 4K_2\hat{j} - K_2\hat{k} + 3K_3\hat{i} + 2K_3\hat{j} - K_3\hat{k} = 0$$

$$\Rightarrow (K_1 + 2K_2 + 3K_3)\hat{i} + (-3K_1 - 4K_2 + 2K_3)\hat{j} + (2K_1 - K_2 - K_3)\hat{k} = 0$$

This component can be written as —

$$\begin{bmatrix} 1 & 2 & 3 \\ -3 & -4 & 2 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = 0$$

We will make this matrix to echelon form by row reducing —

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 11 \\ 0 & -5 & -7 \end{bmatrix} \quad \begin{aligned} r_2' &= r_2 + 3r_1 \\ r_3' &= r_3 - 2r_1 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 11 \\ 0 & 0 & \frac{41}{2} \end{bmatrix} \quad r_3' = r_3 + \frac{5}{2} r_2$$

Now we get →

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 11 \\ 0 & 0 & \frac{41}{2} \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = 0$$

which gives →

$$k_1 + 2k_2 + 3k_3 = 0 \quad \text{—— (i)}$$

$$2k_2 + 11k_3 = 0 \quad \text{—— (ii)}$$

$$\frac{41}{2} k_3 = 0 \Rightarrow k_3 = 0 \quad \text{—— (iii)}$$

putting the value of $k_3$ in (ii)

$$k_2 = 0$$

putting the value of $k_2$ & $k_3$ in eqn (i)

$$k_1 = 0$$

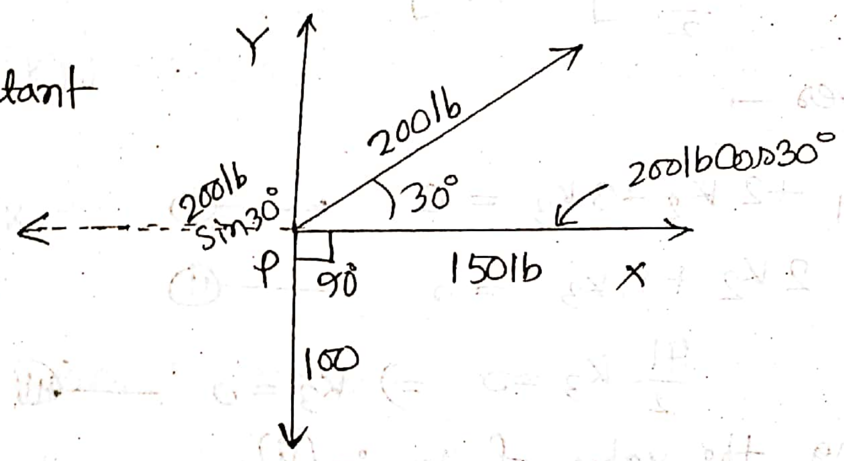
∴ vectors are linearly independent.

2nd edition

Q-1.41 An object p is acted upon by three coplanar forces as shown in the fig. Find the force needed to prevent p from moving.

Soln:

Let Resultant R.




$$\sum R_x = 200lb \cos 30^\circ + 150 = 323.21lb$$

$$\sum R_y = 200lb \sin 30^\circ - 100 = 0$$

$\therefore R = \sqrt{R_x^2 + R_y^2} = 323.21$ lb

So 'p' to be stationary if we apply the force R along opposite direction of 150lb

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Q.55|| Evaluate: (a) $k\cdot(i+j)$ (b) $(\hat{i}-2k)(\hat{j}+3k)$
(c) $(2\hat{i}-\hat{j}+3k)(3\hat{i}+2\hat{j}-\hat{k})$

Sol$^n$:

(b) $(\hat{i}-2\hat{k})(\hat{j}+3\hat{k})$

$=(\hat{i}\cdot\hat{j})+3\cdot(\hat{i}\cdot\hat{k})-2(\hat{k}\cdot\hat{j})-6(\hat{k}\cdot\hat{k})$

$=0+0-0-6 \quad =-6$

Ans

Q.58|| For what values of a, The vectors $A=a\hat{i}-2\hat{j}+k$ & $B=2a\hat{i}+a\hat{j}-4\hat{k}$ perpendicular?

Sol$^n$: For perpendicular $\longrightarrow \bar{A}\cdot\bar{B}=0$

$\Rightarrow (a\hat{i}-2\hat{j}+k)\cdot(2a\hat{i}+a\hat{j}-4\hat{k})=0$

$\Rightarrow 2a^2-2a-4=0$

$\Rightarrow a^2-a-2=0$

$\Rightarrow a^2-2a+a-2=0$

$\Rightarrow a(a-2)+1(a-2)=0 \Rightarrow a=2,-1$

Ans

Q.59// Find the acute angles which the line joining the points (1, -3, 2) & (3, -5, 1) makes with the co-ordinate axes.

Solⁿ: let A(1, -3, 2) & B(3, -5, 1)

$\overline{AB} = \overline{B} - \overline{A}$

$= (3-1)\hat{i} + (-5+3)\hat{j} + (1-2)\hat{k}$

$= 2\hat{i} - 2\hat{j} - \hat{k}$

There are 3-co-ordinate axes. x, y & z.
The unit vectors along x, y & z are $\hat{i}, \hat{j}, \hat{k}$

Now, for angle with x axis:

$\hat{i} \cdot \overline{AB} = |\overline{AB}|\,|\hat{i}| \cos\theta$

$\Rightarrow i \cdot (2\hat{i} - 2\hat{j} - k) = \sqrt{2^2 + (2)^2 + (-1)^2} \cdot 1 \cdot \cos\theta$

$\Rightarrow 2 = 3\cos\theta \Rightarrow \cos\theta = 2/3$

$\Rightarrow \theta = \arccos\left(\frac{2}{3}\right)$

similarly for Y & Z axis.

Ans.

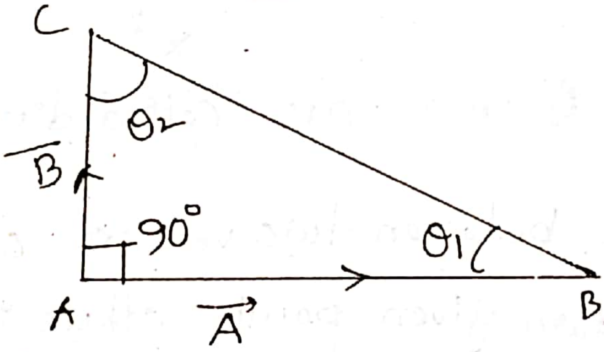
**Q.61** Two sides of a triangle are formed by the vectors $\bar{A} = 3\hat{i}+6\hat{j}-2\hat{k}$ & $\bar{B} = 4\hat{i}-\hat{j}+3\hat{k}$. Determines the Angles of the triangle.

**Sol$^n$:** we see that $\bar{A}\cdot\bar{B} = (3\hat{i}+6\hat{j}-2\hat{k}).(4\hat{i}-\hat{j}+3\hat{k})$

$= 12-6-6 = 0$

$\therefore \bar{A} \perp \bar{B}$ (A & B are perpendicular)

So $\angle BAC = 90°$.




length of AB $= |\vec{A}| = 7$, length of AC $= \sqrt{26}$

$\therefore BC^2 = AB^2 + AC^2$

$= 49+26$

$= 75 \quad \therefore BC = \sqrt{75}$

Now ABC is a right angle triangle.

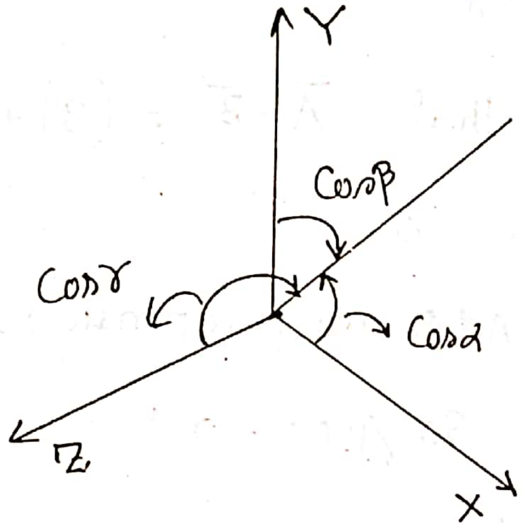
$\therefore \cos\theta_1 = \frac{AB}{BC} = \frac{7}{\sqrt{75}} \Rightarrow \theta_1 = \cos^{-1}\frac{7}{\sqrt{75}} = 36°4'$

& $\cos\theta_2 = \frac{AC}{BC} = \frac{\sqrt{26}}{\sqrt{75}} \Rightarrow \theta_2 = \cos^{-1}\frac{\sqrt{26}}{\sqrt{75}} = 53°56'$

Ans.

Q.No.60|| Find the direction cosines of the line joining the points $(3, 2, -4)$ & $(1, -1, 2)$.

Soln:




$\cos\alpha$, $\cos\beta$ & $\cos\gamma$ are called direction cosines.

$\cos\alpha$ – angle between two vectors. one is the joining vector of the two given points other is the $x$ axis.

So this problem is similar to Q.59

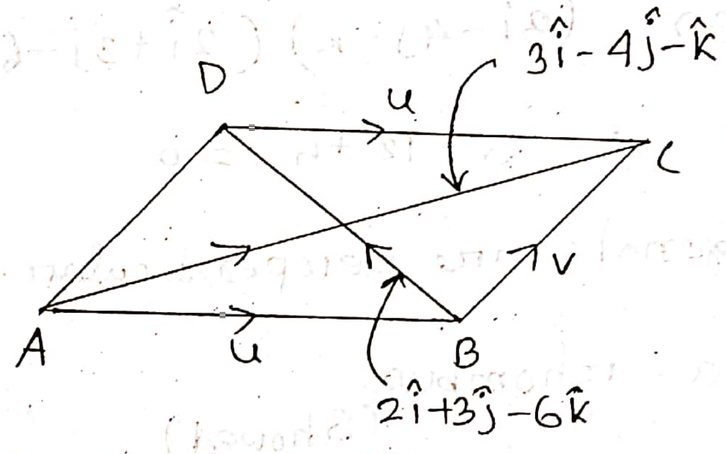
We can take $A(3, 2, 4)$ & $B(1, -1, 2)$

or alternate $A(1, -1, 2)$ & $B(3, 2, 4)$

That will gives us $\cos\alpha = 2/7$ or $-2/7$ respectively.

**Q.62||** The diagonals of a parallelogram are given by $\bar{A} = 3\hat{i} - 4\hat{j} - \hat{k}$ & $\bar{B} = 2\hat{i} + 3\hat{j} - 6\hat{k}$. Show that parallelogram is a rhombus and determine the length of it's sides & it's angles.

**Solⁿ:**




Let $\overline{AC} = \bar{A}$ & $\overline{BD} = \bar{B}$.

$\overline{AB} = u$ ; $\overline{BC} = \bar{V}$

From Triangle ABC $\longrightarrow$ $\overline{AB} + \overline{BC} = \overline{AC}$

$$\Rightarrow \bar{u} + \bar{v} = 3\hat{i} - 4\hat{j} - \hat{k} \quad \text{——} (1)$$

Similarly from $\triangle BCD \longrightarrow$ $\overline{BC} + \overline{CD} = \overline{BD}$

$$\Rightarrow \bar{V} - \bar{u} = 2\hat{i} + 3\hat{j} - 6\hat{k} \quad \text{——} (ii)$$

(as $\overline{CD} = -\bar{u}$)

(1) + (ii),

$$2\bar{v} = 5\hat{i} - \hat{j} - 7\hat{k}$$

$$\Rightarrow \bar{v} = \frac{1}{2}(5\hat{i} - \hat{j} - 7\hat{k})$$

(1) - (ii),

$$2\bar{u} = \hat{i} - 7\hat{j} + 5k$$

$$\Rightarrow \bar{u} = \frac{1}{2}(\hat{i} - 7\hat{j} + 5k)$$

So we get the sides of parallelogram.

Now $|\overline{AB}| = |\overline{BC}| = \frac{1}{2}\sqrt{5^2+(-1)^2+(-7)^2} = \frac{1}{2}\sqrt{1^2+(-7)^2+5^2}$

$= \frac{1}{2}\sqrt{25+1+49} = \frac{5\sqrt{3}}{2}$ Ans

$\therefore$ All sides are equal.

Now, $\overline{AC}\cdot\overline{BD} = (3i-4j-k)(2\hat{i}+3\hat{j}-6\hat{k})$

$= 6-12+6 = 0$

$\therefore$ Diagonals are perpendicular.

So it's a rhombus.
(Showed)

Q.63// Find the projection of the vector $2\hat{i}-3\hat{j}+6\hat{k}$ on the vector $\hat{i}+2\hat{j}+2\hat{k}$.

Soln: Let $\overline{A} = 2\hat{i}-3\hat{j}+6\hat{k}$ & $\overline{B} = \hat{i}+2\hat{j}+2\hat{k}$

unit vector of $\overline{B} = \frac{\overline{B}}{|\overline{B}|} = \frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$

$\therefore$ projection of $\overline{A}$ on $\overline{B}$ $= \frac{(\hat{i}+2\hat{j}+2k)}{3}\cdot(2\hat{i}-3\hat{j}+6k)$

$= \frac{1}{3}(2-6+12) = 8/3$

Ans

Q.64|| Find the projection of the vector $4\hat{i}-3\hat{j}+\hat{k}$ on the line joing through the points $(2,3,-1)$ & $(-2,-4,3)$.

Solⁿ: Similar to the Q.63. except here $\bar{B}$ is not given.

$$\bar{B} = (2+2)\hat{i} + (3+4)\hat{j} + (-1-3)\hat{k}$$

$$= 4\hat{i} + 7\hat{j} - 4\hat{k}$$

$$\& \ \bar{A} = 4\hat{i} - 3\hat{j} + \hat{k}$$

Result can be negative. But projection is always positive. So negative value can be ignored & except it as positive.

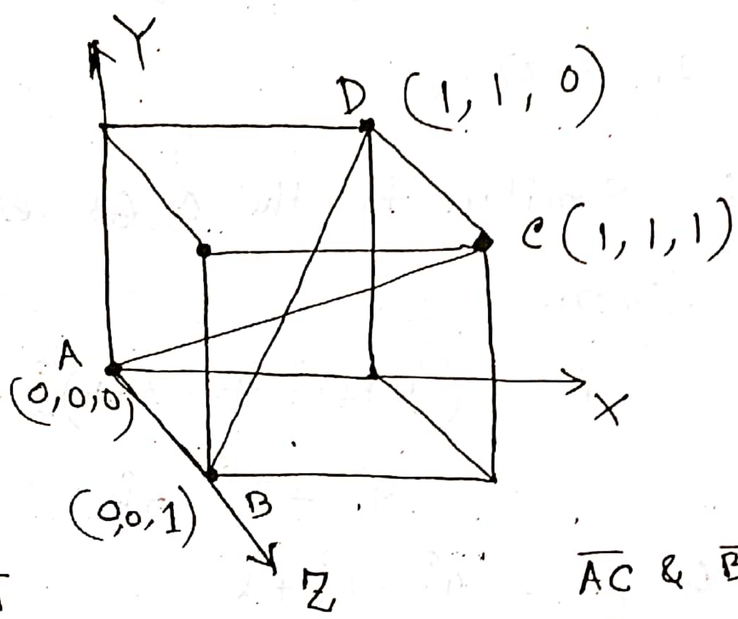
Q.65|| If $\bar{A} = 4\hat{i} - \hat{j} + 3\hat{k}$ & $\bar{B} = -2\hat{i} + \hat{j} - 2\hat{k}$, find the a unit vector perpendicular to both A & B.

Solⁿ: Similar to Q. 32

Q.66// Find the acute angle formed by two diagonals of a cube.

Soln:

Let, each edge of the cube is 1 unit.




$\overline{AC}$ & $\overline{BD}$ are the diagonals

$$\overline{AC} = \overline{C} - \overline{A}$$
$$= (1-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$$
$$= \hat{i} + \hat{j} + \hat{k}$$

$$\overline{BD} = \overline{D} - \overline{B} = (1-0)\hat{i} + (1-0)\hat{j} + (0-1)\hat{k}$$
$$= \hat{i} + \hat{j} - \hat{k}$$

$$\overline{AC} \cdot \overline{BD} = |\overline{AC}| \cdot |\overline{BD}| \cdot \cos\theta$$

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k})(\hat{i}+\hat{j}-\hat{k}) = \sqrt{1^2+1^2+1^2} \cdot \sqrt{1^2+1^2+(-1)^2} \cos\theta$$

$$\Rightarrow 1 = \sqrt{3} \cdot \sqrt{3} \cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\frac{1}{3} = 70^\circ 32'$$

Ans

Q.67 Find a unit vector parallel to the xy plane & perpendiculare to the vector $4\hat{i} - 3\hat{j} + \hat{k}$

**Sol$^n$:** Any vectors parallel to xy-plane can be written as $x\hat{i}+y\hat{j}$.

it is perpendicular to $4\hat{i}-3\hat{j}+\hat{k}$.

$\therefore (x\hat{i}+y\hat{j}).(4\hat{i}-3\hat{j}+\hat{k}) = 0$

$\Rightarrow 4x - 3y = 0$

$\Rightarrow x = \frac{3}{4}y$

$\therefore$ The vector is $= \frac{3}{4}y\hat{i} + y\hat{j}$

$= 3y\hat{i} + 4y\hat{j}$

$\therefore$ unit vector $= \dfrac{3y\hat{i}+4y\hat{j}}{\sqrt{(3y)^2+4y^2}}$

$= \dfrac{(3\hat{i}+4\hat{j})}{5}$

Ans

**Q.69||** Find the work done in moving an object along a straight line from (3, 2, -1) to (2, -1, 4) in a force ~~force~~ field ~~along~~ given by $F = 4\hat{i} - 3\hat{j} + 2\hat{k}$

**Soln:** Work done, $W = F \cdot \Delta r$

$$\Delta r = r_2 - r_1 = (2-3)\hat{i} + (-1-2)\hat{j} + (4+1)\hat{k}$$

$$= -\hat{i} - 3\hat{j} + 5\hat{k}.$$

$$\therefore W = F \cdot \Delta r$$

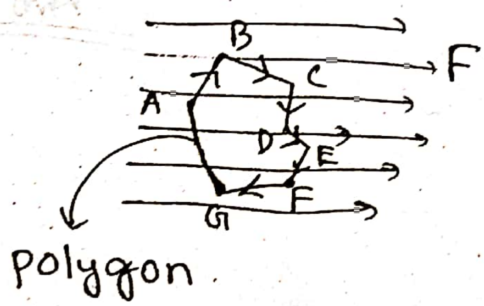
$$= (4\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} - 3\hat{j} + 5\hat{k})$$

$$= -4 + 9 + 10 = 15$$

Ans

**Q.70||** Let F be a constant vector force Field. Show that the work done in moving an object around any closed polygon in this force is zero.

**Soln:**



$\therefore$ work done, $W = \sum \bar{F} \cdot \Delta r$

$$= \bar{F} \cdot \overline{AB} + \bar{F} \cdot \overline{BC} + \bar{F} \cdot \overline{CD} + \cdots + \bar{F} \cdot \overline{GA}$$

$$= \bar{F}\left(\overline{AB} + \overline{BC} + \overline{CD} + \overline{DE} + \overline{EF} + \overline{FG} + \overline{GA}\right)$$

$$= \bar{F}\left(\bar{B} - \bar{A} + \bar{C} - \bar{B} + \bar{D} - \bar{C} + \bar{E} - \bar{D} + \bar{F} - \bar{E} + \bar{G} - \bar{F} + \bar{A} - \bar{G}\right)$$

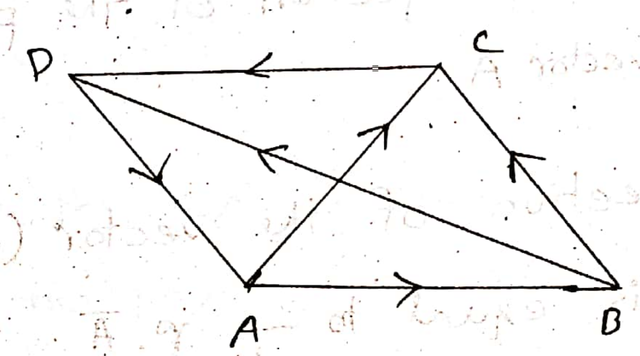
$$= \bar{F} \cdot 0 \qquad \left[\because \overline{AB} = \bar{B} - \bar{A}\right]$$

$$= 0$$

Q.72 Let ABCD be a parallelogram. prove that,

$$\overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2 = \overline{AC}^2 + \overline{BD}^2.$$

Soln:




From $\triangle ABC \longrightarrow \overline{AB} + \overline{BC} = \overline{AC}$

$$\Rightarrow (\overline{AB} + \overline{BC})^2 = (\overline{AC})^2 \quad [\text{Take square}]$$

$$\Rightarrow \overline{AB}^2 + \overline{BC}^2 + 2 \cdot \overline{AB} \cdot \overline{BC} = \overline{AC}^2 \quad \text{--- (1)}$$

From, $\triangle BCD \longrightarrow \overline{BC} + \overline{CD} = \overline{BD}$

$$\Rightarrow (\overline{BC} + \overline{CD})^2 = (\overline{BD})^2$$

$$\Rightarrow \overline{BC}^2 + \overline{CD}^2 + 2\,\overline{BC}\cdot\overline{CD} = \overline{BD}^2$$

$$\Rightarrow (-\overline{DA})^2 + \overline{CD}^2 + 2\cdot\overline{BC}\cdot(-\overline{AB}) = \overline{BD}^2$$

$$\Rightarrow \overline{DA}^2 + \overline{CD}^2 - 2\,\overline{BC}\cdot\overline{AB} = \overline{BD}^2 \quad \text{——— (ii)}$$

(i) + (ii), $\quad \overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2 = \overline{AC}^2 + \overline{BD}^2$

Q.74 || (a) Find an eqn. of a plane perpendicular to a given vector A & distance p from the origin.

Soln: Let r be position vector $(x\hat{i} + y\hat{j} + z\hat{k})$ of any point on the plane.

p means projection of the point $(x\hat{i} + y\hat{j} + z\hat{k})$ on the vector A.

$\therefore$ projection of the vector $(x\hat{i} + y\hat{j} + z\hat{k})$ or $\overline{r}$ on A is equal to – $r.\dfrac{\overline{A}}{|\overline{A}|}$

$\therefore$ according to the question – $r.\dfrac{\overline{A}}{|\overline{A}|} = p$

Ans

(b) Express the eqn of (a) in rectangular co-ordinates.

Soln :
$$\frac{r \cdot \bar{A}}{|\bar{A}|} = p$$

$$\Rightarrow r \cdot \bar{A} = p\,|\bar{A}|$$

Let $\bar{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$$|\bar{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$$

$$\therefore r \cdot (A_1\hat{i} + A_2\hat{j} + A_3\hat{k}) = p\sqrt{A_1^2 + A_2^2 + A_3^2}$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k})\,(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}) = p\sqrt{A_1^2 + A_2^2 + A_3^2}$$

$$\Rightarrow A_1x + A_2y + A_3z = p\sqrt{A_1^2 + A_2^2 + A_3^2}$$

Ans.

Q. 76 Let a be the position vector of a given point $(x_1, y_1, z_1)$ & r the position vector of any point $(x, y, z)$. Determine the Locus of r if

(a) $|r - a| = 3$

Soln :
$$|x\hat{i} + y\hat{j} + z\hat{k} - x_1\hat{i} - y_1\hat{j} - z_1\hat{k}| = 3$$

$$\Rightarrow |(x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k}| = 3$$

$$\Rightarrow \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2} = 3$$

$$\Rightarrow (x-x_1)^2+(y-y_1)^2+(z-z_1)^2 = 9$$

$\therefore$ It's a sphere with centre $(x_1, y_1, z_1)$ & radious 3.

(b) $(r-a).a = 0$

From Q.(a) $\longrightarrow$

$$\Rightarrow \left[(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k}\right](x_1\hat{i}+y_1\hat{j}+z_1\hat{k}) = 0$$

$$\Rightarrow (x-x_1)x_1+(y-y_1)y_1+(z-z_1)z_1 = 0$$

$\therefore$ It's a eqn of plane passing through a point 'a' & perpendicular to the position vector of 'a'.

(c) $(r-a).r = 0$

by putting the value of $\bar{r}$ & $\bar{a}$

$$\Rightarrow \left[(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k}\right](x\hat{i}+y\hat{j}+z\hat{k}) = 0$$

$$\Rightarrow (x-x_1)x+(y-y_1)y+(z-z_1)z = 0$$

$$\Rightarrow x^2 - xx_1 + y^2 - yy_1 + z^2 - zz_1 = 0$$

$$\Rightarrow x^2 - 2.x.\frac{x_1}{2} + \frac{x_1^2}{4} + y^2 - 2y.\frac{y_1}{2} + \frac{y_1^2}{4} + z^2 - 2.z.\frac{z_1}{2} + \frac{z_1^2}{4}$$

$$-\frac{x_1^2}{4} - \frac{y_1^2}{4} - \frac{z_1^2}{4} = 0$$

$$\Rightarrow \left(x-\frac{x_1}{2}\right)^2 + \left(y-\frac{y_1}{2}\right)^2 + \left(z-\frac{z_1}{2}\right)^2 = \frac{x_1^2}{4} + \frac{y_1^2}{4} + \frac{z_1^2}{4}$$

$$\Rightarrow \left(x-\frac{x_1}{2}\right)^2 + \left(y-\frac{y_1}{2}\right)^2 + \left(z-\frac{z_1}{2}\right)^2 = \left(\frac{\sqrt{x_1^2+y_1^2+z_1^2}}{2}\right)^2$$

$\therefore$ It's a eq$^n$ of sphere with center —

$\left(\frac{x_1}{2}, \frac{y_1}{2}, \frac{z_1}{2}\right)$ & radious $= \frac{\sqrt{x_1^2+y_1^2+z_1^2}}{2}$

**Q.77||** Given that, $A = 3\hat{i}+\hat{j}+2\hat{k}$ & $B = \hat{i}-2\hat{j}-4\hat{k}$ are the position vectors of points P and Q respectively.

(a) Find an eq$^n$. for the plane passing through Q & perpendicular to line PQ.

**Sol$^n$:** Let $r$ be the position vector of any point on the plane $(x\hat{i}+y\hat{j}+z\hat{k})$

$$\overrightarrow{PQ} = \vec{Q} - \vec{P} = (1-3)\hat{i} + (-2-1)\hat{j} + (-4-2)\hat{k}$$
$$= -2\hat{i} - 3\hat{j} - 6\hat{k}$$

Now plane passing through the point Q and perpendicular to $\overrightarrow{PQ}$ is ⟶

$$(r-Q)\cdot\overrightarrow{PQ} = 0$$

$$\Rightarrow \left[(x-1)\hat{i} + (y+2)\hat{j} + (z+4)\hat{k}\right](-2\hat{i}-3\hat{j}-6\hat{k}) = 0$$

$\Rightarrow (x-1)(-2) + (y+2)(-3) + (z+4)(-6) = 0$

$\Rightarrow 2x-2+3y+6+6z+24=0$

$\Rightarrow 2x+3y+6z+28=0$

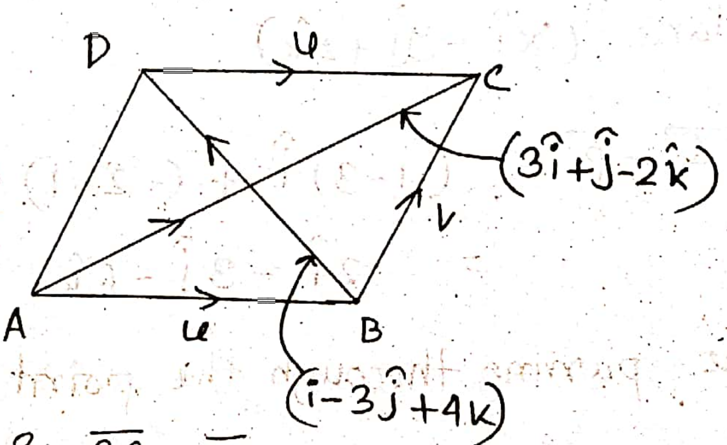
(b) What is the distance from the point $(-1,1,1)$ to the plane?

Soln:
$$\text{distance} = \frac{-2+3+6+28}{\sqrt{2^2+3^2+6^2}}$$
$$= \frac{35}{7} = 5$$

Ans.

Q.82|| Find the area of parallelogram having diagonals $A = 3\hat{i}+\hat{j}-2\hat{k}$ & $B = \hat{i}-3\hat{j}+4k$.

Soln:




let $\overline{AB} = \bar{u}$ & $\overline{BC} = \bar{v}$

$\triangle ABC \longrightarrow \quad \bar{u}+\bar{v} = \overline{AC}$ —— (i)

$\bar{v}-\bar{u} = \overline{BD}$ —— (ii) (i)+(ii)

$$2\bar{v} = \overline{AC} + \overline{BD}$$

$$\Rightarrow \bar{v} = \frac{1}{2}(3\hat{i}+\hat{j}-2\hat{k}+\hat{i}-3\hat{j}+4\hat{k})$$

$$= \frac{1}{2}(4\hat{i}-2\hat{j}+2\hat{k})$$

$$= \frac{1}{2}\times 2(2\hat{i}-\hat{j}+\hat{k})$$

$$= 2\hat{i}-\hat{j}+\hat{k}$$

(i)−(ii)

$$2\bar{u} = \overline{AC} - \overline{BD}$$

$$\Rightarrow \bar{u} = \frac{1}{2}(3\hat{i}+\hat{j}-2\hat{k}-\hat{i}+3\hat{j}-4\hat{k})$$

$$= \frac{1}{2}(2\hat{i}+4\hat{j}-6\hat{k})$$

$$= \hat{i}+2\hat{j}-3\hat{k}$$

$\therefore$ Area $= |\bar{u}\times\bar{v}|$

$$\bar{u}\times\bar{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(2-3)+\hat{j}(-6-1)+\hat{k}(-1-4)$$

$$= -\hat{i}-7\hat{i}-5\hat{k}$$

$\therefore$ Area, $A = |\bar{u}\times\bar{v}| = \sqrt{(-1)^2+(-7)^2+(-5)^2}$

$$= \sqrt{75}$$

$$= 5\sqrt{3}$$

Ans

**Q. 83** Find the Area of a triangle with vertices at $(3,-1,2), (1,-1,-3)$ & $(4,-3,1)$.

**Solⁿ:**

Let $A(3,-1,2)$, $B(1,-1,-3)$ & $C(4,-3,1)$

$$\overline{AB} = \overline{B} - \overline{A} = (1-3)\hat{i} + (-1+1)\hat{j} + (-3-2)\hat{k}$$
$$= -2\hat{i} - 5\hat{k}$$

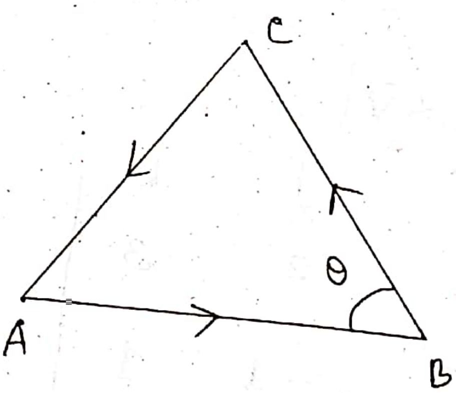
$$\overline{BC} = \overline{C} - \overline{B} = (4-1)\hat{i} + (-3+1)\hat{j} + (1+3)\hat{k}$$
$$= 3\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\overline{CA} = \overline{A} - \overline{C} = (3-4)\hat{i} + (-1+3)\hat{j} + (2-1)\hat{k}$$
$$= -\hat{i} + 2\hat{j} + \hat{k}$$

$|\overline{AB}| = \sqrt{29}$

$|\overline{BC}| = \sqrt{29}$

$|\overline{CA}| = \sqrt{6}$




$$\cos\theta = \frac{|\overline{AB}|^2 + |\overline{BC}|^2 - |\overline{CA}|^2}{2\cdot|AB|\cdot|\overline{BC}|}$$
$$= \frac{29+29-6}{2\cdot\sqrt{29}\cdot\sqrt{29}}$$

$$\cos\theta = \frac{26}{29}$$

$$\therefore \sin\theta = \frac{\sqrt{165}}{29}$$

$$\text{Area} = \frac{1}{2} \times |\overline{AB}| \times |\overline{BC}| \sin\theta$$
$$= \frac{1}{2} \times \sqrt{29} \times \sqrt{29} \cdot \frac{\sqrt{165}}{29}$$
$$= \frac{1}{2}\sqrt{165}$$

**Ans**

**Q.84** If $\overline{A} = 2\hat{i} + \hat{j} - 3\hat{k}$ & $\overline{B} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 5 perpendicular to both A & B.

**Soln:**

$$\overline{A} \times \overline{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(1-6) + \hat{j}(-3-2) + \hat{k}(-4-1)$$

$$= -5\hat{i} - 5\hat{j} - 5\hat{k}$$

$$= -5(\hat{i} + \hat{j} + \hat{k})$$

$\overline{A} \times \overline{B} = -\overline{B} \times \overline{A}$ ∴, $5(\hat{i} + \hat{j} + \hat{k}) = \overline{B} \times \overline{A}$

whic is perpendicular to both $\overline{A}$ & $\overline{B}$

we need a vector of magnitude 5 in the direction of the perpendicular to $\overline{A}$ & $\overline{B}$.

So we first find a unit vector in the direction.

$$\therefore \hat{n} = \frac{\overline{A} \times \overline{B}}{|\overline{A} \times \overline{B}|} = \frac{-5(\hat{i}+\hat{j}+\hat{k})}{\pm\sqrt{5^2+5^2+5^2}} = \pm \frac{5(\hat{i}+\hat{j}+\hat{k})}{5\sqrt{3}}$$

$$= \pm \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$$

∴ it has a magnitude of 1.

$$\therefore \text{vector we want} = 5 \times \pm \frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}$$

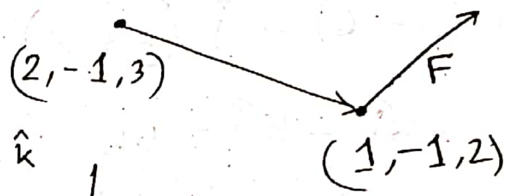
$$= \pm \frac{5(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3} \cdot \sqrt{3}} \sqrt{3}$$

$$= \pm \frac{5\sqrt{3}}{3} (\hat{i}+\hat{j}+\hat{k})$$ Ans

**Q.86//** A force given by $F = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of F about the point $(2, -1, 3)$.

**Soln:**

$$\vec{M} = \vec{F} \times \vec{r}$$




$$\vec{F} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -4 \\ 1 & 0 & 1 \end{vmatrix}$$

$\vec{r} = (2-1)\hat{i} + (-1+1)\hat{j} + (3-2)\hat{k}$
$= \hat{i} + \hat{k}$

$$= \hat{i}(2-0) + \hat{j}(-4-3) + \hat{k}(-2)$$

$$= 2\hat{i} - 7\hat{j} - 2\hat{k}$$

Ans

**Q.87//** The angular velocity of a rotating rigid body about an axis of rotation is given by $\omega = 4\hat{i} + \hat{j} - 2\hat{k}$. Find the linear velocity of a point p on the body whose position vector to a point on the axis of rotation is $2\hat{i} - 3\hat{j} + k$.

**Soln:** Here $\bar{r} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\therefore v = \bar{\omega} \times \bar{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= -5\hat{i} - 8\hat{j} - 14\hat{k}$$

Ans

Q.88|| Simplify $(A+B)\cdot(B+C)\times(C+A)$

Sol$^n$:

$\Rightarrow (A+B)\cdot[(B+C)\times C + (B+C)\times A]$

$= (A+B)\cdot[-C\times(B+C) - A\times(B+C)]$

$= (A+B)\cdot\{-[C\times(B+C) + A\times(B+C)]\}$

$= (A+B)\{-[C\times B + C\times C + A\times B + A\times C]\}$

$= (A+B)\{-\{-(B\times C) + \cancel{|C||C|\sin 0^\circ\cdot\hat{n}}^{0} - B\times A - C\times A\}\}$

$= (A+B)[(B\times C) + (B\times A) + (C\times A)]$

$= A\cdot(B\times C) + A\cdot(B\times A) + A\cdot(C\times A) + B\cdot(B\times C) + B\cdot(B\times A)$
$+ B\cdot(C\times A)$

$(B\times A)$ gives vector perpendicular to both A & B.

$\therefore A\cdot(B\times A) = 0$

Similarly, $A\cdot(C\times A) = B\cdot(B\times C) = B\cdot(B\times A) = 0$

So, we get $\rightarrow$

$= A\cdot(B\times C) + B\cdot(C\times A)$

$= A\cdot(B\times C) + A\cdot(B\times C)$ $[\because A\cdot B\times C = B\cdot C\times A]$

$= 2A\cdot B\times C$

Ans

Q.93 If $A = x_1 a + y_1 b + z_1 c$, $B = x_2 a + y_2 b + z_2 c$ & $c = x_3 a + y_3 b + z_3 c$, prove that

$$A \cdot B \times C = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} (a \cdot b \times c)$$

Soln:

$$B \times C = \begin{vmatrix} a & b & c \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$= a(y_2 z_3 - z_2 y_3) + b(z_2 x_3 - z_3 x_2) + c(x_2 y_3 - y_2 x_3)$$

$$A \cdot B \times C = (x_1 a + y_1 b + z_1 c) \cdot (B \times C)$$

$$= x_1 (y_2 z_3 - z_2 y_3) + y_1 (z_2 x_3 - z_3 x_2) + z_1 (x_2 y_3 - y_2 x_3)$$

which can be written as the form of determinent

$$\rightarrow \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

If we consider a, b, c are 3 perpendicular unit vector.

As a result $a \cdot b \times c$ means $\longrightarrow$

$b \times c$ gives a vector in the direction of of a & magnitude of $|\bar{a}|$.

$\therefore a \cdot b \times c = |a| \cdot |\bar{a}| \cdot \cos 0 = 1$

If we don't consider a, b, c are perpendicular then we will get a determinent which will written as $(a \cdot b \times c)$.

**Q.90|** Find the volume of the parallelepiped whose edges are represented by $A = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $B = \hat{i} + 2\hat{j} - \hat{k}$, $C = 3\hat{i} - \hat{j} + 2\hat{k}$.

**Solⁿ:** Volume of parallelepiped $= A \cdot (B \times C)$

$$\bar{B} \times \bar{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(4-1) + \hat{j}(-3-2) + \hat{k}(-1-6)$$

$$= 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$\therefore \bar{A} \cdot (\bar{B} \times \bar{C}) = (2\hat{i} - 3\hat{j} + 4\hat{k})(3\hat{i} - 5\hat{j} - 7\hat{k})$$

$$= 6 + 15 - 28 = -7$$

As volume cannot be negative

$\therefore$ Ans is 7.

**Q.92|** Find the constant 'a' such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ & $3\hat{i} + a\hat{j} + 5\hat{k}$ are co-planar.

**Solⁿ:** Let $\bar{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\bar{B} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\bar{C} = 3\hat{i} + a\hat{j} + 5\hat{k}$

For co-planar, $\bar{A} \cdot (\bar{B} \times \bar{C}) = 0$

$$\bar{B} \times \bar{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 3 & a & 5 \end{vmatrix}$$

$$= \hat{i}\,(10+3a) + \hat{j}\,(-9-5) + \hat{k}\,(a-6)$$

$$= (10+3a)\hat{i} - 14\hat{j} + \hat{k}\,(a-6)$$

$$\therefore \vec{A}\cdot(\bar{B}\times\bar{C}) = (2\hat{i} - \hat{j} + \hat{k})\left[(10+3a)\hat{i} - 14\hat{j} + \hat{k}\,(a-6)\right]$$

$$= 2(10+3a) + 14 + a - 6$$

For co-planar; $\vec{A}\cdot(\bar{B}\times\bar{C}) = 0$

$$\Rightarrow 20 + 6a + 14 + a - 6 = 0$$

$$\Rightarrow 7a = -28$$

$$\Rightarrow a = -4.$$

Ans

**Q.95** Let P, Q, R have position vectors $r_1 = 3\hat{i} - 2\hat{j} - \hat{k}$ $r_2 = \hat{i} + 3\hat{j} + 4\hat{k}$ & $r_3 = 2\hat{i} + \hat{j} - 2\hat{k}$ relative to an origin 'O'. Find the distance from p to the plane OQR.

**Soln:** let 'o' has position vector $r_4 = 0\hat{i} + 0\hat{j} + 0\hat{k}$ since it is origin.

$eq^n$ of plane OQR is given by - (Q.45)

$$(r - r_1) \cdot (r_2 - r_1) \times (r_3 - r_1) = 0$$

In our problem, $r_1 = r_4$

$r = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore$ $eq^n$ of plane OQR is $\longrightarrow$

$$(r - r_4)\,(r_2 - r_4) \times (r_3 - r_4) = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k} - 0\hat{i} - 0\hat{j} - 0\hat{k}) \cdot (\hat{i} + 3\hat{j} + 4\hat{k} - 0\hat{i} - 0\hat{j} - 0\hat{k})$$
$$\times (2\hat{i} + \hat{j} - 2\hat{k} - 0\hat{i} - 0\hat{j} - 0\hat{k}) = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 3\hat{j} + 4\hat{k}) \times (2\hat{i} + \hat{j} - 2\hat{k}) = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \left[\hat{i}(-6-4) + \hat{j}(8+2) + \hat{k}(1-6)\right] = 0$$

$$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k})\,(-10\hat{i} + 10\hat{j} - 5\hat{k}) = 0$$

$\Rightarrow 2x - 2y + z = 0$ is our plane.

distance of this plane from 'p' is given by -

distance of the plane from point $(3,-2,-1)$

$\therefore$ distance $= \dfrac{2\cdot 3 - 2\cdot(-2) + 1\cdot(-1)}{\sqrt{2^2+(-2)^2+1^2}}$

$= \dfrac{9}{3} = 3$ Ans.

Q.102|| Find a set of vectors reciprocal to the set $2\hat{i}+3\hat{j}-k$, $\hat{i}-\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+2\hat{k}$.

Soln: reciprocal set's are $a', b', c'$

$a' = \dfrac{b\times c}{a\cdot b\times c}$ From Reciprocal set's of vector Theory.

$$b\times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 2 & 2 \end{vmatrix}$$

$= 2\hat{i} + \hat{k}$

$\bar{a}\cdot\bar{b}\times\bar{c} = (2\hat{i}+3\hat{j}-k)(2\hat{i}+k)$

$= 4 - 1 = 3$

$\therefore a' = \dfrac{2\hat{i}+\hat{k}}{3}$

Similarly $b' = \dfrac{c\times a}{a\cdot b\times c}$ & $c' = \dfrac{a\times b}{a\cdot b\times c}$

Ans

Q.97 Given points, P (2, 1, 3), Q (1, 2, 1), R (−1, −2, −2) & S (1, −4,
find the shortest distance between lines PQ and RS.

**Solⁿ:** $-\overline{PQ} = (2-1)\hat{i} + (1-2)\hat{j} + (3-1)\hat{k}$

$= \hat{i} - \hat{j} + 2\hat{k} = \overline{QP}$

$-\overline{RS} = (-1-1)\hat{i} + (-2+4)\hat{j} + (-2-0)\hat{k}$

$= -2\hat{i} + 2\hat{j} - 2\hat{k} = \overline{SR}$

eqⁿ. of Line between P & Q is given by

$$\frac{x-2}{2-1} = \frac{y-1}{1-2} = \frac{z-3}{3-1} = t \text{ (let)}$$

$$\Rightarrow \frac{x-2}{1} = t,\ \frac{y-1}{-1} = t,\ \frac{z-3}{2} = t$$

$\Rightarrow x = 2+t,\ y = 1-t,\ z = 2t+3$ ; $\underset{M}{(x, y, z)}$ are the co-ordinate of any point on PQ line.

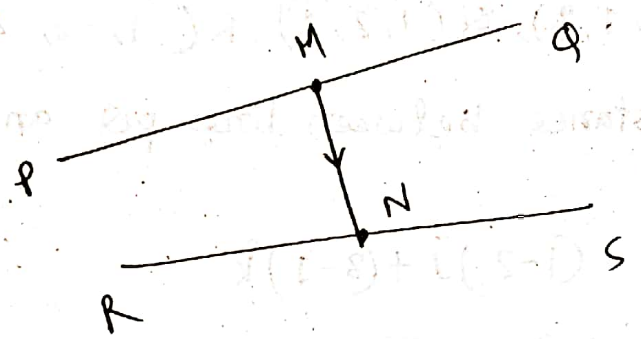
Similarly for R & S ⟶

$$\frac{x+1}{-1-1} = \frac{y+2}{-2+4} = \frac{z+2}{-2-0} = s \text{ (let)}$$

$$\Rightarrow \frac{x+1}{-2} = \frac{y+2}{2} = \frac{z+2}{-2} = s$$

$$\Rightarrow \frac{x+1}{1} = \frac{y+2}{-1} = \frac{z+2}{1} = s$$

$\Rightarrow x = s-1,\ y = -s-2,\ z = s-2$ ; any points on RS line. $\underset{N}{(x, y, z)}$




$$MN = \left((t+2-s+1)\hat{i} + (1-t+s+2)\hat{j} + (2t+3-s+2)\hat{k}\right)$$
$$= \left[(t-s+3)\hat{i} + (-t+s+3)\hat{j} + (2t-s+5)\hat{k}\right)$$

- Now for shortest distance MN must be perpendicular to both PQ & RS.

$$MN \cdot \overline{QP} = 0$$

$$\Rightarrow \left[(t-s+3)\hat{i} + (t+s+3)\hat{j} + (2t-s+5)\hat{k}\right] \cdot \left[\hat{i} - \hat{j} + 2\hat{k}\right] = 0$$

$$\Rightarrow t-s+3+t-s-3+4t-2s+10=0$$

$$\Rightarrow 6t-4s+10=0$$

$$\Rightarrow 3t-2s+5=0 \quad \text{——(i)}$$

& $MN \cdot \overline{SR} = 0$

$$\Rightarrow \left[(t-s+3)\hat{i} + (-t+s+3)\hat{j} + (2t-s+5)\hat{k}\right] \cdot \left[-2\hat{i} + 2\hat{j} - 2\hat{k}\right] = 0$$

$$\Rightarrow -2(t-s+3) + 2(-t+s+3) - 2(2t-s+5) = 0$$

$$\Rightarrow 4t-3s+5=0 \quad \text{——(ii)}$$

Solving (i) & (ii) gives $t = s = -5$

$\therefore\ t = s = -5$.

For this value of $t$ & $s$ MN becomes shortest distance.

$$\overline{MN} = (-5+5+3)\hat{i} + (5-5+3)\hat{j} + (2\cdot(-5)+5+5)\hat{k}$$

$$= 3\hat{i} + 3\hat{j}$$

$$\therefore |\overline{MN}| = \sqrt{3^2+3^2} = 3\sqrt{2}$$

Ans

Q.96|| Find the shortest distance from $(6,-4,4)$ to the line joining $\underbrace{(2,1,2)}_{M}$ & $\underbrace{(3,-1,4)}_{N}$.

Soln: eqn of line between two points $\rightarrow$

$$\frac{x-2}{2-3} = \frac{y-1}{1+1} = \frac{z-2}{2-4} = t$$

$$\Rightarrow \frac{x-2}{-1} = \frac{y-1}{+2} = \frac{z-2}{-2} = t$$

$$\Rightarrow \frac{x-2}{1} = \frac{y-1}{-2} = \frac{z-2}{2} = t$$

$\Rightarrow x = t+2,\ y = 1-2t,\ z = 2+2t$ are coordinate of any point on the line.

let, any point on the line is P & the given point is Q $(6,-4,4)$

$$\therefore \overline{QP} = (t+2-6)\hat{i} + (1-2t+4)\hat{j} + (2+2t-4)\hat{k}$$

$$= (t-4)\hat{i} + (5-2t)\hat{j} + (2t-4)\hat{k}$$

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$\overline{PQ}$ or $\overline{QP}$ is perpendicular to the line for shortest distance.

$$\overline{QP} \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) = 0$$

$$\overline{MN} = (3-2)\hat{i} + (-1-1)\hat{j} + (4-2)\hat{k}$$
$$= \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\Rightarrow t - 4 - 10 + 4t + 4t - 4 = 0$$

$$\Rightarrow 9t - 18 = 0$$

$$\Rightarrow t = 2$$

$\therefore$ $\overline{MN} \perp \overline{PQ}/\overline{QP}$ for $t = 2$.

$$\therefore \overline{QP} = (2-4)\hat{i} + (5-2\cdot2)\hat{j} + (2\cdot2-2)\hat{k}$$

$$= -2\hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore |\overline{QP}| = \sqrt{(-2)^2 + 1^2 + 2^2}$$

$$= 3$$

Ans.

Q.104 || If $a, b, c$ & $a', b', c'$ are such that,

$$a'.a = b'.b = c'.c = 1$$

$$a'.b = a'.c = b'.a = b'.c = c'.a = c'.b = 0$$

prove that if necessarilly follows that $\longrightarrow$

$$a' = \frac{b \times c}{a.b \times c}, \quad b' = \frac{c \times a}{a.b \times c}, \quad c' = \frac{a \times b}{a.b \times c}$$

Sol^n^: Given, $a.a' = 1$

It means that $|a|.|a'|.\cos\theta = 1$

& $a'.b = a'.c = 0$. So $a'$ perpendicular to both b&c

It follows that, $a'$ must be in the direction of a.

$\therefore$ $\cos\theta$ gives the value $-1$

If $\cos\theta = 1$ then, $|a'| = \frac{1}{|a|}$

$b \times c \rightarrow$ gives direction perpendicular to a & the vector a itself.

$$\therefore \quad \bar{a} = \bar{b} \times \bar{c}$$

$$\therefore |a| = \bar{a}.\bar{a} = \bar{a}.\bar{b} \times \bar{c}$$

$$\therefore |a'| = \frac{1}{\bar{a}.\bar{b} \times \bar{c}}$$

as $a'$ has a direction in the direction of a then we write it as $a' = \frac{\bar{b} \times \bar{c}}{\bar{a}.\bar{b} \times \bar{c}}$

Similarity for others.

Q. 103 If $a' = \frac{b \times c}{a \cdot b \times c}$, $b' = \frac{c \times a}{a \cdot b \times c}$, $c' = \frac{a \times b}{a \cdot b \times c}$,

prove that, $a = \frac{b' \times c'}{a' \cdot b' \times c'}$, $b = \frac{c' \times a'}{a' \cdot b' \times c'}$, $c = \frac{a' \times b'}{a' \cdot b' \times c'}$

Soln: In previous problem,

If we replace a, b, c by a′, b′ & c′ respectively we find the result.