GRADE 10 MATHEMATICS MODULE

THIS MODULE IS PREPARED FOR GRADE 10 STUDENTS TO STUDY AT THEIR HOME BECAUSE OF THE IMPOSSIBLITY OF FACE TO FACE TEACHING-LEARNIG PROCESS DUE TO CORONA VIRUS DISEASE (COVID-19) IN 2020.

THE MODULE CONTAINS SHORT NOTES, EXAMPLES, EXERCISES AND PRACTICE QUASTIONS BASED ON GRADE 10 MATHEMATICS FROM UNIT THREE UP TO SEVEN.

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UNIT-3

SOLVING INEQUALITIES

General Objectives:

After Completing this unit, you will be able to:

- Know and apply methods and procedures in solving problems on inequalities involving absolute value.
- Know and apply methods in solving systems of linear inequalities.
- Apply different techniques of solving quadratic inequalities

3.1 Inequalities involving absolute value

Definition: If x is a real number the absolute value of x , denoted by $|x|$, is defined by

$$|x| = \begin{cases} x, if\, x \geq 0 \\ -x, if\ x < 0 \end{cases} \quad \text{or} \quad |x| = \begin{cases} x, if\ x > 0 \\ o, if\ x = 0 \\ -x, if\ x < 0 \end{cases}$$

Example: $|20| = 20$

$|-20| = -(-20) = 20$

Note: i. The absolute value of a number is always non negative.

ii. The absolute value of a number is, in other word the distance of a number from zero without considering the direction.

Properties of absolute value

For any two real numbers *a* and *b*:

1. i. $a \leq |a|$

   ii. $-|a| \leq a \leq |a|$
2. $|ab| = |a||b|$
3. $|a+b| \leq |a| + |b|$ (Triangular inequality)

   i. $|a+b| = |a| + |b|$, if a and b have the same sign (both positive or both negative)

   ii. $|a+b| < |a| + |b|$, if a and b have opposite signs
4. $|a-b| = |b-a|$
5. $\sqrt{a^2} = |a| = |-a|$
6. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $, b \neq 0$

Notation: For fixed real numbers *a* and *b* where *a* < *b*

✓ **(a, b) is an open interval.**
(a, b) = $\{x : a \leq x \leq b\ and\ x \in \Re\}$

✓ **(a, b] and [a, b) are half closed or half open interval.**

(a, b] $=\{x : a < x \le b \ and \ x \in \Re\}$ and

[a, b) = $\{x : a \le x < b \, and \, x \in \Re\}$

✓ **[a, b] is closed interval**.

[a, b] $=\{x : a \le x \le b \, and \, x \in \Re\}$

Example: (4,21) = $\{x : 4 < x < 21 where x \in \Re\}$

[-5, 13) = $\{x : -5 \le x < 13 where x \in \Re\}$

Note: The symbol $"\infty"$ is used to mean positive infinity and $"-\infty"$ is used to mean negative infinity.

Example: $(8, \infty) = \{x : x > 8$ where $x \in \Re\}$

$(-\infty, 10] = \{x : x \le 10 where x \in \Re\}$

Activity: Simplify the following and write in interval notation.

a. $\{x : 3x + 18 \le 30\}$

b. $\{x : -9 > x - 6 > 15\}$

Theorems and Corollaries On Equations and Inequalities, Involving Absolute Values:

Theorem 3.1 **Solutions of the form** $|x| = a$

For any real number a, the equation $|x| = a$ has

i. two solutions ***x = a*** and ***x = -a*** if a > 0
ii. one solution, x = 0 if x = 0
iii. no solution, if a < 0

Example: Solve each of the following absolute value equations.

a. $|-2x - 48| = 4$ b. $|3 - 5x| = -10$

Solution:

a. $|-2x - 48| = 4$, since 4> 0 the equation has two solutions.

$|-2x - 48| = 4$ is equivalent to -2x -48 = 4 or -2x -48 = -4

$\Rightarrow -2x - 48 + 48 = 4 + 48$ or -2x -48 +48 = -4 + 48

$\Rightarrow -2x = 52$ or -2x = 44

$\Rightarrow x = -26$ or x = -22

b. $|3 - 5x| = -10$, since -10 < 0 the equation has no solution.

Theorem 3. 2 **Solution of** $|x| < a$ **and** $|x \le a|$

For any real number a > 0,

i. the solution of the inequality $|x| < a$ is –a < x < a

ii. the solution of the inequality $|x| \le a$ is $-a \le x \le a$

Example: Solve each of the following absolute value inequalities.

a. $|-x+25|<31$ b. $|4x-16|\le 24$ c. $|x+7|<0$ d. $|8x-4|<x-2$

Solution:

a. $|-x+25|<31$ is equivalent to $-31<-x+25<31$

$\Rightarrow -31-25<-x+25-25<31-25$

$\Rightarrow -56<-x<6$ (multiplying both sides of the inequalities by -1 gives)

$\Rightarrow 56>x>-6$ = (-6, 56) is the solution.

b. $|4x-16|\le 24$ is equivalent to $-24\le 4x-16\le 24$

$\Rightarrow -24+16\le 4x-16+16\le 24+16$

$\Rightarrow -8\le 4x\le 40$

$\Rightarrow -2\le x\le 10$ = [-2, 10] is the solution.

c. $|x+7|<0$, since the absolute value of a number is nonnegative the inequality has no solution.

d. $|2x-3|<x-1$ has a solution if x – 1> 0 $\Rightarrow$ x >1 = $(1,\infty)$

By theorem 3.2, -(x-1) < 2x – 3and 2x -3< x – 1

$\Rightarrow$ -x + 1 < 2x – 3&x < 2

$\Rightarrow$ -3x < -4& x <2

$\Rightarrow$ x > $\frac{4}{3}$ & x <2= $\frac{4}{3}<x<2=(-\infty,2)\cap(\frac{4}{3},\infty)=(\frac{4}{3},2)$

Solution set = $(1,\infty)\cap\left(\frac{4}{3},2\right)=(\frac{4}{3},2)$

Theorem 3.3 Solution of $|x|>a$ and $|x|\ge a$

For any real number a, if a > 0, then

i. the solution of the inequality $|x|>a$ is x < -a or x > a

ii. the solution of the inequality $|x|\ge a$ is $x\le -a$ or $x\ge a$

Example: Solve each of the following absolute value inequalities.

a. $|1-2x|\ge 5$ b. $|-4x+27|>-11$

Solution:

a. $|1-2x|\ge 5$ by theorem 3.3 above, $1-2x\le -5$ or $1-2x\ge 5$

$\Rightarrow -2x\le -6$ or $-2x\ge 4$

$\Rightarrow x\ge 3$ or $x\le -2$

$\Rightarrow (-\infty,-2]\cup[3,\infty)$ is the solution

b. $|-4x+27|>-11$ since the absolute value of a number is always greater than or equal to zero, the inequality is true for all real numbers. Therefore the solution is the set of all real numbers.

3.2 SYSTEMS OF LINEAR INEQUALITIES IN TWO VARIABLES

- A first degree (linear) equation in two variables has a form ax +by = c, where a,b and c are real numbers and a and b are non zero.
- Two or more linear equations involving the same variables is called system of linear equations in two variables.
- The set of all ordered pairs satisfying the system (a region satisfying all equations, geometrically) is the solution of the system.
- A system of linear equations in two variables has a form $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$
- In a system of equation, if " = " is replaced by "<",">","≤"or "≥", the system becomes a system of linear inequalities.
- The solution of a system of linear inequalities is the intersection of the shaded regions.

Steps For solving system linear inequalities graphically

Step-1 Change each linear inequality to equation. Which is the equation of the boundary line

Step-2 For equation in step-1 prepare table of values

Step-3 Plot the points on the on the coordinate plane

Step-4 Using a ruler connect the points by solid line (_________) if the inequality "≤"or "≥" and by broken line (--------) if the inequality is "<" or ">"

Step-5 By taking a test point for each inequality shade the plane above or below the boundary line for which the inequality is true

The intersection of the shaded regions is the solution of the system.

Activity: Solve the following systems of linear inequalities graphically

a. $\begin{cases} y + x > 0 \\ y - x \le 1 \\ x \le 3 \end{cases}$

b. $\begin{cases} y - x \le 2 \\ y + x \ge 3 \\ x < 5 \end{cases}$

Note: To determine

i. the domain from graph use a vertical line test

ii. the range from graph use a horizontal line test

3.3 QUADRATIC INEQUALITIES

Definition: An inequality that can be reduced to any one of the following forms

$ax^2+bx+c\le 0$ or $ax^2+bx+c<0$,

$ax^2+bx+c\ge 0$ or $ax^2+bx+c>0$, where a, b and c are constants and $a\neq 0$ is called a *quadratic inequality*.

Example: $x^2-3x+2<0$, $(x+1)(x-1)\ge x-5$

Solving Quadratic Inequalities Using Product Properties

Product properties

a. $m.n>0$, if and only if
   i. m > 0 and n >0 or
   ii. m <0 and n < 0

b. $m.n<0$, if and only if
   i. m > 0 and n < 0 or
   ii. m < 0 and n > 0

Example: 1. Solve each of the following inequalities using product properties.

a. x(x+5) > 0

b. $(1+x)(3-2x)\le 0$

c. (5x-3)(x+7) <0

d. $(5-x)(1-\frac{1}{3}x)\ge 0$

Solution:

a. x(x+5) > 0 if and only if

<u>**case -I**</u> : x > 0 and x+5 >0

$\Rightarrow$x > 0 and x >-5

$\Rightarrow(0,\infty)\cap(-5,\infty)$

$\Rightarrow(0,\infty)$

<u>**Case –II**</u>: x < 0 and x + 5 < 0

$\Rightarrow$x < 0 and x < -5

$\Rightarrow(-\infty,0)\cap(-\infty,-5)$

$\Rightarrow(-\infty,-5)$

Therefore the solution is $(-\infty,-5)\cup(0,\infty)$

b. $(1+x)(3-2x)\le 0$ if and only if

<u>**Case-I:**</u> $(1+x)\ge 0$ and $(3-2x)\le 0$

$\Rightarrow x\ge -1$ and $-2x\le -3$

$\Rightarrow x\ge -1$ and $x\ge\frac{3}{2}$

$\Rightarrow[-1,\infty)\cap[\frac{3}{2},\infty)$

$\Rightarrow[\frac{3}{2},\infty)$, **therefore the solution is** $(-\infty,-1]\cup[\frac{3}{2},\infty)$

<u>**Case- II:**</u> $(1+x)\le 0$ & $(3-2x)\ge 0$

$\Rightarrow x\le -1$ & $x\le\frac{3}{2}$

$\Rightarrow(-\infty,-1]\cap(-\infty,\frac{3}{2}]$

$\Rightarrow(-\infty,-1]$

c and d are for exercise

Activity: Factorize and solve the following inequalities using product properties.

a. $x^2 + 5x + 4 < 0$ c. $x^2 + 5x + 6 \le 0$

b. $3x^2 + 4x + 1 \ge 0$ d. $2x^2 - 7x + 3 > 0$

Solving Quadratic Inequalities Using The Sign Chart Method

The quantity of a given quadratic inequality can be positive or negative or zero for a given values of x. To solve the inequality, we find the values of x for which the expression is negative or positive.

Example : Solve each of the following inequalities using the sign chart method

a. $(2x-3)(x+1) \le 0$ b. $x^2 + 7x + 12 > 0$ c. $x^2 - 5 > 0$

Solution:

a. $(2x-3)(x+1) \le 0$

The zeros are $\frac{3}{2}$ and -1

We plot these zeros on a number line

-1 3/2

(2x-3)	-----	------	+++
(x+1)	-----	+++	+++
(2x-3)(x+1)	+++	-----	+++

(2x-3)(x+1) ≤ 0 in the interval [-1,3/2]

There fore the solution is [-1, 3/2]

b and c -- exercise

2. For what values of k does the following quadratic equations has

i. only one real root? ii. two distinct real roots? iii. No real roots?

a**.** $(k+1)x^2 - kx - 1 = 0$ b. $4x^2 - (k+6)x + 9 = 0$

Solving Quadratic Inequality not factorizable into linear factors

If $b^2 - 4ac < 0$, then $ax^2 + bx + c = 0$ has no real roots and the curve $y = ax^2 + bx + c$ does not meet the x-axis.

Note: $b^2 - 4ac < 0$ is the indicator for $ax^2 + bx + c$ is not factorizabled. In this case, either $ax^2 + bx + c > 0$ for all values x or $ax^2 + bx + c < 0$ for all values of x.

Thus, the solution set of $ax^2 + bx + c > 0$ is

i. For $a > 0$, is $(-\infty, \infty)$ ii. For $a < 0$, is ϕ

And the solution set of $ax^2 + bx + c < 0$ is

i. ϕ for $a > 0$ ii. $\Re$, for $a < 0$

Activity:

Solve each of the following inequalities

a. $2x^2 + 2 > -3x$ b. $-3x^2 + 2x < 1$ c. $-6x + 1 \ge 9x^2$

Solving Quadratic Inequalities Graphically

For any quadratic function $f(x) = ax^2 + bx + c < 0, a \neq 0$

i. If $a > 0$, then f(x) has a minimum value. ii. If $a < 0$, f(x) has a maximum value

- The value of x at which the quadratic function $f(x) = ax^2 + bx + c, a \neq 0$ attains its maximum or minimum point is $x = \frac{-b}{2a}$
- To solve a quadratic inequality graphically, find the values of x for which the part of the graph of the corresponding quadratic function is above the x-axis
- , below the x-axis on the x-axis.

Activity: Solve the following quadratic inequalities graphically.

a. $-x^2 + 5x \leq 4$ b. $x^2 < -2$ c. $4x^2 + 9 - 12x > 0$

PRACTICE QUESTIONS -I

GENERAL DIRECTION: THE FOLLOWING QUESTIONS ARE PREPARED BASED ON CHAPTER THREE OF GRADE 10 MATHEMATICS AS A PRACTICE QUESTIONS FOR GRADE 10 STUDENTS. BEFORE DOING THE QUESTIONS READ THE GIVEN SHORT NOTES AND OTHER RELATED MATERIALS.

I. FOR EACH OF THE FOLLOWING QUESTIONS THERE ARE FOUR ALTERNATIVE CHOISES. CHOOSE THE BEST ANSWER.

1. Which of the following is not quadratic inequality?

   A. $2x^2 \geq x(5x+1)-2$ C. $x^2 > 3(x+x^2)-8$
   B. $x(1-x) \leq (x+4)(2-x)$. D. $(x+5)(x-1) \leq 3-2x$

2. For what values of x does $-2x^2+9x+5 \geq 0$?

   A. $\left[\frac{-1}{2},5\right]$ B $\left(-\infty,\frac{-1}{2}\right)\cup(5,\infty)$ C. $\left(-\infty,\frac{-1}{2}\right)$ D. $(5,\infty)$

3. What must be the value(s) of k, so that $(2k-3)x^2-kx+1=0$ has no real roots?

   A. $(-\infty,2)\cup(6,\infty)$ B. $[2,6]$ C. $(-\infty,\infty)$ D. $(2,6)$

4. The interval notation of $x \in \Re/\{1,2\}$ is;

   A. $(-\infty,1]\cup[2,\infty)$ B. $(1,2)$ C. $(-\infty,1)\cup(1,2)\cup(2,\infty)$ D. $[1,2]$

5. The solution set of $|3x-4| \leq 0$ is;

   A. $\left\{\frac{4}{3}\right\}$ B. ϕ C. $\left(-\infty,\frac{4}{3}\right)\cup\left(\frac{4}{3},\infty\right)$ D. $(-\infty,0)\cup(0,\infty)$

6. What is the value of k, so that $kx^2+4x+k<0$ for all x?

   A. $(-\infty,-2)\cup(2,\infty)$ B. $(1,2)$ C. $[-2,2]$ D. $(-2,\infty)$

7. If $|2x-1|<x+3$,then what is the value of x?

   A. $\left[\frac{1}{2},4\right)$ B. $\left(\frac{-2}{3},\frac{1}{2}\right)$ C. $\left(\frac{-2}{3},4\right)$ D. $\left[\frac{1}{2},\frac{2}{3}\right]$

8. For what value(s) of x does the graph of $-2x^2-4x-3$ is above the x-axis?

   A. $\Re/\{2\}$ B. $\Re$ C. $(-\infty,-3)\cup(4,\infty)$ D. No value of x

9. If x is a non positive real number , then what is the solution of $|x|=x$?

   A. 0 B. $\{\ \}$ C. $\Re$ D. negative real numbers

10. What is the truth set of $-x-3>-2$ & $x-3>-5$?

    A. $\{x: x<-1\}$ B. $\{x: x>-2\}$ C. $\{x: -2<x<-1\}$ D. $\{x: x>-5\}$

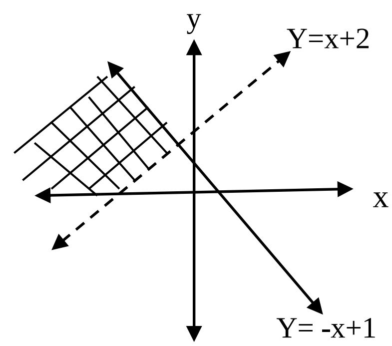
11. Which of the following is a quadratic inequality?

    A. $x(1-x) \leq (x+2)(1-x)$ C. $(x^2+1)(x+1)<-1$
    B. $6x-2x^2>-2(x^2+x)+5$ D. $(x-2)(x+1) \leq -2x+2$

12. The solution set of $|6x-7|=|4x-3|$ is;

A. $\{2\}$ B. $\{-2,2\}$ C. $\{1,2\}$ D. $\{-2,-1,1,2\}$

13. The solution of the system of linear inequalities given below in graph is;



A. $\left\{(x,y): x\leq -2 \ \& \ 0\leq y\leq -x+1 or -2\leq x\leq -\frac{1}{2} \ \& \ x+2\leq y\leq -x+1\right\}$

B. $\left\{(x,y): x< -2 \ \& \ 0\leq y\leq -x+1 \ or -2< x< -\frac{1}{2} \ \& \ x+2< y\leq -x+1\right\}$

C. $\left\{(x,y): x< -2 \ \& \ 0< y\leq -x+1 \ or -2< x\leq -\frac{1}{2} \ \& \ x+2\leq y\leq -x+1\right\}$

D. $\left\{(x,y): x\leq -2 \ \& \ 0< y\leq -x+1 \ or -2\leq x< -\frac{1}{2} \ \& \ x+2< y\leq -x+1\right\}$

14. What is the solution of $x+6-x^2\geq 0$?

A. $(-\infty,-2)\cup(3,\infty)$ B. $[-2,3]$ C. $[-2,\infty)$ D. $(-\infty,3]$

UNIT-4

COORDINATE GEOMETRY

General Objectives

After completing this unit, you will be able to:

- Apply distance formula to find the distance b/n any two given points in a coordinate plane
- Formulate and apply section formula to find a point that devises a given line segment in a given ratio.
- Describe parallel or perpendicular line in terms of their slopes

4.1 DISTANCE BETWEEN TWO POINTS

Distance between points in a plane

Suppose P(x_1,y_1) and Q(x_2,y_2) are two distinct points on the x y- coordinate . We can find the distance b/n P and Q by considering three cases.

Case-I When P and Q are on a line parallel to the x-axis. $\overline{PQ}$ is a horizontal segment. P and Q have the same y-coordinate (Ordinate).The distance b/n P and Q is the difference of x-coordinates.

i.e $PQ = |x_2 - x_1| = |x_1 - x_2|$

Case-II: When P and Q are on a line parallel to the y-axis. In this case the two points have the same x-coordinate (abscissa) and the distance b/n P and Q is the difference y-coordinates.

i.e $PQ = |y_2 - y_1| = |y_1 - y_2|$

Case-III: When $\overline{PQ}$ is neither vertical nor horizontal. The distance b/n P and Q is

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ which is called *distance formula.*

Example: Find the distance b/n the following given two points.

a. A(2,-3) and B(6,4) b. $C(\sqrt{3},-1)$ and $D(2\sqrt{3},5)$

Solution:

a. $AB = \sqrt{(6-2)^2 + (4-(-3))^2} = \sqrt{4^2 + 7^2} = \sqrt{16+49} = \sqrt{65}$

b. Exercise

4.2 DIVISION OF A LINE SEGMENT

- The point R(x_0,y_0) dividing the line segment PQ internally in the ratio *m:n* is given by

$R(x_0,y_0) = \left(\frac{nx_1 + mx_2}{n+m}, \frac{ny_1 + my_2}{n+m}\right)$, where P(x_1,y_1) and Q(x_2,y_2) . This is called *section formula.*

Example:1. Find the coordinates of the point R that divides the line segment with end points ***A(-6,1)*** and ***B(-2,-4)*** internally in the ratio **2:5**

Solution: Let (x_1,y_1) = (-6,1) and (x_2,y_2) = (-2, -4)

m: n = 2: 5 $\Rightarrow$ m = 2 and n=5

$R(x_0, y_0) = \left(\frac{5(-6) + 2(-2)}{2+5}, \frac{5(1) + 2(-4)}{2+5}\right)$ $\Rightarrow$ R(x_0,y_0) = $\left(\frac{-30-4}{7}, \frac{5-8}{7}\right)$ = $\left(\frac{-34}{7}, \frac{-3}{7}\right)$

Activity: Find the coordinates of the points that divides the segment with end points M(-1,4) and N(6,8) into four equal parts.

Hint: The first point divides segment MN in the ratio 1:3, the second point in the ratio 2:2 and the third point in the ratio 3:1

The Mid- point Formula

- ✓ A point that divides a line segment into equal parts is called mid-point.
- ✓ The mid- point of the line segment joining the points (x_1,y_1) and (x_2,y_2) is given by

M(x$_0$,y$_0$) $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$, which is called *mid-point formula*

Activity: Find the coordinates of the mid- point of the line segments joining the points

a. E(-9,12) and F(21,-18) b. $G(1+\sqrt{3},-\sqrt{3})$ and $H(4-\sqrt{3},\sqrt{27})$

4.3 EQUATION OF A LINE

4.3.1 Gradient (Slope) of a line

- ❖ Gradient (slope) of a line is the ratio of vertical increment to horizontal increment
- ❖ Slope = $\frac{y_2-y_1}{x_2-x_1}=\frac{Change\,in\,y}{Change\,in\,x}$, where (x_1,y_1) and (x_2,y_2) are any two points on the given line.

OR Slope = $\tan\theta$, where θ is the angle of inclination of the line measured from the positive x-axis to a line in anticlockwise direction. θ is always less than 180^0

Note: a. A line making an acute angle of inclination θ with the positive direction of the x-axis has positive slope.

b. A line with obtuse angle of inclination θ has negative slope

c. If $90^0<\theta<180^0$, then $\tan\theta=-\tan(180^0-\theta)$

- ❖ Gradient of a horizontal line is 0.
- ❖ Gradient for a vertical line is not defined
- ❖ Three or more points that lie on the same line are called **collinear points**.
- ❖ Three or more lines that passes through one point are called **concurrent lines.**
- ❖ There are different forms of equation of a line. These are;

i. The point- slope form $\Rightarrow y-y_1=m(x-x_1)$

ii. The slope-intercept form $\Rightarrow y=mx+b$

iii. The two point form $\Rightarrow \frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$

iv. General form $\Rightarrow Ax+By+C=0$

Note: All the different forms of equations can be expressed in the general form Ax + By + C = 0

4.4 PARALLEL AND PERPENDICULAR LINES

- If two non vertical lines l_1 and l_2 are parallel to each other, then they have the same slope.
- Two lines with undefined slopes are also parallel.
- If two lines l_1 & l_2 have the same slope then they are parallel
- The equation of the line through P(x_0,y_0) that is parallel to the line Ax +By + C = 0 is given by: $A(x-x_0)+B(y-y_0)=0$, (x, y) is any arbitrary point on the line.
- Two non vertical lines having slopes m_1 & m_2 are perpendicular, if and only if $m_1.m_2=-1$
- A line with slope of 0 and a line with undefined slope are also perpendicular.

PRACTICE QUESTIONS-II

GENERAL DIRECTION: THE FOLLOWING QUESTIONS ARE PREPARED BASED ON CHAPTER FOUR OF GRADE 10 MATHEMATICS AS A PRACTICE QUESTIONS FOR GRADE 10 STUDENTS. BEFORE DOING THE QUESTIONS READ THE GIVEN SHORT NOTES AND OTHER RELATED MATERIALS.

I. FOR EACH OF THE FOLLOWING QUESTIONS THERE ARE FOUR ALTERNATIVE CHOISES. CHOOSE THE BEST ANSWER.

1. If a line $l: y=-2x+6$ crosses the x-axis and y-axis at A(3,0) and B(0,6) respectively, what is AB?
   A. 3 B. $3\sqrt{5}$ C. 6 D. 5
2. What is the coordinate of the midpoint of a line segment with end points P(-3,9) and Q(7,-3)?
   A. (-3,-2) B. (3,2) C. (-2,3) D. (2,3)
3. Suppose a line $l_1: 2x-y+4=0$ and $l_2: \frac{-1}{2}x-y+9=0$ are perpendicular lines. What is the distance of a point (4,12) on l_1 from the point of intersection of l_1 *and* l_2?
   A. $4\sqrt{10}$ B. $4\sqrt{5}$ C. $2\sqrt{5}$ D. $2\sqrt{10}$
4. If an equilateral triangle has vertices A(0,-2) and B(0,2) ,which of the following could be the coordinate of the third vertex?
   A. (0,-2) B. $(2\sqrt{3},0)$ C. $(0,-2\sqrt{3})$ D. $(-2\sqrt{3},2\sqrt{3})$
5. What is the distance of a point (-2,5) from a line $l: x-y=0$?
   A. $\frac{7\sqrt{2}}{2}$ B. 3 C. $\sqrt{29}$ D. 7
6. A line through which of the following points is perpendicular to a line through P(1,8) and Q(-4,2)?
   A. R(-3,5) and Q(-5,8) C. M(-1,-5) and N(-2,-1)
   B. G(0,6) and H(5,0) D. E(12,0) and F(0,10)
7. A triangle with slope of sides -2, $\frac{4}{3}$ and $\frac{1}{2}$ is;
   A. Equilateral triangle B. Isosceles triangle C. Right angled triangle D. Obtuse angled triangle
8. The following pairs of points are points on a line. Through which of the points a line rises up from the right and falls down to the left?
   A. (-2,1) and (2,-2) B. (4,3) and (-4,-3) C. (2,2) and (8,-10) D. (-1,3) and (2,-4)
9. If the midpoint of a line segment with one end point (3,-1) is $(\frac{1}{2},\frac{5}{2})$, then what is the coordinate of the other end point? A. (-2,6) B. (4,2) C. (-2,10) D. (3,7)
10. What is the equation of the perpendicular bisector of the segment joining the points (4,-3) and (2,1)?
    A. $y=-2x+5$ B. $y=\frac{1}{2}x+5$ C. $y=2x-\frac{3}{2}$ D. $y=\frac{1}{2}x-\frac{5}{2}$

11. If a line with x-intercept -4 and y-intercept 6 are given, then what is the slope of the line?

A. $\frac{-2}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. $\frac{-5}{2}$

12. Which one of the following lines is perpendicular to a line with equation x=-10?

A. $x=\frac{1}{10}$ B. $y=-10$ C. $y=x$ D. $y=\frac{1}{10}x$

13. What is the equation of a line which passes through (3,5) that is parallel to $2x-y=14$?

A. $y=x-1$ B. $y=2x-1$ C. $y=x-2$ D. $y=-2x+1$

14. What is the value of k so that P(3,0), Q(k,2) and R(2,4) are collinear?

A. $\frac{3}{2}$ B. $\frac{2}{5}$ C. $\frac{5}{6}$ D. $\frac{5}{2}$

15. What is the coordinates of M on $\overline{ST}$ for S(-3,0), T(0,-3) such that $\frac{SM}{MT}=\frac{2}{3}$?

A. $(\frac{9}{5},\frac{6}{5})$ B. $(\frac{-9}{5},\frac{-6}{5})$ C. $(\frac{-6}{5},\frac{-9}{5})$ D. $(\frac{6}{5},\frac{9}{5})$

16. For what value(s) of k is the distance between E(k+2, 3) and F(-2, k-1) is $\sqrt{40}$?

A. -2 B. -4 C.4 D.3

17. What is the angle of inclination of a line which passes through the points A(-2,11) and B(2,7)?

A. 45^0 B. 120^0 C.150^0 D.135^0

18. What is the equation of a line whose angle of inclination is 120^0 that passes through a point P(-4, 9)?

A. $y+9=\sqrt{3}x+4\sqrt{3}$

B. $y-4=-\frac{\sqrt{3}}{3}(x-9)$

C. $y-9=-\sqrt{3}(x+4)$

D. $y-9=-\frac{\sqrt{3}}{3}(x-4)$

19. If A(2,-2) and B(5,2) are two opposite vertices of square, then what is the area of the square?

A. 5 B. 25 C.12.5 D. 20.5

20. What is the equation of a line which x-intercept 5 and y-intercept -3?

A. $-3y+5x=-15$

B. $5y-3x=1$

C. $\frac{x}{5}-\frac{y}{3}=1$

D. $y=-3x-4$

II. Work out questions (Do the following questions and compare your answer with your partner if possible)

1. If A(-1,2) and B(5,-3) are points on the coordinate plane so that R is mid-point of $\overline{AB}$,then find the coordinates of R.
2. Given three points E(1,0), F(-3,0) and G(1,2).Then find the equation of a line through E and perpendicular to $\overline{FG}$.
3. Find the coordinates of M on $\overline{ST}$ for S(-3,0), T(0,-3) such that $\frac{SM}{MT}=\frac{2}{3}$.
4. Find the value of k, if A(k,3), B(2k-1,k) and C(-1,1) are vertices of right angled triangle ΔABC with hypotenuse $\overline{AB}$.
5. Find the coordinate of a point on the y-axis which is equidistant from A(-2,4) and B(7,9).
6. If A(2,-2) and B(5,2) are two opposite vertices of square, then determine;
   a. The length of its diagonal
   b. The area of square
7. Find the equation of a line which passes through (3,5) that is;
   a. Parallel to $2x-y=14$
   b. Perpendicular to $-3x+4y=6$
8. What is the angle of inclination of a line which passes through the points A(-2,5) and B(2,7)?
9. What is the equation of a line whose angle of inclination is 120^0 that passes through a point P(-4, 9)?
10. If points A(3,1), B(-1,1) and C(1,3) are vertices of a triangle and R, S, and T are mid points side AB,BC and AC respectively ; then what is the equation of a line containing a line segment ST?

UNIT-5

TRIGONOMETRIC FUNCTIONS

After completing this unit, you should be able to:

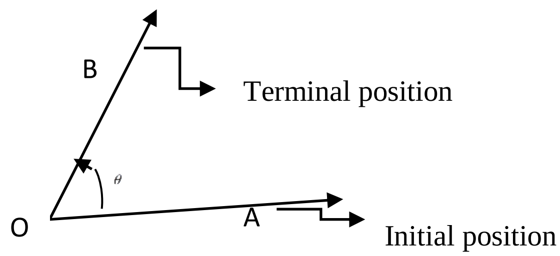
- ❖ know principles and methods for sketching graphs of basic trigonometric functions.
- ❖ understand important facts about reciprocals of basic trigonometric functions.
- ❖ identify trigonometric identities.
- ❖ solve real life problems involving trigonometric functions.

5.1 BASIC TRIGONOMETRIC FUNCTIONS

5.1.1.**The Sine, Cosine and Tangent Functions**

Basic Terminologies

Angle: An angle is the amount of rotation about a fixed point O from an initial position to new position.



The angle formed by a ray rotating anticlockwise is taken to be a positive angle.
An angle formed by a ray rotating clockwise is taken to be a negative angle.

Angles in standard position

An angle in the coordinate plane is said to be in standard position, if

1. its vertex is at the origin, and 2, its initial side lies on the positive x-axis.

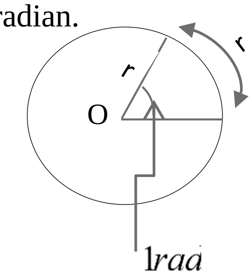
Quadrantal angles

If the terminal side of an angle in standard position lies along the x-axis or the y-axis, then the angle is called a *quadrantal angle.*

Example: …-180^0, -90^0, 0^0, 90^0. 180^0,…

Radian measure of angles

- The angle θ subtended at the center of a circle by an arc equal in length to the radius is 1 radian.
  That is θ = r/ r= 1 radian.
- In general, if the length of the arc is s units and the radius is r units, then θ = s/r radians.



Relationship Between Degrees and Radians

- ➢ A circle with radius r units has a circumference of $2\pi r$
- ➢ $\theta = \frac{2\pi r}{r} = 2\pi$ radians

$\Rightarrow 180^0 = \pi$ radians

1. To convert degrees to radians, multiply by $\frac{\pi}{180^0}$

i.e., radians = degrees × $\frac{\pi}{180^0}$

2. To convert radians to degrees, multiply by $\frac{180^0}{\pi}$

. i.e., degrees = radians × $\frac{180^0}{\pi}$

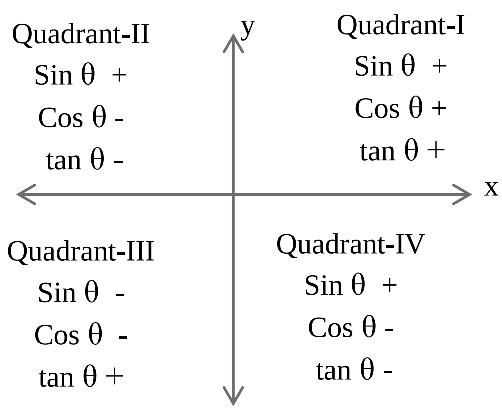
Reference angle(θ_R)

- If θ is an angle in standard position whose terminal side does not lie on either coordinate axis, then a reference angle θ_R for θ is the acute angle formed by the terminal side of θ and the x-axis.
- In different quadrants we have different formulas for θ_R

1. In first quadrant $\theta = \theta_R$ (If $0^0 < \theta < 90^0$)
2. In second quadrant $\theta_R = 180^0 - \theta$. In this quadrant,
   i. $\sin\theta = (180^0 - \theta)$ *ii.* $\cos\theta = -(180^0 - \theta)$ *iii.* $\tan\theta = -(180^0 - \theta)$
3. In third quadrant $\theta_R = \theta - 180^0$. In this quadrant,
   i. $\sin\theta = -(\theta - 180^0)$ *ii.* $\cos\theta = -(\theta - 180^0)$ *iii.* $\tan\theta = (\theta - 180^0)$
4. In fourth quadrant $\theta_R = 360^0 - \theta$. In this quadrant,
   i. $\sin\theta = -(360^0 - \theta)$ *ii.* $\cos\theta = (360^0 - \theta)$ *iii.* $\tan\theta = -(360^0 - \theta)$

Note: The above formula for θ_R in all quadrants is for rotation to anticlockwise direction and $0^0 < \theta < 360^0$

Question: How do you find θ_R for angles greater than 360^0? For negative angles?



Note: 1. If the sum of the measures of two acute angles is 90^0, then the two angles is said to be Complementary angles.

Example: 2^0 & 88^0, 31^0 & 59^0

2. For two complementary angles α & β,

$$\sin\alpha = \cos\beta, \cos\alpha = \sin\beta \ \& \ \tan\alpha = \frac{1}{\tan\beta}$$

3. If the sum of the measures of two angles is 180^0, then the two angles is said to be Supplementary angles.

   Example: 129^0 & 51^0 , 170^0 & 10^0

Co–terminal angles

- Co–terminal angles are angles in standard position that have a common terminal side.

  Example: The four angles $60^0, -300^0, 420^0, 780^0$ are co-terminal angles.

- Given an angle θ, all angles which are co-terminal with θ are given by the formula $\theta \pm n(360^0)$, where n = 1,2, 3, …

  Example: Find two co-terminal angles with

  a. -95^0 b. 230^0 c. -126^0

  Solution:

a. -95^0

If n = 2, $-95^0 + 2(360^0) = -95^0 + 720^0 = 625^0$ or

$-95^0 - 2(360^0) = -95^0 - 720^0 = -815^0$

b. & c ---- exercise

Note: Co- terminal angles have the same trigonometric values for the same trigonometric function.

Example: $\sin 30^0 = \sin 390^0 = \sin(-330^0) = 0.5$

5.1.2. Graphs of the Sine, Cosine and Tangent Functions

- A function that repeats its values at regular intervals is called a periodic function.

1. **Graph of the sine function** $(y = \sin\theta)$

a. The sine function repeats after every *360°* or *2π* radians. Therefore the period of sine function is *360°* or *2π*

b. The domain of sine function is $\Re$.

c. The range of sine function is $-1 \le y \le 1$

Reading assignment: Read graphs of the sine function from your text book on page 194-196 and study its properties.

2. **Graph of the cosine function** $(y = \cos\theta)$

a. The cosine function repeats after every *360°* or *2π* radians. Therefore the period of cosine function is *360°* or *2π*

b. The domain of cosine function is $\Re$.

c. The range of cosine function is $-1 \le y \le 1$

Reading assignment: Read graphs of the cosine function from your text book on page 196-197 and study its properties.

3. **Graph of the tangent function**

a. The tangent function repeats after every *180°* or π radians. Therefore the period of tangent function is *180°* or π

b. The domain of tangent function = $\left\{\theta : \theta \neq n\frac{\pi}{2}, \textit{where n is an odd integer}\right\}$

i.e when $\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$, etc

c. The range tangent function is $\Re$

Reading assignment: Read graphs of the tangent function from your text book on page 197-199 and study its properties.

5.2. SIMPLE TRIGONOMETRIC IDENTITIES

Let θ be an angle in standard position and P(x, y) be a point on the terminal side of θ and r is the distance of p from O(0,0).

1. **Pythagorean Identities**

i. $(\cos\theta)^2 + (\sin\theta)^2 = 1$ ii. $(\tan\theta)^2 + 1 = (\sec\theta)^2$ iii. $(\cot\theta)^2 + 1 = (\csc\theta)^2$

Note: $(\sin\theta)^2 = \sin^2\theta$, $(\cos\theta)^2 = \cos^2\theta$, $(\tan\theta)^2 = \tan^2\theta$

$(\csc\theta)^2 = \csc^2\theta$, $(\sec\theta)^2 = \sec^2\theta$, $(\cot\theta)^2 = \cot^2\theta$

2. **Reciprocal Identities** i. $\csc\theta = \frac{1}{\sin\theta}$ ii. $\sec\theta = \frac{1}{\cos\theta}$ iii. $\cot\theta = \frac{1}{\tan\theta}$

3. **Quotient Identities** i. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ii. $\cot\theta = \frac{\cos\theta}{\sin\theta}$

❖ An identity is an equation that is true for all values of the variable for which both sides of the equation are defined.

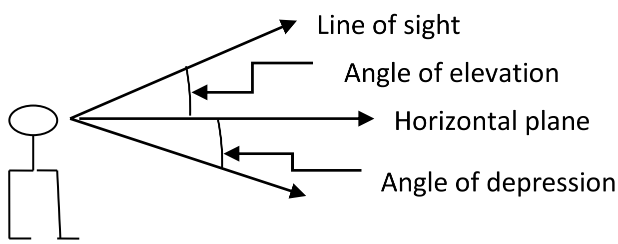
Note: 1. Sine, Cosine and tangent functions are the basic trigonometric functions.

2. Cosecant is the reciprocal of sine, secant is the reciprocal cosine and cotangent is the reciprocal of tangent.

3, Sine and cosine, Secant and Cosecant and tangent and cotangent are co-functions.

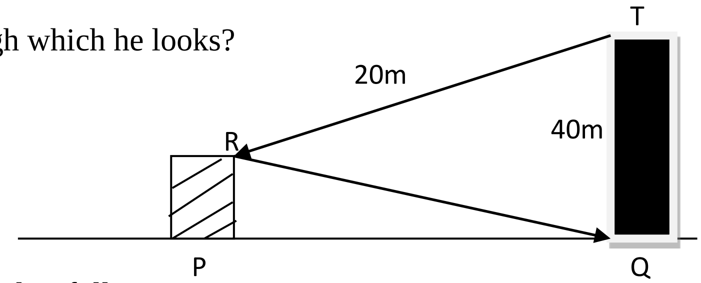
Angle of Elevation and Angle of Depression

- Angle of elevation is an angle between the horizontal line and line of sight
- The line of sight of an object is the line joining the eye of an observer and the object.
- If the object is above the horizontal line (plane) through the eye of the observer the angle between the line of sight and the horizontal plane is called *angle of elevation* .
- If the object is below the horizontal line (plane) through the eye of the observer the angle between the line of sight and the horizontal plane is called *angle of depression* .

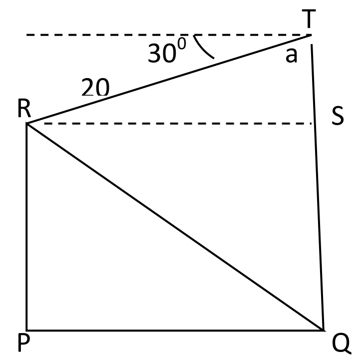


Example 1:A 20m cable from T, on top of building is joined to R, the top of another building across the street PQ, making an angle of depression of 30^0. The building TQ is 40mhigh.An observer at R looks at Q, the base of building across the street.

a. What is the distance RQ through which he looks?
b. How high is the building?



Solution: The problem can be represented as follows.



In ΔTRQ, a = $90^0 - 30^0 = 60^0$

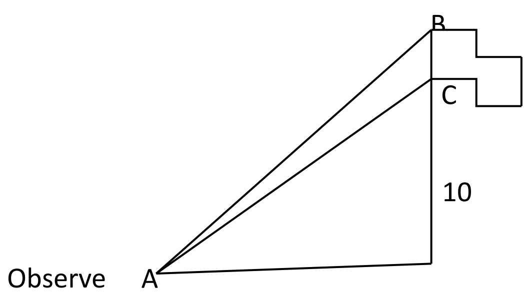
$$RQ^2 = 20^2 + 40^2 - 2(20)(40\cos 60^0)$$
$$= 400 + 1600 - 1600\cos 60^0$$
$$= 2000 - 1600(0.5)$$
$$= 2000 - 800 = 1200$$
$$RQ = \sqrt{1200} = 20\sqrt{3}m$$

In $\Delta RST, \cos 60^0 = \frac{TS}{20} \Rightarrow TS = 20\cos 60^0$

$$= 20(0.5) = 10m$$

Thus PR = TQ – TS = 40 – 10 = 30m

Exercise 1: An observer looks through an angle of elevation of 60^0,at the top B of a flag on a flagpole BD. He then looks through an angle of 45^0 at the bottom C of flagon the pole. The bottom of the flag is 10m above the ground. Find the width BC of the flag and the and the distance of the observer from the pole.



Exercise 2:An observer walks towards a church tower. At B, the angle of elevation of the top of the tower is 20^0. He walks 100m closer to A and notes that the angle of elevation is now 50^0. Determine

a. The height of the tower CD
b. The distance from the tower at A.

PRACTICE QUESTIONS-III

GENERAL DIRECTION: THE FOLLOWING QUESTIONS ARE PREPARED BASED ON CHAPTER FIVE OF GRADE 10 MATHEMATICS AS A PRACTICE QUESTIONS FOR GRADE 10 STUDENTS. BEFORE DOING THE QUESTIONS READ THE GIVEN SHORT NOTES AND OTHER RELATED MATERIALS.

I. FOR EACH OF THE FOLLOWING QUESTIONS THERE ARE FOUR ALTERNATIVE CHOISES. CHOOSE THE BEST ANSWER.

1. Which of the following angles is in quadrant 3?
   A. -231^0 B. 555^0 C. -445^0 D. -641^0
2. The following angles are quadrantal angles except;
   A. 810^0 B. -1260^0 C. 1010^0 C D. -990^0
3. Which of the following is not co terminal angle with $\frac{-8\pi}{9}$?
   A. $\frac{-17\pi}{9}$ B. $\frac{28\pi}{9}$ C. $\frac{10\pi}{9}$ D. $\frac{-26\pi}{9}$
4. From the given below which one is different?
   A. $\cos 105^0$ B. $-\sin 15^0$ C. $\cos 75^0$ D. $\sin(-15^0)$
5. If a line with equation $l: x-2y=0$ is on the terminal side of an angle θ in quadrant III, then what is $\csc\theta$?
   A. $\frac{-\sqrt{5}}{5}$ B. $\frac{-1}{2}$ C. $-\sqrt{2}$ D. $-\sqrt{5}$
6. The simplified form of $\frac{\sin 35^0 \cos 50^0}{\cos 55^0 \sin 40^0}$ is; A. 0 B. -1 C. 1 D. cannot be determined
7. In which interval does cosine function decreases?
   A. $\left[-\pi, \frac{-\pi}{2}\right]$ B. $\left[\frac{\pi}{2}, \pi\right]$ C. $\left[\frac{3\pi}{2}, 2\pi\right]$ D. $\left[\pi, \frac{3\pi}{2}\right]$
8. $\tan(-240^0) =$ ____
   A. $\sqrt{3}$ B. $\frac{\sqrt{3}}{3}$ C. $\frac{-\sqrt{3}}{2}$ D. $-\sqrt{3}$
9. For which values of θ, $y = \sec\theta$ is undefined?
   A. -3π B. $\frac{5\pi}{2}$ C. 5π D. -6π
10. If $\sin(-330^0) = \frac{1}{2}$, then what is $\cot(-330^0)$? A. $\frac{-\sqrt{3}}{3}$ B $-\sqrt{3}$ C. $\frac{\sqrt{3}}{3}$ D. $\sqrt{3}$
11. $\sec(\frac{-10\pi}{3}) =$ ____ A. $\sqrt{2}$ B. $-\sqrt{2}$ C. 2 D. -2
12. If $\cos 2\theta = \sin 3\theta$, then what is the value of θ? A. 30^0 B. 18^0 C. 15^0 D. 25^0

13. Which of the following statements is false?
    A. The graph of $y = \tan\theta$ is increasing in its domain.
    C. Domain of $y = \cot\theta$ is $\Re / \pi k$,where k is an integer.
    B. The graph of $y = \cot\theta$ is decreasing in its domain.
    D. The range of $y = \csc\theta$ is $[-1,1]$.
14. For θ an angle in standard position, if $\tan\theta > 0$ and $\sin\theta < 0$,then in which quadrant does the terminal side of θ lies? A. I B. II C. III D. IV
15. If θ is an angle in standard position whose terminal side is parallel to the line with equation $l: y-5x=-2$ and θ is in quadrant III, what is $\csc\theta$? A. $\frac{-5}{\sqrt{26}}$ B. 5 C. $\frac{-\sqrt{26}}{5}$ D. $\frac{1}{5}$
16. Which of the following is has a negative value? A. $\sin(-475^0)$ B. $\cos 275^0$ C. $\cot(-95^0)$ D. $\tan(-155^0)$
17. The exact value of $\cos\frac{-19\pi}{6}$ is; A. $\sqrt{3}$ B. $\frac{-\sqrt{3}}{2}$ C. $-\sqrt{3}$ D. $\frac{1}{2}$
18. The period of cotangent function is ; A. 2π B. π C. $\frac{\pi}{2}$ D. Not known
19. Which of the following angles does not have equal reference angle to -118^0?
    A. -298^0 B. 242^0 C. 478^0 D. -738^0
20. For a non quadrantal angle θ, what is the simplified form of $\cot^2\theta - \csc^2\theta$?
    A. 1 B. $2\cot^2\theta$ C. -1 D. $-2\csc^2\theta$
21. If $\cos 33^0 = \frac{1}{\csc\theta}$,then what is θ? A. 33^0 B. 43^0 C. 53^0 D. 57^0
22. If $\csc 20^0 = a$, what is $\sin 200^0$? A. –a B. $\frac{-1}{a}$ C. a^2 D. $\frac{1}{a}$
23. From the top of a building the angle of depression of a point on the ground 12m away from the base of the building is 30^0. What is the height of the building?
    A. $4\sqrt{3}m$ B. $2\sqrt{3}m$ C. $3\sqrt{2}m$ D. $6m$
24. Which of the following is true?
    A. The range of cotangent function is the set of real number.
    B. The domain of secant function is the set of real number.
    C. The range of cosecant function is [-1,1]
    D. Secant function is periodic function with period of length π .
25. Let $f(\theta) = 1 + \sin\theta$. Which one of the following is not true about $f(\theta)$?
    A. Range of $f(\theta)$ is [0,2]
    C. The domain of $f(\theta)$ is $(-\infty, \infty)$
    B. The period of $f(\theta)$ is 2π.
    D. $f(\theta) > o$ for each value of θ
26. If $\cot\theta = \frac{-1}{2}$ and $\sec\theta < 0$, then which one of the following is true?
    A. $\cos\theta = -\sqrt{5}$ B. $\tan\theta = \frac{1}{2}$ C. $\sin\theta = \frac{2\sqrt{5}}{5}$ D. $\csc\theta = \sqrt{5}$
27. A plane having an angle of depression of 30^0 to object on ground moves horizontally to the object for 4km changes its angle of depression to 60^0. How high is the plane from the ground perpendicularly?
    $A.3\sqrt{2}km$ $B.2\sqrt{5}km$ C $C.2\sqrt{3}km$ $D.2km$

UNIT-6
PLANE GEOMETRY

6.1. THEOREMS ON TRIANGLES

- A triangle is a polygon with three sides and the simplest type of polygon
- A line that divides an angle into two congruent angles is called angle bisector.
- A line that divides a line segment into two congruent line segments is called a bisector of the line segment.
- When a bisector of a line segment forms a right angle with line segment , then it is called the perpendicular bisector of the line segment.

Median of a triangle

- ✓ A median of a triangle is a line segment drawn from any vertex to the mid point of the opposite side.
- ✓ The medians of a triangle are concurrent at a point 2/3 of the distance from each vertex to the mid point of the opposite side.
- ✓ The point of intersection of the medians of a triangle is called the centroid of the triangle.
- ✓ The medians of an equilateral triangle are equal in length.
- ✓ In a right angled triangle the length of the median to the hypotenuse is equals one half the length of the hypotenuse.

Altitude of a triangle

- ❖ The altitude of a triangle is a line segment drawn from a vertex perpendicular to the opposite side, or to the opposite side produced.
- ❖ The altitudes of a triangle are concurrent at a point called Orthocenter of the triangle.
- ❖ For acute angled triangle Orthocenter lies inside the triangle.
- ❖ For right angled triangle Orthocenter is the vertex of a right angle.
- ❖ For obtuse triangle the Orthocenter lies outside the triangle.

Bisector of a triangle

- A line segment that passes through the mid-point of a side of a triangle and perpendicular to the side is called perpendicular bisector of a triangle.
- The perpendicular bisectors of the sides of any triangle are concurrent at a point which is equidistant from the vertices of the triangle.
- The point of intersection of the perpendicular bisectors of a triangle is called Circumcenter of the triangle.
- The three perpendicular side bisector of a triangle for,
    - i. An acute triangle lies inside the triangle.
    - ii. An obtuse triangle lies outside the triangle.
    - iii. A right triangle lies on hypotenuse of the triangle.
- If a circle passes through the three vertices of a given triangle, then the center of the circle lies at Circumcenter.

Angle bisector of a triangle

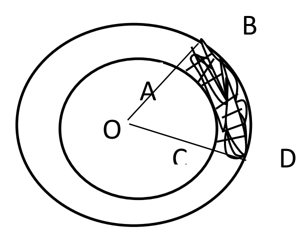
- The angle bisectors of any triangle are concurrent at a point which is equidistant from the sides of the triangle.
- The point of intersection of the angle bisectors of a triangle is called the incentre of the triangle
- For all types of triangles incentre is inside the triangle.
- Incentre is the center of an inscribed circle of the triangle.

PRACTICE QUESTIONS-IV

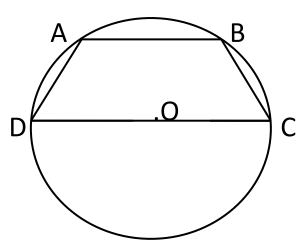
GENERAL DIRECTION: THE FOLLOWING QUESTIONS ARE PREPARED BASED ON CHAPTER SIX OF GRADE 10 MATHEMATICS AS A PRACTICE QUESTIONS FOR GRADE 10 STUDENTS. BEFORE DOING THE QUESTIONS READ THE GIVEN SHORT NOTES AND OTHER RELATED MATERIALS.

I. FOR EACH OF THE FOLLOWING QUESTIONS THERE ARE FOUR ALTERNATIVE CHOISES. CHOOSE THE BEST ANSWE.

1. The area of a regular 12 sided polygon of radius 6units long is;
   A. 98 unit^2 B. 108 unit^2 C. 48 unit^2 D. 144unit^2
2. If ABCD is a quadrilateral inscribed in a circle such that $\overline{AB} // \overline{CD}$, then which of the following is necessarily true?
   A. ABCD is a square C. ABCD is a rectangle
   B. ABCD is rhombus D. ABCD is trapezium
3. The area of regular hexagon is $96\sqrt{3}cm^2$. What is its perimeter?
   A. 54cm B. 48cm C. 60cm D. 42cm
4. Which one of the following statement is not correct?
   A. The line segment drawn from the center of circle to the mid points of the chords is perpendicular to the chord.
   B. If two chords of the circle are equidistance from the center, then they are equal in length.
   C. If two tangent line are drawn to the circle external point, then the tangents are equal in length.
   D. If two chords of the circle are equal in length, then they may not have equidistance from the center of the circle.
5. Suppose two concentric circles of radius 3cm and 6cm with shaded region(ABDC) are drawn as shown in the figure below. If m(<BOD) = 80^0, then what is the area of the shaded region?
   A. $6\pi cm^2$ B. $4\pi cm^2$ C. $3\pi cm^2$ D. πcm^2

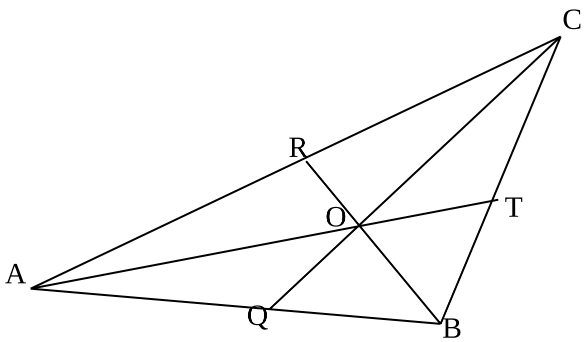


6. What is the length of side a regular hexagon whose area is $18\sqrt{3}cm^2$?
   A. 3cm B. $\sqrt{3}cm$ C. $\sqrt{2}cm$ D. $2\sqrt{3}cm$
7. The area A of an equilateral triangle inscribed in a circle of radius r is
   A. $\frac{3\sqrt{3}}{2}r^2$ B. $\frac{3\sqrt{2}}{4}r^2$ C. $\frac{3\sqrt{6}}{4}r^2$ D. $\frac{3\sqrt{3}}{4}r^2$
8. The circumference of a circle is 16π cm. What is the area of a sector whose central angle measures 120^0
   A. $\frac{8}{3}\pi cm^2$ B. $\frac{64}{3}\pi cm^2$ C. $\frac{16}{3}\pi cm^2$ D. $\frac{32}{3}\pi cm^2$
9. In the following figure ABCD is an isosceles trapezium which is inscribed in a circle O with $\overline{AB} // \overline{CD}$.
   If $m(\angle ADC) = 70^0$,then what is $m(\widehat{BC})$?



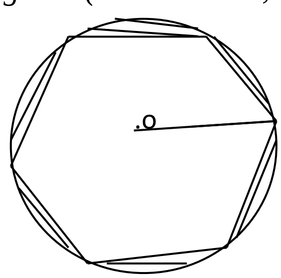
   A. 40^0 B. 60^0 C. 80^0 D. 100^0

10. In the figure below , $\overline{AT}, \overline{CQ}$ and $\overline{BR}$ are medians of ΔABC . If AT=18cm,OQ=3cm, then what is CO?



A. 6cm B. 4cm C. 3cm D.8cm

11. Which of the following statements is false?
   A. The point of intersection of angle bisectors of a triangle is always inside a triangle.
   B. The point of intersection of perpendicular bisectors of sides of a triangle is equidistant from each vertex of a triangle.
   C. The point of intersection of perpendicular bisectors of sides of a triangle is the center of circle inscribed in a triangle.
   D. The point of intersection of the altitudes of a triangle can be outside of a triangle.
12. Which of the following statements is false?
   A. A rhombus is an equilateral quadrilateral.
   B. If the diagonals of a parallelogram are perpendicular bisectors of each other, then the parallelogram is a rhombus.
   C. A parallelogram formed by joining the mid points of a rectangle is a square.
   D. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
13. The following figure shows a regular hexagon inscribed in a circle of radius 4cm. What is the area of the shaded *region? (use* $\pi \approx 3.14, \sqrt{3} \approx 1.73 and \sqrt{2} \approx 1.41$ *if necessary)*



$A.41.52cm^2$ $B.50.24cm^2$ $C.20.32cm^2$ D. $.8.72cm^2$

14. Which one of the following is **FALSE**?

   A. Each interior angle of a rectangle and a square is a right angle

   B. A square is an equilateral polygon

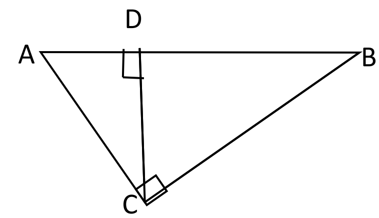
   C. The four triangles formed by the diagonals of the rhombus are equal and isosceles triangles

   D. A rectangle has all the properties of a square

15, Two squares have a total area of $208cm^2$ and the sum of their perimeters is $80cm$. What are the lengths of the sides of the squares?

$A.10cm$ and $8cm$ B .$8cm$ and $12cm$ $C.10cm$ and $12cm$ $D.6cm$ and $14cm$

16. The following figure is a right angled triangle with $\overline{CD}$ altitude to hypotenuse $\overline{AB}$ such that AD=16cm and BD=9cm. Then what is the length of $\overline{BC}$?

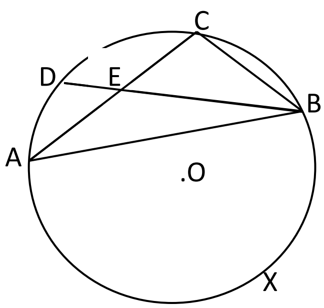


A. 12cm B. 20cm C. 15cm D. 18cm

17. For what values of k, the points (1,-1), (3,5) and (k,-10) are collinear?

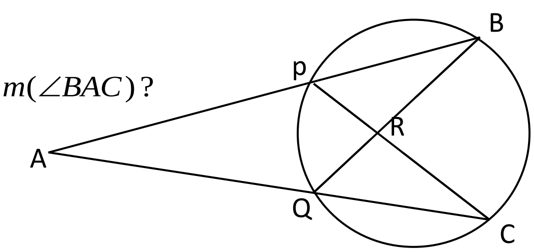
A. -2 B.2 C. -4 D. -3

18. *In the following figure O is the center,* $m(\angle CAB) = 40^0$,and $m(A\hat{X}B) = 220^0$ *,m(<ABD)=10;* What is $m(D\hat{C})$?



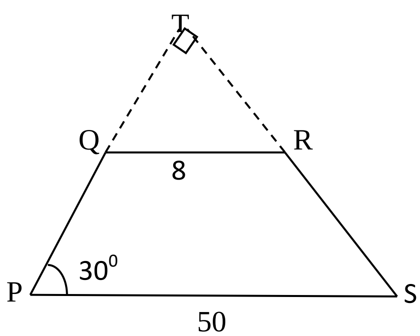
$A.30^0$ $B.20^0$ C $C.40^0$ $D.60^0$

19. In the figure below if $m(\angle QBP) = 20^0, m(\angle BRP) = 112^0$. What is $m(\angle BAC)$?



$A.56^0$ $B.40^0$ $C.28^0$ $D.68^0$

20. PQRS is a trapezium with $QR = 8cm$ *and* $m(\angle P) = 30^0$. *What is the length of the altitude drawn from R to PQ produced?*



A. 6cm B. 4cm C.12cm D. 10cm

Reading assignment: Read about special quadrilaterals, circles and regular polygons from your text book and other reference materials;

UNIT -7

MEASURMENT

General Objectives:

After Completing this unit, you will be able to:

- Solve problems involving surface area and volume of solid figures.
- Know basic facts about frustums of cones and pyramids.

INTRODUCTION

We know that plane Geometry (sometimes called Euclidian Geometry) is a branch of Geometry which deals about the properties of flat surfaces and plane figures, such as polygons, circles and so on.

Geometrical figures that have three dimensions (length, width and height) are called solid figures. For example prisms, cylinders, cones, pyramids, spheres, hemispheres etc. These solid figures have a volume. A branch of Geometry which deals about the surface areas and volume of these solid figures is called Solid Geometry. In this unit you will learn more about surface areas and volumes of solid figures. You will also study about surface areas and volumes of composed solids and frustums of pyramids and cones.

7.1 Prism

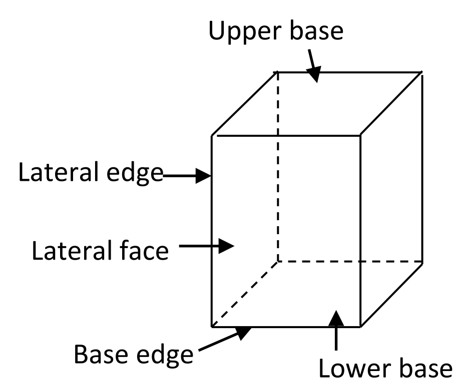
- A prism is a solid figure which is bounded by two congruent polygons called the bases (upper and lower bases) .
- A prism is named by its bases (Triangular prism, rectangular prism, pentagonal prism,…)
- If the lateral faces are perpendicular to the bases, then the prism is called a Right-prism.(the lateral faces are rectangles)
- If the lateral faces are not perpendicular to the bases, then the prism is called an Oblique prism.(the lateral faces are parallelograms)
- The perpendicular distance, between the plane containing the bases is called Altitude of the prism.
- The union of the lateral faces and bases is called total surface (or surface)

Note:- a. In a prism

- Lateral edges are equal and parallel
- Lateral faces are parallelograms

b. In right prism

- Altitude is equal to lateral edges and perpendicular to bases.
- Lateral faces are rectangles.



b. In oblique prisms

- Altitude is shorter than lateral edges

c. A right prism with bases of regular polygon is called Regular prism

d. A right square prism whose altitude equals to length of edge of bases is called a **Cube.**

e. If bases are n-sided polygon prism then the prism has **3n** edges and **2n** vertices.

7.1.1 Surface area and Volume of Prisms

For a prism if its height = h Lateral surface area = A_L

Total surface area = A_T Base area = A_B

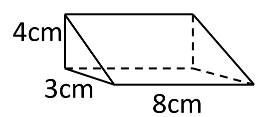
Base Perimeter = p and Volume = V, then

1. $A_L = ph$ (Lateral surface area = Base perimeter x Altitude)
2. A_B has no fixed formula because the base is different polygons (can be triangle, rectangle, square, pentagon,…). But we have to remember that there are two congruent polygons.
3. $A_T = A_L + 2A_B$ (Total surface area = Lateral surface area + 2(One Base area))
4. $V = A_B \cdot h$ (Volume = One base area x height)

Example1:Find the total surface area and volume of the following right triangular prism.

Solution: The base is right triangle. So,

$$A_B = (\frac{1}{2}x4cmx3cm) = 6cm^2$$



Let the third side of the base be x

Hence, $x^2 = (3cm)^2 + (4cm)^2 = 25cm^2$

$$\Rightarrow x = 5cm$$

Therefore, $A_L = ph = (3cm + 4cm + 5cm)x8cm = 96cm^2$

Hence, $A_T = A_L + 2A_B$

$$= 96cm^2 + 2(6cm^2)$$

$$= 108cm^2$$

$V = A_B.h$

$$= 6cm^2 x8cm = 48cm^3$$

Example 2:The bases of a right prism are an equilateral triangle of length 3cm and its lateral surfaces are rectangular region, and if its altitude is 7cm, then find

a. A_T b. v

Solution: Since the prism is a right prism, the altitude is equal to the lateral edges.

P = 3cm + 3cm +3cm

=9cm

AL = Ph

$$\Rightarrow A_L = 9cmx7cm = 63cm^2$$

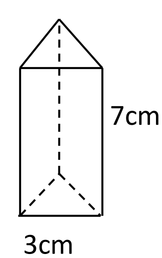
$$A_B = \frac{1}{2}(3cm)(3cm)(\sin 60^0)$$

$$= \frac{9\sqrt{3}}{4}cm^2 \quad v = A_B.h \quad \Rightarrow \quad v = \frac{9\sqrt{3}}{4}cm^2 x7cm = \frac{63\sqrt{3}}{4}cm^3$$

$$A_T = 2A_B + A_L$$

$$= 2(\frac{9\sqrt{3}}{4}cm^2) + 63cm^2$$

$$= (\frac{9\sqrt{3}}{2} + 63)cm^2$$



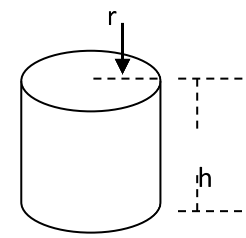
Exercise 1: The base of a prism is a regular hexagon one side 10cm and height 8cm. Find:

a. A_T b. V

Exercise 2: If the diagonal of a cube is $\sqrt{27}cm$, then find its A_T and V.

Hint: If the length of side a cube is b, then the length of its diagonal is $b\sqrt{3}$

7.2. CYLINDER



A cylinder is a solid figure whose two bases are congruent circles.

7.2.1 **Surface area and volume of cylinder**

a. $A_L = ch$,where c is circumference of the base.

$A_L = 2\pi rh$, since $c = 2\pi r$ $\quad$ *c.* $A_T = A_L + A_B$ $\quad$ *d. Volume (v) =* $A_B.h$

a. $A_B = 2\pi r^2$ $\quad$ $A_T = 2\pi rh + 2\pi r^2$ $\quad$ $v = \pi r^2 h$

$A_T = 2\pi r(h+r)$

Example1: The radius of the base of a right circular cylinder is 5cm and its altitude is 8cm. Find A_T & V

Solution:

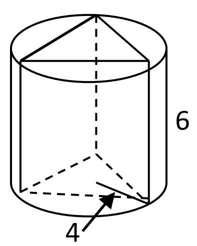
$A_L = 2\pi rh$ $\quad$ *a.* $A_T = A_L + A_B$ $\quad$ a. $A_T = A_L + A_B$ $\quad$ b. $v = \pi r^2 h$

$A_L = 2\pi(5cm)(8cm) = 80\pi cm^2$ $\quad$ $A_T = 80\pi cm^2 + 50\pi cm^2$ $\quad$ $v = \pi(5cm)^2(8cm)$

$A_B = 2\pi r^2 h$ $\quad$ $A_T = 130\pi cm^2$ $\quad$ $v = 200\pi cm^3$

$A_B = 2\pi(5cm)^2 = 50\pi cm^2$

Example 2: If an equilateral triangle of radius 4 unit is drilled through the center of a right circular cylinder of radius of base 4 units and height of 6 units, then find A_B , A_L ,A_T and v of the resulting .



Solution: Let base area of the resulting solid = A_b

Base area of cylinder=A_c

Base area of triangle = A_t

a. $A_b = A_c - A_t$

$$A_b = \pi r^2 - \frac{1}{2}(n)(r)^2 \sin(\frac{360^0}{n})$$

$$A_b = \pi(4)^2 - \frac{1}{2}(3)(4)^2 \sin(\frac{360^0}{3})$$

$$A_b = 16\pi - \frac{1}{2}(48)\sin(120^0)$$

$$A_b = (16\pi - 12\sqrt{3}) \text{ unit sq.}$$

Therefore , $2(16\pi - 12\sqrt{3})$ unit sq. is base area of the resulting solid

a. $A_L = A_{Lc} + A_{Lt}$

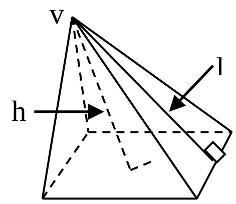
$$A_L = 2\pi(r)(h) + 2(n)(r)\sin(\frac{180^0}{n})(h)$$

$$A_L = 2\pi(4)(6) + 2(3)(4)\sin(\frac{180^0}{3})(6) \qquad \Rightarrow A_L = (48\pi + 72\sqrt{3}) \text{ unit sq.}$$

A_T & V------- Exercise

Exercise: A cylinder of base area 225cm^2 and height 15cm has the same volume as a cube. What is A_T of the cube?

7.3. PYRAMID



- ✓ Pyramid is a solid figure formed when each vertex of a polygon is joined to the same point not in the plane of the polygon.
- ✓ The altitude of a pyramid is the length of the perpendicular from the vertex to the plane containing the base
- ✓ The slant height of a regular pyramid is the altitude of any of its lateral faces.
- ✓ A regular pyramid is a pyramid whose base is a regular polygon and whose altitude passes through the center of the base.
- ✓ If the base of the pyramid is a triangular region, it is called a tetrahedron.

7.3.1 Surface area and volume of pyramid

a. $A_L = \frac{1}{2}pl$, where *p* – perimeter of the base

l--- slant height

b .For A_B no fixed formula b/c the base different polygons.

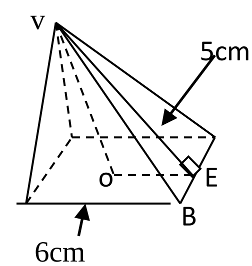
c, $A_T = A_L + A_B$ *d.* $v = \frac{1}{3}A_B.h$

$A_T = \frac{1}{2}pl + A_B$

Example 1: An edge of a right square pyramid is 6cm long, if the length of the slant height is 5cm, then find:

a. A_L b. A_T c. V

Solution:



a. $A_L = \frac{1}{2}pl$ (since l= 5cm; p = (6 +6 + 6 +6)cm =24cm)

$A_L = \frac{1}{2}x24cmx5cm = 60cm^2$

b. $A_T = A_B + A_L$, (A_B = 6cm x 6cm = 36cm^2)

$A_T = 36cm^2 + 60cm^2 = 96cm^2$

c. To find the volume we need to find the altitude ' h'.

$(VO)^2 + (OE)^2 = (VE)^2$

$h^2 + 3^2 = 5^2 \Leftrightarrow h^2 = 25 - 9 = 16$

$\Leftrightarrow h = 4cm$

$v = \frac{1}{3}A_B.h$

$v = \frac{1}{3}x36cm^2x4cm$

$v = 48cm^3$

Exercise 1:Find the altitude 'h', A_L and A_T area of a right pyramid with volume of 72cm3 having a rectangular base of dimensions 3cm by 4cm.

Exercise 2:The volume a regular square pyramid is 120cm3. If its altitude is 10cm long, find the length of one edge of the base and its total surface area.

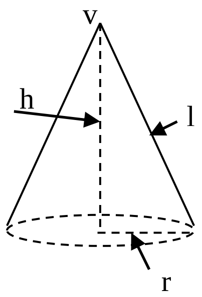
Exercise 3: If a lateral edge of a regular tetrahedron is x cm, find the measure of

a. Altitude b. surface area c. volume

7.4. Cone

Cone a solid figure formed by joining all points of a circle to a point not on the plane of the circle.

7.4.1 Surface area and volume of cone



a. $A_L = \frac{1}{2}cl$,where *c =circumference of the base and*

l = *slant height and*

$l = \sqrt{h^2 + r^2}$

$A_L = \frac{1}{2}(2\pi r)l = \pi rl$

b. $A_B = \pi r^2$

c. $A_T = A_L + A_B$

$A_T = \pi rl + \pi r^2$

$A_T = \pi r(l + r)$

d. $v = \frac{1}{3}A_B h$

$v = \frac{1}{3}\pi r^2 h$

Example 1:Calculate the total surface area and volume a right circular cone with length of altitude 8cm and radius 6cm.

Solution: h = 8cm, r = 6cm

$l = \sqrt{(8cm)^2 + (6cm)^2} = \sqrt{100cm^2} = 10\text{cm}$

a. $A_T = \pi r(l + r)$

$A_T = \pi(6cm)(10cm + 6cm)$

$A_T = (6cm)\pi(16cm) = 96\pi cm^2$

$v = \frac{1}{3}\pi r^2 h$

$v = \frac{1}{3}\pi(6cm)^2(8cm)$

$v = 96\pi cm^3$

Example 2: The volume of a right circular cone is $128\pi cm^3$. The radius is 4cm.Find the length of the perpendicular height.

Solution: $v = \frac{1}{3}\pi r^2 h$

$128\pi cm^3 = \frac{1}{3}\pi(4cm)^2.h = \frac{16}{3}\pi cm^2.h$

$h = 24cm$

Exercise 1: Calculate the total surface area and volume of a circular cone whose altitude and diameter of base are equally x cm.

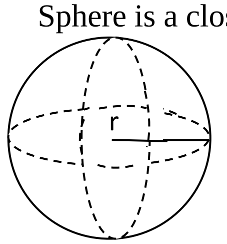
Exercise 2:If the slant height of a circular cone is 13cm and the radius of a base is 12cm, then calculate,

a. h b. A_L c. A_T d. v

7.5 Sphere and hemisphere

7.5.1. Surface area and volume of Sphere

Sphere is a closed surface, all points of which are equidistant from a point called the center.



a. Area of sphere (A) $= 4\pi r^2$

b. $Volume(v) = \frac{4}{3}\pi r^3$

Example 1: The diameter of a sphere is 6cm, find the area and volume of the sphere.

Solution: diameter (d) $= 2r \Rightarrow 6cm = 2r$

$$\Rightarrow r = 3cm$$

a. $A = 4\pi r^2$

$A = 4\pi(3cm)^2$

$A = 36\pi cm^2$

b. $Volume(v) = \frac{4}{3}\pi r^3$

$Volume(v) = \frac{4}{3}\pi(3cm)^3$

$Volume(v) = \frac{4}{3}\pi(27)cm^3 = 36\pi cm^3$

Example 2: The radius of one sphere is twice as long as the radius of another sphere. If the volume of the smaller sphere is 12 cubic units, then calculate the volume of the larger sphere.

Solution: Let R be the radius of the larger sphere and

r be the radius of the smaller sphere

$\Rightarrow R = 2r$

$V_{smaller} = \frac{4}{3}\pi r^3 \Rightarrow 12 = \frac{4}{3}\pi r^3$

$\Rightarrow (\frac{3}{4\pi})(12) = (\frac{3}{4\pi})(\frac{4}{3})\pi r^3 \quad \Rightarrow r = \sqrt[3]{\frac{9}{\pi}}$ unit and $R = 2\sqrt[3]{\frac{9}{\pi}}$ unit

$V_{l\arg er} = \frac{4}{3}\pi R^3 \quad = \frac{4}{3}\pi(2\sqrt[3]{\frac{9}{\pi}})^3 \quad = 96$ cubic units

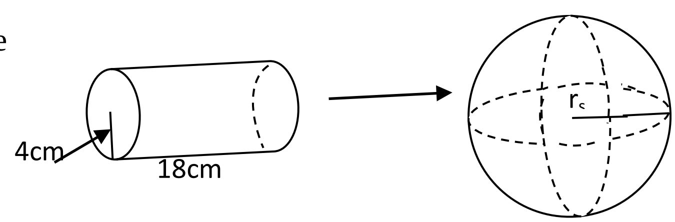
Example 3: If 18cm long wire whose radius of circular thickness is 4cm is melted to form a sphere, find the surface area and volume of the sphere.

Solution: To change in shape may change the surface area but doesn't change the volume.

Then let r_s = radius of the sphere, A = area of the sphere

r_c = radius of cylinder, h = altitude of cylinder

v_c = volume of cylinder, v_s = volume of sphere



$V_s = V_c =$ (equal volume)

$\frac{4}{3}\pi r_s^3 = \pi r_c^2 h$

$\frac{4}{3}\pi r_s^3 = \pi(4cm)^2(18cm) = 288\pi cm^3$

$r_s^3 = (288\pi)(\frac{3}{4\pi})cm^3 = 216cm^3 \qquad \Rightarrow r_s = 6cm$

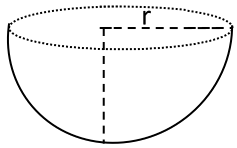
Therefore, $A = 4\pi r^2 = 4\pi(6cm)^2 = 144\pi cm^2$

$Volume$---- for exercise

Exercise 1: A cylindrical container of base radius 8 cm has enough water in it. An iron ball of radius 3 cm is inserted in the cylinder. Assuming that the ball is completely immersed, how high does the water level rise?

Exercise 2:If a spherical stone with radius 6ocm is submerged in a cylindrical water tank whose base radius is 2m then how much is the level of water raised?

7.5.2. **Surface area ad volume of a hemisphere (Half sphere)**



a. $A = \frac{1}{2}(4\pi r^2) + \pi r^2$ ----- (A $_{\text{Hemisphere}}$ =1/2(A $_{\text{sphere}}$) + A $_{\text{Circle}}$)

$A = 3\pi r^2$

b. $Volume(v) = \frac{1}{2}(\frac{4}{3}\pi r^3)$ ------ (V $_{\text{Hemisphere}}$ =1/2(V $_{\text{Sphere}}$)

$Volume(v) = \frac{2}{3}\pi r^3$

Example: Find the surface area and volume a hemispherical metal of radius 12cm.

Solution: The radius of the hemisphere is 12cm.

a. $A = 3\pi r^2$

$A = 3\pi(12cm)^2$

$A = 432\pi cm^2$

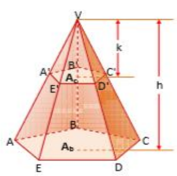
b. $v = \frac{2}{3}\pi r^3$

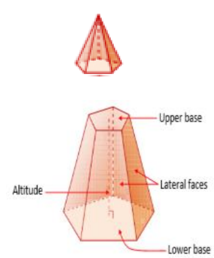
$v = \frac{2}{3}\pi(12cm)^3$

$v = 1152\pi cm^3$

7.6. Frustum of pyramid and cone

- **A** frustum of a pyramid a part of the pyramid included between the base and a plane parallel to the base.
- The lateral faces of a frustum of a pyramid are trapeziums.
- The lateral faces of a frustum of a regular pyramid are congruent isosceles trapezium.
- The slant height of a frustum of a regular pyramid is the altitude of any one of the lateral faces.
- The lateral surface area of a frustum of a pyramid is the sum of the areas of the lateral faces.





Note: $\frac{A'_B}{A_B} = \frac{k^2}{h^2}$

a. $A_L = \frac{1}{2}l(p + P')$, p – perimeter of the lower base

P' – perimeter of the upper base

l – slant height of frustum

b. $V_{frustum} = V_{bigge\ pyramid} - V_{smaller\ pyramid}$

$V_{frustum} = \frac{1}{3}A_B h - \frac{1}{3}A_B' k$, where A_B = Area of lower base

A'_B = Area of upper base

h= altitude of the pyramid (bigger pyramid)

k = altitude of smaller pyramid

OR $V_{frustum} = \frac{1}{3}h^{'}(A_B + A^{'}{}_B + \sqrt{A_B A^{'}{}_B})$,where A_B—Area of lower base

A'_B –Area of upper base

h'—height of frustum

Example: The side of the upper and lower bases of frustum of a regular square pyramid are 4cm and 6cm respectively. If the slant height is 10cm, then find: a. A_L b. v c. A_T

Solution:

a. $A_L = \frac{1}{2}(P + P^{'})l$

$A_L = \frac{1}{2}(6x4 + 4x4)x10cm$

$A_L = \frac{1}{2}(24cm + 16cm)x10cm$

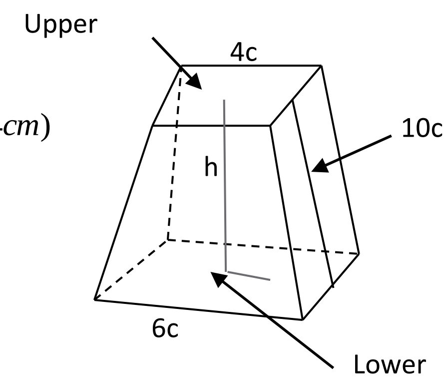
$A_L = \frac{1}{2}(40cm)x10cm = 200cm^2$

c. $A_T = A_L + A_B + A^{'}{}_B$

$A_T = 200cm^2 + (6cmx6cm) + (4cm + 4cm)$

$A_T = 200cm^2 + 36cm^2 + 16cm^2$

$A_T = 252cm^2$



b. To find the volume first find the height of frustum

$h^2 + (1cm)^2 = (10cm)^2$

$h^2 = 100cm^2 - 1cm^2 = 99cm^2 \Rightarrow h = \sqrt{99}cm$

Hence, $v = \frac{h^{'}}{3}(A_B + A^{'}{}_B + \sqrt{A_B A^{'}{}_B})$

$V = \frac{\sqrt{99}}{3}(36cm^2 + 16cm^2 + \sqrt{(36cm^2)(16cm^2)})$

$V = \frac{\sqrt{99}}{3}(52cm^2 + \sqrt{576cm^4})$

$V = \frac{\sqrt{99}}{3}(52cm^2 + 24cm^2)$

$V = \frac{\sqrt{99}}{3}(76cm^2) \quad \Rightarrow V = (\frac{76\sqrt{99}}{3})cm^2$

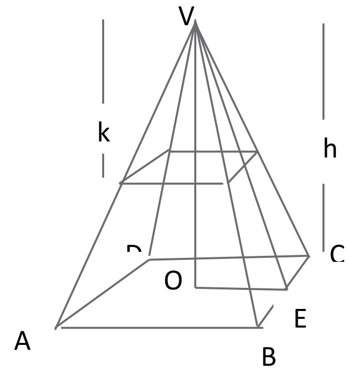
Example 2: A frustum of a regular square pyramid whose lateral faces are equilateral triangles of side 8cm has altitude is 4cm. Find the volume of the frustum.

Solution:

Let V_c = Volume of smaller pyramid

V = Volume of frustum

V_b = Volume of bigger pyramid



In ΔVEB, $VE^2 = VB^2 - BE^2$

$l^2 = 8^2 - 4^2 \Rightarrow l = 4\sqrt{3}cm$

$\Delta VOE, VO = \sqrt{VE^2 - OE^2} \quad \Rightarrow \sqrt{l^2 - (\frac{1}{2}AB)^2}$

$\Rightarrow h = \sqrt{(4\sqrt{3})^2 - 4^2} = 4\sqrt{2}cm$

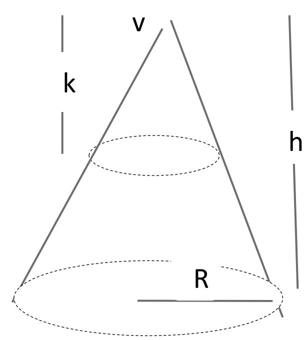
$V_b = \frac{1}{3}A_b h = \frac{1}{3}(8)^2(4\sqrt{2}) = \frac{256}{3}\sqrt{2}cm^3$

$\Rightarrow \frac{V_c}{V_b} = \frac{k^3}{h^3} \Rightarrow \frac{V_b - V}{V_b} = \frac{(h-4)^3}{h^3} \Rightarrow V = V_b(1-(1-\frac{4}{h})^3)$

$\Rightarrow V = \frac{256}{3}(1-(1-\frac{4}{4\sqrt{2}})^3) = \frac{640-448\sqrt{2}}{3}cm^3$

Exercise: A frustum of height 4cm is formed from regular square pyramid of lateral faces are equilateral triangles of side $4\sqrt{3}m$. Find the volume of: a. the frustum b. the pyramid c. surface area of the frustum

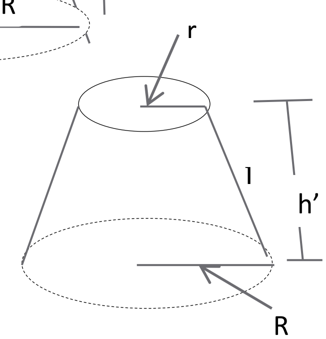
Frustum of Cone



a. $A_L = \frac{1}{2}l(c + c')$, where, c =circumference of lower base

$A_L = \frac{1}{2}l(2\pi R + 2\pi r)$ c' = circumference of upper base

$A_L = l\pi(R + r)$ l – slant height



b. $A_B = \pi(R^2 + r^2)$

c. $A_T = A_L + A_B$

$A_T = l\pi(R + r) + \pi(R^2 + r^2)$

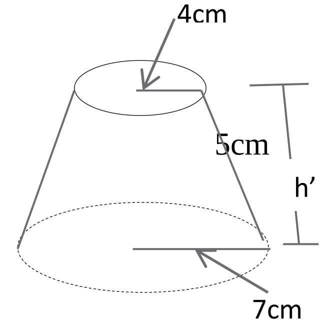
$A_T = \pi[l(R + r) + R^2 + r^2]$

d. $V_f = V_{bigger\,cone} - V_{smaller\,cone} \quad \Rightarrow V_f = \frac{1}{3}\pi R^2 h - \frac{1}{3}\pi r^2 k \;\Rightarrow V_f = \frac{1}{3}\pi(R^2 h - r^2 k)$

OR $V_f = \frac{1}{3}h'(A_B + A'_B + \sqrt{A_B A'_B})$

$$\Rightarrow V_f = \frac{\pi}{3}h'(R^2 + r^2 + R.r)$$

Example 1: Calculate the surface area and volume of frustum of cone with length of bases radii 7cm and 4cm that has a slant height 5cm.



Solution:

a. $A_T = A_L + A_B$, R= 7cm, r = 4cm , l = 5cm

$$A_T = \pi l(R + r) + \pi(R^2 + r^2)$$

$$A_T = \pi(5cm)(7cm + 4cm) + \pi[(7cm)^2 + (4cm)^2$$

$$A_T = \pi(5cm)(11cm) + \pi[49cm^2 + 16cm^2]$$

$$A_T = 55\pi cm^2 + 65\pi cm^2$$

$$A_T = 120\pi cm^2$$

b. To calculate volume we need to find height of frustum
   By Pythagoras $h'^2 + (3cm)^2 = (5cm)^2 \Rightarrow h' = 4cm$

$$V = \frac{\pi}{3}h'(R^2 + r^2 + R.r)$$

$$V = \frac{\pi}{3}(4cm)[(7cm)^2 + (4cm)^2 + (7cm)(4cm)]$$

$$V = \frac{4\pi}{3}cm[49cm^2 + 16cm^2 + 28cm^2] \qquad \Rightarrow V = \frac{4\pi}{3}cm(93cm)^2$$

$$\Rightarrow V = 124\pi cm^3$$

Example 2:A frustum of right circular cone with volume of 2 litters having radii of 6cm and 12cm, find:

a. Height of frustum b. slant height of frustum c. altitude of the cone

Solution: Note that 2litters = 2000cm^3

a. $V = \frac{\pi}{3}h'(R^2 + r^2 + R.r)$

$$2000cm^3 = \frac{\pi}{3}h'[(12cm)^2 + (6cm)^2 + (12cm)(6cm)]$$

$$2000cm^3 = \frac{\pi}{3}h'[144cm^2 + 36cm^2 + 72cm^2] \Rightarrow 2000cm^3 = \frac{\pi h'}{3}(252cm^2) \qquad \Rightarrow h' = \frac{6000}{252\pi}cm$$

b. $h'^2 + (R-r)^2 = l^2$

$(\frac{6000}{252\pi}cm)^2 + (12cm - 6cm)^2 = l^2$

$\Rightarrow (\frac{36000000}{63504\pi}cm^2 + 36cm^2) = l^2$

$\Rightarrow (\frac{36000000 + 2286144\pi}{63504\pi})cm^2 = l^2$

$\Rightarrow \frac{36000000 + 5334336}{148176}cm^2 = l^2$

c. Left as exercise

Exercise 1 : A frustum formed from the right circular cone of diameter 20cm . If the height of the frustum is 8cm and the ratio of the heights of frustum and the cone is 2:3, find the volume the frustum.

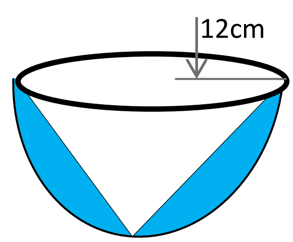
Exercise 2:From frustum of right circular cone of altitude 10cm and radii of bases 6cm and 8cm, if cone of the same altitude with vertex on larger base of the frustum is drilled calculate the surface area and volume of the resulting solid, for the drilled cone has the base radius 4cm.

7.7. SURFACE AREA AND VOLUME OF COMPOSED SOLIDS

Composed solids means combination of two or more solid figures.

Example1: From a hemispherical solid of radius 12cm, a conical part is removed. Calculate the volume and surface area of the resulting solid**.**

Solution:



$r^2 + r^2 + l^2$

$2r^2 = l^2 \Rightarrow l = 12\sqrt{2}cm$

a. $A_T = A_{Hemisphere} + A_{LCone}$ (A Hemisphere = only half the sphere)

$A_T = \frac{1}{2}(4\pi r^2) + \pi r l$

$A_T = [2\pi(12cm)^2] + \pi(12cm)(12\sqrt{2}cm)$

$A_T = 288\pi cm^2 + 144\sqrt{2}\pi cm^2$

$A_T = 144\pi(2 + \sqrt{2})cm^2$

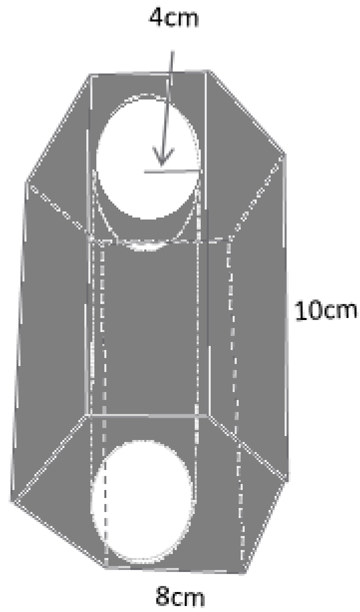
b. $V = V_{Hemispher} - V_{Cone}$

$V = \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^2 h$

$V = \frac{2}{3}\pi(12cm)^3 - \frac{1}{3}\pi(12cm)^2 12cm)$

$V = \frac{2}{3}\pi(1728cm^3) - \frac{1728\pi cm^3}{3} \Rightarrow V = 576\pi cm^3$

Example 2: Through a regular hexagonal prism of length of base 8cm and altitude 10cm is drilled a circular cylinder hole of radius 4cm. Calculate the total surface area and volume of the resulting solid.



Solution: In regular hexagon length of side = length of radius

a. $A_T = A_{L\ prism} + A_{B\text{Prism}} + A_{Lcylinder} - A_{Bcylider}$

$$A_T = 6(8cm)(10cm) + 2[\frac{1}{2}(6)(8cm)^2 \sin(\frac{360^0}{6}) + 2\pi(8cm)(10cm) - 2[\pi(8cm)^2]$$

$$A_T = (480 + 192\sqrt{3} + 160\pi - 128\pi)cm^2$$

$$A_T = (480 + 192\sqrt{3} + 32\pi)cm^2$$

b. $V = V_{\text{Prism}} - V_{Cylinder}$

$$V = A_B.h - \pi r^2, h$$

$$V = \frac{1}{2}(6)(8cm)^2(\sin\frac{360^0}{6})(10cm) - \pi(8cm)^2(10cm)$$

$$V = (960\sqrt{3} - 640\pi)cm^3$$

PRACTICE QUESTIONS-V

GENERAL DIRECTION: THE FOLLOWING QUESTIONS ARE PREPARED BASED ON CHAPTER SEVEN OF GRADE 10 MATHEMATICS AS A PRACTICE QUESTIONS FOR GRADE 10 STUDENTS. BEFORE DOING THE QUESTIONS READ THE GIVEN SHORT NOTES AND OTHER RELATED MATERIALS.

I. FOR EACH OF THE FOLLOWING QUESTIONS THERE ARE FOUR ALTERNATIVE CHOISES. CHOOSE THE BEST ANSWE.

1. The lower and upper bases of a frustum of a regular pyramid are squares with measures of sides 4m and 3m respectively. If the altitude of the frustum is 6m, then what is its volume ?

   A. $60m^3$ B. $50+4\sqrt{3}\ m^3$ C. $74m^3$ D. $30m^3$

2.. If a frustum formed from a regular pyramid, then which one of the following statements is **TRUE** about its lateral faces?

   A. They are isosceles trapezium C. They are rhombus

   B. Their altitudes are not equal D. They are parallelograms.

3. A right circular cylinder has radius of the base 5cm and height 8cm. What is the total surface area of this cylinder? A. $80\pi cm^2$ B. $130\pi cm^2$ C. $200\pi cm^2$ D. $105\pi cm^2$

4. A right prism with base right angled triangle of two lengths of legs 5cm and 12cm long. If the height of the prism is 20cm, what is the total surface area of the prism?

   $A.800cm^2$ B $B.660cm^2$ $C.580cm^2$ $D.720cm^2$

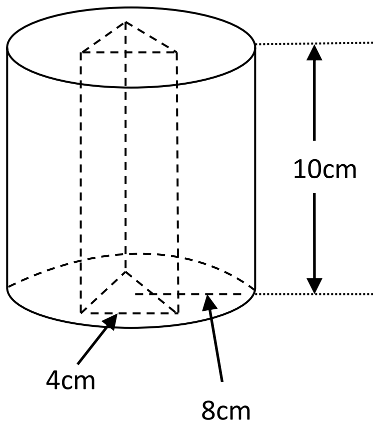
5. If the length of each edge of a regular tetrahedron is 4cm, then what is its altitude?

   $A.\frac{16\sqrt{6}}{3}cm^3$ A $B.20\sqrt{3}cm$ $C.25\sqrt{3}cm$ $D.\frac{18\sqrt{3}}{11}cm$

6. What is the volume for question number 5 above?

   $A.15\sqrt{3}cm^3$ $B.16\sqrt{6}cm^3$ $C.\frac{16}{3}\sqrt{2}cm^3$ $D.\frac{4}{2}\sqrt{3}cm^3$

7. Trough a right circular cylinder whose base radius 8cm and whose height 10cm is drilled an equilateral triangular prism hole whose length of base side 4cm. What is the total surface area of the remaining solid?



A. $8(36\pi + 15 - \sqrt{3})cm^2$

B. $8(36\pi + 15)cm^2$

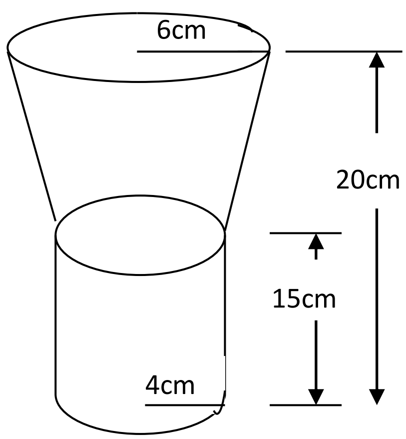
C. $8(20\pi + 15 - \sqrt{3})cm^2$

D. $8(15\pi + 15 - \sqrt{3})cm^2$

8.A glass in a shape of frustum of a right circular cone has slant height of 13cm and bases radii 4cm and 9cm. What is the volume of the glass?

$A.325\pi cm^3$ $B.476\pi cm^3$ C $C.532\pi cm^3$ $D.331\pi cm^3$

9. A torch 20cm long is in the form of a right circular cylinder of height 15cm and radius 4cm, joined to it is a frustum of cone of radius 6cm. What is its total surface area?



$A.152\pi cm^2$ C $C.(172 + 10\sqrt{29})\pi cm^2$

$B.(160 + 3\sqrt{3})\pi cm^2$ $D.(168 + 5\sqrt{2})\pi cm^2$

10. What is the volume of the figure in question number 9 above?

A. $A.76\pi cm^3$ $B.\frac{562}{3}\pi cm^3$ $C.252\pi cm^3$ $D.\frac{380}{3}\pi cm^3$

11. From your text book do exercise 7.3, 7.4 and review exercises

"PRACTISE MAKES PERFECT!!"

THE END!!

ANSWER FOP PRCTISE QUESTIONS

PRACTISE QUESTION- I

1. B
2. A
3. D
4. C
5. A
6. A
7. C
8. D
9. A
10. C
11. D
12. C
13. B
14. B

PRACTISE QUESTION- II

1. B
2. D
3. C
4. C
5. A
6. D
7. C
8. B
9. A
10. D
11. C
12. B
13. B
14. D
15. B
16. A
17. D
18. C
19. C
20. C

PRACTISE QUESTION- III

I.

1. B
2. C
3. A
4. C
5. D
6. C
7. B
8. D
9 B
10. D
11.D
12.B
13. D
14.C
15.C
16. A
17. B
18.B
19.D
20.C
21.D
22.B
23.A
24.A
25.D
26. C
27.C

II.1(2,-1/2)

2.y=2x-2

3.(-9/5 ,-6/5)

4. $k = -1 \pm \sqrt{2}$

5. (0,11)

6. a, 5

b. 5/2

7. a, y =-1/2 x+13/2

b, y =-4/3 x +9

PRACTISE QUESTION- IV

1.B
2. C
3. B
4. D
5. A
6. D
7. D
8. B
9.A
10. A
11. C
12. C
13. D
14. D
15. B
16. C
17. A
18.C
19. C
20. B

PRACTISE QUESTION- V

1.C
2.A
3.B
4.B
5. A
6. C
7. A
8. C
9. C
10. D