UNIT 4

CURRENT ELECTRICITY

At the end of this unit you will be able to:

- Appreciate the flow of electric charges in a metallic conductor.
- Know what electric current is.
- Understand Ohm's law and electrical resistance.
- Enjoy parallel and series combination of resistors.
- Apply appropriate equations to find equivalent resistance.
- Realize electromotive force (e.m.f) and internal resistance of a cell.
- Realize electrical energy and power in an electrical circuit.
- Understand the nature of the generation and supply of electricity in Ethiopia.

Introduction

Up to now, we have considered primarily static charges. In this unit, we study the electric current through a material, where the electric current is the rate of flow of charge.

We also examine a characteristic of materials known as the resistance. Resistance is a measure of how much a material impedes the flow of charge, and it will be shown that the resistance depends on temperature. In general, a good conductor, such as copper, gold, or silver, has very low resistance. We also define electrical power. Power is the rate at which energy is moved.

4.1 Electric current

At the end of this section you will be able to:

- Explain static and current electricity.
- Define electric current and its SI unit.
- Explain the flow of electric charges in a metallic conductor.
- Explain the direction of current flow.
- Use Galvanoscope to test the presence of current.

- Calculate the number of electrons that pass a point in a given length of time when the current in the wire is known.
- Define drift velocity.
- State how to get electric energy from chemicals.
- Use thermocouple thermometer to generate electric current.

Electricity

Electricity is a form of energy resulting from the existence of charged particles (such as electrons or protons), either statically as an accumulation of charge or dynamically as a current.

There are two types of Electricity: Static Electricity and Current Electricity.
The difference between the two is based simply on whether the electrons are at rest (static) or in motion (dynamic).

Static electricity is a buildup of an electrical charge on the surface of an object. It is considered “static” due to the fact that there is no current flowing. It is usually caused by rubbing materials together. The result of a build-up of static electricity is that objects may be attracted to each other or may even cause a spark to jump from one to the other.

Current electricity is named for the way electrons move. They “flow” in one direction - like a river current. The study of electrons in motion like this is called **Electrodynamics**. Materials that can conduct electricity are able to have an electric current flowing through them. **Current electricity** is the electricity that powers our homes and electrical devices. Electricity is used to operate your cell phone, power trains and ships, run your refrigerator, and power motors in machines like food processors. Electric energy must be changed to other forms of energy such as heat, light or mechanical in order to be useful.

There are two kinds of current electricity: **direct current (DC) and alternating current (AC)**. With direct current, electrons move in one direction. Batteries produce direct current. In alternating current, electrons flow in both directions.

Electric current and its SI unit

Electric current is defined as the rate of flow of electric charges. It is produced by moving electrons and it is measured in amperes. The ampere is one of the fundamental units of the SI system. Unlike static electricity, current electricity must flow through a conductor, usually copper wire. Current with electricity is just like current when you think of a river. The river flows from one spot to another, and the speed it moves is the speed of the current. With electricity, current is a measure of the amount of energy transferred over a period of time. That energy is called a flow of electrons. One of the results of current is the heating of the conductor. When an electric stove heats up, it's because of the flow of current.

The size of an electric current is equal to the rate of flow of charge.

$$\mathrm{I} = \frac{\mathrm{Q}}{\mathrm{t}}$$

Where:

Q = quantity of charge in coulombs, C

I = current in amperes, A

t = time in seconds, s

Worked example 4.1

120 C of charge passes in 1 minute. What is the current?

$$I = \frac{Q}{t} = \frac{120C}{60s} = \frac{2C}{s} = 2A$$

Worked example 4.2

How long will a current of 5A take to pass 100 C of charge?

Q = It, so $t = \frac{Q}{I} = \frac{100C}{5A} = 20s$

What are the charges that flow round a circuit?

To answer this question we must first look in rather more detail at the structure of an atom. All its positive charges are located in the central part, the nucleus. Each chemical element has a different number of positive charges in its nucleus, from one to nearly 100.

Take copper as an example:

- It has 29 positive charges in the nucleus. This means that an uncharged copper atom must have 29 negative electrons as well. They orbit round the nucleus, and each one has its own path.
- The first two electrons orbit in the innermost shell, the next eight fill the second shell and the following 18 just complete the third shell. That leaves a solitary electron as the first member of a new fourth shell.

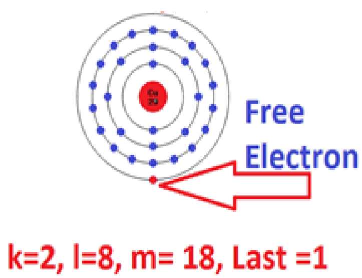

Figure 4.1.1*: copper orbit structure*

- It is this electron that accounts for the behavior of copper as a conductor. It is comparatively easy to remove this electron, changing an uncharged copper atom into what we call a copper **ion** with an overall single positive charge (Figure 4.1.2). An atom bearing unequal numbers of electrons and protons is known as an **ion**.

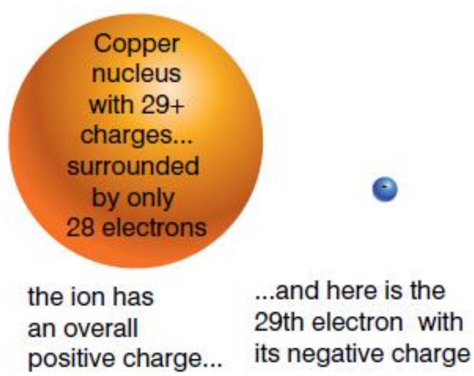

Figure 4.1.2: *The copper atom can be changed into a positive copper ion plus a negative electron.*

In a copper wire the atoms are packed close together just as in any other solid. More precisely, what are packed together are the positive ions; the complete atoms except for those single outer electrons. They are there as well, so the metal as a whole is uncharged.

In each copper atom 28 of the electrons are still firmly bound in orbit around their nucleus, fixed in its place in the solid. The 29th electrons we call the **conduction electrons**. Conduction electrons are electrons in the conduction band of a solid, free to move under the influence of an electric field. They remain trapped within the metal as a whole, but otherwise are free to drift about inside it. They are the charges that move when an electric current flows down the wire. Being negative they will be repelled from the cell's negative terminal and attracted to the positive one (Figure 4.1.3).

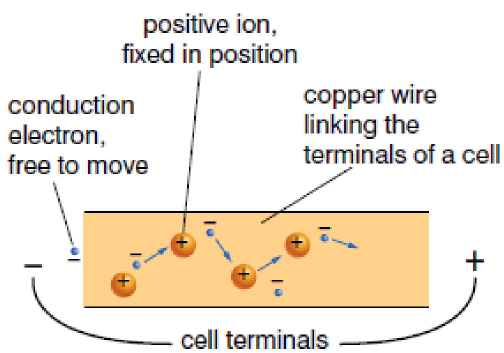

Figure 4.1.3: *The unattached electrons in the copper wire move when it is connected to the terminals of a cell.*

All metals have these extra one or two outer electrons, so they will all conduct electricity. With most other elements, every electron without exception is tightly bound to its own nucleus, so most other elements are **insulators**. Insulator is a material that resists the flow of electric charge.

When electrons move through a conducting wire, they do not move at a constant velocity, that is, the electrons do not move in a straight line at a constant speed. Rather, they interact with and collide with atoms and other free electrons in the conductor. Thus, the electrons move in a zigzag fashion and drift through the wire.

Electrical signals move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a light switch is moved to the ‘on’ position. The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in Figure 4.1.4, the incoming charge pushes other charges ahead of it due to the repulsive force between like charges. These moving charges push on charges farther down the line.

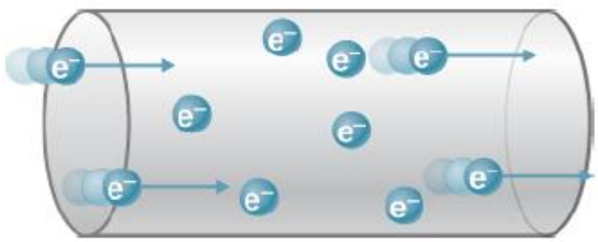

Figure 4.1.4: *When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave.*

The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Conventional current

Conventional current or simply current, behaves as if positive charge carriers cause current flow. Conventional current flows from the positive terminal to the negative. Perhaps the clearest way to think about this is to pretend as if movement of positive charge carriers constituted current flow.

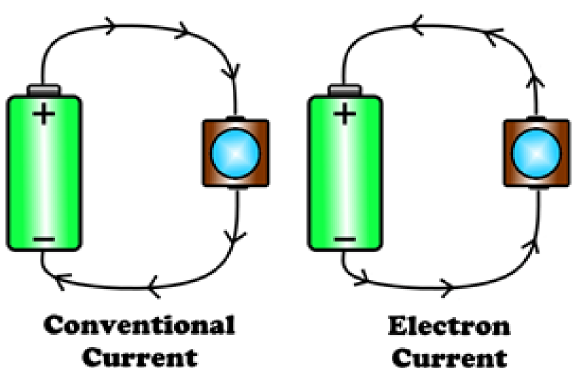

Figure 4.1.5: *Conventional and electron current*

Electric currents have been investigated since the first cell was invented around 1800.
It soon became obvious that charges were flowing round the circuit, but there was no way of telling whether they were positive charges going one way or negative charges going the other.

It was agreed to picture the flow as positive charges repelled from the positive plate, and this was referred to as conventional current.

When the electron was discovered in 1895, it was realized that the guess had been wrong. However, so firmly fixed was the idea that even today we still mark on our circuits the 'conventional current' flowing from the positive pole of the cell to the negative.

You must understand, however, that the electron flow is really negative charge going the opposite way.

Galvanoscope

Galvanoscope is an instrument that detects the presence of an electric current.
You can use a compass needle to make a simple **galvanoscope.** Simply wrap a few turns of thin insulated copper wire around a compass, connect the ends as shown in Figure 4.1.6 to a 1.5 V cell and a 1.5 V light bulb. When the wires are connected as shown, the bulb will light and the compass needle will move around in one direction. If you want to investigate this further, disconnect the cell, turn it round and reconnect it the other way round. The needle should move in the opposite direction.

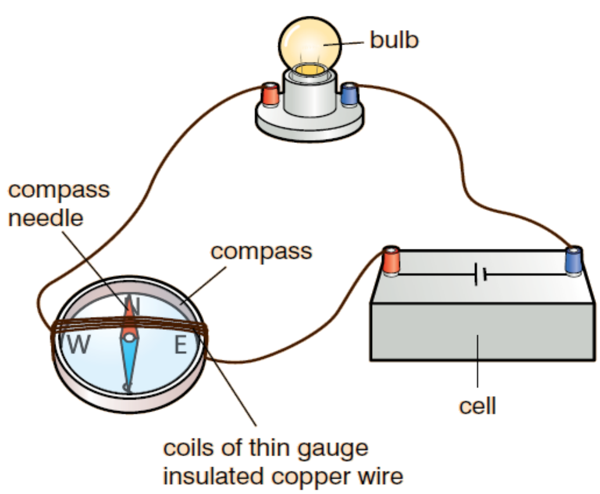

Figure 4.1.6: *current testers.*

Activity 4.1: Testing conductivity

Not all materials have conduction electrons in their structure, so not all materials will conduct electricity. Use your simple galvanoscope to test which materials conduct electricity and which do not.

Connect your galvanoscope to a copper rod as shown in Figure 4.1.7a. Observe that the compass needle moves, showing that the rod conducts electricity, allowing an electric current to flow.

Connect your galvanoscope to a glass or plastic rod, as in Figure 4.1.7b.
Observe that the compass needle does not move, showing that the rod does not allow an electric current to flow. Repeat this for a selection of different materials of the same size, and rank your materials in order of the size of the deflection of the compass needle.

You will see that some materials conduct electricity better than others, while some materials do not conduct electricity at all.

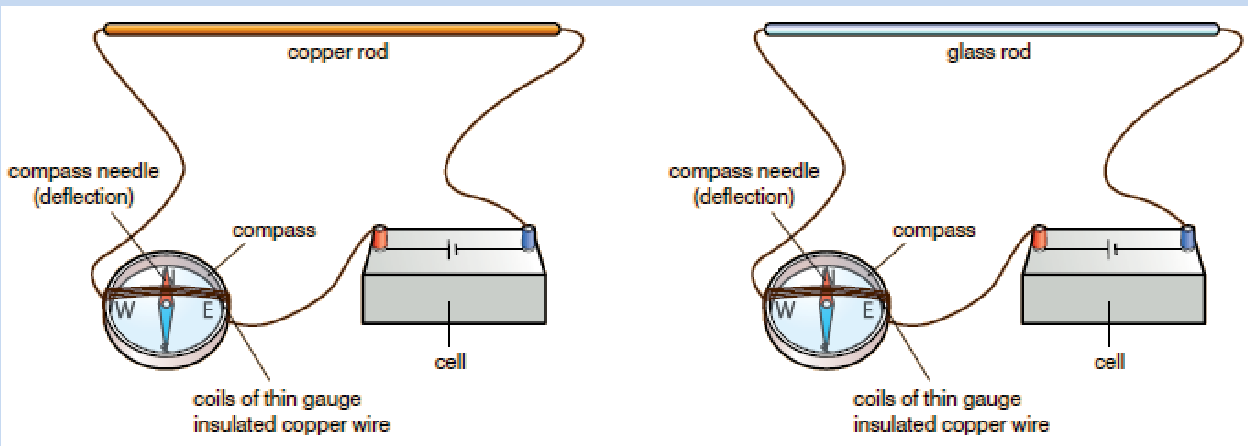

Figure 4.1.7a Figure 4.1.7b

***Figure 4.1.7**: Testing conductivity*

Drift Velocity

Good conductors have large numbers of free charges. In metals, the free charges are free electrons. In fact, good electrical conductors are often good heat conductors too, because large numbers of free electrons can transport thermal energy as well as carry electrical current.

Figure 4.1.8 shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electrical field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** $\overrightarrow{v_d}$ is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges.

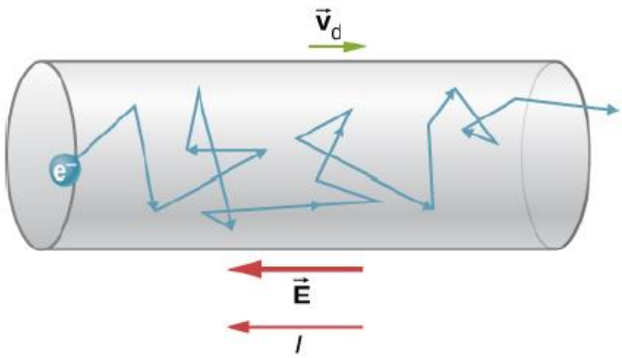

Figure 4.1.8: *Free electrons moving in a conductor*

The free electrons moving in a conductor make many collisions with other electrons and other particles. A typical path of one electron is shown. The average velocity of the free charges is called the drift velocity $\overrightarrow{v_d}$ and for electrons, it is in the direction opposite to the electrical field. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Calculating the number of electrons that pass a point at a given length of time when the current in the wire is known

The wire shown in Figure 4.1.9 carries a current of **I** amperes. There are **n** free electrons per cubic meter of wire. Each electron carries a charge of **e** coulombs, the cross-sectional area of the wire is **A** square meters and the average drift speed of the electrons in this material is **v** meters per second.

Drift speed is the average speed that an electron attains in an electric field.

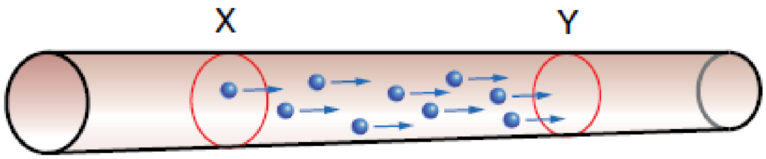

Figure 4.1.9

Assuming that it takes **t** seconds for an electron to pass from X to Y (and using the formula distance = speed × time), distance X–Y is **vt** metres. The volume of wire between X and Y is therefore **Avt** cubic metres. The number of electrons between X and Y is therefore **nAvt**.

As each electron carries a charge of **e** coulombs, the total charge passing point Y in **t** seconds is **nAevt**. The current, **I**, is the charge per time:

$I=\frac{nAevt}{t}$

or I = nAev

As the charge carried by an electron is known to be 1.6×10^{-19} C, the number of electrons passing through a wire can be calculated if the current in the wire is known.

Worked example 4.3

A current of 5 A is flowing through a 2 mm diameter wire. The drift speed of electrons in the wire is 10^{-5} m/s and the charge on the electron is 1.6×10^{-19} C. Calculate the number of electrons passing a point in the wire in a second.

The cross sectional area (A) of the wire is πr^2.

$$A = \pi \times \frac{d}{2} \times \frac{d}{2} = \pi \times 0.001 \times 0.001 = \pi \times 10^{-6} m^2$$

using $n = \frac{I}{Aev}$

$$number\ of\ electrons = \frac{5}{\pi \times 10^{-6} \times 1.6 \times 10^{-19} \times 10^{-5}} = 9.947 \times 10^{29}$$

Worked example 4.4

In the conductor shown in the following figure, if a current of 15A flowed for 1hr,

a) What amount of charge would move through the cross-section in this time?

b) How many electrons pass across the cross-section?

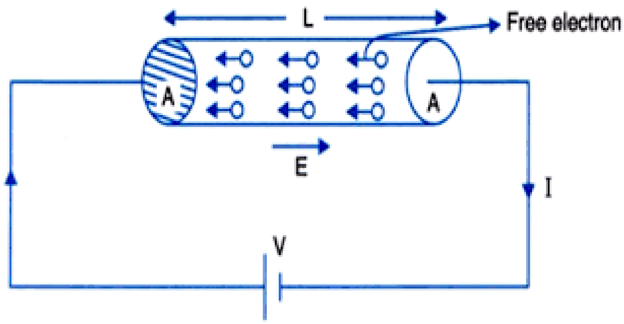

Figure 4.1.10

Solution: I=15A, t =1hr = 3600s

a) Q = It = 15A×3600s = 54000C

b) To find number of electrons (n) we use the relation:

$$n = \frac{\text{Total charge transferred}(Q)}{\text{Charge on an electron}(e)} = \frac{5.4 \times 10^{4}\text{C}}{1.6 \times 10^{-19}} = 3.38 \times 10^{23} \text{ electrons}$$

Study of electric charges in a metallic conductor

In Activities 4.1 you detected an electric current. Where does this electric current keep coming from and where does it keep going to? To start investigating the answer to this, look at Figure 4.1.11 which shows the top view of a corridor that ends up where it started. The whole place is filled with people who form a kind of endless queue. The door is shut, which means that nobody in the queue can move forward. Once the door is opened (once the circuit is switched on), all the people in that corridor can immediately start moving. They can keep circulating round and round the corridor, thus forming a continual current.

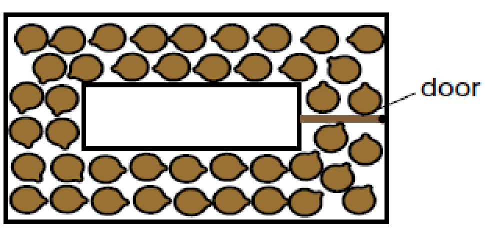

Figure 4.1.11

Of course when the door was opened the people filling the corridor found themselves free to move round the circuit, but they did not have to do so. Likewise charges will not circulate to give a current unless there is something that keeps 'pumping' them round. That is the job of the cell, as we shall see below.

Electric energy from chemicals

An electrochemical cell is one device that makes current flow round a circuit. It is a device capable of deriving electrical energy from chemical reactions.

Such cells have two **electrodes** made of two different **conductors**, an **electrolyte** solution (a solution that conducts electricity) which reacts with the electrodes, and a conductive wire through which electrons can flow. **3**

Activity 4.2: Make your own electrochemical cell

Roll a lemon, orange, grapefruit, or other citrus on a firm surface to break the internal membranes. Insert two rods – one copper, one zinc – into the fruit and connect as shown in Figure 4.1.12. Observe the size of the deflection of the needle on the instrument.

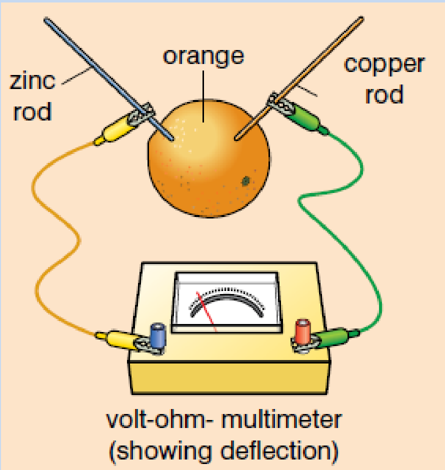

Figure 4.1.12

The electrodes in this cell are the copper rod and the zinc rod.
The electrolyte here is the juice inside the orange.

Repeat the experiment using rods made from different materials and observe the deflection of the needle on the instrument. Do some pairs of materials produce a greater deflection than others?

Some cells are more powerful than others. An ordinary torch cell (usually referred to as a battery) is rated at 1.5 volts. We call this the **electromotive force** of the cell, usually shortened to **e.m.f**. **Electromotive force** is a source of energy causing current to flow in an electrical circuit. It is the cell that pumps the charge round the circuit, and for the time being it is sufficient to think of the e.m.f. as its ‘electrical pumping strength’. All other things being equal, the greater the e.m.f. of the cell the greater the current that it will drive round a circuit. You will have got the right idea when you do not like to hear people talking about volts flowing round a circuit. Volts do not flow. They cause coulombs of charge to flow round a circuit at a rate of amperes.

There are many different types of cells. Figure 4.1.13 illustrates the principles of one common form, which we call a **dry cell**.

Dry cell is an electrochemical cell containing electrolyte in the form of a paste. Experiment shows that the carbon plate is the positive one, the zinc is the negative, and the e.m.f. is very close to 1.5 V.

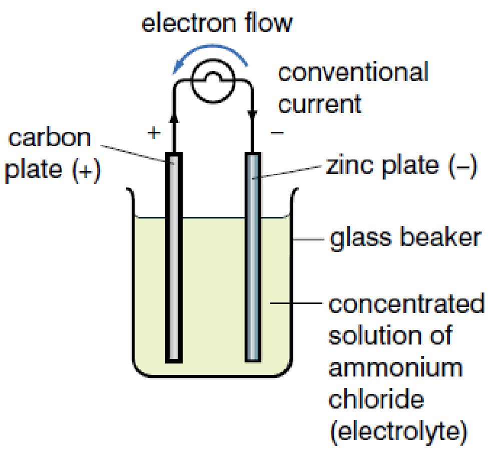

Figure 4.1.13: *The principle of a common sort of cell.*

Such a cell is about as portable as a glass of water, and we now see how it has been converted into the tough flashlight cell (commonly known as a battery) we are familiar with.

The negative zinc plate is shaped to form the casing of the cell (Figure 4.1.14). The carbon runs down the centre in the form of a rod (with a brass cap on the top, because carbon is easily damaged). The concentrated solution of ammonium chloride that has to go into the space between them is made much less runny by forming it into a paste or jelly. It is for this reason that the design is called a dry cell.

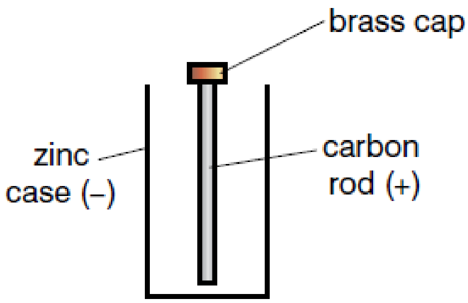

Figure 4.1.14: *The zinc casing forms the negative plate*.

This leaves the complete cell as shown in Figure 4.1.15.

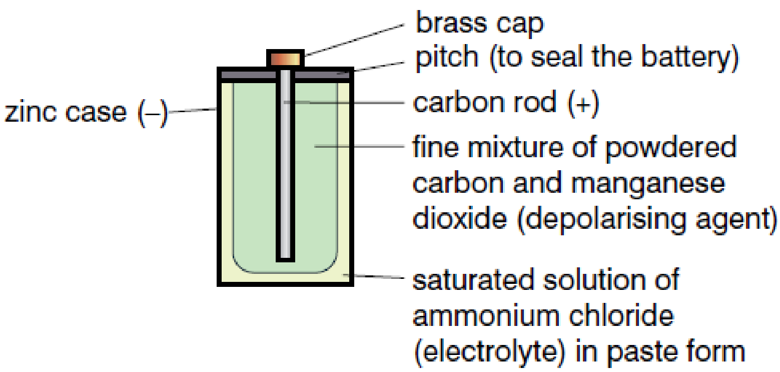

Figure 4.1.15: *The complete dry cell.*

The positive plate is both the carbon rod down the centre and the thick layer of powdered carbon that surrounds it.

Mixed in with the powdered carbon is manganese dioxide, another chemical in powder form. While the cell is giving a current it suffers from **polarisation**: the formation of a film of hydrogen gas on the positive plate which ‘clogs’ the cell up. The manganese dioxide is there to deal with this problem by supplying oxygen to convert this unwanted hydrogen to water. Sometimes the mixture of black powders is held in place round the central carbon rod with a cloth bag, but in other batteries it is just the ‘stiffness’ of the ammonium chloride paste that keeps it there.

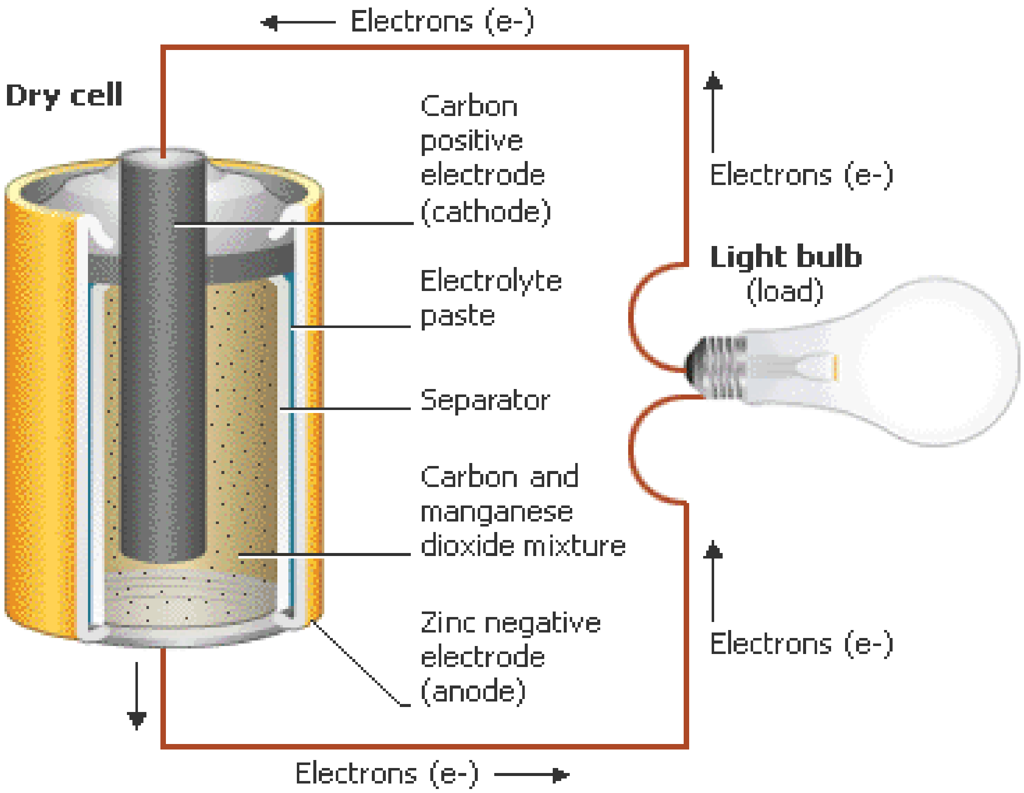

Figure 4.1.16*: Dry Cell and light bulb*

This simple dry cell contains a negative electrode (a zinc sheath which encloses the battery materials) and a positive electrode (the carbon rod and the carbon and manganese dioxide mixture that surrounds the rod). An electrolyte paste separates the two electrodes and facilitates a chemical reaction between them. This reaction causes a current to flow (that is, makes the electrons move) through a conductor that connects the positive and negative electrodes.

As the cell runs down, the zinc casing gets thinner and thinner, and when it gets very old the ammonium chloride may seep out from holes in the casing as a white corrosive paste.

There is nothing unique about carbon, zinc and ammonium chloride solution: any combination of two different metals (or one metal and carbon) placed in a solution of an electrolyte will produce a voltage. Other combinations are available, such as the alkaline cell, which uses potassium hydroxide for the electrolyte.

What is Cell and Battery?

Cell is a smaller unit while battery is larger. We many times use term battery in place of cell as working of both is same. Both convert chemical energy into electrical energy. The difference between cell and battery is that cell is a single unit which converts chemical energy into electrical energy while battery is a group of cells which all converts chemical energy into electrical energy. Figure of a cell and battery is given below for your better understanding. We use different symbols to represent cell and battery. Symbols of both are given below.

***Figure 4.1.17**: cell and Battery*

Primary cells and secondary cells

Cells that produce their voltage directly from chemical energy stored in the ingredients which make them up are known as **primary cells**. When all their chemicals have reacted, you have to buy a new one. The dry cell is one of these.

There are also **secondary cells**, which have first to be charged up by forcing a current 'backwards' through them. The commonest types of these are the increasingly popular rechargeable cells and the lead–acid accumulators that make up the battery in a car.

- The major difference between a primary cell and the secondary cell is that **primary cells are the ones that cannot be charged** but secondary cells are the ones that are rechargeable.
- Internal resistance of primary cells is very high, whereas secondary cells possess low internal resistance.

Activity 4.3: The direction of current flow

Part A

Connect the cell, motor and switch (open) as shown in Figure 4.1.18 on the edge of a table or bench. Clamp the motor to the edge of the table. Attach weights to the spindle of the motor, to hang over the edge of the table or bench. Close the switch. The motor will turn and the weights will move. Observe the movement of the weights; one weight will rise and one weight will fall.

Part B

Attach the cell the opposite way round. Close the switch and observe the movement of the motor and the weights. (x rises, y falls)

You will see that the motor turns in the opposite direction in Part B, causing the weight which rose in Part A to fall in Part B, and the one which fell in Part A to rise in

Part B.

This demonstrates that the electrons in the circuit flowed in one direction in Part A and in the opposite direction in Part B.

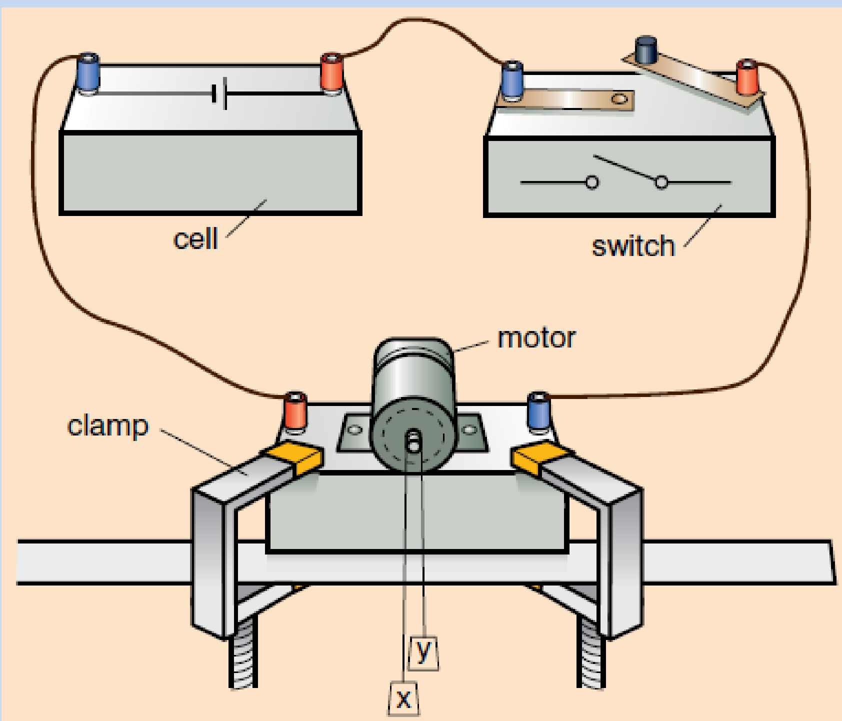

Figure 4.1.18

Electric energy from heat

We have seen how electric energy can come from chemicals and we now see that it can also come from heat; this is called **thermoelectricity**. Thermoelectric materials generate power directly from the heat by converting temperature differences into electric voltage.

If two different metals are joined in a circuit, and if one junction between the metals is hotter than the other, a very small voltage (a few millivolts, mV) is generated. This effect is known as the Seebeck effect after the German physicist Thomas Seebeck (1770–1831) who discovered it.

The circuit must be made from two different metals, any two, although some pairs may be better than others.

The thermocouple thermometer

The Seebeck effect is used in the thermocouple thermometer to measure temperature. It consists of a circuit made from any two different metals and includes a sensitive meter to measure small currents (Figure 4.1.19). If one of the junctions between the metals is hotter than the other, a small voltage is generated, which drives a tiny current round the circuit. The larger the temperature difference, the greater the current.

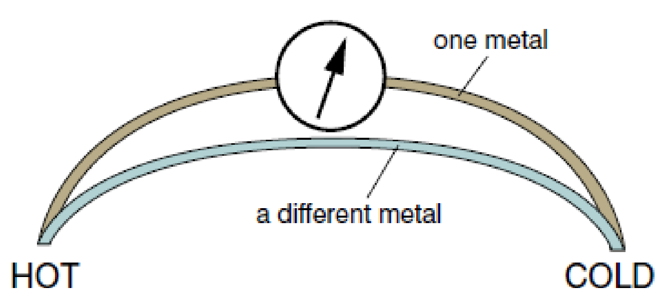

Figure 4.1.19: *The thermocouple thermometer.*

When using a thermocouple thermometer, it is usual to hold one junction at 0 °C by keeping it immersed in melting ice. The other junction acts as the probe to investigate the temperature to be measured, as in Figure 4.1.20.

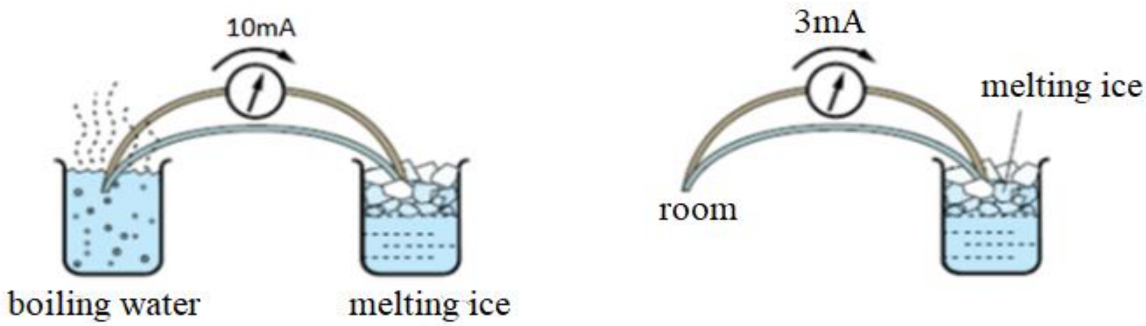

Figure 4.1.20: *The thermocouple thermometer in use.*

In the first part of Figure 4.1.20, a temperature difference of 100 °C between boiling water and melting ice shows a current of 10 mA (milliamperes, thousandths of an ampere). Each 1 mA therefore indicates a difference of 10 °C.

Thus when the probe is at room temperature and a current of 3 mA is measured, the junction at room temperature must be 30 °C hotter than the cold junction, which is at 0 °C. Room temperature is therefore 30 °C.

Worked example 4.5

One junction of a thermocouple thermometer is immersed in melting ice and the other in boiling water. A current of 20 mA is recorded. The thermocouple thermometer is then used to measure the temperature of a liquid. One junction is immersed in melting ice and the other in the liquid whose temperature is to be measured. A current of 5.5 mA is recorded. What is the temperature of the liquid?

Solution

In the first, calibration, measurement, 20 mA represents a temperature difference of 100 °C.

Therefore 1 mA represents a temperature difference of 5 °C. In the temperature measurement, the temperature difference between the junctions is $5.5 \times 5 = 27.5$ °C.

The junction immersed in melting ice is at 0 °C. The temperature of the liquid being measured is therefore $0 + 27.5 = 27.5$ °C.

Activity 4.4: Making a thermocouple thermometer

Create thermocouple junctions at both ends of a section of iron wire by twisting the ends together with copper wires.

Place one copper–iron junction in a beaker with ice water and leave the other junction outside. The two remaining ends of the copper wires should be connected to a sensitive galvanometer.

Heat the exposed junction with a Bunsen burner or match and record the current.

Does the current increase or decrease if the heat source is removed? Is the change in current immediate? Discuss these questions.

Summary

- **Electricity** is a form of energy resulting from the existence of charged particles.
- There are two types of Electricity: Static Electricity and Current Electricity.
- Current is a flow of electric charge.
- Conduction electrons are electrons in the conduction band of a solid, free to move under the influence of an electric field.
- Coulombs of charge flow at a rate of amperes.
- Conventional current behaves as if positive charge carriers cause current flow. It flows from the positive terminal to the negative.
- We need a voltage supply to cause the charge to circulate round the circuit.
- Galvanoscope is an instrument that detects the presence of an electric current.
- **Drift speed** is the average speed that an electron attains in an electric field.
- As each electron carries a charge of **e** coulombs, the total charge passing a given point in **t** seconds is **nAevt**. The current, **I**, is the charge per time:

$$I=\frac{nAevt}{t}$$

or $I = nAev$

- An electrochemical cell is one device that makes current flow round a circuit. It is a device capable of deriving electrical energy from chemical reactions.
- Electromotive force is a source of energy causing current to flow in an electrical circuit. It is the cell that pumps the charge round the circuit
- A primary cell uses the chemicals in it to supply electrical energy; a secondary cell has to be charged up first.
- Thermoelectric materials generate power directly from the heat by converting temperature differences into electric voltage.

Review questions

1. Charge flows along a wire at rate of 3 A. How many coulombs of charge will pass a given point in the circuit in:

   a) 1 s b) 12 s c) 2 minutes?

2. How many electrons flow through a point in a wire in 3.0 s if there is a constant current of $I = 4.0$ A ?
3. When a small torch is switched on, the current drawn from the cell is 0.4 A.
   a) Calculate the charge passing a point in the bulb filament when the torch is switched on for 10 minutes.
   b) How many electrons drift past the point in this time?
   (The charge on one electron is 1.6×10^{-19} C.)
4. The current in a lightning strike is 6000 A. The strike lasts for 180 ms. Calculate
   a) the charge, in C, which flows in the strike to the ground
   b) the number of electrons transferred to the ground.
5. A very sensitive ammeter records a current of 30 µA. How long will it take before 6 µC of charge has flowed through it?
6. In an experiment to copperplate a coin, a current of 100 mA is passed through a solution of copper sulphate. If 30 C of charge must pass before the coin has enough copper deposited on it, for how long must you leave the current switched on?
7. a) A single electron has a charge of 1.6×10^{-19} C. How many electrons must pass round the circuit of question 6 to achieve the 30 C?
   b) For every two electrons that arrive at the coin which is being plated, one copper atom is deposited. If the mass of a copper atom is 1.1×10^{-25} kg, what is the total mass of the copper which now plates the coin?
8. A current of 3A is flowing through a 1.5 mm diameter wire. The drift speed of electrons in the wire is 2×10^{-5} m/s and the charge on the electron is 1.6×10^{-19} C. What is the number of electrons passing a point in the wire in a second?
9. Calculate the number of charge carriers per unit volume in a copper wire of cross-sectional area 4 mm^2 carrying a current of 1A if the drift velocity of the charge carriers is 0.0002 m/s. (The elementary charge is 1.6×10^{-19} C.)

10. In the conductor shown in the figure, if a current of 10A flowed for 90min,

    a) What amount of charge would move through the cross-section in this time?

    b) How many electrons pass across the cross-section?

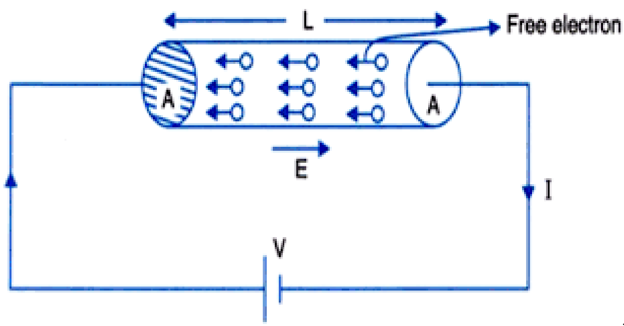

Figure 4.1.21

11. What name do we give to the unit ‘ampere second’ (A s)? What physical quantity would be measured in such a unit?
12. You have two batteries, a large one and a tiny one, each consisting of a single dry cell. In terms of their performance, what would you expect to be the same for the two batteries and what would you expect to be different about them?
13. What is the difference between a primary cell and a secondary cell?
14. What temperature does the thermocouple in Figure 4.1.22 indicate for:

    a) air b) liquid B c) liquid C?

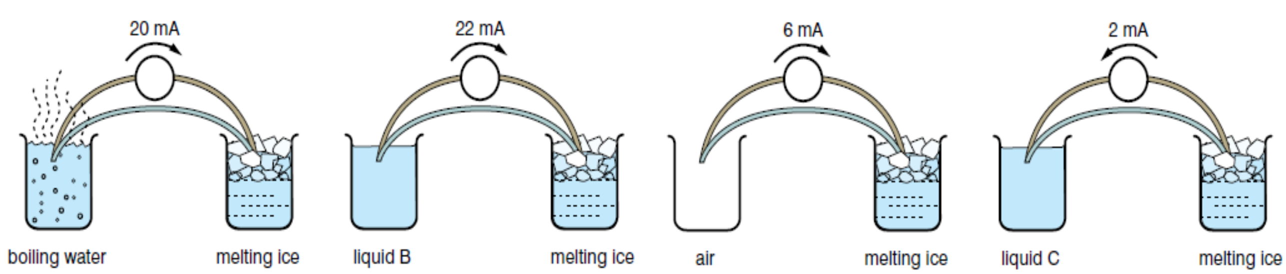

Figure 4.1.22

15. A thermocouple junction will respond to changes in the temperature of its surroundings more quickly than a mercury in- glass thermometer would. There are two reasons for this:
    a) the junction is smaller than the bulb full of mercury
    b) the junction is not encased in glass.

Explain why each of these factors helps the thermocouple to respond quickly.

16. What temperature does the thermocouple in Figure indicate for liquid **B**?

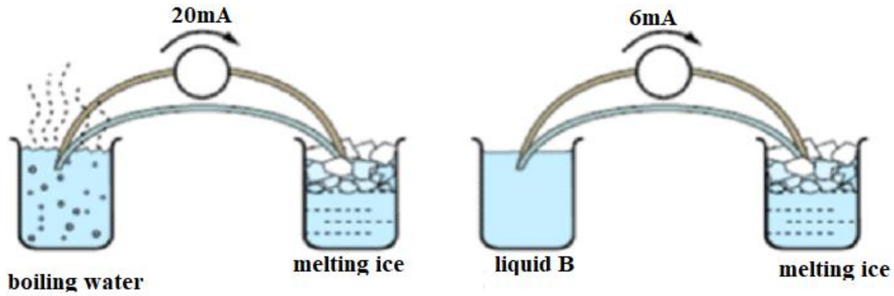

Figure 4.1.23

17. A metal wire is uncharged. Explain how it is possible for a current to flow through it.
18. Consider the following figure. A current of one ampere is flowing through the area *A* each second. Calculate the number of electrons passing through the given area A each second.

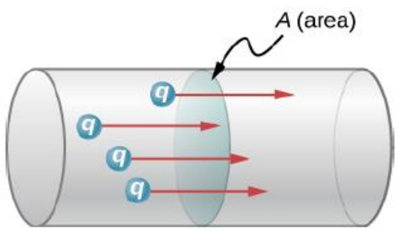

Figure 4.1.24

4.2 Ohm's law and electrical resistance

By the end of this section you should be able to:

- Describe factors affecting the resistance of a conductor.
- Write the relationship between resistance *R*, resistivity *ρ*, length *l* and cross-sectional area *A* of a conductor.
- Calculate the resistance of a conductor using the formula $R = \rho l/A$.
- Find the relationship between resistivity and conductivity.
- Construct and draw an electric circuit consisting of a source, connecting wires, resistors, a switch and a bulb using their symbols.
- Explain why an ammeter should be connected in series with a resistor in a circuit.
- Explain why a voltmeter should be connected in parallel across a resistor in a circuit.

- Do experiments using an ammeter and a voltmeter to investigate the relationship between current and p.d. for metallic conductors at constant temperature.

Electrical resistance

Resistance is defined as the measure of the opposition to the flow of current through the conductor.

Resistance is a property of a material that controls the amount of current that flows through it. It is measured in **ohms** (Ω).

A **resistor** is a passive component in a circuit which provides resistance to the flow of current. In electronic circuits, resistors are used to reduce current flow.

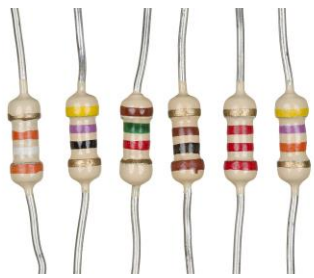

Figure 4.2.1: electrical resistors

The size of the current a cell will pump round a circuit depends on two things:

- ✓ One is the electromotive force, or e.m.f., of that cell, measured in volts. As we shall see in Section 4.4, adding a second cell **in series** (the current passes through one cell then the other) with the first makes a battery of cells, which has twice the e.m.f. of a single cell, and the charge will be pumped round the circuit at twice the rate. In other words, the current will double.
- ✓ The second factor determining the current is the circuit through which the cell must drive the charge. Is it an easy circuit made of thick pieces of a good conductor, or is it a more difficult circuit consisting of thin wire made from a metal that does not conduct electricity so well? It is this second factor we are going to consider now. Does the circuit have a low **resistance**, or does it have a high resistance?

We specify the resistance of a circuit by the number of volts of 'battery power' we would need to get a current of 1 A to flow round it. A low resistance circuit might need only 2 V per ampere (2 V/A), say. This suggests that a 1 V battery would produce a flow rate of 0.5 A round it, or that to get a current of 3 A going you would need to provide an e.m.f. of 6 V. A less easy circuit might have a resistance of 200 V/A; in other words, as many as 200 V would be needed to establish a current of just 1 A. It is important to study the behavior of resistors in electrical circuits.

Measuring the resistance of a resistor

The principle is simple: apply a voltage across the resistor, and measure the size of the resulting current. A suitable circuit is shown in Figure 4.2.2 where *R* is the resistor to be measured.

Figure 4.2.2 also shows the symbol of a variable resistor (sometimes called a 'rheostat'). By moving a slider or rotating a knob you can alter its resistance. This in turn will alter the total resistance of the whole circuit, and will therefore control the current drawn from the battery which then passes through the ammeter.

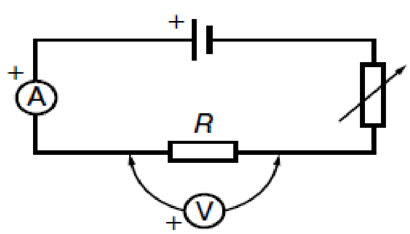

Figure 4.2.2: *Measuring the resistance of a resistor.*

In a lighting circuit a variable resistor would act as a dimmer switch to make the bulb fainter or brighter.

The ammeter may be placed anywhere in the circuit since the same current flows all the way round. The voltmeter is not part of the circuit itself; it is placed alongside to measure the drop in voltage between the two ends of *R*. The variable resistor is not essential, but it enables you to alter the voltage drop across *R* and the current through it so as to get check readings.

If you try this activity for yourself, you should choose a resistor to measure that is not significantly warmed by the current you send through it. A length of resistance wire open to the air should do, but avoid using a light bulb. You will find out why in the next section.

Your results should be recorded in a table like that in Figure 4.2.3.

p.d. V across R (volts)	current I through R (amperes)	$R = \frac{V}{I}$ (ohms)
	Average	

Figure 4.2.3

As you repeat the experiment for different values of *V* and *I*, you should find that the value for *R* remains constant. This illustrates **Ohm's law**.

Ohm's law

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points. Introducing the constant of proportionality, the **resistance**, one arrives at the usual mathematical equation that describes this relationship:

$$I = \frac{V}{R}$$

Where **I** is the current through the conductor in units of amperes, **V** is the voltage measured *across* the conductor in units of volts, and ***R*** is the resistance of the conductor in units of ohms. More specifically, Ohm's law states that the *R* in this relation is constant, independent of the current. If the resistance is not constant, the previous equation cannot be called *Ohm's law*.

For a metal wire at a constant temperature, the current that flows through it is proportional to the potential difference (the voltage drop) between its ends.

In other words, if the voltage drop across the wire doubles, charge will flow through it at exactly twice the rate (that is, the current doubles too). Put yet another way, $\frac{V}{I}$ stays constant: the wire's resistance does not change. Notice that this will apply only if the temperature of the conductor does not change.

The ohm (Ω)

The unit for resistance is the ohm (Ω). It is named after Georg Ohm(1787 1854), a German physicist who was one of the first to investigate how currents flowed in circuits.
The abbreviation for ohm should really be a capital ‘O’, but that could be confused with a zero. Luckily there is a letter in the Greek alphabet called ‘omega’, which provides the ideal replacement: the abbreviation for ohms, therefore, is ‘Ω’, which is a capital omega.

Worked example 4.6

A 12 V car battery is connected to a circuit for which total resistance is 6 Ω. What current will flow?

$$I = \frac{V}{R} = \frac{12}{6} = 2A$$

Worked example 4.7

A 12 V battery is connected to a circuit. If the current is 2 mA, what must the resistance of the circuit be? (Notice here that the current is given as 2 mA. Before it can be entered into the formula it must be converted to 0.002 A (or to 2×10^{-3} A.)

$I = \frac{V}{R}$, so rearranging we get $R = \frac{V}{I} = \frac{12}{0.002} = 6000\Omega$

A thousand ohms is one kilohm (rather than ‘kiloohm’), so the answer could be expressed as 6 kΩ.

Worked example 4.8

What voltage battery would be needed to send a current of 3 A round a circuit for which the total resistance is 4 Ω?

Here I = 3 A and R = 4 Ω.

$V = IR = 3 \times 4 = 12$ V

Factors affecting the resistance of a conductor

1. Effect of heat on resistance

If you display the current and voltage readings measured when studying a resistor at constant temperature in the form of a graph of the current through the resistor plotted against the voltage drop applied across it, you should get a straight line (Figure 4.2.4). Double the potential difference (p.d.) across the wire and you will double the current flowing through it.

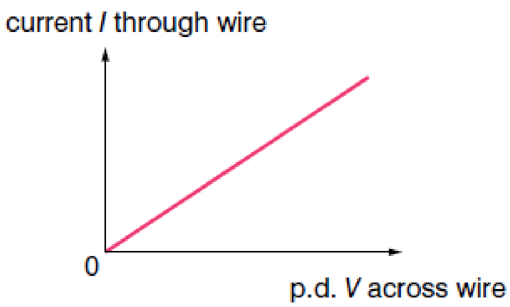

Figure 4.2.4: *A graph of current against voltage drop for a resistor, R.*

However, if you use a light bulb as your resistor and take a range of readings such that the filament of the bulb is varied from not even red-hot to brilliantly white-hot, the graph of your readings will form a curve (Figure 4.2.5). As you increase the voltage and the lamp glows more brightly, the current does not rise as rapidly as expected.

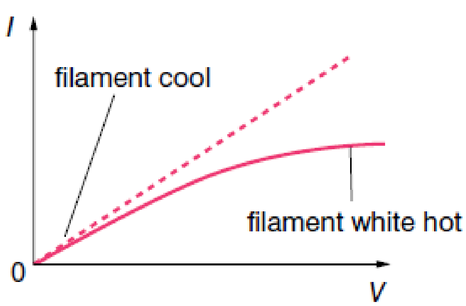

Figure 4.2.5: *As the temperature of the filament rises, so does its resistance.*

As the filament becomes white hot, its temperature increases by at least several hundred degrees. The resistance of metals rises with temperature, and that is why a hot bulb does not conduct as well as a cool one. Instead of taking current and voltage readings, it is useful to measure the resistance of a light bulb at as wide a range of currents as possible. At one extreme, readings should be taken when the current is so small that the filament is not even glowing feebly red; at the other extreme the bulb should be brighter than normal.

Use these readings to plot a graph of resistance against current, as in Figure 4.2.6. If the temperature had not changed, the resistance would have been constant (line (a)). As it is, the resistance rises markedly (line (b)).

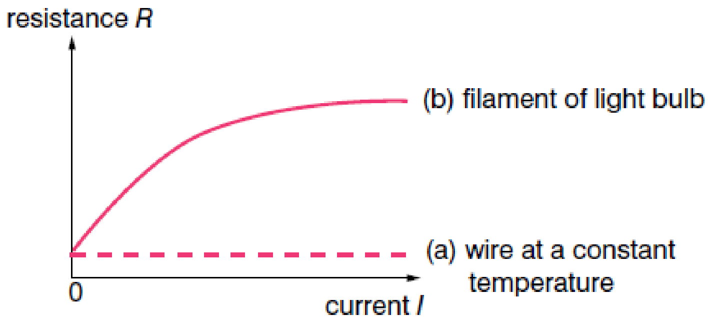

Figure 4.2.6: *Resistance–current graph.*

Notice that the line will not pass through the origin. If you continue the curve back so it approaches or equals zero, you will have the resistance of the bulb when it is cold (that is, at room temperature). At its working temperature the resistance of the bulb may well have risen to ten times that value.

The resistance of a metal increases in an approximate straight line if you plot it against its temperature. The increase in the value of the resistance as it warms up depends on:

- The rise in the wire's temperature, ΔT. The greater the rise, the bigger the increase.
- How many ohms of resistance it possesses. Since this will vary with its temperature, we work on the basis of its resistance at 0 °C and call it R_0. The greater the number of ohms at the start, the greater the increase in resistance will be.
- The metal it is made from.

Putting these together, we can say:

- The increase in resistance = $\alpha R_0 \Delta T$
- $\Delta R = R_0 \alpha \Delta T$, where R_0 is the original resistance.
- $R - R_0 = R_0 \alpha \Delta T$, *where* R is the resistance after a temperature change.
- $R = R_0 + R_0 \alpha \Delta T$
- $R = R_0(1 + \alpha \Delta T)$

Alpha (α) is a constant, which varies from one metal to another. We call it the metal's temperature coefficient of resistance. The units of α have to be K^{-1} (kelvin^{-1}) in order to give an answer that is in ohms.

You will probably be working in degrees Celsius rather than kelvin, but there is no need to worry, remember that a rise in temperature of 10 °C is exactly the same as a rise of 10 K.

Worked example 4.9

A copper resistor ($\alpha = 4.3 \times 10^{-3}$ K^{-1}) whose value is 10.0 Ω at 0 °C is warmed from that temperature up to 150 °C. Work out its resistance when hot.

Increase in resistance = $\alpha R_0 \Delta T = 4.3 \times 10^{-3} \times 10 \times 150 = 6.45\ \Omega$

Resistance when hot = 10 + 6.45 = 16.45 Ω

Worked example 4.10

A carbon resistor at room temperature (20 °C) is attached to a 9.0V battery and the current measured through the resistor is 3.0 mA.

a) What is the resistance of the resistor measured in ohms?
b) If the temperature of the resistor is increased to 60 °C by heating the resistor, what is the current through the resistor?

Solution

a) The resistance can be found using Ohm's law. Ohm's law states that $V = IR$, so the resistance can be found using $R = \frac{V}{I} = \frac{9.0V}{3.0\times10^{-3}A} = 3.0 \times 10^{3}\Omega = 3k\Omega$.
b) The resistance at 60 °C can be found using $R = R0\ (1 + \alpha\Delta T)$ where the temperature coefficient for carbon is $\alpha = -0.0005$. $R = R_0\,(1 + \alpha\Delta T)$

$$R = 3.00 \times 10^{3}(1 - 0.0005(60\ °C - 20\ °C)) = 2.94\ k\ \Omega\ .$$

The current through the heated resistor is:

$$I = \frac{V}{R} = \frac{9.0V}{2.94 \times 10^{3}\Omega} = 3.06 \times 10^{-3}A = 3.06mA$$

2. Effect of length and diameter of resistor on resistance

Connect a light bulb (1 to 2 V) in series with a pencil lead (pencil 'lead' is made of carbon, an element frequently used in resistors) and a 6 V battery (four 1.5 V cells connected in series). Turn off the room lights and observe the brightness of the bulb.

Move one wire contact along the *length* of the pencil lead and observe the change in the intensity of light. The light should become brighter as the resistor becomes shorter, as there is less resistance impeding the circuit.

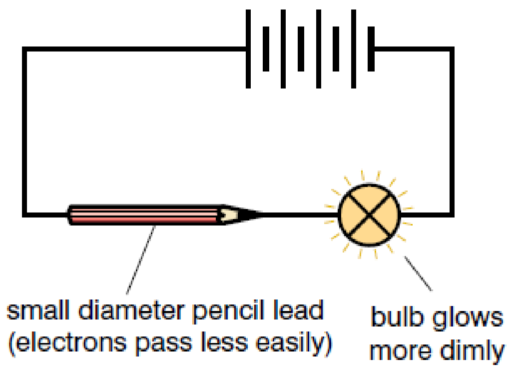

Figure 4.2.7: *A small diameter pencil lead, dim bulb*

Repeat the activities with a pencil lead of different *diameter* and observe the changes in the intensity of light. The light should become brighter when the diameter of the resistor is larger as it is easier for the current to flow.

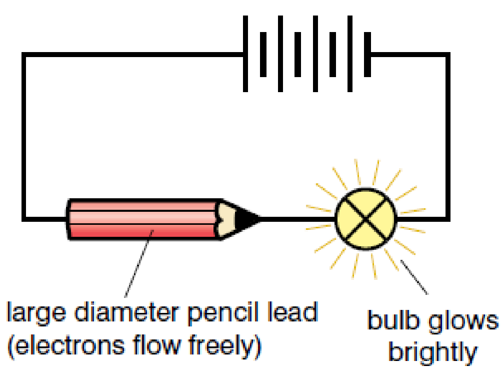

Figure 4.2.8: *A large diameter pencil lead, bright bulb*

The relationship between resistance *R*, resistivity *ρ*, length *l* and cross-sectional area *A* of a conductor

The resistance of a metal wire at a given temperature is determined by three factors:

- Its length l, in metres – the resistance is proportional to l, so if the length doubles so does the resistance.
- Its area of cross-section A, in m^2 – the resistance is inversely proportional to A, so a wire with twice the cross-sectional area will have only half the resistance.
- The material from which the wire is made – copper, for example, is a better conductor than iron.

Thus the resistance R of a wire can be expressed in the form:

$$R = \frac{\rho l}{A}$$

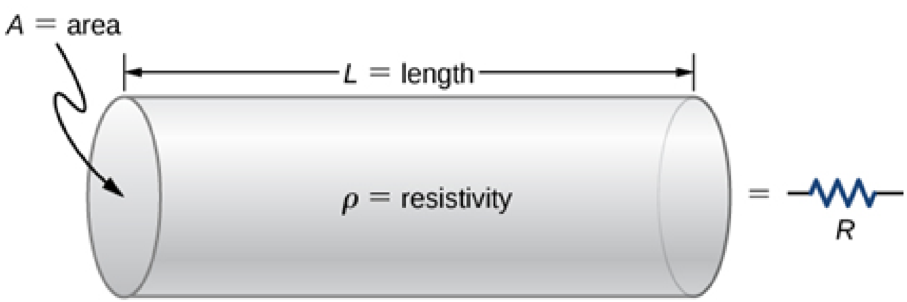

Figure 4.2.9*: Resistance and resistivity*

The symbol ρ (the Greek letter rho) is a constant, the value of which depends on the material from which the wire is made – the value for iron will be greater than the value for copper. We call ρ the resistivity of the material: it is defined by the equation above.

The units of resistivity are Ω m – ohms multiplied by metres. To see this, consider the units of the right-hand side. They will be Ω m × m divided by m^2, which works out correctly to give the resistance in ohms.

Worked example 4.11

What is the resistance of a copper cable that has a cross sectional area of 1 cm^2 and a length of 2 km? The resistivity of copper is 2×10^{-8} Ω m.

Solution

Be careful over the units.

l = 2 km = 2×10^3 m

A = 1 cm^2 = 1×10^{-4} m^2 (since there are 100 × 100 centimetre squares in a metre square).

$$R = \frac{\rho l}{A}$$

Putting in the values we get

$$R = \frac{2 \times 10^{-8} \times 2 \times 10^{3}}{1 \times 10^{-4}} = 0.4\Omega$$

The relationship between resistivity and conductivity

The resistivities of most metals are in the range 10^{-7} to 10^{-8} Ω m.

Those with larger resistivities conduct electricity less well.

The resistivity of an insulator such as dry polythene may be as high as 10^{15} Ω m.

Conductors and insulators are two extremes, but there are only a few materials in between these extremes: germanium at room temperature, for instance, may display a resistivity of around 0.001 Ω m. We call such materials **semiconductors**. **Semiconductors** are materials that have an electrical conductivity between that of a conductor and an insulator.

The capacitor (Figure 4.2.10) is an electrical component made of a combination of resistor and insulator. Many circuits use the capacitor to store and release charge. While there are many types of capacitor, they all have the same basic features – two conducting plates separated by a layer of insulating material.

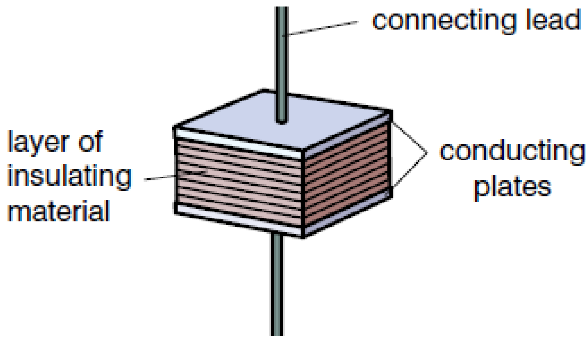

Figure 4.2.10: *The capacitor.*

Constructing and drawing electric circuits

An electric current is a flow of charge. A circuit is made of conducting materials, with a device such as a cell, which keeps pumping the charges round the circuit.

Figure 4.2.11a shows the symbol for a cell. We often refer to it as a battery, though strictly speaking this is incorrect: 'battery' is the term reserved for a whole collection of cells acting together. The plus and minus signs are not usually marked on the drawing, so you must remember which is which. Remember too that **conventional current flows round the circuit from positive to negative**.

Figure 4,2,11b represents a battery of four cells. Notice that in order for them all to be pumping the charge the same way, the '+' terminal of one cell has to be joined to the '–' terminal of the next one.

Figure 4.2.11: *a) The symbol for a cell; b) a battery of four cells.*

Some circuit symbols

Figure 4.2.12 shows the symbols for a range of electrical components.

If you have access to equipment, test what you have learnt so far by using some of these components to make circuits.

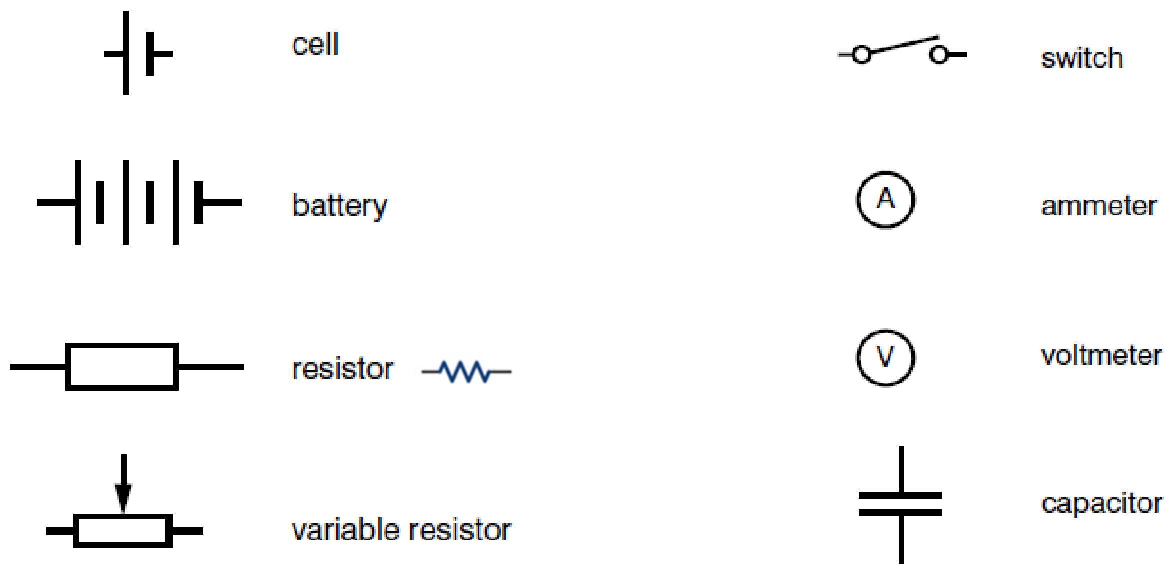

Figure 4.2.12: *Some circuit symbols.*

If you have access to equipment − such as resistors, variable resistors, ammeter, voltmeter, capacitors − use these and observe what happens when you construct a circuit. Using the symbols for each component, draw a circuit diagram.

The correct position for an ammeter in a circuit

Ammeter is an instrument that measures the size of a current. To measure the current through a device or component, the ammeter is placed in series with the device or component. A series connection is used because objects in series have the same current passing through them.

The symbols for a cell, a light bulb, an ammeter and a switch are shown in the circuit in Figure 4.2.13. Ammeter is fitted into the circuit so the current to be measured flows through it. In order that the ammeter does not reduce the current it has been put in to measure, it is essential that the instrument allows the current to flow through it freely (has a low resistance).

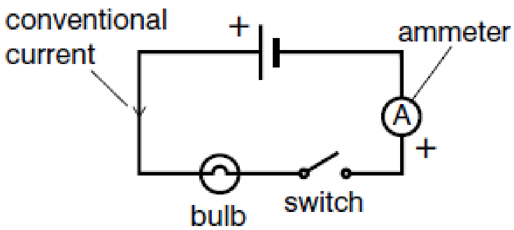

Figure 4.2.13: *A simple circuit.*

It is important that you connect the ammeter the correct way around; otherwise the current would try to twist it backwards. The terminal on it marked '+' needs to be joined to the positive side of the battery, as shown in Figure 4.2.13. (Sometimes the terminals are colour-coded. The '+' terminal will be red, and the '–' terminal black.)

The correct position for a voltmeter in a circuit

Voltmeter an instrument used to measure potential difference in a circuit.

A voltmeter is connected in parallel with whatever device it is measuring. A parallel connection is used because objects in parallel experience the same potential difference.

Consider the circuit of Figure 4.2.14, which shows two resistors in series in a circuit.

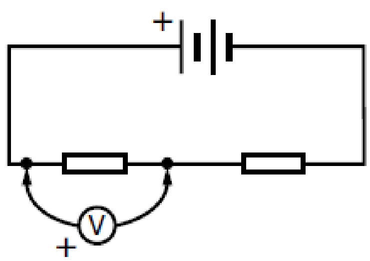

Figure 4.2.14: *A voltmeter connected in a circuit.*

Unlike an ammeter, the voltmeter is not part of the circuit. If you wish to measure a voltage drop, you place the voltmeter on the bench nearby. One lead comes from the voltmeter to see what conditions are like at one point in the circuit, while the other lead goes to a second point in the circuit to find out what things are like there: the voltmeter then indicates the difference between the two points. We call it the **potential difference** (p.d.) between the ends of the resistor, and measure it in volts. Sometimes we speak of it as the voltage drop (or just the voltage) across the resistor.

The voltmeter shows the difference in something – but the difference in what? It might help you to think of a comparison with heat: a temperature difference makes thermal energy flow, and a potential difference makes charge flow.

Ideally a voltmeter should draw no current from the circuit it is investigating. It must therefore have a very high resistance.

Summary

- In a series circuit, the current is the same all the way round.
- Ohm's law can be summarized using the equation V = IR. This can be used to analyze circuits and solve circuit problems involving potential difference, current and resistance.
- You calculate current using $I = \frac{V}{R}$.
- You can measure the value of a resistor with an ammeter and voltmeter by applying Ohm's law.
- The resistance of a metal increases as its temperature rises. Its temperature coefficient of resistance (α) provides a numerical measurement of this.
- You can work out the resistance of a wire from $= \frac{\rho l}{A}$, where ρ is the resistivity of the material of the wire.
- There are several factors that affect the resistance of a conductor;
    - material, eg copper, has lower resistance than steel
    - length - longer wires have greater resistance
    - thickness - smaller diameter wires have greater resistance
    - temperature - heating a wire increases its resistance
- **Ammeter** is an instrument that measures the size of a current.
- **Voltmeter** an instrument used to measure potential difference in a circuit.

Review questions

1. A battery of e.m.f. 6 V is connected to a circuit with a total resistance of 12 Ω. What current would you expect to flow?
2. A flashlight bulb has '2.4 V 0.3 A' marked on it. This means 'If you connect it to a 2.4 V battery a current of 0.3 A will be driven through it; this will make it white-hot'.

   a) Estimate the resistance of that flashlight bulb when it is lit up.

   b) If the bulb was connected to the mains (either 110 V or 220V) instead, would the current initially be 0.3 A, more than 0.3 A or less than 0.3 A? Give your reason.

c) What would happen next? Explain.

3. The resistivity of iron is 1.0×10^{-7} Ω m. Find the resistance of a 12 km length of a railway line with a cross-sectional area of 200 cm^2.
4. A tungsten resistor ($\alpha = 5.8 \times 10^{-3}$ K^{-1}) with a value of 10.0 Ω at 0 °C is warmed from that temperature up to 150 °C. Work out its resistance when hot.
5. Figure 4.2.15 shows a circuit with a cell, a variable resistor and a lamp.

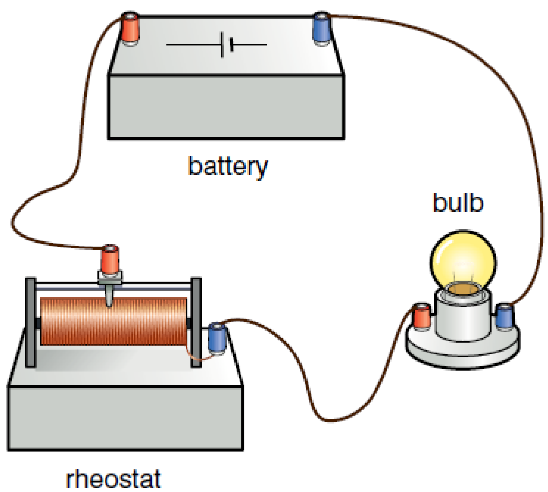

Figure 4.2.15

   a) Draw a circuit diagram of this arrangement.
   b) Redraw the circuit diagram adding an ammeter to measure the current in the circuit, and a voltmeter to measure the p.d. across the lamp.
   c) Describe how you would use this circuit to measure the current through the lamp for various p.d.s across it and describe the shape of a graph of p.d. against current that would be obtained.
   d) What would you change in this circuit to demonstrate Ohm's law?
6. A student designs the circuit shown in Figure 4.2.16 in order to try to test Ohm's law.

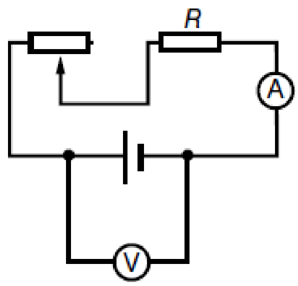

Figure 4.2.16

a) What is wrong with this circuit?

b) Draw the circuit diagram the student should have used.

7. What is the difference between a cell and a battery?
8. A carbon resistor at room temperature (22 °C) is attached to a 6.0V battery and the current measured through the resistor is 2.0 mA.

   a) What is the resistance of the resistor measured in ohms?

   b) If the temperature of the resistor is increased to 60 °C by heating the resistor, what is the current through the resistor?
9. A resistor R is connected between the terminals of a 6 V battery. A charge of 1 C passes through the resistor in 100 s. What is the resistance of the resistor?
10. What is the resistance of a copper cable that has a cross sectional area of 1 cm^2 and a length of 500m? The resistivity of copper is 2×10^{-8} m.
11. Wire B has twice the length and twice the radius of wire A. Both wires are made from the same material. If wire A has a resistance R, what is the resistance of wire B?
12. A copper wire of cross-sectional area 7.4×10^{-7} m^2 carries a current of 1 A. Copper contains 8.4×10^{28} free electrons/m^3. Find the electron drift speed.
13. A nichrome wire is 1 m long and 1×10^{-6} m^2 in cross-sectional area. When connected to a potential difference of 2 V, a current of 4 A exists in the wire. Determine the resistivity of this nichrome.
14. Constantan has a resistivity of 47×10^{-8} Ω m. How much length of this wire is needed to make a 10 Ω resistance if the diameter is 0.5 mm?

4.3 Combinations of resistors

By the end of this section you should be able to:

- Identify combinations of resistors in series, parallel and series–parallel connection.
- Derive an expression for the effective resistance of resistors connected in series.
- Derive an expression for the effective resistance of resistors connected in parallel.
- Calculate the effective resistance of resistors connected in series.
- Calculate the effective resistance of resistors connected in parallel.
- Calculate the current through each resistor in simple series, parallel and series–parallel combinations.
- Calculate the voltage drop across each resistor in simple series, parallel and series–parallel connections.

Resistors in different combinations

We shall now look at **resistors** connected in electric circuits in different ways − in series and in parallel. In both cases, it might help to think of an electrical circuit as a large crowd of people trying to get into a stadium.

In Current and Resistance, we described the term ‘resistance’ and explained the basic design of a resistor. Basically, a resistor limits the flow of charge in a circuit and is an ohmic device where V = IR. Most circuits have more than one resistor. If several resistors are connected together and connected to a battery, the current supplied by the battery depends on the equivalent resistance of the circuit.

The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections (Figure 4.3.1). In a series circuit, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor. In a parallel circuit, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor. The sum of the individual currents equals the current that flows into the parallel connections.

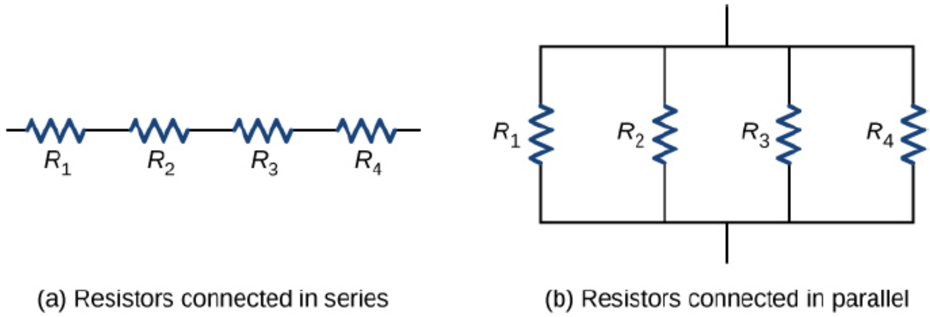

Figure 4.3.1: (a) For a series connection of resistors, the current is the same in each resistor.
(b) For a parallel connection of resistors, the voltage is the same across each resistor.

Combination of resistors in series

Resistors are said to be in series whenever the current flows through the resistors sequentially. Consider **Figure 4.3.2**, which shows three resistors in series with an applied voltage equal to $\mathbf{V_{ab}}$. Since there is only one path for the charges to flow through, the current is the same through each resistor. The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.

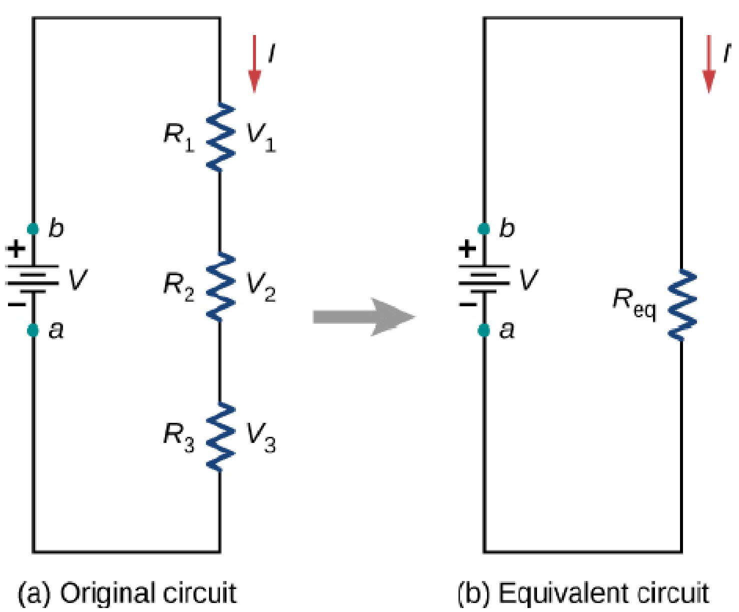

Figure 4.3.2: (a) Three resistors connected in series to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

In **Figure 4.3.2**, the current coming from the voltage source flows through each resistor, so the current through each resistor is the same. The current through the circuit depends on the voltage supplied by the voltage source and the resistance of the resistors. For each resistor, a potential drop occurs that is equal to the loss of electric potential energy as a current travels through each resistor. According to Ohm's law, the potential drop *V* across a resistor when a current flows through it is calculated using the equation *V* = *IR*, where *I* is the current in amps (A) and *R* is the resistance in ohms (Ω). Since energy is conserved, and the voltage is equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be the same.

$V = V_1 + V_2 + V_3$

$IR_{eq} = IR_1 + IR_2 + IR_3$

$R_{eq} = R_1 + R_2 + R_3$

Worked Example 4.12

Consider the circuit shown in Figure 4.3.3.

a) Calculate the total resistance of the whole circuit.

b) Calculate the current in the circuit.

Since the circuit is a series one, the same current must flow through all three resistors.

Consider the 3.5 Ω resistor.

c) Calculate the voltage drop necessary to send 2 A through 3.5 Ω.

d) Calculate the p.d. across the other two resistors.

Solution

You will need to use the relationship $R = \frac{V}{I}$ in the form of V = IR and $I = \frac{V}{R}$.

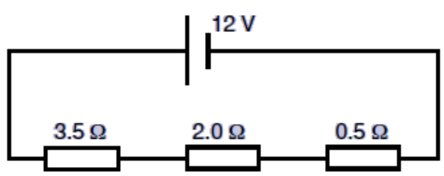

Figure 4.3.3

a) Total resistance in the circuit (series circuit) = 3.5 + 2 + 0.5 = 6 Ω

b) Voltage supplied to the circuit is 12 V

Resistance in the circuit is 6 Ω

From Ohm's Law, $I = \frac{V}{R} = \frac{12}{6} = 2A$

c) Resistance = 3.5 Ω, Current = 2 A

$V = IR$

$V = 2 \times 3.5 = 7$ V

The voltage drop across the 3.5 Ω resistor is 7 V.

d) For the 2 Ω resistor: resistance = 2 Ω, current = 2 A; $V = IR = 2 \times 2 = 4$ V

For the 0.5 Ω resistor: resistance = 0.5 Ω, current = 2 A; $V = IR = 0.5 \times 2 = 1$ V

Worked Example 4.13

Calculate the size of resistor X in Figure 4.3.4.

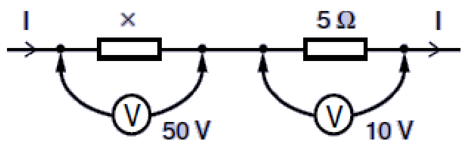

Figure 4.3.4

Solution

First look at the 5 Ω resistor. From $= \frac{V}{R}$, a p.d. of 10 V across it must mean that a current I is flowing through it, given by:

$$I = \frac{V}{R} = \frac{10}{5} = 2A$$

Now look at resistor 'X'. Since they are in series the same 2 A is flowing through it. To achieve this there is a p.d. of 50 V across it.

$$R = \frac{V}{I} = \frac{50}{2} = 25A$$

Combination of Resistors in parallel

Figure 4.3.5 shows resistors in parallel, wired to a voltage source. Resistors are in parallel when one end of all the resistors are connected by a continuous wire of negligible resistance and the other end of all the resistors are also connected to one another through a continuous wire of negligible resistance. The potential drop across each resistor is the same. Current through each resistor can be found using Ohm's law $I = V/R$, where the voltage is constant across each resistor.

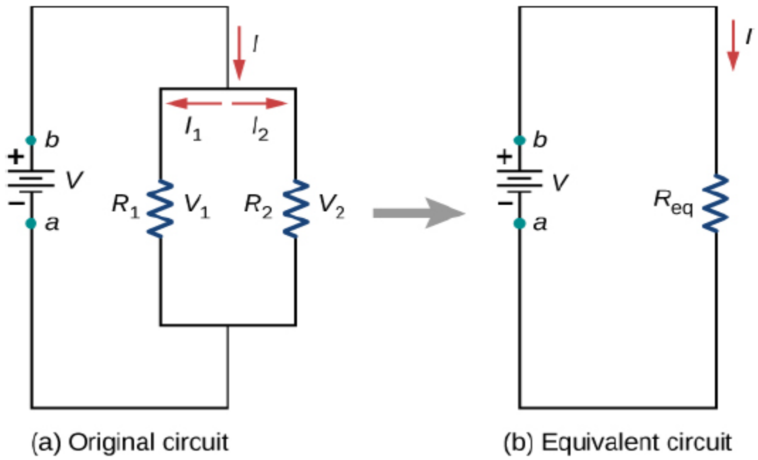

Figure 4.3.5 (a) two resistors connected in parallel to a voltage source. (b) The original circuit is reduced to an equivalent resistance and a voltage source.

The current flowing from the voltage source in Figure **4.3.5** depends on the voltage supplied by the voltage source and the equivalent resistance of the circuit. In this case, the current flows from the voltage source and enters a junction, or node, where the circuit splits flowing through resistors R_1 and R_2 . As the charges flow from the battery, some go through resistor R_1 and some flow through resistor R_2. The sum of the currents flowing into a junction must be equal to the sum of the currents flowing out of the junction:

$I = I_1 + I_2 \; and \; I = \frac{V}{R_{eq}}, I_1 = \frac{V_1}{R_1}, I_2 = \frac{V_2}{R_2}$

$$\frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

For parallel combination: $V = V_1 = V_2$

$$So \; \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

This relationship results in an equivalent resistance R_{eq} that is less than the smallest of the individual resistances. When resistors are connected in parallel, more current flows from the source than would flow for any of them individually so, the total resistance is lower.

Worked Example 4.14

A circuit consists of a 2 Ω resistor and a 3 Ω resistor in parallel. A 3 V battery is connected to the circuit. How big a current is drawn from it?

Solution

We must first calculate the effective resistance of the circuit using the formula for resistors in parallel.

Here,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

That gives us 'one over R', so to find R we must turn it over.

$$R = \frac{6}{5} = 1.2\Omega$$

The question now becomes 'What current will a 3 V battery send round a 1.2 Ω circuit?'

$$I = \frac{V}{R} = \frac{3}{1.2} = 2.5A$$

The voltage drop across resistors in parallel

Look at the circuit of Figure 4.3.6. Remembering that there is no voltage drop down a conducting lead, you should be able to see that the left-hand end of all three resistors is at +6 V and their right hand ends are at 0 V.

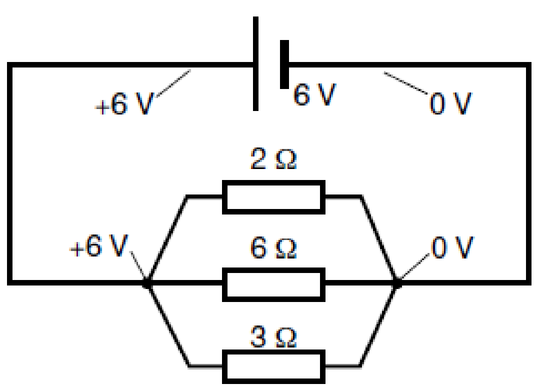

Figure 4.3.6

That illustrates what happens in this case:

If resistors are connected in parallel, they all have the same voltage drop across them.

This provides us with a means to work out the current in each branch of the circuit. Take the 2 Ω resistor as an example. The full 6 V of the battery is dropped across it, so the current I through it is given by:

$$I = \frac{V}{R} = \frac{6}{2} = 3A$$

Worked Example 4.15

For the circuit shown in Figure 4.3.7:

a) Calculate the voltage drop across each resistor

b) Calculate the current through each resistor

c) Calculate the effective resistance of the circuit.

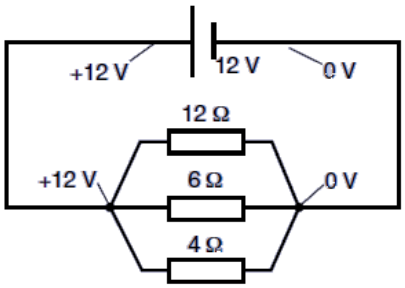

Figure 4.3.7

Solution

a) As the resistors are connected in parallel, the voltage drop will be the same across each, and will be equal to the full 12 V of the battery.

b) The voltage across the 12 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{12} = 1A$$

The voltage across the 6 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{6} = 2A$$

the voltage across the 4 Ω resistor is 12 V,

$$I = \frac{V}{R} = \frac{12}{4} = 3A$$

c) Total resistance of the circuit is R.

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4} = \frac{6}{12} = \frac{1}{2}$$

$$R = 2\Omega$$

Worked Example 4.16

Two resistors of 10 Ω and 15 Ω are connected. What is their combined resistance if they are connected?

a) in series

b) in parallel?

Solution

a) In series

$R = R_1 + R_2 = 10 + 15 = 25\ \Omega$

b) In parallel:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30}$$

$$R = \frac{30}{5} = 6\Omega$$

Activity 4.5: Series and parallel circuits

Do these activities in groups.

Connect one bulb in series with a battery and note its brightness. Now connect a second and third bulb in series with the bulb.

Is there any change in the brightness of the first bulb when the second and third bulbs are added?

Once all bulbs are lit, remove one of the bulbs. What happens to the brightness of the others?

Connect one bulb to a battery and note its brightness. Now connect a second and third bulb in parallel with this bulb.

Is there any change in brightness of the first bulb when the second or third ones are added?

Once all bulbs are lit, remove one. Is there any change in the brightness of the remaining bulbs?

Summary

- Resistors in series simply add.
- Resistors in parallel add by a '$\frac{I}{R}$' formula. Two resistors in parallel will conduct better than either one on its own.
- The resistance of the whole circuit includes the resistance within the battery itself, and this may not always be negligible.
- Where the circuit branches in a parallel circuit, the sum of the currents approaching the junction equals the sum of the currents leaving it.
- In a series circuit, the separate voltage drops (potential differences) across all the resistors add up to the voltage of the battery.
- In a parallel circuit each branch will have the same voltage drop across it.

Review questions

1. What should ammeters (a), (b) and (c) read in Figure 4.3.8?

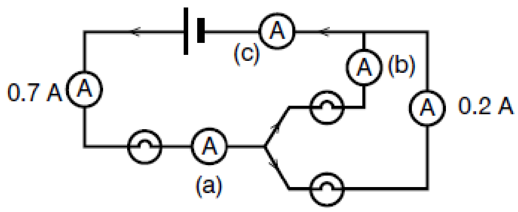

Figure 4.3.8

   Which bulb should be the brightest? Why?

2. Give the missing ammeter readings a and b in Figure 4.3.9. Suggest why more current flows through some bulbs than through others.

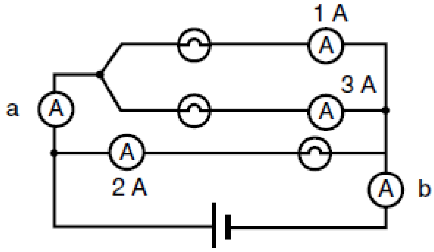

Figure 4.3.9

3. Give the missing ammeter readings c and d in Figure 4.3.10. Assume all the bulbs are identical ones. How would you expect the brightness of the different bulbs to compare?

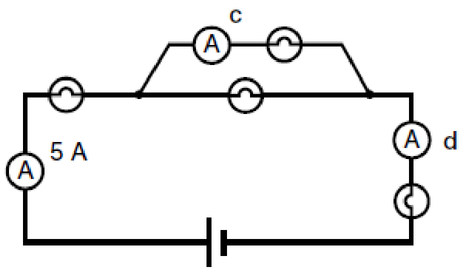

Figure 4.3.10

4. Draw a circuit diagram to show a battery of two cells with three light bulbs connected in parallel. Include two switches: one must turn one of the bulbs on and off, and the other must do the same for the other two bulbs together.
5. For the circuit diagram in Figure 4.3.11:

   a) What is the effective resistance of the two 4 Ω resistors in parallel?

   b) What is the total resistance of the circuit?

   c) Work out the current *I*.

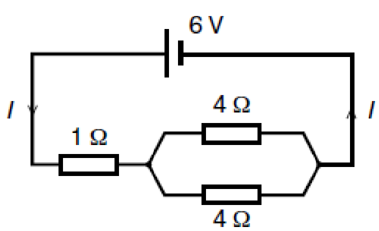

Figure 4.3.11

6. What single resistor could be placed in the box to replace the two shown in Figure 4.3.12?

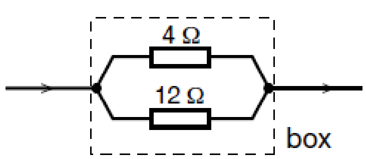

Figure 4.3.12

7. Calculate the effective resistance of a 25 Ω resistor and a 100 Ω resistor in parallel. What current will be drawn from a 50 V supply connected to the circuit?
8. a) Work out the total resistance of the circuit in Figure 4.3.13.

   b) What current will be drawn from the battery?

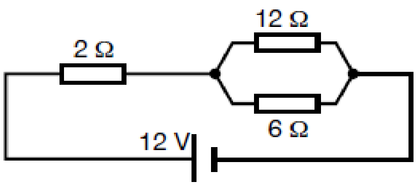

Figure 4.3.13

9. In the circuit of Figure 4.3.14 the battery has negligible internal resistance. Calculate:
   a) the total resistance in the circuit
   b) the current flowing in the circuit
   c) the potential difference across the 50 Ω resistor.

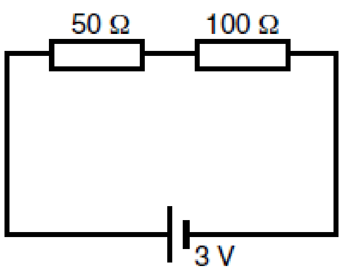

Figure 4.3.14

15. In the circuit shown in the Figure 4.3.15, the voltage across the 5Ω resistor is V_1 and the voltage across the 4Ω resistor is V_2. What is the ratio $\frac{V_1}{V_2}$?

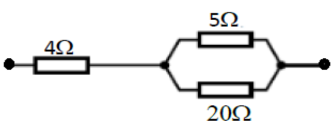

Figure 4.3.15

16. The circuit diagram shown below is a combination of resistors and a voltage source.

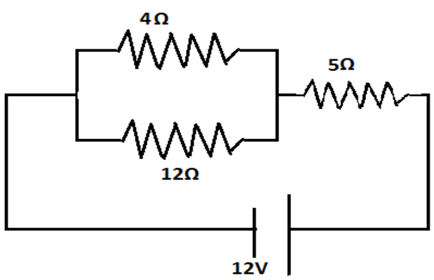

Figure 4.3.16

What is the voltage drop across the 5Ω resistor?

17. Two resistors connected in series have an equivalent resistance of 7Ω. When they are connected in parallel, their equivalent resistance is $\frac{10}{7}\Omega$, what is the resistance of each resistor?
18. The equivalent resistance of two resistors R and 3R when they are connected in series is 32Ω. What is the equivalent resistance of the two resistors when they are connected in parallel?
19. What is the current in the 5.0Ω resistor in the circuit shown in Figure 4.3.17?

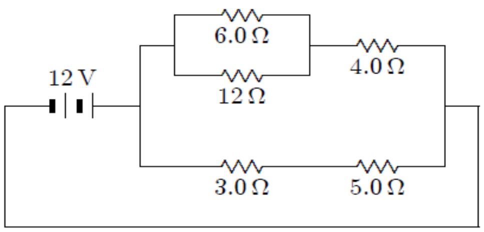

Figure 4.3.17

20. In the following circuit, find the current at points A, B, C and D.

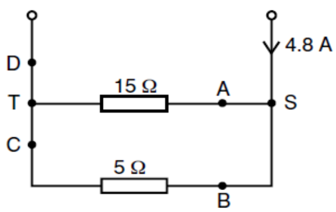

Figure 4.3.18

21. A total resistance of 3.0 **Ω** is to be produced by combining an unknown resistor R with 12 **Ω** resistor. What is the value of R and how is it to be connected to the 12 **Ω** resistor?
22. Each of the resistors in the diagram has a resistance of 12 **Ω**. Calculate the resistance of the entire circuit.

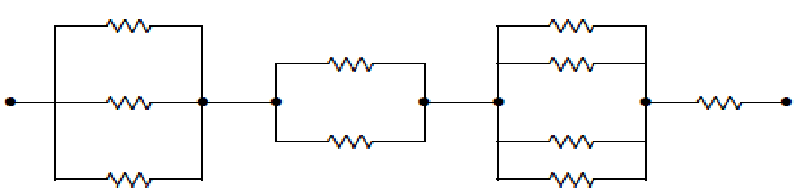

Figure 4.3.19

23. In the diagram, the current in the 3-Ω resistor is 4A. What is he potential difference between points 1 and 2?

Figure 4.3.20

24. Find the values of the ammeter and voltmeter readings in this circuit. Assume that the ammeter and cell have negligible internal resistance and that the voltmeter has an infinite resistance.

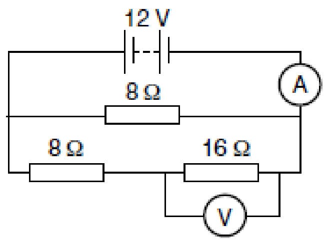

Figure 4.3.21

4.4 E.m.f. and internal resistance of a cell

By the end of this section you should be able to:

- Define the electromotive force (e.m.f.) of a cell.
- Distinguish between e.m.f. and terminal p.d. of a cell.
- Write the relationship between e.m.f., p.d., current and internal resistance in a circuit.
- Use the equation $V = E - Ir$ to solve problems in a circuit.
- Identify cell combinations in series and parallel.
- Compare the e.m.f. of combinations of cells in series and parallel.

The electromotive force of a cell

The e.m.f or electromotive force is **the energy supplied by a battery or a cell per coulomb (Q) of charge passing through it**. The magnitude of e.m.f is equal to V (potential difference) across the cell terminals when there is no current flowing through the circuit.

Electromotive force is a source of energy causing current to flow in an electrical circuit.
As we saw in Section 4.1, a cell pumps charge round a circuit. Its effectiveness at doing this is called its electromotive force (e.m.f.), in volts. A **voltmeter** is an instrument that can measure the e.m.f. of a cell: it has two leads going to it, and one is connected to each terminal of the cell.

Volts do not flow round a circuit. Volts cause coulombs of charge to flow at a rate of amperes. If we know the resistance R of a circuit, we can predict the current I that a battery will send round it, from $I = \frac{V}{R}$.

The resistance inside a cell

Internal Resistance is the resistance which is present within the battery that resists the current flow when connected to a circuit. Thus, it causes a voltage drop when current flows through it. It is the resistance provided by the electrolyte and electrodes which is present in a cell.

Suppose you short-circuit a cell. This means that you join its two terminals by a circuit that effectively has no resistance – a short piece of very thick copper wire, for instance. The cell has an e.m.f. *V*, but the circuit apparently has no resistance *R*. What happens then? Does the current increase without limit?

The point we are forgetting is that the cell has to pump the charge round the whole circuit, and that includes the part within the cell as well as the external circuit. Internal resistance varies between different cells, but it is what finally sets a limit to the current a cell can supply.

A 1.5 V torch cell (represented in Figure 4.4.1 by the dotted line) typically has an internal resistance of up to an ohm. This means that even if you short-circuit the cell, there is still an ohm of resistance. The biggest current it can deliver is given by $I = \frac{V}{R} = \frac{1.5}{1} = 1.5A$

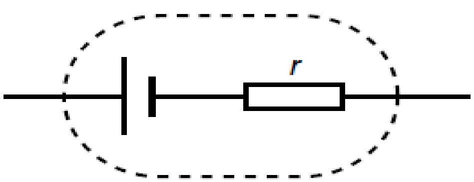

Figure 4.4.1: *Representation of a cell indicating its internal resistance r.*

So far, we have taken it for granted that if you double the e.m.f. in a circuit, you will double the current. This should be checked experimentally, but it needs some careful thought in order to give it a fair test.

The obvious thing to do is to add cells one at a time to a circuit, measure the current and see if it increases accordingly. A fair test requires that you change only one thing at a time, which means that you must not change the resistance of the circuit.

There are two problems here:

1. Adding another cell adds a bit more resistance to the circuit. All you can do about that is to choose cells that have a very low internal resistance (such as the lead–acid cells that make up a car battery) or to use a circuit for which the resistance is so high that another ohm or two makes virtually no difference.
2. The resistance of a metal wire changes as it heats up. This rule out circuits containing light bulbs, because as you add more batteries their temperature changes from less than red-hot to strongly white-hot: a change of perhaps 1000 degrees!

The difference between e.m.f. and terminal p.d. of a cell

A high resistance voltmeter connected across the terminals of a cell can give a good reading of the cell's e.m.f. However, if the cell is then connected in a circuit where the current is high, the reading on the voltmeter drops. This drop is caused by the cell's internal resistance.

Figure 4.4.2 shows the cell connected in a circuit.

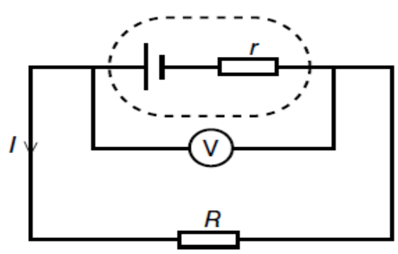

Figure 4.4.2

If the e.m.f. of this cell is E and it is connected to a resistor R, the potential difference across its terminals (V) will now be less than E because some of its energy is used to drive the current (I) through the internal resistance (r) in the cell. If R decreases, the current (I) increases and the terminal p.d. (V) of the cell will decrease.

The relationship between e.m.f., current and internal resistance in a circuit

The relationship between a cell's e.m.f. (E) and internal resistance (r) and the current (I) and resistance (R) in a circuit is given by the expression:

E = p.d. across R + p.d. across r

$E = IR + Ir$

$E = I(R + r)$

Worked example 4.17

In the circuit shown in Figure 4.4.3, the cell's e.m.f. is 10 V and its internal resistance is 2 Ω. Find the p.d. across the terminals of the cell when it is connected to a 3 Ω resistor.

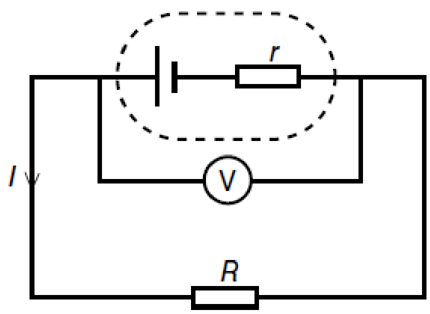

Figure 4.4.3

Solution

First find the current in the circuit:

$E = I(R + r)$

$$I = \frac{E}{R + r} = \frac{10}{3 + 2} = 2A$$

The p.d. across the cell terminals is equal to the p.d. across the resistor (3 Ω). Find the p.d. across the resistor:

$V = IR = 2 \times 3 = 6$ V

Therefore the p.d. across the terminals of the cell is 6 V.

Activity 4.6: Measuring electromotive force and terminal voltage of a cell

Set up a circuit consisting of a dry cell, a bulb, a switch and a voltmeter. Connect the voltmeter as shown in Figure 4.4.4.

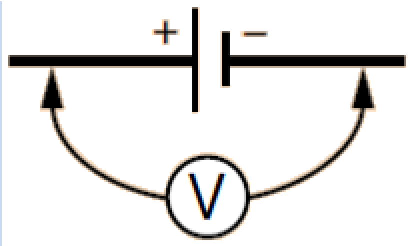

Figure 4.4.4

Take the reading in the voltmeter while the switch is on.

Then another reading while the switch is off. Compare the two readings.

Which one is larger? Which reading is the e.m.f.? Discuss with your group.

Combinations of identical cells in series and parallel

Combination of cells in series

When external resistance is negligible in comparison to the internal resistance, then the cells are connected in parallel to get maximum current.

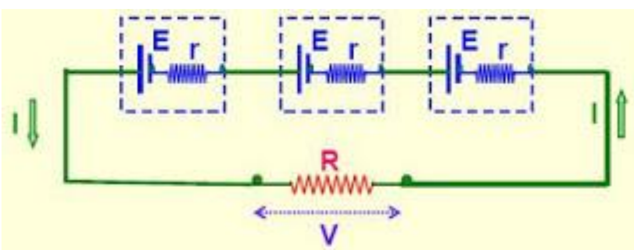

Figure 4.4.5

Cells are connected in series when they are joined end to end so that the same quantity of electricity must flow through each cell.

1. The emf of the battery is the sum of the individual emfs
2. The current in each cell is the same and is identical with the current in the entire arrangement.
3. The total internal resistance of the battery is the sum of the individual internal resistances.

Total emf of the battery = nE (for n no. of identical cells)

Total Internal resistance of the battery = nr

Total resistance of the circuit = nr + R

You know that E=I(R+r)

For the above three identical cells: 3E=I(R+3r)

$$so\ I = \frac{3E}{R + 3r}$$

For **n** identical cells:

$$I = \frac{nE}{R + nr}$$

When internal resistance is negligible in comparison to the external resistance, then the cells are connected in series to get maximum current.

Combination of identical cells in Parallel

Cells are said to be connected in parallel when they are joined positive to positive and negative to negative such that current is divided between the cells.

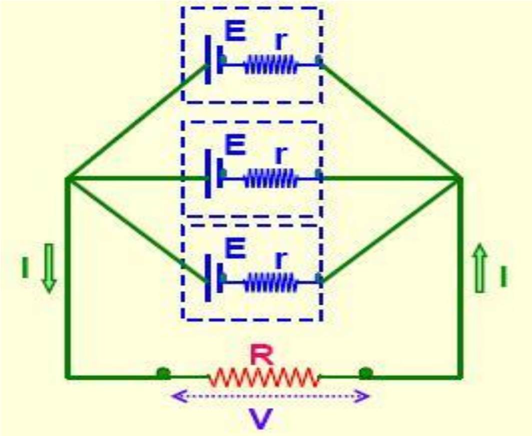

Figure 4.4.6

1. The emf of the battery is the same as that of a single cell.
2. The current in the external circuit is divided equally among the cells.
3. The reciprocal of the total internal resistance is the sum of the reciprocals of the individual internal resistances.

   Total emf of the battery = E

   Total Internal resistance of the battery = r / n

   Total resistance of the circuit = (r / n) + R

   $E=I(R+r_T)$

   $E = I(R + \frac{r}{n})$

   By rearranging the above formula:

$$nE = I(nR + r)$$

$$So\ for\ \boldsymbol{n}\ identical\ cells\colon I = \frac{nE}{nR + r}$$

In Figure 4.4.7, (a), (b) and (c) show cells connected in series, (d) shows cells connected in parallel. Each cell has an e.m.f. of 1.5 V.

The total e.m.f. of cells connected in series is the sum of the e.m.f. of each cell: $E = E_1 + E_2 + \ldots + E_n$. Equipment requiring high power uses cells arranged in series.
The e.m.f. of cells connected in parallel is the e.m.f. of an individual cell: $E = E_1 = E_2 = \ldots = E_n$. The current supplied by a parallel arrangement of cells can be maintained for far longer than that supplied by a single cell, so equipment requiring a steady current for long periods will use cells arranged in parallel.
(a) 1 cell total e.m.f. = 1.5 V
(b) 2 cells in series total e.m.f. = 1.5 + 1.5 = 3 V
(c) 3 cells in series total e.m.f. = 1.5 + 1.5 + 1.5 = 4.5 V
(d) 3 cells connected in parallel total e.m.f. = 1.5 V

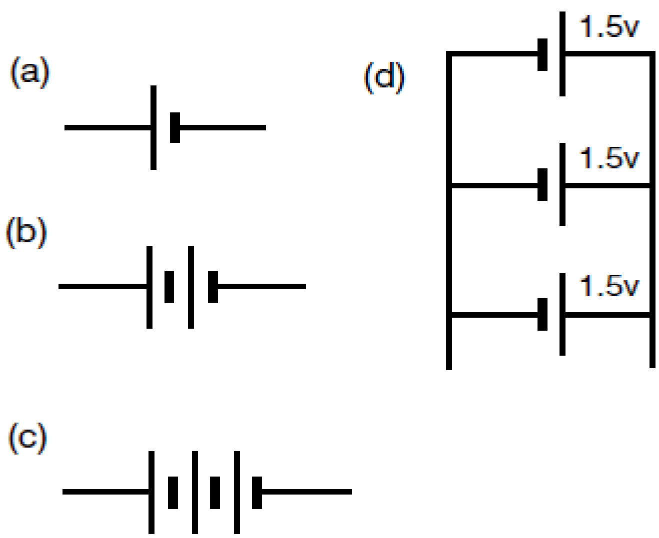

Figure 4.4.7: *Combining cells.*

Activity 4.7: Connecting cells together

For this activity you can use either the fruit cells made in Activity 3.3 or purchased 1.5 V cells. Connect a small light bulb in a circuit with one cell. Observe its brightness. Now take another cell and connect it in the circuit as shown in Figure 4.4.8a. Observe the brightness of the bulb. You can add further cells in this way if you wish.

The cells in Figure 4.4.8a are connected in series. Now connect two cells together in parallel, as shown in Figure 4.4.8b. Now connect these cells to a light bulb as shown in Figure 4.4.8c. Observe its brightness. How does the brightness of the bulb in the circuit with one cell compare with its brightness in the circuit with two cells in series?

How does the brightness of the bulb in the circuit with the cells in parallel compare to the brightness when the cells are in series?

You can see that the cells are connected negative to negative, positive to positive in Figure 4.4.8b. See what happens if you connect them negative to positive and positive to negative before connecting them in a circuit.

What do you find?

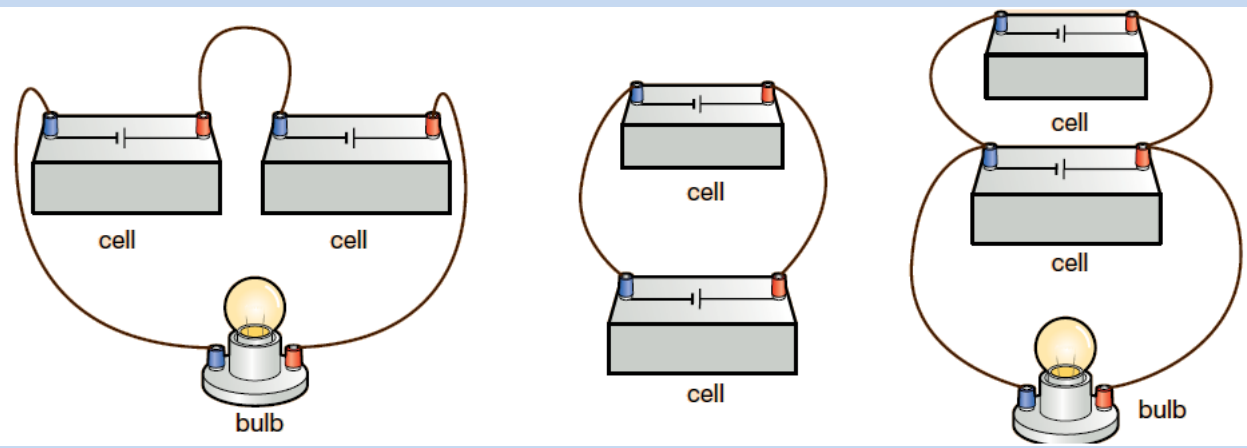

Figure 4.4.8a ***Figure 4.4.8b*** ***Figure 4.4.8c***

Summary

- **Electromotive force** is a source of energy causing current to flow in an electrical circuit.
- A cell pumps charge round a circuit. Volts do not flow round a circuit. Volts cause coulombs of charge to flow at a rate of amperes.
- E.m.f. (E), terminal p.d. (V), and internal resistance (r) are related by the equation:

$E = V + Ir$.

- The total e.m.f. of cells connected in series is the sum of the e.m.f. of each cell: $E = E_1 + E_2 + \ldots + E_n$. Equipment requiring high power uses cells arranged in series.
- The e.m.f. of cells connected in parallel is the e.m.f. of an individual cell: $E = E_1 = E_2 = \ldots = E_n$.

Review questions

1. Imagine you have a brother two years younger than you are. Explain to him why:
   a) the '+' terminal of one cell in a circuit should be connected to the '–' terminal of a second cell, but
   b) the '+' terminal of an ammeter needs to be connected to the '+' terminal of the battery.
2. A circuit consists of three resistors in series: one of 3 Ω, one of 7 Ω and one of 10 Ω. A battery of e.m.f. 12 V and negligible internal resistance is connected to the circuit. What size current will it supply?
3. A battery has an e.m.f. of 3 V and an internal resistance of 1 Ω. What current will it give if it is connected to a circuit of resistance?

   a) 2 Ω b) 4 Ω?

   What current will it give if its terminals are short-circuited?

4. In Figure 4.4.9, A and B represent the terminals of a battery of e.m.f. 4 V and internal resistance 0.5 Ω. R is the total resistance of the circuit to which it is connected.
   a) Explain why a voltmeter connected as shown across R would read the same as a voltmeter connected across the terminals of the battery.
   b) Calculate the current which flows round the circuit if the resistance of R is 1.5 Ω.
   c) What then is the p.d. between the terminals of the battery?

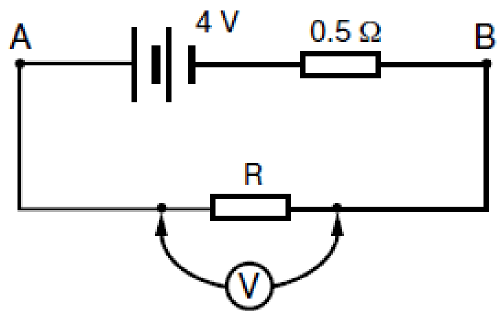

Figure 4.4.9

5. a) What current is flowing through the 3 Ω resistor in Figure 4.4.10?

   b) If its internal resistance is 2 Ω, what is the e.m.f. of the battery?

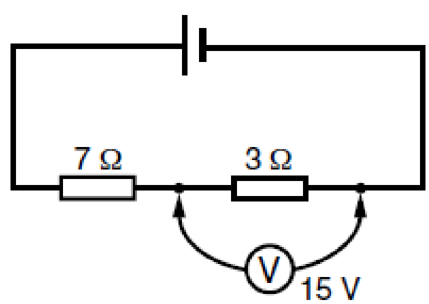

Figure 4.4.10

6. If a battery with ***e.m.f*** of 12V is connected to a 3Ω resistor and as a result, a current of 2A exists in the resistor, what is the internal resistance of the battery?
7. A cell supplies a current of 1A through a 2Ω resistor and a current of 0.5A through a 6Ω resistor. If a cell has internal resistance r, what is the e.m.f of the cell?
8. If a cell supplies a current of 0. 9 A when connected to a 2Ω resistor and a current of 0.3A when connected to a 7Ω resistor, then what will be the internal resistance of the cell?

4.5 Electric energy and power

By the end of this section you should be able to:

- Define electrical energy and power in an electrical circuit.
- $Use\ P = VI = \frac{V^2}{R} = I^2R$ to solve problems in electric circuits.

- $Use\ W = VIt = I^2Rt = \frac{V^2t}{R}$ to calculate electric energy dissipated in an electric circuit.
- Calculate the cost of electrical energy expressed in KWh.

Definitions for electrical energy and power

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as:

Electrical energy = qV, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through).

Power is the rate at which energy is moved, and so electric power is:

$$Power = \frac{electrical\ energy}{time} = \frac{qV}{t}$$

Recognizing that current $\mathrm{I} = \frac{q}{t}$, the expression for power becomes:

$$P = \frac{qV}{t} = \frac{ItV}{t}$$

$$P = IV$$

Three expressions for electric power are listed together here for convenience:

$$P = IV$$

$$P = \frac{V^2}{R}$$

$$P = I^2R$$

The word 'power' here means the rate at which energy is being supplied or converted. Power is measured in the units of joules per second (J/s), or watts (W).

The electrical energy produced by a current of I amperes flowing through a p.d. of V volts for a time t seconds is given by:

Energy = VIt J.

Power is the energy produced by an electrical appliance in one second. Thus, if $t = 1$:

Power = $VI \times 1 = VI$ W.

Worked example 4.18

A 60 W light bulb is switched on for 2 minutes. How many joules of electrical energy does it convert into heat and light in that time?

The power of the light bulb is 60 W; this means that it uses 60 J of energy every second.

It is switched on for 2 minutes = 120 s.

Energy = VIt (and Power =VI)

Total energy supplied = 60 J/s × 120 s = 7200 J

Worked example 4.19

If the potential difference across a working electrical motor is

50 V and the current is 2 A, calculate the power of the motor.

Power = $VI = 50 \times 2 = 100$ W

Consider a resistor that converts all the electrical energy supplied to heat.

The power produced = VI

This can be rewritten (using Ohm's law, $V = IR$) as $IR \times I = I^2R$

Therefore, for a resistor heat produced per second = power = I^2R.

If V and R are known, then the equation can be written (again using Ohm's law)

$$Power = VI = V \times \frac{V}{R} = \frac{V^2}{R}$$

Worked example 4.20

Two heating coils A and B produce heat at a rate of 1 kW and 2 kW, respectively, when connected to 250 V mains.

a) Calculate the resistance of each resistor.

b) Find the power they would produce when connected in series to the mains.

Solution

a) For the first resistor:

Power = VI

$1000 = 250 \times I, so\ I = \frac{1000}{250} = 4A$

Therefore: $R = \frac{V}{I} = \frac{250}{4} = 62.5\Omega$

For the second resistor: Power = *VI*

$2000 = 250 \times I, so\ I = \frac{2000}{250} = 8A$

Therefore: $R = \frac{V}{I} = \frac{250}{8} = 31.25\Omega$

b) If the resistors are wired in series, their total resistance is

$R_1 + R_2 = 62.5 + 31.25 = 93.75\ \Omega$.

Using $R = \frac{V}{I}$

$93.75 = \frac{250}{I}, giving\ I = \frac{250}{93.75} = 2.67A$

So power = *VI* = 250 × 2.67 = 666 W

Notice that this is less than the power produced when either resistor is connected to the mains separately. Can you see why?

Worked example 4.21

Calculate the power of a water heater that draws a current of 10 A from a 220 V supply.

Power = *VI* = 10 × 220 = 2.2 kW

Cost of electrical energy

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. For example, the more light bulbs burning, the greater ***P*** used; the longer they are on, the greater ***t*** is.

The energy unit on electric bills is the kilowatt-hour (kW · h) , consistent with the relationship

$E = Pt$. It is easy to estimate the cost of operating electrical appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted into joules.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This not only reduces the cost but also results in a reduced impact on the environment.

To calculate the cost of electrical energy we use the following equation:

The total cost $=$ *number of units (in kWh)* $\times$ *cost of each unit*

Electricity is distributed to homes and businesses and the quantity supplied is measured in kilowatt hours (KWh). A kilowatt hour is the energy used by a 1 kW appliance working for 1 hour.

Consumers of electricity are charged for each kWh used.

How many joules are there in 1 kWh?

One kWh = energy transformed by 1 kW (1000 W) for 1 hour (3600 s) = energy when 1000 J are transformed each second for 3600 s = 1000 × 3600 = 3 600 000 J = 3.6 MJ

Worked example 4.22

How many units (kWh) are used by:

a) a 3 kW electric fire used for 2 hours,

b) a 100 W light bulb used for 15 hours?

a) Number of units = 3 × 2 = 6 kWh

b) Number of units = 0.1 × 15 = 1.5 kWh

Worked example 4.23

A 1.5 kW electric fire is accidentally left on overnight for eight hours. The cost of a unit of electrical energy is 0.273 Birrs.

How much money has been wasted?

The number of units (kWh) = power in kW × time in hours = 1.5 × 8 = 12 kWh

The total cost = 12 × cost of each unit = 12 × 0.273 = 3.28 Birrs

Worked example 4.24

A current of 4 A flows through an electric fire for 1 hour. The supply voltage is 240 V. What energy is transformed by the fire in 1 hour? (1 hour = 60 × 60 seconds)

$W = VIt = 240 \times 4 \times 3600 = 3.456$ megajoules (MJ)

Worked example 4.25

The Ethiopian Power Corporation is distributing power saving lamps to the public free of cost. If an 11 W (0.011 kW) power saving bulb is used in place of the equivalent 60 W (0.06 kW) conventional lamp, and each type of bulb is used for 10 hours a day for four weeks (seven days a week), how much would the customer of the Ethiopian Power Corporation save?

Number of hours = 10 × 7 × 4 = 280

Cost of one unit of electricity is 0.273 Birrs

Conventional lamp

Number of units (kWh) = power in kW × time in hours = 0.06 ×

280 = 16.8 kWh

Total cost of using conventional bulb = 16.8 × 0.273 = 4.59 Birrs

Power-saving lamp

Number of units = 0.011 × 280 = 3.08 kWh

Total cost of using power-saving lamp = 3.08 × 0.273 = 0.84 Birrs

The customer would save 3.75 Birrs.

Summary

- Electric energy depends on both the voltage involved and the charge moved. This is expressed as: Electrical energy = qV.
- Power is the rate at which energy is moved. Electric power = IV
- The power dissipated in an electrical component is $P = IV = I^2 R$.
- The energy unit on electric bills is the kilowatt-hour (kW · h).
- To calculate the cost of electrical energy we use the following equation:

  The total cost = number of units (in kWh) × cost of each unit

Review questions

1. Work out the sources of energy used at home. Can you suggest any economies or improvements?
2. Carry out the same analysis for your school.
3. In the circuit of Figure 4.5.1 (i) calculate:

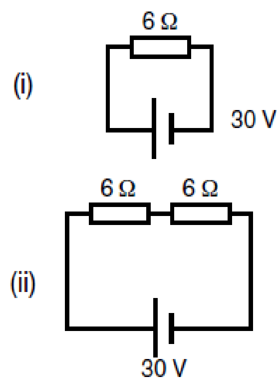

Figure 4.5.1

a) the current flowing
b) the number of joules per second (power) of heat produced in the 6 resistor. Another 6 Ω resistor is added in series with the first (Figure 3.51(ii)). Calculate:
c) the current flowing now
d) the power produced in the two 6 Ω resistors together
e) the power produced in a single 6 Ω resistor. Comment on the reasons for the difference in the heat produced in a 6 Ω resistor in the two circuits.

4. The two ends of a 6Ω resistor are connected to a 12V battery. What is the total power delivered by the battery to the circuit?
5. A stove that draws 10A current from 220V line is used for 5 hours. If the electrical energy company demands 0.35Birr per kilowatt hour (kWh), what is the cost of the electrical energy consumed?
6. A current of 0.6A is passed through a lamp for 4 minutes using a 12V power supply. Calculate the energy dissipated by this lamp in 4 minutes.
7. An ordinary light bulb is marked “60W, 120V”. What is its resistance?
8. A student kept her 60watt, 120volt study lamp turned on from 2:00 PM until 2:00 AM. How many coulombs of charge went through it?

9. A certain resistor dissipates 0.5W power when connected to a 3 V potential difference. When connected to a 1 V potential difference, what amount of power this resistor will dissipate?

4.6 Electric installation and safety rules

By the end of this section you should be able to:

- Understand the dangers of mains electricity.
- Have some awareness of safety features incorporated in mains electrical installations.
- Understand the nature of the generation and supply of electricity in Ethiopia.
- Consider employment prospects in Ethiopia's electricity industry.

In Ethiopia, while traditional sources of energy, such as wood, are still of great importance, the provision of mains electricity is becoming increasingly significant. Mains electricity is generated, transmitted and distributed by the Ethiopian Electric Power Corporation (EEPCO).

While Ethiopia uses some fossil fuels for electricity generation, the country is fortunate in having access to sources of hydroelectric power at Melka Wakena, Finchaa, Koka, Awash, Tis Abay, Gilgel Gibe and other planned sites, for electricity generation. Ethiopia also has potential for using other renewable energy sources, such as solar, wind and geothermal energy, for generating electricity.

Work is underway to increase the availability of mains electricity in Ethiopia. Mains electricity is much more powerful than the electricity we have studied in this unit, and great care must be taken when using it. Any work being undertaken on mains electrical installations must only be undertaken by those who have qualified through the Electrical Professional Competence Certification Scheme (operated by the Ethiopian Electricity Agency).

Various safety features are incorporated into mains electrical installations; one of these features is the **fuse**, which uses the heating effect studied earlier in this unit. A fuse is a small, thin piece of wire in a glass tube that has metal connectors on each end.

It is designed to be used in a circuit to allow a current of a certain size to pass through it. If the current in the circuit increases beyond this point, the fuse heats up and then breaks, causing the current in the circuit to stop. The fuse is a very useful safety device, protecting appliances from surges of current.

Electrical safety is also increased if appliances are **earthed**. This means that if there were a fault in an appliance, any dangerous large current would run harmlessly to the earth and cause the fuse to blow, thus preventing injury. **Earthed** circuit connected to the earth, allowing any dangerously large current to be safely discharged.

Miniature circuit breakers (MCBs) can now be used in place of fuses in some appliances. They cut off the power if there is a fault that causes the appliance to overheat. They can be re-set when the appliance cools down.

Earth circuit leakage breakers (ECLBs) are used in place of fuses in domestic electricity supply boards, where they are useful in cutting off the supply very quickly if a fault occurs.
Small, plug-sized ECLBs (also called residual current circuit breakers) are also useful in preventing injury when using electrical equipment outdoors. They will cut off the power supply very quickly if the power cable is cut accidentally.

Calculating the Fuse rating

Using the guide that the fuse is rating at 125% of the normal operating current, then it can be calculated as follow.

Power=Current x voltage

Current=$\frac{\text{Power}}{\text{Voltage}}$ or I = $\frac{P}{V}$

Fuse rating = $\frac{P}{V}$ x125%

Finally use the next highest fuse rating after the calculation.

For example, a) If you want to calculate the fuse rating for 500W appliance in Ethiopia, then

Fuse rating =$\frac{P}{V}$ x125% (In Ethiopia, the main supply provided to our home is 110v or 220v)

= $\frac{500\text{W}}{220\text{v}}$ x125% =2.841A

Then the next highest fuse rating after calculation is 3A.

b) If you want to calculate the fuse rating for 1000W appliance in Ethiopia, then

Fuse rating =$\frac{P}{V}$ x125%

$= \frac{1000\text{W}}{220\text{v}}$ x125% =5.682A

Then the next highest fuse rating after calculation is 6A.

Some safety rules while working with electrical instruments and tools

- ✓ Avoid water at all times when working with electricity. Never touch or try repairing any electrical equipment or circuits with wet hands. It increases the conductivity of the electric current.
- ✓ Disconnect the power source before servicing or repairing electrical equipment.
- ✓ Use only tools and equipment with non-conducting handles when working on electrical devices.
- ✓ Never use metallic pencils or rulers, or wear rings or metal watchbands when working with electrical equipment.
- ✓ When it is necessary to handle equipment that is plugged in, be sure hands are dry and, when possible, wear nonconductive gloves, protective clothes and shoes with insulated soles.
- ✓ If an individual comes in contact with a live electrical conductor, do not touch the equipment, cord or person. Disconnect the power source from the circuit breaker or pull out the plug using a leather belt.
- ✓ If water or a chemical is spilled onto equipment, shut off power at the main switch or circuit breaker and unplug the equipment.
- ✓ Never handle electrical equipment when hands, feet, or body are wet or perspiring, or when standing on a wet floor

Activity 4.8: Electrical safety

Take five 1.5 V cells, a 2 A fuse and a selection of connecting wires. Connect the circuit shown in Figure 4.6.1.

Look carefully at the fuse, and while observing the fuse closely, close the switch. The fuse consists of a very fine length of wire inside a transparent tube with metal connectors at each end.

When the switch is closed, the wire in the tube quickly becomes red and then separates into two pieces. The wire becomes dark again as it cools down. At the end of the experiment, the wire in the fuse has a gap in the centre.

There might be a small lump of metal on one of the broken ends of wire. This is a result of the metal becoming very hot and melting as the current passes through it, before the fuse wire breaks and causes the current to stop.

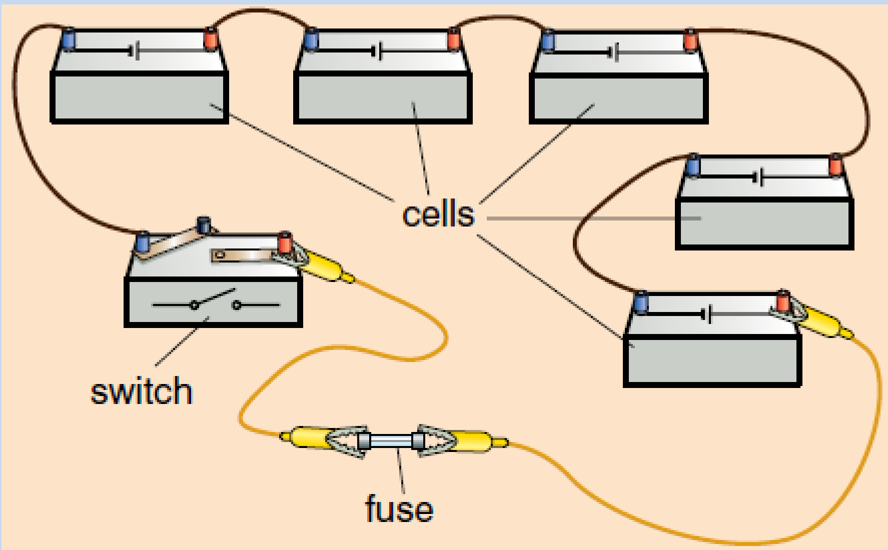

Figure 4.6.1

Review questions

1. For safety, the circuit for a 3 kW metal-bodied electric kettle is both earthed and contains a fuse.
   a) Why is the kettle earthed?
   b) Why does the circuit contain a fuse?
2. In a house you might find i) MCBs, ii) an ECLB.
   a) Say where each of these may be found.
   b) Explain the purpose that each might be serving.
3. List some safety rules while working with electrical instruments and tools.

End of unit questions

1. If 36 C of charge passes through a wire in 4 s, what current is it carrying?
2. An appliance has a resistance of 5 Ω and requires a current of 0.5 A for it to work. The only battery available has an e.m.f. of 12 V and negligible internal resistance. What extra resistance will you need to provide to limit the current to its correct value? Draw the circuit you would assemble.
3. A student incorrectly set up the circuit shown in Figure 4.1

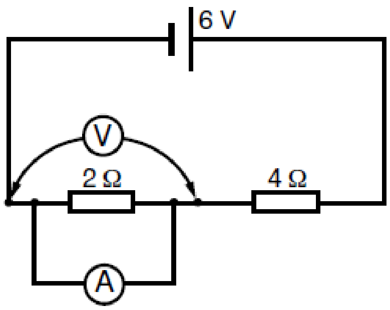

Figure 4.1

   a) What is wrong with it?
   b) Assume that both the ammeter and the battery have negligible resistance. Starting from one terminal of the battery, what route will all the current take as it flows round the circuit? How much resistance will this circuit have? What will the ammeter read?
   c) What will the voltmeter read?
   d) The student then reconnects the circuit so that the two resistors, the ammeter and the voltmeter are all in series with the battery. What would you expect the ammeter to read now? Give a reason for your answer.
4. A 12 V light bulb is put under test, with the following results:

Current through bulb (A)	0.4	0.6	0.8	1.0	1.2	1.4	1.6
p.d. across bulb (V)	1.4	2.6	3.9	5.5	7.4	9.7	12.6

   a) Draw the circuit you would use to obtain these readings.
   b) Copy the table out, and for each set of readings work out the resistance *R* of the bulb.
   c) Plot a graph of *R* against *I*. Why is it not valid to assume the graph goes through the origin?

d) Use your graph to estimate the resistance of the bulb at room temperature. (Hint: in what circumstance will the bulb not be heated at all?)

5. Constantan is the name of an alloy that is sometimes used for making resistors in the laboratory. Its resistivity is 4.9×10^{-7} Ω m. Calculate the resistance of a 3 m long constantan wire with 1 mm2 cross-sectional area.
6. Draw a labelled diagram of a dry cell. State the function of each of its parts.
7. The resistivity of copper is 2×10^{-8} Ω m. Work out the resistance of a copper wire of 1 mm^2 cross-sectional area and 3 m long.
8. A battery sends a current of 3 A through a 4 Ω resistor.
   a) What must the e.m.f. of the battery be?
   b) How many coulombs of charge will pass through the resistor in 1 minute?
9. Explain why it is wrong to write the units of resistivity as ohm/metre (Ω/m) with the stroke between the two quantities.
10. Draw a circuit diagram to show a battery of two cells and three light bulbs connected in series. Include a switch to turn the lights on and off. Does it matter where the switch is placed in the circuit?
11. a) What current is flowing round the circuit in Figure 4.2?

    b) How great must the p.d. be across resistor R?

    c) What is the resistance of resistor R?

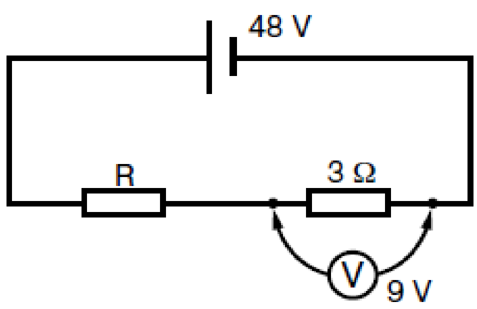

Figure 4.2

12. What should be the reading of ammeters A_1 and A_2 in Figure 4.3?

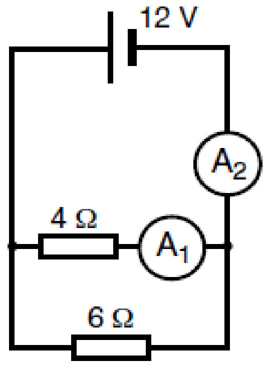

Figure 4.3

13. These questions refer to the circuit drawn in Figure 4.4.

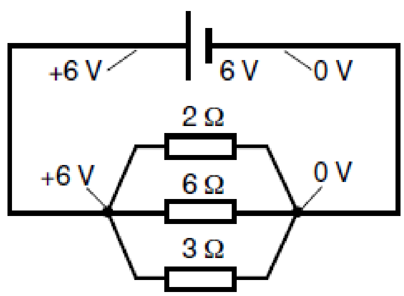

Figure 4.4

a) What are the currents through the 6 Ω branch, through the 3 Ω branch and through the 2 Ω branch?

b) What is the total current drawn from the battery?

c) As far as the battery is concerned, what is the total resistance of its whole circuit?

d) Compare this with the value obtained from

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

14. In Figure 4.5, A and B represent the terminals of a battery of e.m.f. 4 V and internal resistance 0.5 Ω. *R* is the total resistance of the circuit to which it is connected. Calculate the current that flows round the circuit and the p.d. between the terminals of the battery if the resistance *R* is: a) 7.5 Ω b) 0 Ω.

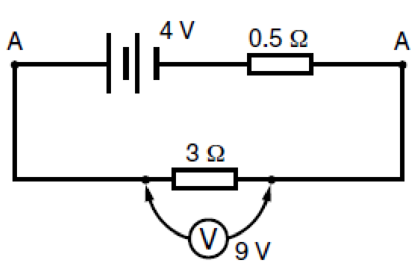

Figure 4.5

15. Figure 4.6 shows a series circuit with an internal resistance, *r*.

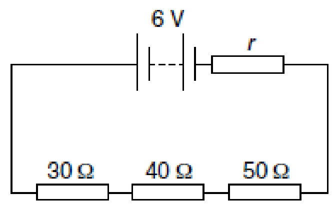

Figure 4.6

The battery has an e.m.f. of 6 V and an internal resistance, *r*, of 1.5 Ω. Calculate:

a) the current it supplies to the external resistors
b) the p.d. across each resistor.

16. Figure 4.7 shows a 12 V battery of negligible internal resistance connected to three resistors and an ammeter.

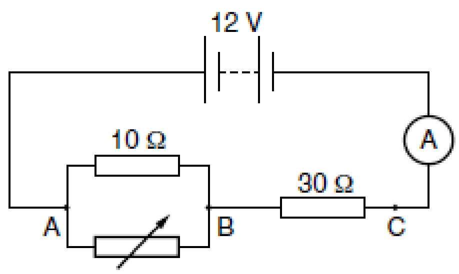

Figure 4.7

a) The variable resistor is set to it maximum value of 15 Ω. Calculate the resistance between points i) A and B; ii) A and C.
b) Calculate the maximum and minimum readings on the ammeter when the variable resistor is set to 0 Ω and 15 Ω.

17. Calculate the total resistance of the network of resistors shown in Figure 4.8.

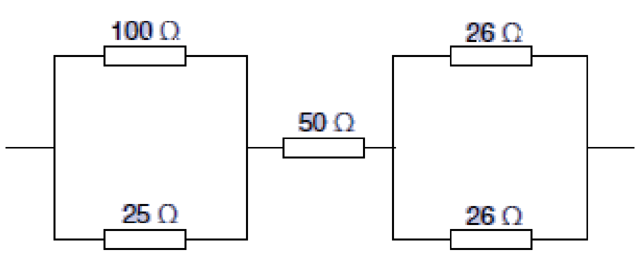

Figure 4.8

18. The diagram shows a series circuit with an internal resistance, *r*.

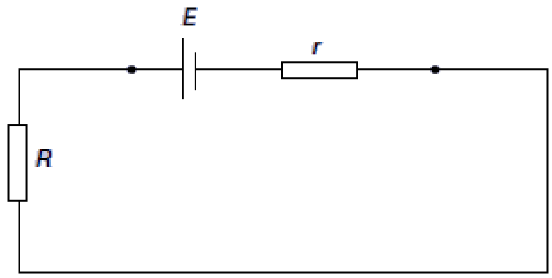

Figure 4.9: *A circuit with internal resistance*

The battery has an e.m.f. of 12 V and an internal resistance of 3 Ω. Calculate:

a) the current it supplies to the resistor, *R*, with value 12 Ω
b) the power used in the external resistor
c) the percentage of the total power wasted in the internal resistance.

19. Two resistors of resistance 20 Ω and 40 Ω are connected in series to a 6.0 V cell. Calculate:
a) the total resistance in the circuit
b) the current in the circuit
c) The p.d. across the 40 Ω resistor.

20. Find the current that flows in each of the resistors in the circuit shown in Figure 4.10.

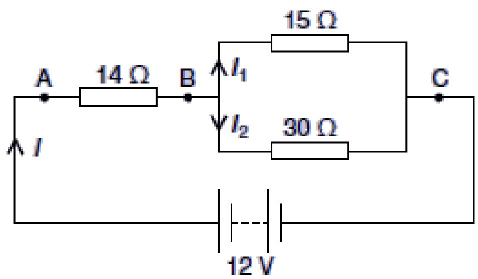

Figure 4.10