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ML Assignment 01:

Question 01:

Answer:

Here,

$f(z) = \log_e(1+z)$, where $z = X^T X$, $x \in \mathbb{R}^d$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \text{ and } X^T = \begin{bmatrix} x_1 & x_2 & \dots & x_d \end{bmatrix}$$

$$\therefore X^T X = [x_1^2 + x_2^2 + x_3^2 + \dots + x_d^2]$$

By Applying chain rule we find,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}(\log_e(1+z)) \cdot \frac{d}{dx}(X^T x)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dz}(z) \cdot (2x_1 + 2x_2 + \dots + 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \dots + x_d)$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

Answer for question 02:

$$f(z) = e^{-z/2} \quad , \text{where } z = g(y)$$
$$g(y) = y^T . S^{-1} y$$
$$y = h(x) = x - \mu$$

By Applying chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx} \quad \ldots (i)$$

Here,

$$\frac{df}{dz} = \frac{d}{dz}\left(e^{-z/2}\right) = -\frac{1}{2} . e^{-z/2}$$

$$\frac{dz}{dy} = \frac{d}{dy} g(y)$$
$$= \frac{d}{dy}\left(y^T . S^{-1} y\right)$$
$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$
$$= \lim_{h \to 0} \frac{(y^T + h) . S^{-1} (y+h) - y^T S^{-1} y}{h}$$
$$= \lim_{h \to 0} \frac{(y^T S^{-1} + h S^{-1})(y+h) - y^T S^{-1} y}{h}$$
$$= \lim_{h \to 0} \frac{y^T S^{-1} y + y^T S^{-1} h + h S^{-1} y + h S^{-1} h - y^T S^{-1} y}{h}$$
$$= \lim_{h \to 0} \frac{y^T S^{-1} h + h S^{-1} y + h^2 S^{-1}}{h}$$

$$= \lim_{h \to 0} \frac{h(y^T s^{-1} + s^{-1} y + h s^{-1})}{h}$$

$$= \lim_{h \to 0} (y^T s^{-1} + s^{-1} y + h s^{-1})$$

$$= y^T s^{-1} + s^{-1} y + 0$$

$$= y^T s^{-1} + s^{-1} y$$

And, $\frac{dy}{dx} = (x - \mu) = 1$

So, from equation (1), we find,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{1}{2} e^{-z/2} \cdot (y^T s^{-1} + s^{-1} y) \cdot 1$$

$$= -\frac{1}{2} e^{-z/2} \cdot \frac{1}{s} (y^T + y)$$

(Ans)