Averages

Average is defined as the ratio of sum of all terms in a group to the number of items in the group

$$Average = \frac{Sum\ of\ observations}{Number\ of\ observations}$$

NOTE: Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$

Ex 1.

Find the average of the following set of numbers: 354, 281, 623, 518, 447, 702, 876

Ans.

$$average = \left(\frac{354+281+623+518+447+702+876}{7}\right)$$

$$= \frac{3801}{7} = 543$$

Ex 2.

There are six numbers 30, 72, 53, 68, x and 87, out of which x is unknown. The average of the numbers is 60. What is the value of x ?

Ans.

$$average = \left(\frac{30 + 72 + 53 + 68 + x + 87}{6} \right)$$

$$= \frac{310 + x}{6}$$

But, the average is 60 then

$$\frac{310 + x}{6} = 60 \Rightarrow x = 50.$$

Ex 3.

13 chairs and 5 tables were bought for Rs. 8280. If the average cost of a table be Rs. 1227, what is the average cost of a chair ?

Ans.

Total cost of 5 tables $= 1227 \times 5 = 6135$

Total cost of 13 chairs $= 8280 - 6135 = 2145$

Therefore, average cost of a chair $= \dfrac{2145}{13} = 165$

Ex 4.

Of the three numbers, second is twice the first and is also thrice the third. If the average of three numbers is 44, what is the largest number?

Ans.

Let the numbers be x, y, z

Given $y = 2x, y = 3z$

$$\Rightarrow \frac{x + y + z}{3} = 44$$

$$\Rightarrow \frac{y}{2} + y + \frac{y}{3} = 132 \Rightarrow y = 72.$$

Therefore, the numbers are $x = 36, y = 72, z = 24$

Hence, largest number is 72

Ex 5.

The average of 11 numbers is 30. If the average of first six numbers is 17.5 and that of last six is 42.5, then what is the sixth number?

Ans.

Step 1: Calculate total of 11 numbers by multiplying it by average value 30 = 11 x 30 = 330

Step 2: Calculate total of first six members by multiplying it by average value $17.5 = 17.5 \times 6 = 105$

Step 3: Calculate total of last six members by multiplying it by average value $42.5 = 42.5 \times 6 = 255$

Therefore, we can find sixth number by adding value of first six and last six numbers and subtracting it from the total value of 11 numbers.

Sixth number $= (105 + 255) - 330 = 30$

Ex 6.

A person covers a distance of 60 km from P to Q at a speed of 20 km/hr and returns from Q to P at a speed of 30 km/hr. Find the average speed of person?

Ans.

V1 and V2 are the speeds at which the person travels. The person travels P to Q at a speed of 20 km/hr and Q to P at a speed of 30 km/hr. That is,

$V1 = 20 \, km/hr$ and $V2 = 30 \, km/hr.$

Therefore, Average Speed $= \dfrac{2 \times V_1 \times V_2}{V_1 + V_2} = \dfrac{1200}{50} = 24 \, km/hr$

Ex 7.

If the arithmetic mean of 0, 5, 4, 3 is a, that of -1, 0, 1, 5, 4, 3 is b and that of 5, 4, 3 is c , then the relation between a, b and c is?

Ans.

We have

$$a = \frac{0+5+4+3}{4} = 3;$$

$$b = \frac{-1+0+1+5+4+3}{6} = 2;$$

$$c = \frac{5+4+3}{3} = 4;$$

Clearly, $a: b: c = 3: 2: 4$

Ex 8.

The following table shows the number of working hours and the number of employees employed in a small scale industry? The average number of working hours of an employee is ?

No. of working hours	No. of employees
3-5	7
5-7	10
7-9	18
9-11	57
11-13	14
13-15	8

Ans.

We have:

Mean working hours	4	6	8	10	12	14
No. of employees	7	10	18	57	14	8

Sum of working hours of all the employees
$= 4 \times 7 + 6 \times 10 + 8 \times 18 + 10 \times 57 + 12 \times 14 + 14 \times 8 = 1082$

Total No. of employees $= 7 + 10 + 18 + 57 + 14 + 8 = 114$

Average number of working hours $= \dfrac{1082}{114} = 9.49 \approx 9.5$

Ex 9.

The average of x_1, x_2, x_3 and x_4 is 16. Half the sum of x_2, x_3, x_4 is 23. What is the value of x_1?

Ans.

Given

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = 16 \text{ and}$$

$$\frac{x_2 + x_3 + x_4}{2} = 23$$

From the above two equations, $x_1 = 18$

The mean of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$ is ?

Ans.

We know,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Here, $n = 7$,

So, $\sum_{i=1}^{7} i^2 = \frac{7(7+1)(2\times7+1)}{6} = 140$

Therefore, mean of $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2$ is $\frac{140}{7} = 20$

The average of 7 consecutive numbers is 20. The largest of these numbers is?

Ans.

Let the numbers be $x, (x + 1), (x + 2), (x + 3), (x + 4), (x + 5)$ and $(x + 6)$

Then,

$$\frac{x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6)}{7} = 20$$

or

$7x + 21 = 140$

$\Rightarrow x = 17$

Therefore, largest number is $(x + 6) = 17 + 6 = 23$

The average of the two-digit numbers, which remain the same when the digits interchange their positions, is ?

Ans.

The numbers in which remain the same when the digits interchange their positions are 11,22,33,44,55,66,77,88 and 99

So, the average is $\dfrac{11+22+33+44+55+66+77+88+99}{9} = \dfrac{495}{9} = 55$

Ex 13.

The average price of three items of furniture is $Rs.\,15000$. If their prices are in the ration $3:5:7$, the price of the cheapest item is?

Ans.

Let the prices be $3x, 5x$ and $7x$ respectively.

Then,

$$\Rightarrow \frac{3x + 5x + 7x}{3} = 15000$$

$$\Rightarrow x = 3000$$

Therefore, cost of cheapest item $3000 \times 3 = 9000$

Ex 14.

There are four consecutive odd numbers (x_1, x_2, x_3, x_4) and three consecutive even numbers (y_1, y_2, y_3). The average of the odd numbers is 6 less than the average of the even numbers. If the sum of the three even numbers is 16 less than the sum of the four odd numbers, what is the average of x_1, x_2, x_3, x_4?

Ans.

Given,

$$\frac{x_1+x_2+x_3+x_4}{4} = \frac{y_1+y_2+y_3}{3} - 6 \qquad \text{---------------- (1)}$$

and

$$y_1 + y_2 + y_3 = x_1 + x_2 + x_3 + x_4 - 16 \qquad \text{----------------(2)}$$

From (1) and (2),

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = \frac{4}{3}(y_1 + y_2 + y_3) - 24$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = \frac{4}{3}(x_1 + x_2 + x_3 + x_4 - 16) - 24$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = 136$$

Thank You