

MPI: Microprocessors and Interfacing

Academic Year: 2022 - 23

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Write recursive functions in assembly language program to compute the following (use NASM assembler).

- 1 Factorial of a number.
- 2 $F(n) = F(n - 1) + n$, where $n \geq 2$, and $F(1) = 1$.
- 3 Sum of all elements in an array A.
- 4 Find the smallest element in an array A

You are given with n positive Integers. Write assembly language functions to compute the following (use NASM assembler).

- 1 To find number of even numbers
- 2 To find number of odd numbers
- 3 To find number of prime numbers
- 4 To find GCD of n numbers
- 5 To find LCM of n numbers

Write assembly language functions to compute the following (use NASM assembler). Assume that all elements are integers.

- 1 Find the maximum and the minimum of n numbers.
- 2 Assume that you are given with a list of n elements and **key**, search the **key** using linear search. If the required element is found, return its position. Otherwise return -1.
- 3 Evaluate $1 + x + x^2 + x^3 + \dots + x^n$
(Assume that x and n are positive integers)
- 4 Assume that you are given with a list of n elements then find **mode** and **median**.

Installations.

- 1 Install NASM in your laptop and run a sample code.
- 2 Install MASM in your laptop and run a sample code.

Develop C-Programs for the following problem statements.

- 1 Print the internal representation of data stored in primary data types: **int**, **float**, and **double**.
- 2 **Bonus Problem** Perform addition and multiplication of two 32-bit numbers (Please remember that the input is 32-bit binary number). The numbers can either be integers or real numbers. Assume that the integers are represented in 2's complement form and the real numbers are represented using single-precision IEEE 754 standard.

Submission Guide Lines

- ▶ **Max. team size is 4.**
- ▶ Mail-ID: cs3106.mpi@gmail.com
- ▶ Sub:TEAM_NUM_LAB_NUM
- ▶ Attach.Name and Type: (Sub.).zip
- ▶ Late Submission \leq 3-Days:50%.
- ▶ Write a readme file to understand your solutions.

Number Systems

- ▶ Representation of Integer Numbers
 - ▶ Signed Magnitude Representation
 - ▶ 1's Complement Representation
 - ▶ 2's Complement Representation
- ▶ Representation of Real Numbers
 - ▶ Fixed Point Representation
 - ▶ Floating Point Representation

Resolution is difference between two successive numbers.

Representation of Integer Numbers

Let $A = a_{n-1}a_{n-2}a_{n-3}\dots a_0$ is an n-bit binary number

if A is an **unsigned integer**, then value of A is : $\sum_{i=0}^{n-1}(2^i \times a_i)$.

if A is a **signed integer**:

▶ Signed Magnitude Representation:

▶ $A = \sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 0$

▶ $A = -\sum_{i=0}^{n-2}(2^i \times a_i)$, if $a_{n-1} = 1$

▶ 1's Complement Rep.: $A = -(2^{n-1} - 1) \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$

▶ 2's Complement Rep.: $A = -2^{n-1} \times a_{n-1} + \sum_{i=0}^{n-2}(2^i \times a_i)$

Resolution: 1

Range of Numbers

Let $A = a_{n-1}a_{n-2}a_{n-3}\dots a_0$ is an n bit binary number

if A is an **unsigned integer**, then range of A is : 0 to $(2^n - 1)$.

if A is a **signed integer**:

- ▶ Signed Magnitude Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- ▶ 1's Complement Rep., range of A is : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$.
- ▶ 2's Complement Rep., range of A is : -2^{n-1} to $(2^{n-1} - 1)$.

Expansion of Bit Length

Add additional bit positions to the left and fill in with value of the sign bit.

Let $A = 1\ 0\ 1\ 0$ is a 4-bit binary number,

Representation of A using 8-bits (i.e. B): $1\ 1\ 1\ 1\ 1\ 0\ 1\ 0$.

is $A=B$?

- ▶ In 2's Complement Rep.: **Yes**.
- ▶ In 1's Complement Rep.: **Yes**.
- ▶ In Signed Magnitude Rep.: **No**.

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Real Numbers

❶ $(4.5)_{10} = (100.1)_2$

❷ $(8.25)_{10} = (1000.01)_2$

❸ $(16.125)_{10} = (10000.001)_2$

❹ $(0.875)_{10} = (0.111)_2$

❺ $(4.5)_{10} = (1.001)_2 \times 2^2$

❻ $(8.25)_{10} = (1.00001)_2 \times 2^3$

❼ $(16.125)_{10} = (1.0000001)_2 \times 2^4$

❽ $(0.875)_{10} = (1.11)_2 \times 2^{-1}$

Representations 5 to 8 are called Normalized Representations.

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**,
'xxxxxx' is a **Fraction/Mantissa**.

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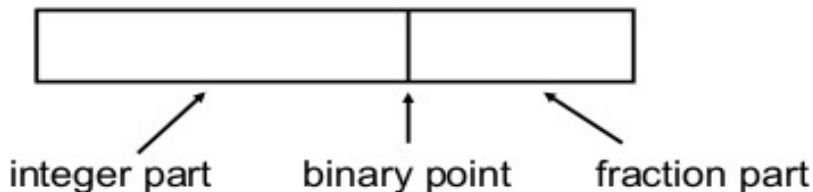
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Fixed Point (FP) Representation



Find the decimal equivalent of the following binary numbers. Assume that the binary numbers are represented using FP representation (6,2), i.e., 6 bits for integer part and 2 bits for fractional part.

▶ $(00101011)_2 = ?$

▶ $(11111011)_2 = ?$

Smallest +ve number that can be represented using FP (6,2) rep.: ?

Biggest +ve number that can be represented using FP (6,2) rep.: ?

Fixed Point Arithmetic

Consider a 16-bit binary representation, in which least significant 8 bits are used for precision then give the range of values that can be represented using the 16-bit representation.

- ▶ +ve Values: 2^{-8} to $2^7 - 2^{-8}$
- ▶ -ve Values: -2^7 to -2^{-8}
- ▶ Zero
- ▶ Resolution is ?

Advantages of FP Arithmetic:

- ▶ Easy to implement and occupies less space.
- ▶ If performance is important than precision.
- ▶ Once can choose a trade off between range and precision.

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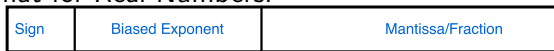
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IEEE 754 Representation of Real Numbers

▶ IEEE 754 format for Real Numbers.



Single Precision N=32	1 bit	8 bits	23 bits	Bias Value: +127
Double Precision N=64	1 bit	11 bits	52 bits	Bias Value : +1023

Biased Exponent = True Exponent + Bias Value

▶ Rep. of $(4.5)_{10}$ using Single Precision

$$(4.5)_{10} = (100.1)_2 = (1.001)_2 \times 2^2$$

Normalized Rep.: $(\pm 1.xxxxxx)_2 \times 2^E$, Where 'E' is a **True Exponent**, 'xxxxxx' is a **Fraction/Mantissa**.

▶ Biased Exponent = $2 + 127 = 129 = 1000\ 0001$

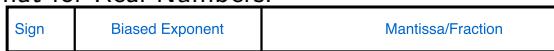
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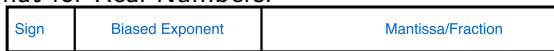
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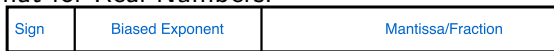
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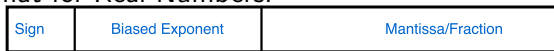
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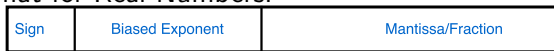
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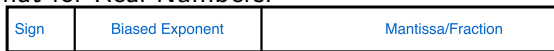
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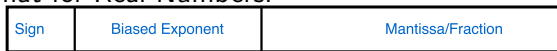
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- ▶ Biased Exponent = True Exponent + Bias Value,
where $1 \leq \text{Biased Exponent} \leq (2^{\text{Length of Biased Exponent}} - 2)$.
- ▶ Single Precision (N=32), $1 \leq \text{Biased Exponent} \leq 254$.
- ▶ Biased Exponent = 0,
 - ▶ Mantissa = ± 0 , then Value is ± 0 .
 - ▶ Mantissa $\neq 0$, then Value is **not a normalized number**.
- ▶ Biased Exponent = 255,
 - ▶ Mantissa = ± 0 , then Value is $\pm \infty$.
 - ▶ Mantissa $\neq 0$, then Value is **NAN**.
- ▶ Range of positive values: $[1.0 \times 2^{-126}, (2 - 2^{-23}) \times 2^{127}]$
- ▶ Range of negative values: $[-(2 - 2^{-23}) \times 2^{127}, -1.0 \times 2^{-126}]$
- ▶ Single Precision Number Resolution: $2^{-23} \times 2^{\text{True Exponent}}$

BCD Representation

BCD: Binary Coded Decimal

It uses a 4-bit binary number to represent each decimal digit.

Decimal Number	BCD Rep.
0	0000
1	0001
2	0010
3	0011
9	1001
10	0001 0000
25	0010 0101
99	1001 1001

Table 1: BCD equivalent of a decimal number.

All the best 😊