AE606/AE457 : Flight Dynamics and Control

Assignment - 1

The aircraft parameters that can be used for the simulation are as follows.

Table 1: *Geometry, Mass and Inertia Characteristics*

Mean Aerodynamic Chord, $\bar{c}$	1.211 m
Wing Span, b	10.47 m
Aspect Ration, AR	8.8
Wing Area, S	12.47 m^2
Mass, m	750 kg
Moment of Inertia, I_{xx}	873 $kg-m^2$
Moment of Inertia, I_{yy}	907 $kg-m^2$
Moment of Inertia, I_{zz}	1680 $kg-m^2$
Moment of Inertia, I_{xz}	1144 $kg-m^2$
Engine Thrust, T	1136 N

The aerodynamic model for the aircraft is given in the equations as follows.

Longitudinal Aerodynamic Model

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_q}\frac{q\bar{c}}{2V} + C_{L_{\delta_e}}\delta_e$$

$$C_D = C_{D_0} + C_{D_\alpha} abs(\alpha) + C_{D_{\delta_e}} abs(\delta_e)$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}\frac{q\bar{c}}{2V} + C_{m_{\delta_e}}\delta_e$$

Lateral-Directional Aerodynamic Model

$$C_Y = C_{Y_0} + C_{Y_\beta}\beta + C_{Y_p}\frac{pb}{2V} + C_{Y_r}\frac{rb}{2V} + C_{L_{\delta_r}}\delta_r$$

$$C_l = C_{l_0} + C_{l_\beta}\beta + C_{l_p}\frac{pb}{2V} + C_{l_r}\frac{rb}{2V} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r$$

$$C_n = C_{n_0} + C_{n_\beta}\beta + C_{n_p}\frac{pb}{2V} + C_{n_r}\frac{rb}{2V} + C_{n_{\delta_r}}\delta_r$$

The initial condition for the simulation are:

- Free stream velocity, $V_\infty = 60$ m/s
- Altitude, H = 2000 m

Aerodynamic Derivatives

(a) *Longitudinal*

C_{D_0} = 0.036	C_{L_0} = 0.365	C_{m_0} = 0.05
C_{D_α}= 0.041	C_{L_α} = 4.2	C_{m_α} = -0.59
e = 0.8	C_{L_q} = 17.3	C_{m_q} = -9.3
	$C_{L_{\dot{\alpha}}}$ = 8.3	$C_{m_{\dot{\alpha}}}$ = -4.3
$C_{D_{\delta_e}}$ = 0.026	$C_{L_{\delta_e}}$ = 0.26	$C_{m_{\delta_e}}$ = -1.008

(b) *Lateral-Directional*

C_{Y_0} = -0.013	C_{l_0} = 0.0015	C_{n_0} = 0.001
C_{Y_β}= -0.431	C_{l_β} = -0.051	C_{n_β} = 0.071
C_{Y_p} = 0.269	C_{l_p} = -0.251	C_{n_p} = -0.045
C_{Y_r} = 0.433	C_{l_r} = 0.36	C_{n_r} = -0.091
$C_{Y_{\delta_r}}$ = 0.15	$C_{l_{\delta_r}}$ = 0.005	$C_{n_{\delta_r}}$ = -0.049
$C_{Y_{\delta_a}}$ = 0	$C_{l_{\delta_a}}$ = -0.153	$C_{n_{\delta_a}}$ = 0

The steps to follow:

1. Formulate and identify the Eigen Values for the fourth order and 2 sets of reduced order Longitudinal Dynamics
2. Find all the characteristics of the above said dynamics and comment on them.
3. Plot the locus of the eigen values of the dynamics mentioned in Part 1 for the following cases
   (a) Variation of speed : ±25 m/s.
   (b) Varation of Altitude: 0 - 5000 m.
   (c) Variation of Mass and Moment of Inertia: upto ±40%.
   (d) Varation of C.G : ±10%.

The report should emphasize on the effects of the variations on the aircraft design parameters and provide your comments are the limits of the same varation that can be allowed during flight. Proper justifications are required for your comments without which they doesn't provide any feedback to the designer. Extra plots are allowed if they prove the point you were trying to make. Try to use the plots in a proper way to support your comments. Try not to cluster everything in one plot. The codes have to submitted along with the report in a zip file, with file name as "SCnumber_FirstName.zip"

Copying in any aspect will result in NO MARKS for all assignments.

2

Assignment 1: Flight Dynamics and Controls

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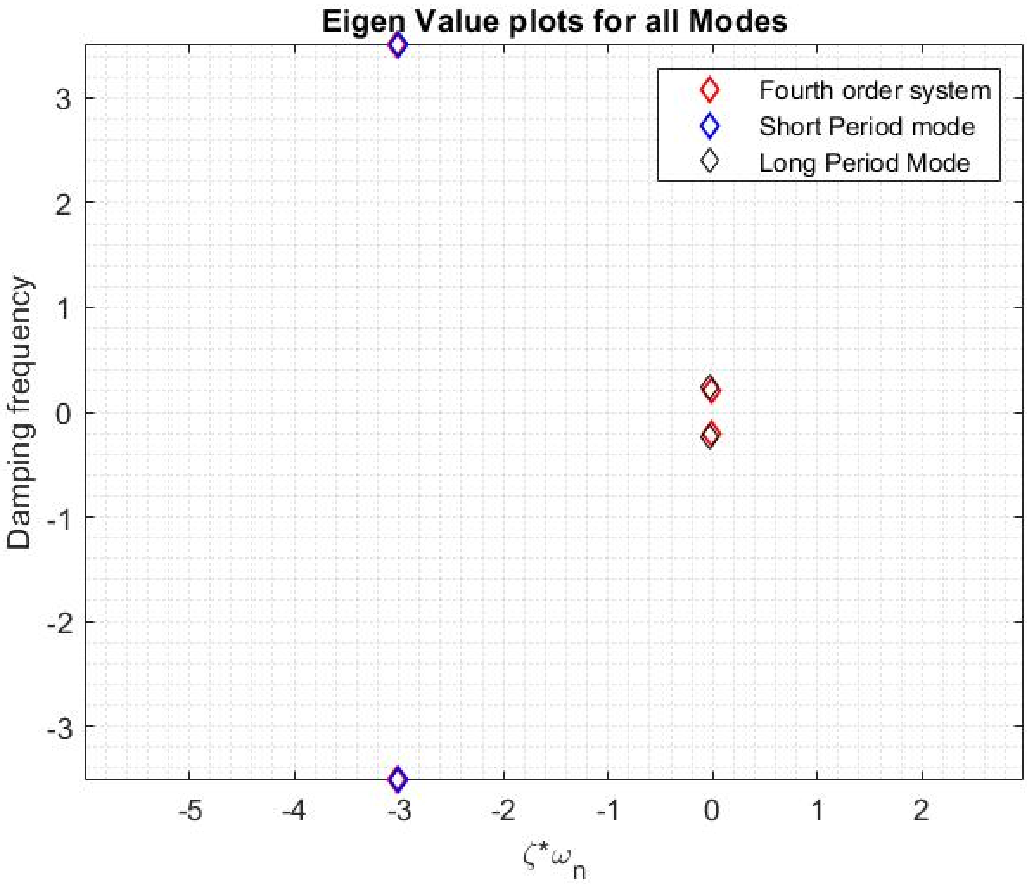
April 18, 2021

1 Eigen Values and Characteristics for the given Aircraft

For aircraft flying at an altitude of 2000m and at V=60 m/s and for the given specifications.The Eigen values obtained are:

1.1 From the fourth order approximation the eigen values are:

-3.0115 + 3.5094i
-3.0115 - 3.5094i
-0.018988 + 0.20576i
-0.018988 - 0.20576i



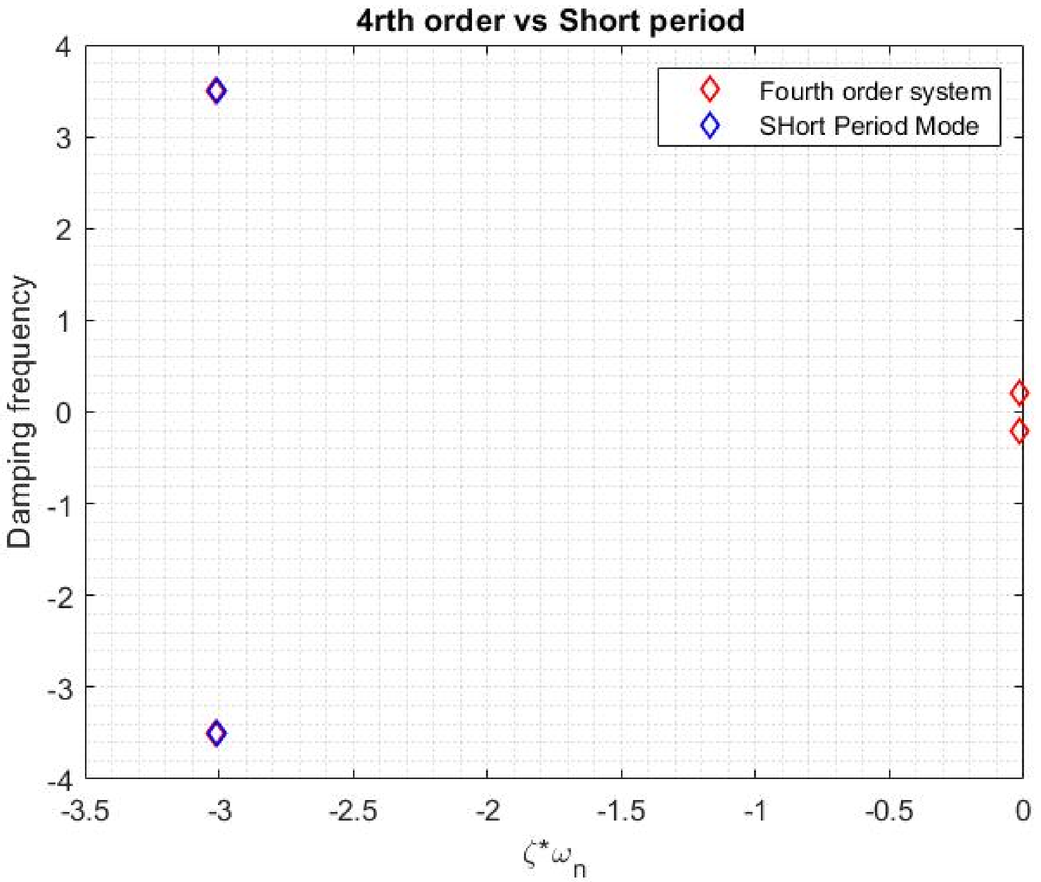
Based on the two set of complex conjugates. Two longitudinal modes are presented. The short period mode has large damping as well as natural frequency, where as the longperiod mode has small damping ratio and natural frequency. In the next section, I have shown the variation between the fourth order model and the approximate second order systems.

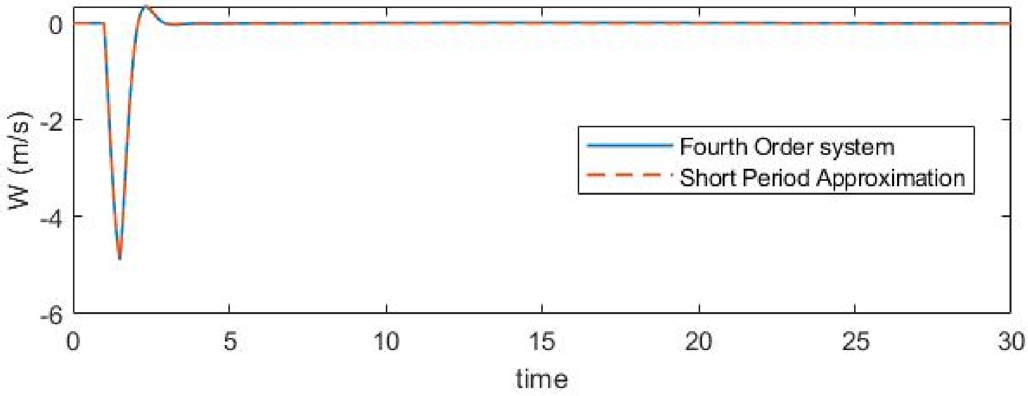
1.2 Short period Mode

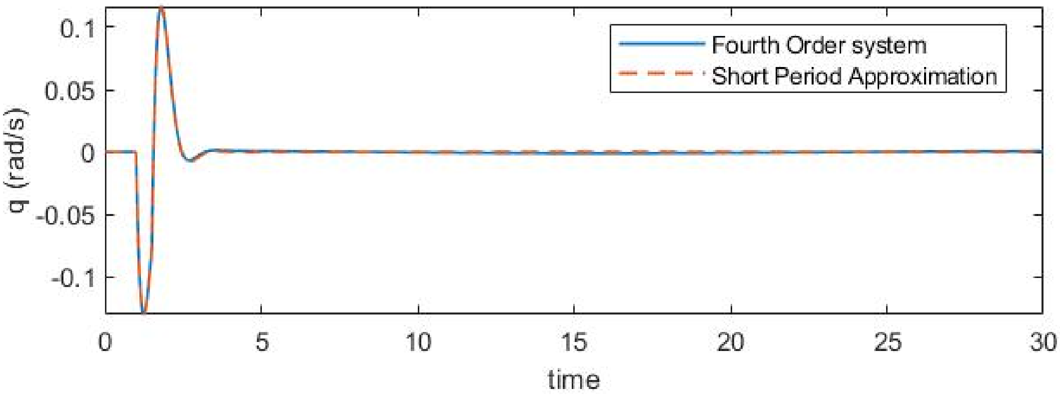
For the second order short period approximation the eigen values obatined are as follows:

$$-3.01 + 3.5079i$$
$$-3.01 - 3.5079i$$

Characteristics of Short Period approximation:
1.Damping ratio=0.6512
2.Frequency=3.5079 / s
3.T Halftime=0.23028 s
4.TimePeriod=1.7912 s







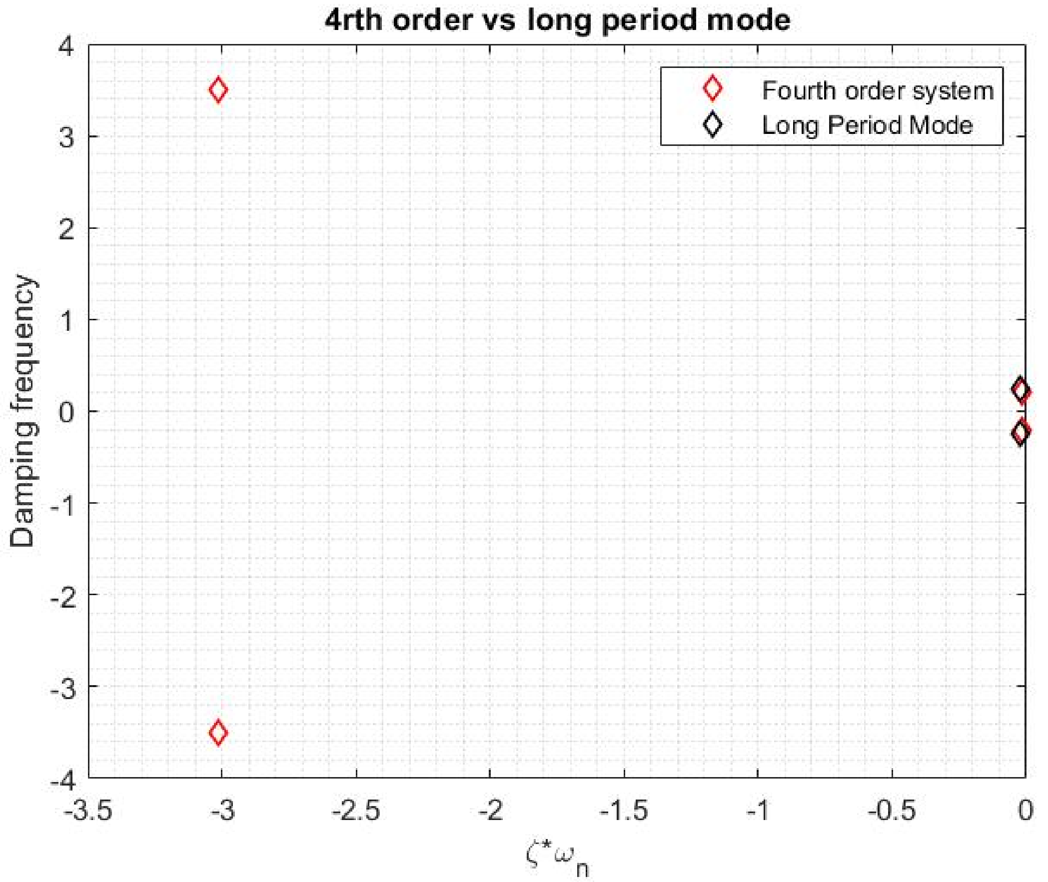
In the above system,I have given disturbance of w=5 m/s for 0.5 sec. We can see that the short period mode quickly damps and it also approximately matches with the fourth order system. From the Eigenvalues & Graph of we can say that the Short period mode approximately matches with the fourth order system.

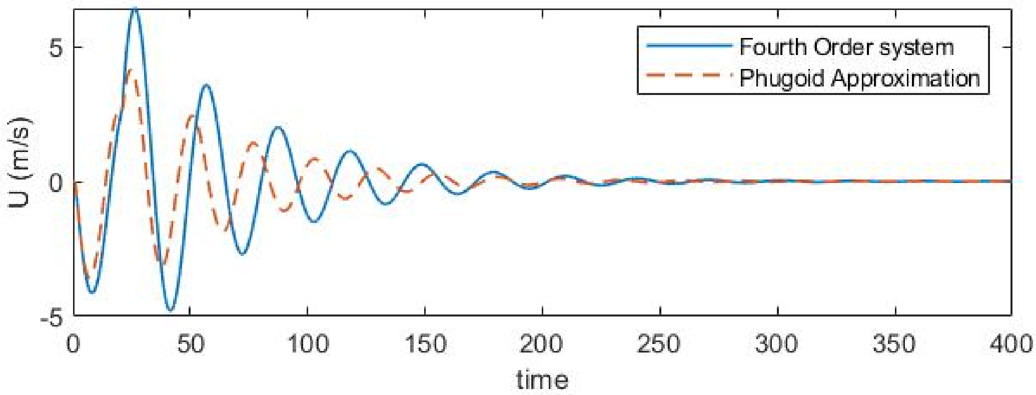
1.3 Phugoid Mode.

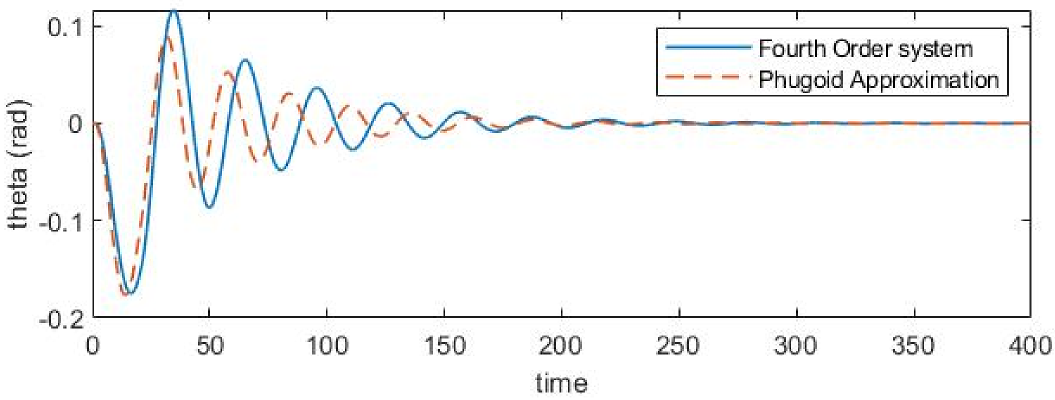
The second order phugoid approximation has the following eigen values:

$$-0.020481 + 0.20121i$$
$$-0.020481 - 0.20121i$$

Characteristics of phugoid approximation:
1.Damping ratio=0.084605
2.Frequency=0.23161/s
3.T Halftime=33.8436 s
4.TimePeriod=27.1284 s







In this system, the disturbance is given in u =10m/s for 20 seconds, the aircraft has the oscillatory motion for long period and damps gradually.
From the Eigenvalues & Graph of we can say that the Phugoid mode has small deviation from the

fourth order model. However, for initial estimate we can use these reduced order models effectively.

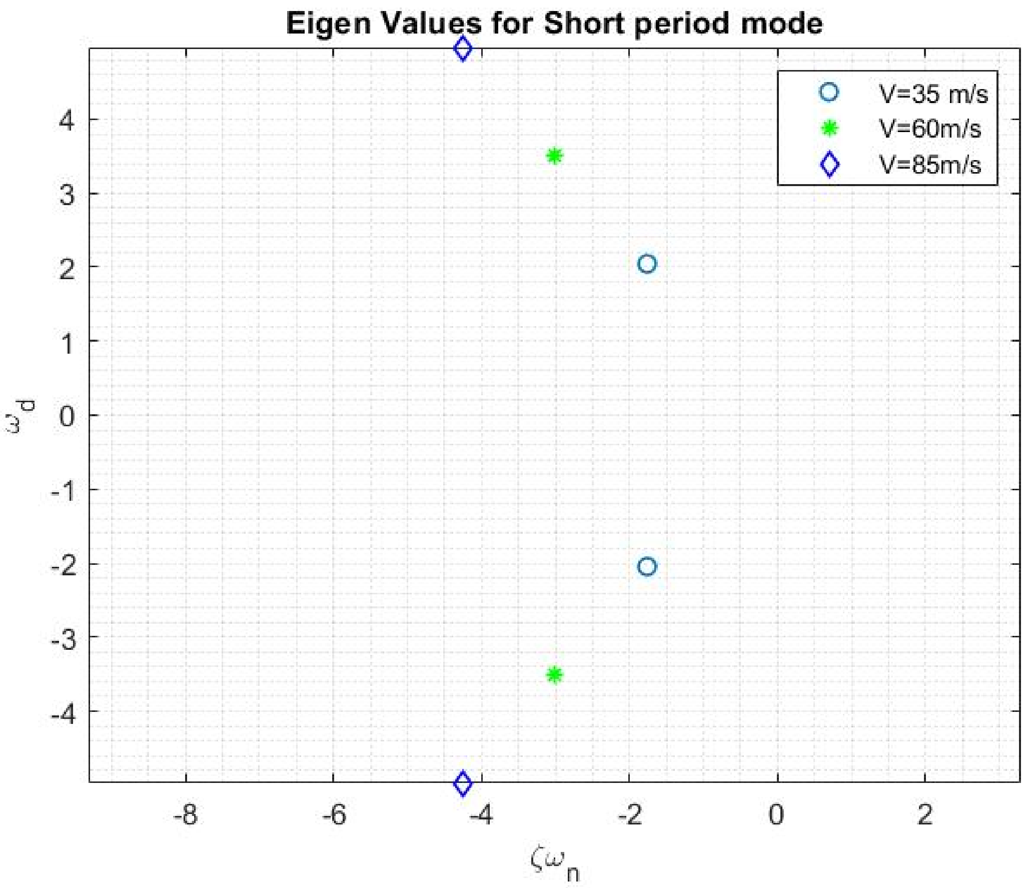
2 Locus of Eigen Values for various parameters.

2.1 Variation in Velocity

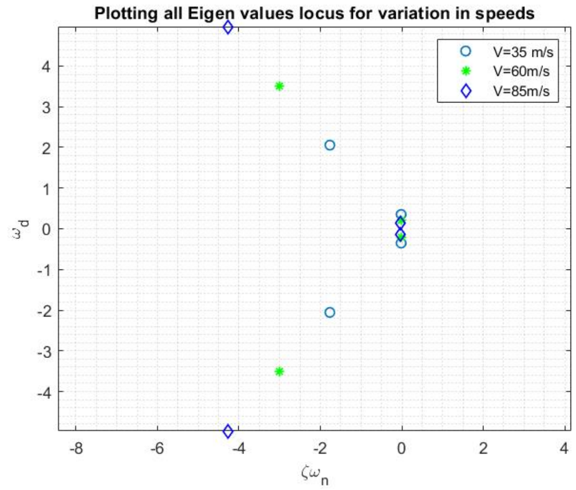
1. For the variation in speed by $+/-25m/s$. As observed from the plots, as the speed increases, the aircraft damps quickly in the short period mode and also the natural frequency also gets reduced. This is also true if we look at the aprroximations, for

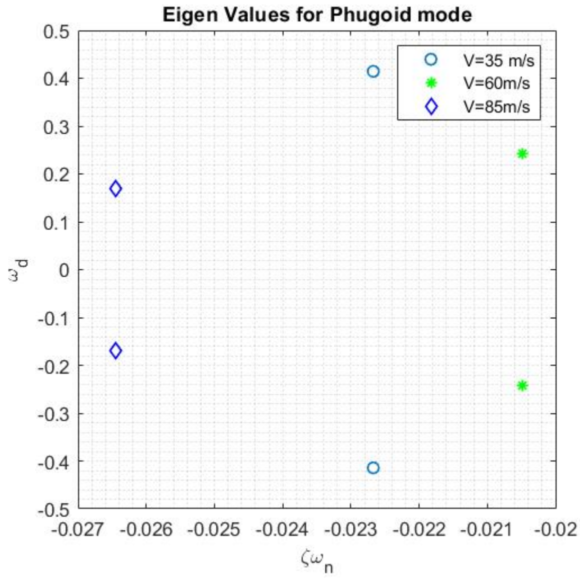
$$\zeta_s = -[(M_q + M_{\dot{\alpha}} + Z_{\dot{\alpha}}/Uo)/2 * \omega_s]$$
$$\omega_s = -(M_\alpha + Z_\alpha * M_q/U0)^{(}1/2)$$

2. For **Short Period approximation** ,Since all the values are increasing in the $-ve$ direction as the speed increases the damping ratio will get increased and frequency will also increase, so the eigen values shift towards the left half of the complex plane. In other words the stability of the aircraft increases as the speed increases.

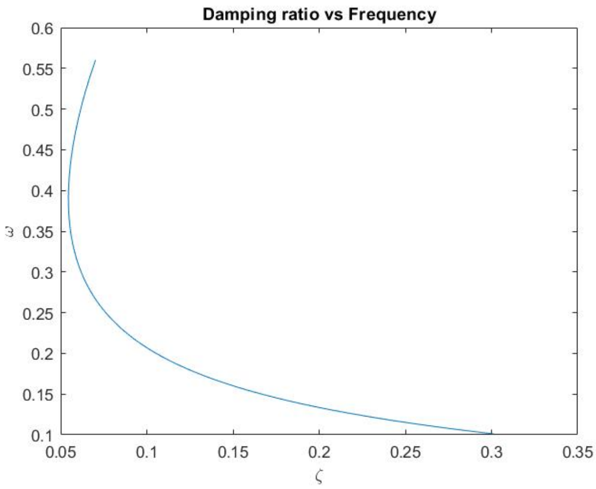


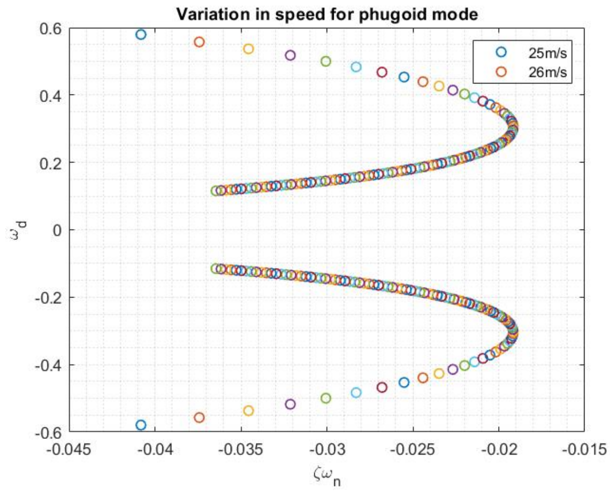
3. For **Phugoid approximation** the variationof ζ is very small compared to short period mode, however as the speed increases from 35m/s first the aircrraft goes towards the unstable reagion upto some time and as we increase the speed further, the aircraft goes to stable region. This is because as per the approximations both ζ and ω are function of $U0$ in opposite sense, i.e as ζ increases ω starts decreasing, so at one particular point both reach their minima condition. Due to this we can observe the aircraft goes initially towards unstable region, and again starts going back towards stability. From our example, we can see this phenomenon as the aircraft from 35 m/s approaches towards 60 m/s and after that as it goes to 85m/s it again starts going to stable zone.





To explain this phenomenon, I have plotted a graph for ζ and ω variying velocity from 25-120m/s from this we can see this trade off easily.



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From our previous observation we have seen that aircraft has a limiting region where both Damping
ratio and Natural frequency gets minimized.However, we should also consider the fact that the
aerodynamic efficiency of the aircraft is not sacrificed. In that aspect keeping all othr parameters
constant. Taking Maximum Velocity to 42 m/s will give us Aerodynamic Efficiency 16.3117.

Wingloading= 590.016
ThrusttoWeight_ratio= 0.037397

Velocity=    35m/s 40m/s     42m/s
Aerodynamics_efficiency= 20.9947      17.4993      16.3117

for V=42m/s
        variables          T4sp       Tsp        error_sp
    _________________    ______    _______    __________

    {'zeta'          }    0.6515    0.65168    0.00017543
    {'Timeperiod'    }    2.5533     2.5589     0.0056731
    {'Omega_damping'}    2.4608     2.4554     0.0054556
    {'T Half'        }     0.328    0.32857      0.000575

        variables           T4lp        Tlp        error_lp
    _________________    ________    _______    _________
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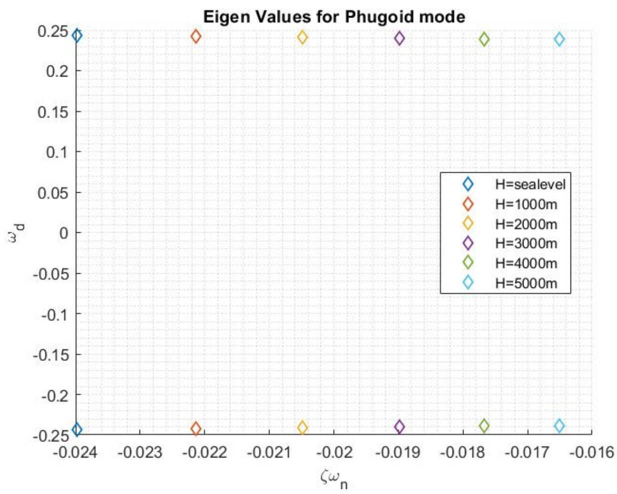
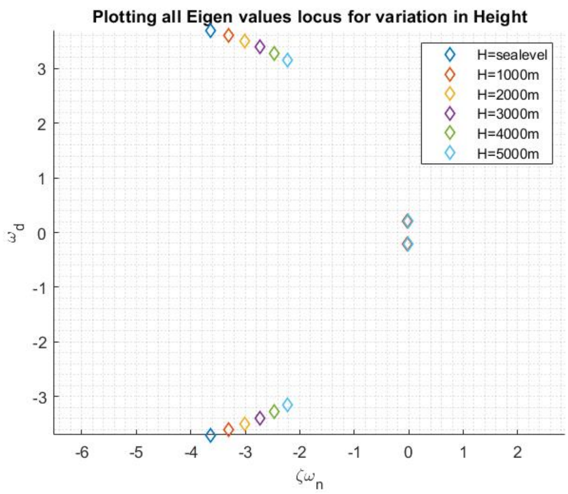
{'zeta'          }     0.054219     0.05688     0.0026613
{'Timeperiod'    }        21.36      18.709        2.6515
{'Omega_damping'}      0.29415     0.33584      0.041689
{'T Half'        }       43.397      35.238        8.1592
```

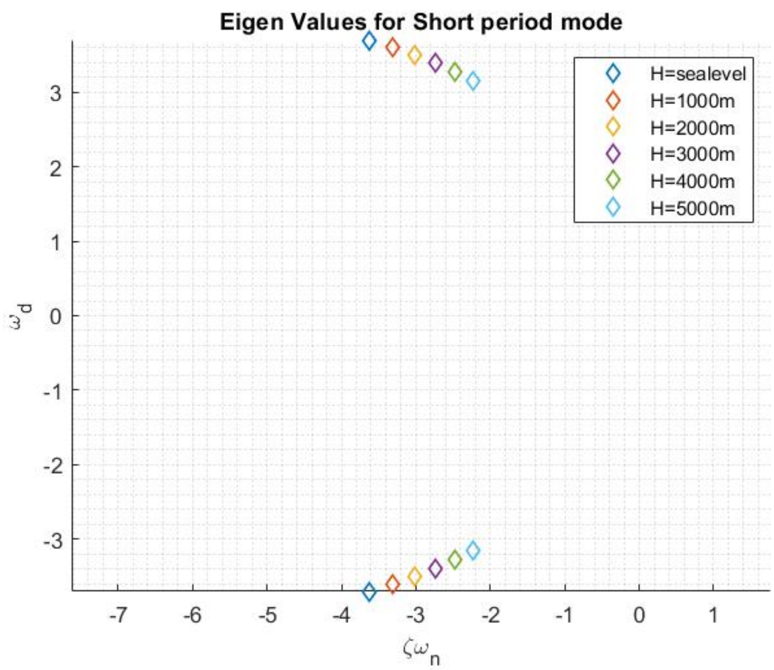
2.2 Variation in Altitude:

1. **In Phugoid mode**, the altitude is varying, the density varies and due to which CL, CD values keep on varying however, the CL/CD value remains to be constant. However, as per the results obtained, the values of Xu and Zq vary, the real part of the Eigen value starts varying which we can see as we zoom in; there are small variation in the graph.





2. **In short period approximation**, the effect of stability derivatives of M and Z is huge, due to this the eigen values change as the height in increased. We can observe that as the Altitude increases the stability of the aircraft in short period mode also gets reduced due to this derivatives.



For the variation in altitude we can observe that at 5000 m the variation in T half for fourth order model is 5.194 seconds greater than Phugoid mode.To maintain the error upto

```
5 seconds. I have restricted the Maximum Height upto 5000m only.

Wingloading= 590.016
Height = 2000m      3500m 5000m
ThrusttoWeight_ratio= 0.037397    0.036072    0.034747
Aerodynamics_efficiency= 6.4767      6.4767      6.4767

At H = 5000 m
        variables          T4sp        Tsp        error_sp
    -----------------    -------    -------    ----------

    {'zeta'          }    0.57583    0.57571    0.00012234
    {'Timeperiod'    }     1.9865     1.9875    0.00099037
    {'Omega_damping'}     3.1629     3.1614     0.0015761
    {'T Half'        }    0.31115     0.3114    0.00025409

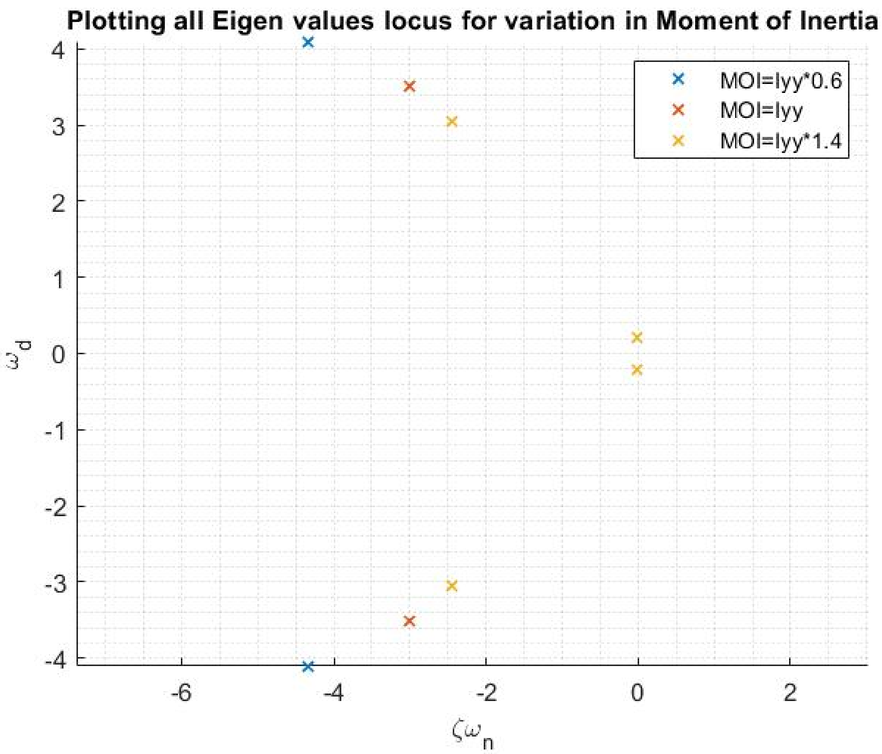
        variables          T4lp         Tlp         error_lp
    -----------------    --------    --------    ---------

    {'zeta'          }    0.069178    0.069077    0.0001013
    {'Timeperiod'    }      29.655      27.246       2.4088
    {'Omega_damping'}     0.21188     0.23061     0.018732
    {'T Half'        }      47.177      41.983        5.194
```

2.3 Variation in Mass and MOI

2.3.1 Variation in MOI

1. Varying the MOI will effect the short period mode since, the Iyy is the only factor which effect in longitudinal modes and that too in the short period mode, since short period mode is effected by M derivatives.
2. In phugoid mode, the X and Z derivatives are not a function of Iyy, so the eigen values remain constant which means the stability is not effected in the phugoid mode for variation in MOI.



We can observe that, if there is any increase in length or diameter of the fuselage, the Aircraft's stability in the short period mode gets reduced and vice versa, assuming the CG location is fixed for all the cases.

since $I_y = f(x, z)$

```
For MOI= 1.7 times the actual MOI. I am resticting the MOI because error in Time period to half the
amplitude will increase for lon period mode , so I have restricted up to 6 seconds.
Wingloading= 590.016
ThrusttoWeight_ratio= 0.037397
Aerodynamics_efficiency= 6.4767
```

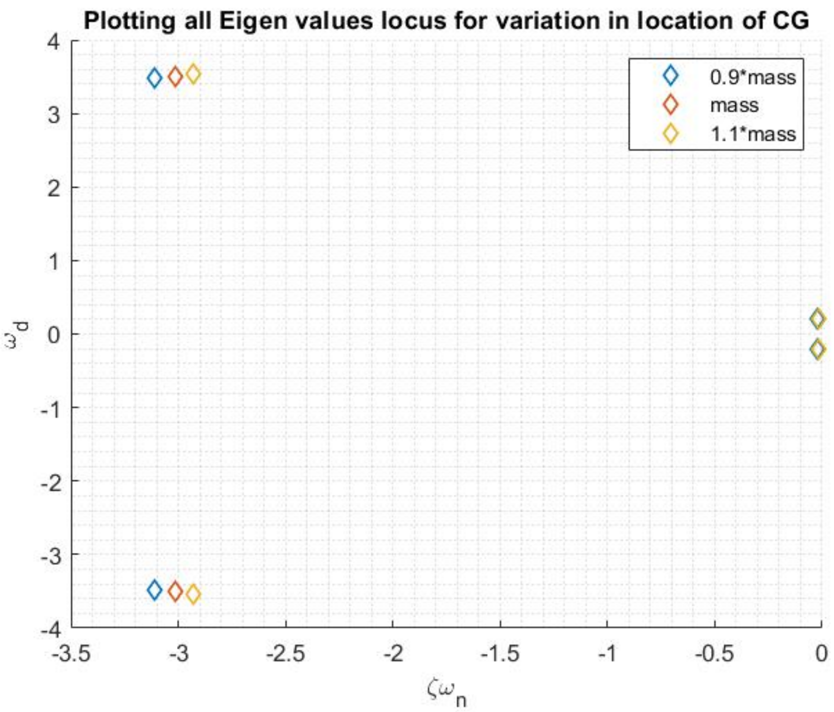
variables	T4sp	Tsp	error_sp
{'zeta' }	0.61825	0.6181	0.00014674
{'Timeperiod' }	2.2525	2.2546	0.0021565
{'Omega_damping'}	2.7895	2.7868	0.0026681
{'T Half' }	0.3159	0.31632	0.00042395

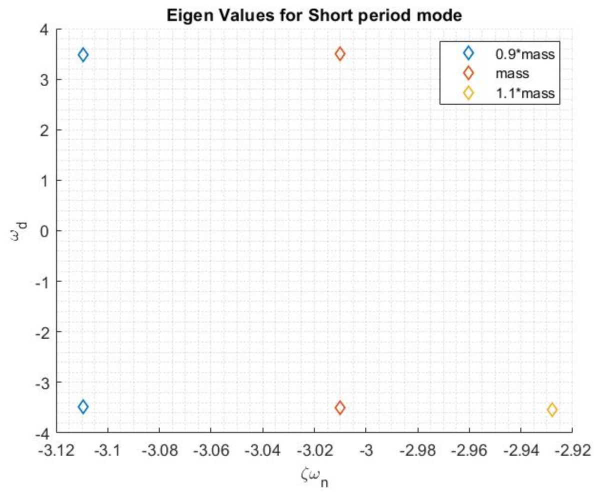
variables	T4lp	Tlp	error_lp
{'zeta' }	0.08494	0.084605	0.00033475
{'Timeperiod' }	30.537	27.128	3.409
{'Omega_damping'}	0.20575	0.23161	0.025855
{'T Half' }	39.518	33.844	5.6742

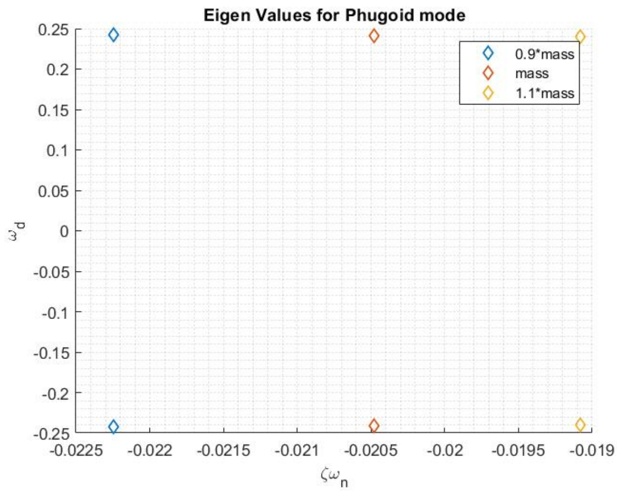
2.3.2 Variation in Mass

1. As the mass of the aircraft increases the Short period modes move towards the right side since, the damping decreases and the frequency starts increasing. Therefore the in overall the time period of mdoe gets reduced as the weight is increased.
2. As the mass increases the CL increases and the aerodynamic efficiency of the aircraft also increase , this reduces the damping characteristics of aircraft in phugoid mode. However, the effect if very small and so we can neglect it.

Limits: We can vary the aircraft mass by a factor of 0.5*W to 70*W, still we will get the approximate values for both the fourth order and reduced order models. However, due to the limitation of Aerodynamic Efficiency we restrict the variation in mass up to (0.9-2.5) times its weight. Since the designer wants tho increase the aerodynamic efficiency as high as possible, but in practical cases as the aerodynamic efficiency of aircraft is limited to (16-17), here I am limiting the variation of weight up to 2.5(All other parameters are constant)







Here the variation in Weight (W) is for 0.9*W, W, 2.5*W
Observation is as the mass increases the Aerodynamic Efficiency(E) increases, and for 2.5 times weight; E is 18.875. We can see the corresponding variation in Fourth order model (T4lp); Short period approximation (Tsp) and Phugoid mode (Tlp).

Wingloading= 531.01443 590.01604 1475.0401

ThrusttoWeight_ratio= 0.041552 0.037397 0.014959

Aerodynamics_efficiency= 7.83632 8.83435 18.8375

variables	T4sp	Tsp	error_sp
{'zeta' }	0.55988	0.55984	4.335e-05
{'Timeperiod' }	1.7269	1.7272	0.00032754
{'Omega_damping'}	3.6384	3.6377	0.00068996
{'T Half' }	0.28193	0.28202	8.5279e-05

variables	T4lp	Tlp	error_lp
{'zeta' }	0.056765	0.056283	0.00048182
{'Timeperiod' }	28.568	27.477	1.0906
{'Omega_damping'}	0.21994	0.22867	0.0087298
{'T Half' }	55.43	52.32	3.1105

2.4 Variation in location of CG

The variation in CG is done by varying the derivative dC_m/dC_l, since this will directly relate to static margin we can change the variation in CG from the static margin taking a fixed neutral point. Since dCm/dCL=(Xcg-Xnp)/c

$$Xnp = Xac/c - Cm_{\alpha fs}\ /Cm_{\alpha w} + \eta * VH * CL * CL_{\alpha t}\ (1-(d\epsilon/d\alpha))$$

Since neutral point is unknown for the given aircraft , assuming the $CL_{\alpha t}\ = 0$ we can take Xnp =0.25.
Now , as we vary the location from 0.9-1.1 times CG the values of Cm_α obtained are:

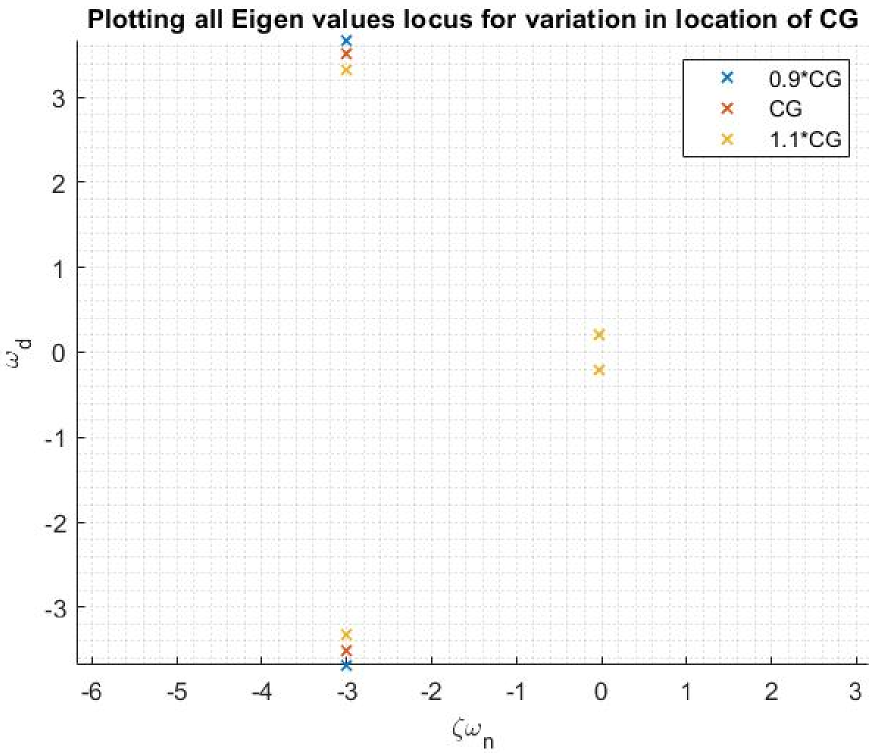
Cm_α=dCm/dCL.*CL_α

Cm_α= [-0.636 -0.59 -0.544]

Since we are taking the aircraft to be varying from trim condition we are always taking the Cm=0; at trim.

Observation

As the Cg location of the aircraft shift towards the right side, it reaches towards neutral stability and the statically stable aircraft becomes neutrally stable at Xcg=Xnp. The variation of CG effects mainly on short period mode ,the time period for damping is seen to be increasing as the Xcg shifts towards the aft of the aircraft. However the long period mode is not effected by the variation in CG.

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The maximum variation of CG is restricted to 1.3 times the actual CG location, since after
 that the variation in time period for fourth order approximation and Phugoid mode will
be more than 5 second. On the other hand, if we increase the CG location towards
the rear of the aircraft, it will be statically more unstable, and during the flight
the fuel consumption should also be taken consideration. So lot of control effort
will be needed to stabilize the aircraft.

Wingloading= 590.016
ThrusttoWeight_ratio= 0.037397
Aerodynamics_efficiency= 8.8337      8.8343      8.8362

for X_CG=1.3* X_CG
        variables          T4sp        Tsp        error_sp
    __________________    _______    _______    __________

    {'zeta'          }    0.71492    0.71504    0.00011388
    {'Timeperiod'    }     2.1335      2.135      0.001578
    {'Omega_damping'}     2.9451     2.9429     0.0021767
    {'T Half'        }    0.23018    0.23028    9.5208e-05

        variables           T4lp        Tlp        error_lp
    __________________    ________    ________    ________

    {'zeta'          }    0.096874    0.084605    0.012268
    {'Timeperiod'    }      31.792      27.128      4.6638
    {'Omega_damping'}     0.19763     0.23161    0.033976
    {'T Half'        }      36.034      33.844      2.1905
```

3 Conclusion:

In the above Configured Aircraft, I have assumed that all the given parameters remain same and the aircraft is initially in steady flight having no pitch rate and without control effect. Furthermore, using the data of Aerodynamic Efficiency, Time delay compared with fourth order approximation to set the limiting conditions for Velocity, weight, MOI, and CG location.
Based on the Above results.

1. Limited the Aircraft to 42 m/s.
2. Mass =1875 kg.
3. Moment of Inertia Variation up to 1.7 times.
4. Location of CG up to 1.3 times the CG initial location.
5. Altitude =5000m.

```
Now for the above limits the modified aircraft parameters, the values obtained are as below:
I have not changed the Thrust in any of the model, however in the actual scenario
thrust/weight will be of the order 0.2-1.2. So due to which again Aerodynamic efficiency varies,
again we need to reconfigure the model.

Wingloading= 1475.0401
ThrusttoWeight_ratio= 0.013899
Aerodynamics_efficiency= 16.1917
        variables           T4sp       Tsp       error_sp
    -----------------     -------    -------    ----------

    {'zeta'          }    0.45224    0.45154    0.00069096
    {'Timeperiod'    }     3.0754     3.0798     0.0043772
    {'Omega_damping'}       2.043     2.0401     0.0029037
    {'T Half'        }    0.66911    0.67135      0.002241

        variables            T4lp        Tlp        error_lp
    -----------------     --------    --------    ---------

    {'zeta'          }    0.076346    0.082759    0.0064131
    {'Timeperiod'    }       19.85      19.605      0.24554
    {'Omega_damping'}      0.31653     0.32049    0.0039643
    {'T Half'        }        28.6      25.029        3.571
```