ISMAT401

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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – MAY / JUNE 2014

Course & Branch : B.E: INFORMATION SCIENCE & ENGG — Semester : IV

Subject : Engineering Mathematics-IV — Max. Marks : 100

Subject Code : ISMAT401 — Duration : 3 Hrs

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT-I

1. a) i) Define intermediate value property. (02)

   ii) Show that r lies between -1 and +1 (03)

   b) Using Newton-Raphson method find a real root of the equation x sinx + cosx = 0 near $x = \pi$. Correct to 4 decimal places.(x is in radians). (08)

   c) Find a law of the form $y = a + \frac{b}{x}$ for the following data. (07)

X	50	47	46	45	44
Y	2	3	4	6	10

2. a) i) Write regression lines of x on y and y on x. (02)

   ii) Derive the normal equation to fit a straight line of the form $y = ax + b$ by the method of least squares. (03)

   b) Find correlation coefficient and the lines of regression for the following data (08)

X	1	2	3	4	5
Y	2	5	3	8	7

   c) Fit a curve of the form $y = ab^x$ for the following data. (07)

X	1	2	3	4	5	6	7
Y	87	97	113	129	202	195	193

UNIT-II

3. a) i) Define Random variable with an example. (02)

   ii) For three events A, B, C, prove that P{A∪B/C} = P(A/C) + P(B/C) – P(A∩B/C). (03)

   b) An assembly plant purchases respectively 50%, 30% and 20% of its voltage stabilizers from firms A, B and C. Only 90%, 75% and 50% of the stabilizers purchased from firms A, B, C respectively work according to specifications. A voltage stabilizer is selected at random from the assembly plant. What is the probability that it works according to specifications? What is the probability that a voltage stabilizer working according to specification came from firm B? (08)

c) The pdf of a random variable X is given by $P(X=x)=\begin{cases} x & , 0\le x\le 1 \\ 2-x & , 1<x\le 2 \\ 0 & , elsewhere \end{cases}$ (07)

Find (i) Cumulative distribution function F(X) and (ii) $P(X \ge 1.5)$.

4. a) i) Define Mathematical expectation of the function $\phi(X)$ of the random variate X in both discrete and continuous cases. (02)

ii) If A and B are any two events then prove that (03)
$P(A) = P(A\cap B) + P(A\cap \bar{B})$

b) A and B throw alternatively a pair of dice. The one who throws 9 first wins. Show that the chances of their winning are 9:8. (08)

c) The probability density function of a variate X is (07)

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

Find (i)$p(x\le 3), p(x\ge 5). p(3<x<6)$ (ii) Mean

UNIT-III

5. a) i) Write pdf of Gamma distribution. (02)

ii) The probabilities of a Poisson variate taking the values 3 and 4 are equal. Calculate the probabilities of the variate taking the values 0 and 1. (03)

b) The joint distribution of random variables X and Y is given by (08)
$P_{ij} = K(i+j)$, i=1,2,3,4 ; j = 1,2,3. Find (i) K, (ii) the marginal distributions of X and Y, (iii) Show that X and Y are stochastically independent.

c) In an examination taken by 500 candidates, the average and S.D of marks obtained are 40% and 10% respectively. Assuming normal distribution find (i)How many will pass if 50% is failed as maximum for passing and (ii) How many have scored above 60% (iii) What should be the minimum for 350 candidates. (07)

6. a) i) Write pdf of Geometrical distribution. (02)
ii)Find mean and variance of exponential distribution (03)

b) The probability that an airplane engine will fail, when in flight, is 1-p, p<1. The airplane will make a successful flight if at least 50% of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane? (08)

c) The daily consumption of milk in a town, in excess of 30,000 litres is distributed as a Gamma distribution with parameters $\alpha = 2$ and $\beta = 10{,}000$. The town has a daily stock of 40,000 litres. Find the probability that the stock is adequate on a particular day. (07)

UNIT-IV

7. a) i)State the condition when t-test is used. (02)

ii) Write short note on i) Type I and Type II errors ii) Testing of hypothesis (03)

b) Two independent samples of sizes 7 and 6 have the following values : (08)

Sample A	28	30	32	33	33	29	34
Sample B	29	30	30	24	27	29	-

Examine Whether the samples have been drawn from normal population having the same variance?

c) Five dice were thrown96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below. (07)

No.of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution ($\chi^2_{0.05} = 11.07$ for 5 d.f.)

8. a) i)Define standard error. (02)

ii) A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure.5,2, 8, -1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure? (03)

b) Fit a Poisson distribution for the following data and test the goodness of fit. (08)

x:	0	1	2	3	4
f:	122	60	15	2	1

c) A sample of 12 measurements of the diameter of a metal ball gave the mean 7.38 mm with standard deviation 1.24mm. Find (i) 95% and (ii)99% confidence limits of the actual diameter. (07)

UNIT-V

9. a) i) Define M|M|1 Queuing model. (02)

ii) Find the fixed probability vector of the matrix $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ (03)

b) A habitual gambler is a member of two clubs A and B. He visits either of the clubs everyday for playing cards. He never visits club A on two consecutive days. But, if he visits club B on a particular day, then the next day he is as likely to visit club B or club A. Find the transitionprobability matrixof thisMarkov chain. a) . Show that the matrix is a regular stochastic matrix and find the unique fixed probability vector. b). If the person had visited club B on Monday, find the probability that he visits club A on Thursday. (08)

c) Customers arrive in a telephone booth at intervals of 12 minutes on the average. The length of a phone call is 4 minutes on the average. (07)

(i) What is the probability that a person arriving at the booth will have to wait?
(ii) What is the average length of the queue that forms from time to time?
(iii) The owner of the booth will instal a second booth when convinced that an arrival would expect to wait atleast five minutes for the phone. By how much must the flow of arrivals be increased in order to justify a second booth?

10. a) i) Define stochastic matrix with an example. (02)

ii) Prove that the Markov chain with t.p.m $\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. (03)

b) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. If the service time for all cars is constant and equal to 10 minutes, determine L_s, L_q, W_s and W_q (08)

c) A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. In the long run, how often does he study? (07)