Karnaugh Map

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Contents

***Abstract*—This manual explains Karnaugh maps (K-map) by finding the logic functions for the incrementing decoder.**

1 Incrementing Decoder

The incrementing decoder takes the numbers $0, 1, \ldots, 9$ in binary as inputs and generates the consecutive number as output. The corresponding truth table is available in Table. 0.

Z	Y	X	W	**D**	**C**	**B**	**A**
0	0	0	0	**0**	**0**	**0**	**1**
0	0	0	1	**0**	**0**	**1**	**0**
0	0	1	0	**0**	**0**	**1**	**1**
0	0	1	1	**0**	**1**	**0**	**0**
0	1	0	0	**0**	**1**	**0**	**1**
0	1	0	1	**0**	**1**	**1**	**0**
0	1	1	0	**0**	**1**	**1**	**1**
0	1	1	1	**1**	**0**	**0**	**0**
1	0	0	0	**1**	**0**	**0**	**1**
1	0	0	1	**0**	**0**	**0**	**0**

TABLE 0

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2 Karnaugh Map

Using Boolean logic, output A in Table 0 can be expressed in terms of the inputs W, X, Y, Z as

$$A = W'X'Y'Z' + W'XY'Z' + W'X'YZ' + W'XYZ' + W'X'Y'Z \quad (2.1)$$

1. K-Map for A: The expression in (2.1) can be minimized using the K-map in Fig. 1. In Fig. 1, the *implicants* in boxes 0,2,4,6 result in $W'Z'$. The implicants in boxes 0,8 result in $W'X'Y'$. Thus, after minimization using Fig. 2.2, (2.1) can be expressed as
$$A = W'Z' + W'X'Y' \quad (2.2)$$
Using the fact that
$$\begin{aligned} X + X' &= 1 \\ XX' &= 0, \end{aligned} \quad (2.3)$$
derive (2.2) from (2.1) algebraically.
2. K-Map for B: From Table 0, using boolean logic,
$$B = WX'Y'Z' + W'XY'Z' + WX'YZ' + W'XYZ' \quad (2.4)$$
Show that (2.4) can be reduced to
$$B = WX'Z' + W'XZ' \quad (2.5)$$
using Fig. 2.
3. Derive (2.5) from (2.4) algebraically using (2.3).
4. K-Map for C: From Table 0, using boolean logic,
$$C = WXY'Z' + W'X'YZ' + WX'YZ' + W'XYZ' \quad (2.6)$$
Show that (2.6) can be reduced to
$$C = WXY'Z' + X'YZ' + W'YZ' \quad (2.7)$$

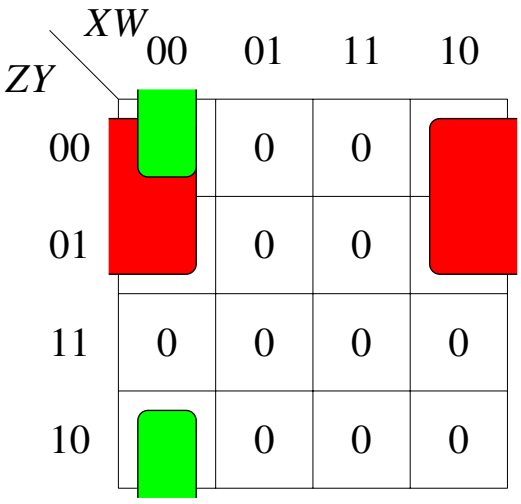

Fig. 1: K-map for A.

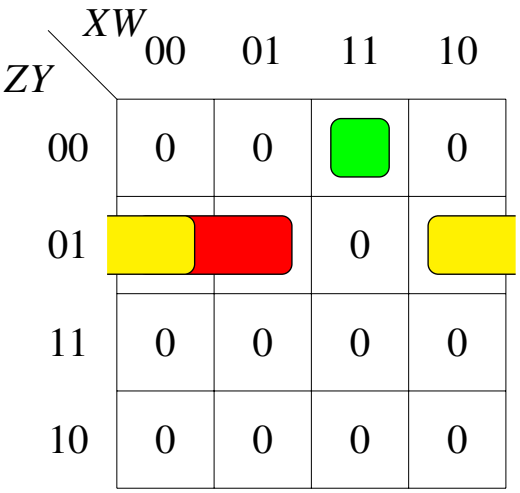

Fig. 4: K-map for C.

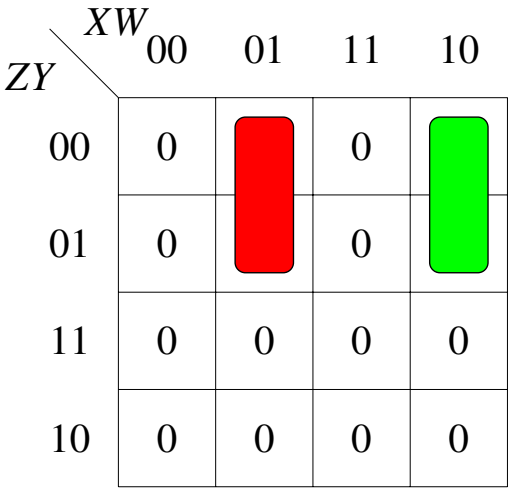

Fig. 2: K-map for B.

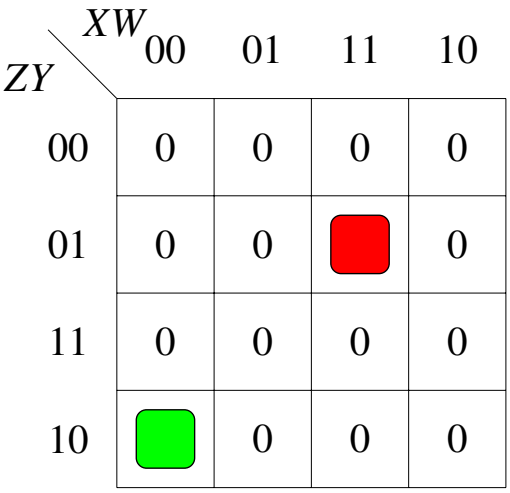

Fig. 6: K-map for D.

using Fig. 4.

5. Derive (2.7) from (2.6) algebraically using (2.3).
6. K-Map for D: From Table 0, using boolean logic,

$$D = WXYZ' + W'X'Y'Z \tag{2.8}$$

7. Minimize (2.8) using Fig. 6.
8. Download the code in

```
wget https://raw.githubusercontent.com/gadepall
    /arduino/master/7447/codes/inc_dec/
    inc_dec.ino
```

and modify it using the K-Map equations for A,B,C and D. Execute and verify.

9. Display Decoder: Table 9 is the truth table for the display decoder in Fig. **??**. Use K-maps to obtain the minimized expressions for a, b, c, d, e, f, g in terms of A, B, C, D with and without don't care conditions.

D	C	B	A	**a**	**b**	**c**	**d**	**e**	**f**	**g**	Decimal
0	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	**1**	0
0	0	0	1	**1**	**0**	**0**	**1**	**1**	**1**	**1**	1
0	0	1	0	**0**	**0**	**1**	**0**	**0**	**1**	**0**	2
0	0	1	1	**0**	**0**	**0**	**0**	**1**	**1**	**0**	3
0	1	0	0	**1**	**0**	**0**	**1**	**1**	**0**	**0**	4
0	1	0	1	**0**	**1**	**0**	**0**	**1**	**0**	**0**	5
0	1	1	0	**0**	**1**	**0**	**0**	**0**	**0**	**0**	6
0	1	1	1	**0**	**0**	**0**	**1**	**1**	**1**	**1**	7
1	0	0	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	8
1	0	0	1	**0**	**0**	**0**	**1**	**1**	**0**	**0**	9

TABLE 9: Truth table for display decoder.