CHAPTER 13

# Properties of Triangle

## Section-A JEE Advanced/ IIT-JEE

### A Fill in the Blanks

1. In a $\Delta ABC$, $\angle A = 90°$ and $AD$ is an altitude. Complete the relation

$$\frac{BD}{BA} = \frac{AB}{(....)}$$ *(1980)*

2. $ABC$ is a triangle, $P$ is a point on $AB$, and $Q$ is point on $AC$ such that $\angle AQP = \angle ABC$. Complete the relation

$$\frac{\text{area of } \Delta APQ}{\text{area of } \Delta ABC} = \frac{(...)}{AC^2}$$ *(1980)*

3. $ABC$ is a triangle with $\angle B$ greater than $\angle C$. $D$ and $E$ are points on $BC$ such that $AD$ is perpendicular to $BC$ and $AE$ is the bisector of angle $A$. Complete the relation *(1980)*

$$\angle DAE = \frac{1}{2}[(\ ) - \angle C]$$

4. The set of all real numbers $a$ such that $a^2 + 2a$, $2a + 3$ and $a^2 + 3a + 8$ are the sides of a triangle is .................. *(1985 - 2 Marks)*

5. In a triangle $ABC$, if $\cot A$, $\cot B$, $\cot C$ are in A.P., then $a^2, b^2, c^2$, are in .................. progression. *(1985 - 2 Marks)*

6. A polygon of nine sides, each of length 2, is inscribed in a circle. The radius of the circle is .................. *(1987 - 2 Marks)*

7. If the angles of a triangle are 30° and 45° and the included side is $(\sqrt{3}+1)$ cms, then the area of the triangle is .................. *(1988 - 2 Marks)*

8. If in a triangle $ABC$, $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ca}$, then the value of the angle $A$ is .................. degrees. *(1993 - 2 Marks)*

9. In a triangle $ABC$, $AD$ is the altitude from $A$. Given $b > c$, $\angle C = 23°$ and $AD = \dfrac{abc}{b^2 - c^2}$ then $\angle B =$ .................. *(1994 - 2 Marks)*

10. A circle is inscribed in an equilateral triangle of side a. The area of any square inscribed in this circle is .................. *(1994 - 2 Marks)*

11. In a triangle $ABC$, $a : b : c = 4 : 5 : 6$. The ratio of the radius of the circumcircle to that of the incircle is .................. *(1996 - 1 Mark)*

### C MCQs with One Correct Answer

1. If the bisector of the angle $P$ of a triangle $PQR$ meets $QR$ in $S$, then *(1979)*
   (a) $QS = SR$ (b) $QS : SR = PR : PQ$
   (c) $QS : SR = PQ : PR$ (d) None of these

2. From the top of a light-house 60 metres high with its base at the sea-level, the angle of depression of a boat is 15°. The distance of the boat from the foot of the light house is *(1983 - 1 Mark)*
   (a) $\left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ 60 metres (b) $\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ 60 metres
   (c) $\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$ metres (d) none of these

3. In a triangle $ABC$, angle $A$ is greater than angle $B$. If the measures of angles $A$ and $B$ satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is *(1990 - 2 Marks)*
   (a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{5\pi}{6}$

4. If the lengths of the sides of triangle are 3, 5, 7 then the largest angle of the triangle is *(1994)*
   (a) $\dfrac{\pi}{2}$ (b) $\dfrac{5\pi}{6}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{3\pi}{4}$

5. In a triangle $ABC$, $\angle B = \dfrac{\pi}{3}$ and $\angle C = \dfrac{\pi}{4}$. Let $D$ divide $BC$ internally in the ratio 1 : 3 then $\dfrac{\sin \angle BAD}{\sin \angle CAD}$ is equal to *(1995S)*
   (a) $\dfrac{1}{\sqrt{6}}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{\sqrt{3}}$ (d) $\sqrt{\dfrac{2}{3}}$

6. In a triangle $ABC$, $2ac\sin\dfrac{1}{2}(A - B + C) =$ *(2000S)*
   (a) $a^2 + b^2 - c^2$ (b) $c^2 + a^2 - b^2$
   (c) $b^2 - c^2 - a^2$ (d) $c^2 - a^2 - b^2$

7. In a triangle $ABC$, let $\angle C = \frac{\pi}{2}$. If $r$ is the inradius and $R$ is the circumradius of the triangle, then $2(r+R)$ is equal to *(2000S)*
(a) $a+b$ (b) $b+c$ (c) $c+a$ (d) $a+b+c$

8. A pole stands vertically inside a triangular park $\Delta ABC$. If the angle of elevation of the top of the pole from each corner of the park is same, then in $\Delta ABC$ the foot of the pole is at the *(2000S)*
(a) centroid (b) circumcentre
(c) incentre (d) orthocentre

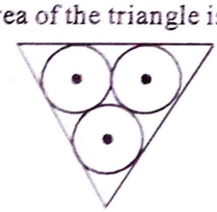
9. A man from the top of a 100 metres high tower sees a car moving towards the tower at an angle of depression of 30°. After some time, the angle of depression becomes 60°. The distance (in metres) travelled by the car during this time is
(a) $100\sqrt{3}$ (b) $200\sqrt{3}/3$ *(2001S)*
(c) $100\sqrt{3}/3$ (d) $200\sqrt{3}$

10. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle $ABC$ ($R$ being the radius of the circumcircle)? *(2002S)*
(a) $a, \sin A, \sin B$ (b) $a, b, c$
(c) $a, \sin B, R$ (d) $a, \sin A, R$

11. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is *(2003S)*
(a) $\sqrt{3}:(2+\sqrt{3})$ (b) $1:6$
(c) $1:2+\sqrt{3}$ (d) $2:3$

12. The sides of a triangle are in the ratio $1 : \sqrt{3} : 2$, then the angles of the triangle are in the ratio *(2004S)*
(a) $1:3:5$ (b) $2:3:4$ (c) $3:2:1$ (d) $1:2:3$

13. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is *(2005S)*



(a) $4+2\sqrt{3}$ (b) $6+4\sqrt{3}$
(c) $12+\frac{7\sqrt{3}}{4}$ (d) $3+\frac{7\sqrt{3}}{4}$

14. In a triangle $ABC$, $a, b, c$ are the lengths of its sides and $A, B, C$ are the angles of triangle $ABC$. The correct relation is given by *(2005S)*
(a) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\frac{A}{2}$
(b) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\frac{B-C}{2}$
(c) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\frac{A}{2}$
(d) $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\frac{B+C}{2}$

15. One angle of an isosceles $\Delta$ is 120° and radius of its incircle $= \sqrt{3}$. Then the area of the triangle in sq. units is *(2006 - 3M, -1)*
(a) $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$
(c) $12+7\sqrt{3}$ (d) $4\pi$

16. Let $ABCD$ be a quadrilateral with area 18, with side $AB$ parallel to the side $CD$ and $2AB = CD$. Let $AD$ be perpendicular to $AB$ and $CD$. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is *(2007 - 3 marks)*
(a) 3 (b) 2 (c) $\frac{3}{2}$ (d) 1

17. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is *(2010)*
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\sqrt{3}$

18. Let $PQR$ be a triangle of area $\Delta$ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$; where $a$, $b$, and $c$ are the lengths of the sides of the triangle opposite to the angles at $P, Q$ and $R$ respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals. *(2012)*
(a) $\frac{3}{4\Delta}$ (b) $\frac{45}{4\Delta}$ (c) $\left(\frac{3}{4\Delta}\right)^2$ (d) $\left(\frac{45}{4\Delta}\right)^2$

19. In a triangle the sum of two sides is $x$ and the product of the same sides is $y$. If $x^2 - c^2 = y$, where $c$ is the third side of the triangle, then the ratio of the in radius to the circum-radius of the triangle is *(JEE Adv. 2014)*
(a) $\frac{3y}{2x(x+c)}$ (b) $\frac{3y}{2c(x+c)}$
(c) $\frac{3y}{4x(x+c)}$ (d) $\frac{3y}{4c(x+c)}$

## D MCQs with One or More than One Correct

1. There exists a triangle $ABC$ satisfying the conditions *(1986 - 2 Marks)*
(a) $b\sin A = a, A < \pi/2$
(b) $b\sin A > a, A > \pi/2$
(c) $b\sin A > a, A < \pi/2$
(d) $b\sin A < a, A < \pi/2, b > a$
(e) $b\sin A < a, A > \pi/2, b = a$

2. In a triangle, the lengths of the two larger sides are 10 and 9, respectively. If the angles are in A.P. Then the length of the third side can be *(1987 - 2 Marks)*
(a) $5-\sqrt{6}$ (b) $3\sqrt{3}$
(c) 5 (d) $5+\sqrt{6}$ (e) none

3. If in a triangle $PQR$, $\sin P$, $\sin Q$, $\sin R$ are in A.P., then *(1998 - 2 Marks)*

(a) the altitudes are in A.P. (b) the altitudes are in H.P.
(c) the medians are in G.P. (d) the medians are in A.P.

4. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments $A_0A_1$, $A_0A_2$ and $A_0A_4$ is *(1998 - 2 Marks)*

(a) $\frac{3}{4}$ (b) $3\sqrt{3}$
(c) 3 (d) $\frac{3\sqrt{3}}{2}$

5. In $\Delta ABC$, internal angle bisector of $\angle A$ meets side $BC$ in $D$. $DE \perp AD$ meets $AC$ in $E$ and $AB$ in $F$. Then *(2006 - 5M, –1)*

(a) AE is HM of $b$ & $c$ (b) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$
(c) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ (d) $\Delta AEF$ is isosceles

6. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of $x$ for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are) *(2010)*

(a) $-(2+\sqrt{3})$ (b) $1+\sqrt{3}$
(c) $2+\sqrt{3}$ (d) $4\sqrt{3}$

7. In a triangle $PQR$, $P$ is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides $PQ$, $QR$ and $RP$ at $N$, $L$ and $M$ respectively, such that the lengths of $PN$, $QL$ and $RM$ are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are) *(JEE Adv. 2013)*

(a) 16 (b) 18
(c) 24 (d) 22

8. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then *(JEE Adv. 2016)*

(a) area of the triangle XYZ is $6\sqrt{6}$

(b) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(c) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(d) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

9. In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE? *(JEE Adv. 2018)*

(a) $\angle QPR = 45°$

(b) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$

(c) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(d) The area of the circumcircle of the triangle PQR is $100\,\pi$.

10. In a non-right angled triangle $\Delta PQR$, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, RS and PE intersect at O. If $p = \sqrt{3}, q = 1$ and the radius of the circumcircle of the $\Delta PQR$ equals 1, then which of the following options is/are correct ?

(a) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

(b) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

(c) Length of OE $= \frac{1}{6}$

(d) Length of RS $= \frac{\sqrt{7}}{2}$

## E Subjective Problems

1. A triangle $ABC$ has sides $AB = AC = 5$ cm and $BC = 6$ cm Triangle $A'B'C'$ is the reflection of the triangle $ABC$ in a line parallel to $AB$ placed at a distance 2 cm from $AB$, outside the triangle $ABC$. Triangle $A''B''C''$ is the reflection of the triangle $A'B'C'$ in a line parallel to $B'C'$ placed at a distance of 2 cm from $B'C'$ outside the triangle $A'B'C'$. Find the distance between $A$ and $A''$. *(1978)*

2. (a) If a circle is inscribed in a right angled triangle $ABC$ with the right angle at $B$, show that the diameter of the circle is equal to $AB + BC - AC$.

(b) If a triangle is inscribed in a circle, then the product of any two sides of the triangle is equal to the product of the diameter and the perpendicular distance of the third side from the opposite vertex. Prove the above statement. *(1979)*

3. (a) A balloon is observed simultaneously from three points $A$, $B$ and $C$ on a straight road directly beneath it. The angular elevation at $B$ is twice that at $A$ and the angular elevation at $C$ is thrice that at $A$. If the distance between $A$ and $B$ is a and the distance between $B$ and $C$ is $b$, find the height of the balloon in terms of $a$ and $b$.

(b) Find the area of the smaller part of a disc of radius 10 cm, cut off by a chord $AB$ which subtends an angle of $22\frac{1}{2}°$ at the circumference. *(1979)*

4. $ABC$ is a triangle. $D$ is the middle point of $BC$. If $AD$ is perpendicular to $AC$, then prove that *(1980)*

$$\cos A\,\cos C = \frac{2(c^2 - a^2)}{3ac}$$

5. $ABC$ is a triangle with $AB = AC$. $D$ is any point on the side $BC$. $E$ and $F$ are points on the side $AB$ and $AC$, respectively, such that $DE$ is parallel to $AC$, and $DF$ is parallel to $AB$. Prove that

$$DF + FA + AE + ED = AB + AC$$ *(1980)*

6. (i) $PQ$ is a vertical tower. $P$ is the foot and $Q$ is the top of the tower. $A, B, C$ are three points in the horizontal plane through $P$. The angles of elevation of $Q$ from $A, B, C$ are equal, and each is equal to $\theta$. The sides of the triangle $ABC$ are $a, b, c$; and the area of the triangle $ABC$ is $\Delta$. Show that the height of the tower is $\dfrac{abc\tan\theta}{4\Delta}$.

(ii) $AB$ is a vertical pole. The end $A$ is on the level ground. $C$ is the middle point of $AB$. $P$ is a point on the level ground. The portion $CB$ subtends an angle $\beta$ at $P$. If $AP = n\,AB$, then show that $\tan\beta = \dfrac{n}{2n^2+1}$ *(1980)*

7. Let the angles $A, B, C$ of a triangle $ABC$ be in A.P. and let $b : c = \sqrt{3} : \sqrt{2}$. Find the angle $A$. *(1981 - 2 Marks)*

8. A vertical pole stands at a point $Q$ on a horizontal ground. $A$ and $B$ are points on the ground, $d$ meters apart. The pole subtends angles $\alpha$ and $\beta$ at $A$ and $B$ respectively. $AB$ subtends an angle $\gamma$ at $Q$. Find the height of the pole. *(1982 - 3 Marks)*

9. Four ships $A, B, C$ and $D$ are at sea in the following relative positions : $B$ is on the straight line segment $AC$, $B$ is due North of $D$ and $D$ is due west of $C$. The distance between $B$ and $D$ is 2 km. $\angle BDA = 40°$, $\angle BCD = 25°$. What is the distance between $A$ and $D$? [Take $\sin 25° = 0.423$] *(1983 - 3 Marks)*

10. The ex-radii $r_1, r_2, r_3$ of $\Delta\,ABC$ are in H.P. Show that its sides $a, b, c$ are in A.P. *(1983 - 3 Marks)*

11. For a triangle $ABC$ it is given that $\cos A + \cos B + \cos C = \dfrac{3}{2}$. Prove that the triangle is equilateral. *(1984 - 4 Marks)*

12. With usual notation, if in a triangle $ABC$;

$\dfrac{b+c}{11} = \dfrac{c+a}{12} = \dfrac{a+b}{13}$ then prove that $\dfrac{\cos A}{7} = \dfrac{\cos B}{19} = \dfrac{\cos C}{25}$. *(1984 - 4 Marks)*

13. A ladder rests against a wall at an angle $\alpha$ to the horizontal. Its foot is pulled away from the wall through a distance $a$, so that it slides a distance $b$ down the wall making an angle $\beta$ with the horizontal. Show that $a = b\tan\frac{1}{2}(\alpha+\beta)$ *(1985 - 5 Marks)*

14. In a triangle $ABC$, the median to the side $BC$ is of length $\dfrac{1}{\sqrt{11-6\sqrt{3}}}$ *(1985 - 5 Marks)* and it divides the angle $A$ into angles 30° and 45°. Find the length of the side $BC$.

15. If in a triangle $ABC$, $\cos A\cos B + \sin A\sin B\sin C = 1$, Show that $a : b : c = 1 : 1 : \sqrt{2}$ *(1986 - 5 Marks)*

16. A sign-post in the form of an isosceles triangle $ABC$ is mounted on a pole of height h fixed to the ground. The base $BC$ of the triangle is parallel to the ground. A man standing on the ground at a distance d from the sign-post finds that the top vertex $A$ of the triangle subtends an angle $\beta$ and either of the other two vertices subtends the same angle $\alpha$ at his feet. Find the area of the triangle. *(1988 - 5 Marks)*

17. $ABC$ is a triangular park with $AB = AC = 100$ m. A television tower stands at the midpoint of $BC$. The angles of elevation of the top of the tower at $A, B, C$ are 45°, 60°, 60°, respectively. Find the height of the tower. *(1989 - 5 Marks)*

18. A vertical tower $PQ$ stands at a point $P$. Points $A$ and $B$ are located to the South and East of $P$ respectively. $M$ is the mid point of $AB$. $PAM$ is an equilateral triangle; and $N$ is the foot of the perpendicular from $P$ on $AB$. Let $AN = 20$ metres and the angle of elevation of the top of the tower at $N$ is $\tan^{-1}(2)$. Determine the height of the tower and the angles of elevation of the top of the tower at $A$ and $B$. *(1990 - 4 Marks)*

19. The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle. *(1991 - 4 Marks)*

20. In a triangle of base a the ratio of the other two sides is $r\,(<1)$. Show that the altitude of the triangle is less than of equal to $\dfrac{ar}{1-r^2}$ *(1991 - 4 Marks)*

21. A man notices two objects in a straight line due west. After walking a distance $c$ due north he orserves that the objects subtend an angle $\alpha$ at his eye; and, after walking a further distance $2c$ due north, an angle $\beta$. Show that the distance between the objects is $\dfrac{8c}{3\cot\beta - \cot\alpha}$; the height of the man is being ignored. *(1991 - 4 Marks)*

22. Three circles touch the one another externally. The tangent at their point of contact meet at a point whose distance from a point of contanct is 4. Find the ratio of the product of the radii to the sum of the radii of the circles. *(1992 - 4 Marks)*

23. An observer at $O$ notices that the angle of elevation of the top of a tower is 30°. The line joining $O$ to the base of the tower makes an angle of $\tan^{-1}(1/\sqrt{2})$ with the North and is inclined Eastwards. The observer travels a distance of 300 meters towards the North to a point A and finds the tower to his East. The angle of elevation of the top of the tower at $A$ is $\phi$, Find $\phi$ and the height of the tower *(1993 - 5 Marks)*

24. A tower $AB$ leans towards west making an angle $\alpha$ with the vertical. The angular elevation of $B$, the topmost point of the tower is $\beta$ as observed from a point $C$ due west of $A$ at a distance $d$ from $A$. If the angular elevation of $B$ from a point $D$ due east of $C$ at a distance $2d$ from $C$ is $\gamma$, then prove that $2\tan\alpha = -\cot\beta + \cot\gamma$. *(1994 - 4 Marks)*

25. Let $A_1, A_2, \ldots\ldots\ldots, A_n$ be the vertices of an $n$-sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, Find the value of $n$. *(1994 - 4 Marks)*

26. Consider the following statements concerning a triangle $ABC$ *(1994 - 5 Marks)*
(i) The sides $a, b, c$ and area $\Delta$ are rational.
(ii) $a, \tan\frac{B}{2}, \tan\frac{C}{2}$ are rational.
(iii) $a, \sin A, \sin B, \sin C$ are rational.
Prove that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i)

27. A bird flies in a circle on a horizontal plane. An observer stands at a point on the ground. Suppose 60° and 30° are the maximum and the minimum angles of elevation of the bird and that they occur when the bird is at the points $P$ and $Q$ respectively on its path. Let $\theta$ be the angle of elevation of the bird when it is a point on the arc of the circle exactly midway between $P$ and $Q$. Find the numerical value of $\tan^2\theta$.(Assume that the observer is not inside the vertical projection of the path of the bird.) *(1998 - 8 Marks)*

28. Prove that a triangle $ABC$ is equilateral if and only if $\tan A + \tan B + \tan C = 3\sqrt{3}$. *(1998 - 8 Marks)*

29. Let $ABC$ be a triangle having $O$ and $I$ as its circumcenter and in centre respectively. If $R$ and $r$ are the circumradius and the inradius, respectively, then prove that $(IO)^2 = R^2 - 2Rr$. Further show that the triangle BIO is a right-angled triangle if and only if $b$ is arithmetic mean of $a$ and $c$. *(1999 - 10 Marks)*

30. Let $ABC$ be a triangle with incentre $I$ and inradius $r$. Let $D,E,F$ be the feet of the perpendiculars from $I$ to the sides $BC$, $CA$ and $AB$ respectively. If $r_1, r_2$ and $r_3$ are the radii of circles inscribed in the quadrilaterals $AFIE$, $BDIF$ and $CEID$ respecitvely, prove that

$$\frac{r_1}{r-r_1} + \frac{r_2}{r-r_2} + \frac{r_3}{r-r_3} = \frac{r_1r_2r_3}{(r-r_1)(r-r_2)(r-r_3)}.$$

*(2000 - 7 Marks)*

31. If $\Delta$ is the area of a triangle with side lengths $a, b, c$, then show that $\Delta \le \frac{1}{4}\sqrt{(a+b+c)abc}$. Also show that the equality occurs in the above inequality if and only if $a = b = c$. *(2001 - 6 Marks)*

32. If $I_n$ is the area of $n$ sided regular polygon inscribed in a circle of unit radius and $O_n$ be the area of the polygon circumscribing the given circle, prove that

$$I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right).$$

*(2003 - 4 Marks)*

## I Integer Value Correct Type

1. Let $ABC$ and $ABC'$ be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30°$. The absolute value of the difference between the areas of these triangles is *(2009)*

2. Consider a triangle $ABC$ and let $a$, $b$ and $c$ denote the lengths of the sides opposite to vertices $A$, $B$ and $C$ respectively. Suppose $a=6$, $b=10$ and the area of the triangle is $15\sqrt{3}$, if $\angle ACB$ is obtuse and if $r$ denotes the radius of the incircle of the triangle, then $r^2$ is equal to *(2010)*

# Section-B JEE Main / AIEEE

1. The sides of a triangle are $3x+4y$, $4x+3y$ and $5x+5y$ where $x, y > 0$ then the triangle is [2002]
(a) right angled
(b) obtuse angled
(c) equilateral
(d) none of these

2. In a triangle with sides $a, b, c, r_1 > r_2 > r_3$ (which are the ex-radii) then [2002]
(a) $a > b > c$
(b) $a < b < c$
(c) $a > b$ and $b < c$
(d) $a < b$ and $b > c$

3. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a , is [2003]
(a) $\frac{a}{4}\cot\left(\frac{\pi}{2n}\right)$
(b) $a\cot\left(\frac{\pi}{n}\right)$
(c) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$
(d) $a\cot\left(\frac{\pi}{2n}\right)$.

4. In a triangle ABC, medians AD and BE are drawn. If AD = 4, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the $\Delta ABC$ is
(a) $\frac{64}{3}$
(b) $\frac{8}{3}$
(c) $\frac{16}{3}$
(d) $\frac{32}{3\sqrt{3}}$ [2003]

5. If in a $\Delta ABC$ $a\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{A}{2}\right) = \frac{3b}{2}$, then the sides $a, b$ and $c$ [2003]
(a) satisfy $a + b = c$
(b) are in A.P
(c) are in G.P
(d) are in H.P.

6. The sides of a triangle are $\sin\alpha$, $\cos\alpha$ and $\sqrt{1+\sin\alpha\cos\alpha}$ for some $0<\alpha<\frac{\pi}{2}$. Then the greatest angle of the triangle is [2004]
(a) $150°$ (b) $90°$ (c) $120°$ (d) $60°$

7. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is $60°$ and when he retires 40 meters away from the tree the angle of elevation becomes $30°$. The breadth of the river is [2004]
(a) $60\,m$ (b) $30\,m$ (c) $40\,m$ (d) $20\,m$

8. In a triangle $ABC$, let $\angle C=\frac{\pi}{2}$. If $r$ is the inradius and $R$ is the circumradius of the triangle $ABC$, then $2(r+R)$ equals [2005]
(a) $b+c$ (b) $a+b$ (c) $a+b+c$ (d) $c+a$

9. If in a $\Delta ABC$, the altitudes from the vertices $A, B, C$ on opposite sides are in H.P, then $\sin A$, $\sin B$, $\sin C$ are in [2005]
(a) G.P. (b) A. P. (c) A.P-G.P. (d) H. P

10. A tower stands at the centre of a circular park. $A$ and $B$ are two points on the boundary of the park such that $AB\,(=a)$ subtends an angle of $60°$ at the foot of the tower, and the angle of elevation of the top of the tower from A or B is $30°$. The height of the tower is [2007]
(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$ (c) $2a/\sqrt{3}$ (d) $2a\sqrt{3}$.

11. $AB$ is a vertical pole with $B$ at the ground level and $A$ at the top. $A$ man finds that the angle of elevation of the point $A$ from a certain point $C$ on the ground is $60°$. He moves away from the pole along the line $BC$ to a point $D$ such that $CD=7$ m. From $D$ the angle of elevation of the point $A$ is $45°$. Then the height of the pole is [2008]
(a) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$ (b) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)m$

(c) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)m$ (d) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$

12. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A *false* statement among the following is [2010]
(a) There is a regular polygon with $\frac{r}{R}=\frac{1}{\sqrt{2}}$
(b) There is a regular polygon with $\frac{r}{R}=\frac{2}{3}$
(c) There is a regular polygon with $\frac{r}{R}=\frac{\sqrt{3}}{2}$
(d) There is a regular polygon with $\frac{r}{R}=\frac{1}{2}$

13. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point $O$ on the ground is $45°$. It flies off horizontally straight away from the point $O$. After one second, the elevation of the bird from $O$ is reduced to $30°$. Then the speed (in m/s) of the bird is [JEE M 2014]
(a) $20\sqrt{2}$ (b) $20\left(\sqrt{3}-1\right)$
(c) $40\left(\sqrt{2}-1\right)$ (d) $40\left(\sqrt{3}-\sqrt{2}\right)$

14. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are $30°$, $45°$ and $60°$ respectively, then the ratio, AB : BC, is : [JEE M 2015]
(a) $1:\sqrt{3}$ (b) $2:3$
(c) $\sqrt{3}:1$ (d) $\sqrt{3}:\sqrt{2}$

15. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC=\beta$, then $\tan\beta$ is equal to: [JEE M 2017]
(a) $\frac{4}{9}$ (b) $\frac{6}{7}$
(c) $\frac{1}{4}$ (d) $\frac{2}{9}$

16. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45°$, $30°$ and $30°$, then the height of the tower (in m) is : [JEE M 2018]
(a) 50 (b) $100\sqrt{3}$
(c) $50\sqrt{2}$ (d) 100