CHAPTER 2

Complex Numbers

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If the expression (1987 - 2 Marks)

$$\frac{\left[\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)+i\tan(x)\right]}{\left[1+2\,i\,\sin\left(\frac{x}{2}\right)\right]}$$

is real, then the set of all possible values of x is ....

2. For any two complex numbers z_1, z_2 and any real number a and b. (1988 - 2 Marks)

$|az_1 - bz_2|^2 + |bz_1 + az_2|^2 =$ ....

3. If a, b, c, are the numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then $a =$ ....and $b =$ .... (1989 - 2 Marks)

4. $ABCD$ is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD = 2AC$. If the points D and M represent the complex numbers $1 + i$ and $2 - i$ respectively, then A represents the complex number ....or......... (1993 - 2 Marks)

5. Suppose Z_1, Z_2, Z_3 are the vertices of an equilateral triangle inscribed in the circle $|Z| = 2$. If $Z_1 = 1 + i\sqrt{3}$ then $Z_2 =$ ...., $Z_3 =$ .... (1994 - 2 Marks)

6. The value of the expression $1\cdot(2-\omega)(2-\omega^2)+2\cdot(3-\omega)(3-\omega^2)+....+(n-1).(n-\omega)(n-\omega^2)$, where ω is an imaginary cube root of unity, is..... (1996 - 2 Marks)

B True / False

1. For complex number $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, we write $z_1 \cap z_2$, if $x_1 \le x_2$ and $y_1 \le y_2$. Then for all complex numbers z with $1 \cap z$, we have $\frac{1-z}{1+z} \cap 0$. (1981 - 2 Marks)

2. If the complex numbers, Z_1, Z_2 and Z_3 represent the vertices of an equilateral triangle such that $|Z_1| = |Z_2| = |Z_3|$ then $Z_1 + Z_2 + Z_3 = 0$. (1984 - 1 Mark)

3. If three complex numbers are in A.P. then they lie on a circle in the complex plane. (1985 - 1 Mark)

4. The cube roots of unity when represented on Argand diagram form the vertices of an equilateral triangle. (1988 - 1 Mark)

C MCQs with One Correct Answer

1. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x-1)^3 + 8 = 0$ are (1979)
   (a) $-1, 1+2\omega, 1+2\omega^2$ (b) $-1, 1-2\omega, 1-2\omega^2$
   (c) $-1, -1, -1$ (d) None of these

2. The smallest positive integer n for which (1980)

   $\left(\frac{1+i}{1-i}\right)^n = 1$ is

   (a) $n=8$ (b) $n=16$
   (c) $n=12$ (d) none of these

3. The complex numbers $z = x + iy$ which satisfy the equation

   $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on (1981 - 2 Marks)

   (a) the x-axis
   (b) the straight line $y = 5$
   (c) a circle passing through the origin
   (d) none of these

4. If $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then (1982 - 2 Marks)
   (a) $\mathrm{Re}(z) = 0$ (b) $\mathrm{Im}(z) = 0$
   (c) $\mathrm{Re}(z) > 0, \mathrm{Im}(z) > 0$ (d) $\mathrm{Re}(z) > 0, \mathrm{Im}(z) < 0$

5. The inequality $|z-4| < |z-2|$ represents the region given by (1982 - 2 Marks)
   (a) $\mathrm{Re}(z) \ge 0$ (b) $\mathrm{Re}(z) < 0$
   (c) $\mathrm{Re}(z) > 0$ (d) none of these

6. If $z = x + iy$ and $\omega = (1 - iz)/(z - i)$, then $|\omega| = 1$ implies that, in the complex plane, (1983 - 1 Mark)
   (a) z lies on the imaginary axis
   (b) z lies on the real axis
   (c) z lies on the unit circle
   (d) None of these

7. The points $z_1, z_2, z_3\, z_4$ in the complex plane are the vertices of a parallelogram taken in order if and only if *(1983 - 1 Mark)*

(a) $z_1+z_4=z_2+z_3$ (b) $z_1+z_3=z_2+z_4$
(c) $z_1+z_2=z_3+z_4$ (d) None of these

8. If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that $c=(1-r)\,a+rb$ and $w=(1-r)u+rv$, where r is a complex number, then the two triangles *(1985 - 2 Marks)*

(a) have the same area (b) are similar
(c) are congruent (d) none of these

9. If $\omega\ (\neq 1)$ is a cube root of unity and $(1+\omega)^7=A+B\,\omega$ then A and B are respectively *(1995S)*

(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) $-1, 1$

10. Let z and ω be two non zero complex numbers such that $|z|=|\omega|$ and $\operatorname{Arg} z+\operatorname{Arg}\omega=\pi$, then z equals *(1995S)*

(a) ω (b) $-\omega$ (c) $\overline{\omega}$ (d) $-\overline{\omega}$

11. Let z and ω be two complex numbers such that $|z|\leq 1$, $|\omega|\leq 1$ and $|z+i\omega|=|z-i\overline{\omega}|=2$ then z equals *(1995S)*

(a) 1 or i (b) i or $-i$ (c) 1 or -1 (d) i or -1

12. For positive integers n_1, n_2 the value of the expression

$(1+i)^{n_1}+(1+i^3)^{n_1}+(1+i^5)^{n_2}+(1+i^7)^{n_2}$, where $i=\sqrt{-1}$ is a real number if and only if *(1996 - 1 Marks)*

(a) $n_1=n_2+1$ (b) $n_1=n_2-1$
(c) $n_1=n_2$ (d) $n_1>0, n_2>0$

13. If $i=\sqrt{-1}$, then $4+5\left(-\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}\right)^{334}+3\left(-\dfrac{1}{2}+\dfrac{i\sqrt{3}}{2}\right)^{365}$

is equal to *(1999 - 2 Marks)*

(a) $1-i\sqrt{3}$ (b) $-1+i\sqrt{3}$ (c) $i\sqrt{3}$ (d) $-i\sqrt{3}$

14. If $\arg(z)<0$, then $\arg(-z)-\arg(z)=$ *(2000S)*

(a) π (b) $-\pi$ (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{2}$

15. If z_1, z_2 and z_3 are complex numbers such that *(2000S)*

$|z_1|=|z_2|=|z_3|=\left|\dfrac{1}{z_1}+\dfrac{1}{z_2}+\dfrac{1}{z_3}\right|=1,$ then $|z_1+z_2+z_3|$ is

(a) equal to 1 (b) less than 1
(c) greater than 3 (d) equal to 3

16. Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form *(2001S)*

(a) $4k+1$ (b) $4k+2$ (c) $4k+3$ (d) $4k$

17. The complex numbers z_1, z_2 and z_3 satisfying $\dfrac{z_1-z_3}{z_2-z_3}=\dfrac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is *(2001S)*

(a) of area zero
(b) right-angled isosceles
(c) equilateral
(d) obtuse-angled isosceles

18. For all complex numbers z_1, z_2 satisfying $|z_1|=12$ and $|z_2-3-4i|=5$, the minimum value of $|z_1-z_2|$ is *(2002S)*

(a) 0 (b) 2 (c) 7 (d) 17

19. If $|z|=1$ and $\omega=\dfrac{z-1}{z+1}$ (where $z\neq -1$), then $\operatorname{Re}(\omega)$ is *(2003S)*

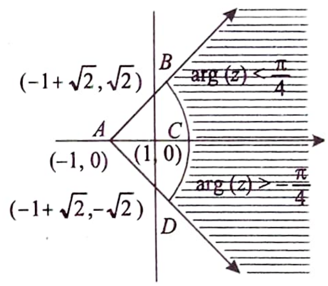
(a) 0 (b) $-\dfrac{1}{|z+1|^2}$

(c) $\left|\dfrac{z}{z+1}\right|\cdot\dfrac{1}{|z+1|^2}$ (d) $\dfrac{\sqrt{2}}{|z+1|^2}$

20. If $\omega\ (\neq 1)$ be a cube root of unity and $(1+\omega^2)^n=(1+\omega^4)^n$, then the least positive value of n is *(2004S)*

(a) 2 (b) 3 (c) 5 (d) 6

21. The locus of z which lies in shaded region (excluding the boundaries) is best represented by *(2005S)*



(a) $z:|z+1|>2$ and $|\arg(z+1)|<\pi/4$
(b) $z:|z-1|>2$ and $|\arg(z-1)|<\pi/4$
(c) $z:|z+1|<2$ and $|\arg(z+1)|<\pi/2$
(d) $z:|z-1|<2$ and $|\arg(z+1)|<\pi/2$

22. a, b, c are integers, not all simultaneously equal and ω is cube root of unity ($\omega\neq 1$), then minimum value of $|a+b\omega+c\omega^2|$ is *(2005S)*

(a) 0 (b) 1 (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{2}$

23. Let $\omega=-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$, then the value of the det.

$\begin{vmatrix}1 & 1 & 1\\ 1 & -1-\omega^2 & \omega^2\\ 1 & \omega^2 & \omega^4\end{vmatrix}$ is *(2002 - 2 Marks)*

(a) 3ω (b) $3\omega(\omega-1)$
(c) $3\omega^2$ (d) $3\omega(1-\omega)$

24. If $\dfrac{w-\overline{w}z}{1-z}$ is purely real where $w=\alpha+i\beta$, $\beta\neq 0$ and $z\neq 1$, then the set of the values of z is *(2006 - 3M, –1)*

(a) $\{z:|z|=1\}$ (b) $\{z:z=\overline{z}\}$
(c) $\{z:z\neq 1\}$ (d) $\{z:|z|=1, z\neq 1\}$

25. A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is *(2007 -3 marks)*
(a) $3e^{i\pi/4}+4i$ (b) $(3-4i)e^{i\pi/4}$
(c) $(4+3i)e^{i\pi/4}$ (d) $(3+4i)e^{i\pi/4}$

26. If $|z|=1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on
(a) a line not passing through the origin *(2007 -3 marks)*
(b) $|z|=\sqrt{2}$
(c) the x-axis
(d) the y-axis

27. A particle P starts from the point $z_0 = 1+2i$, where $i=\sqrt{-1}$. It moves horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i}+\hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by *(2008)*
(a) $6+7i$ (b) $-7+6i$
(c) $7+6i$ (d) $-6+7i$

28. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is *(2009)*
(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$ (c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

29. Let $z = x+iy$ be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation : $z\bar{z}^3 + \bar{z}z^3 = 350$ is *(2009)*
(a) 48 (b) 32 (c) 40 (d) 80

30. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2+z+1$ is real. Then a cannot take the value *(2012)*
(a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

31. Let complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on circles $(x-x_0)^2 + (y-y_0)^2 = r^2$ and $(x-x_0)^2+(y-y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2|z_0|^2 = r^2+2$, then $|\alpha| =$ *(JEE Adv. 2013)*
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

32. Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is *(JEE Adv. 2019)*
(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

D MCQs with One or More than One Correct

1. If $z_1 = a+ib$ and $z_2 = c+id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1\bar{z}_2) = 0$, then the pair of complex numbers $w_1 = a+ic$ and $w_2 = b+id$ satisfies – *(1985 - 2 Marks)*
(a) $|w_1| = 1$ (b) $|w_2| = 1$
(c) $\text{Re}(w_1\bar{w}_2) = 0$ (d) none of these

2. Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1+z_2}{z_1-z_2}$ may be *(1986 - 2 Marks)*
(a) zero (b) real and positive
(c) real and negative (d) purely imaginary
(e) none of these.

3. If z_1 and z_2 are two nonzero complex numbers such that $|z_1+z_2| = |z_1| + |z_2|$, then Arg z_1 − Arg z_2 is equal to *(1987 - 2 Marks)*
(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$
(e) π

4. The value of $\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7} - i\cos\frac{2\pi k}{7}\right)$ is *(1987 - 2 Marks)*
(a) -1 (b) 0 (c) $-i$ (d) i
(e) None

5. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals *(1998 - 2 Marks)*
(a) 128ω (b) -128ω (c) $128\omega^2$ (d) $-128\omega^2$

6. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals *(1998 - 2 Marks)*
(a) i (b) $i-1$ (c) $-i$ (d) 0

7. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x+iy$, then *(1998 - 2 Marks)*
(a) $x=3, y=2$ (b) $x=1, y=3$
(c) $x=0, y=3$ (d) $x=0, y=0$

8. Let z_1 and z_2 be two distinct complex numbers and let $z = (1-t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg (w) denotes the principal argument of a non-zero complex number w, then *(2010)*
(a) $|z-z_1| + |z-z_2| = |z_1-z_2|$
(b) Arg $(z-z_1) =$ Arg $(z-z_2)$
(c) $\begin{vmatrix} z-z_1 & \bar{z}-\bar{z}_1 \\ z_2-z_1 & \bar{z}_2-\bar{z}_1 \end{vmatrix}$
(d) Arg $(z-z_1) =$ Arg (z_2-z_1)

9. Let $w = \frac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, ...\}$. Further $H_1 =$ $\left\{z \in \mathbb{C} : \operatorname{Re} z > \frac{1}{2}\right\}$ and $H_2 = \left\{z \in \mathbb{C} : \operatorname{Re} z < \frac{-1}{2}\right\}$, where c is the set of all complex numbers. If z_1 Î $P Ç H_1$, z_2 Î $P Ç H_2$ and O represents the origin, then $\angle z_1 O z_2 =$ *(JEE Adv. 2013)*

(a) $\frac{p}{2}$ (b) $\frac{p}{6}$ (c) $\frac{2p}{3}$ (d) $\frac{5p}{6}$

10. Let a, b $\in \mathbb{R}$ and $a^2 + b^2 \neq 0$.

Suppose $S = \left\{Z \in C : Z = \frac{1}{a+ibt}, t \in \mathbb{R}, t \neq 0\right\}$, where $i = \sqrt{-1}$. If z = x + iy and z $\in$ S, then (x, y) lies on *(JEE Adv. 2016)*

(a) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for a > 0, b ≠ 0

(b) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for a < 0, b ≠ 0

(c) the x-axis for a ≠ 0, b = 0

(d) the y-axis for a = 0, b ≠ 0

11. Let a, b x and y be real numbers such that a – b = 1 and y ≠ 0. If the complex number z = x + iy satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ? *(JEE Adv. 2017)*

(a) $-1+\sqrt{1-y^2}$ (b) $-1-\sqrt{1-y^2}$

(c) $1+\sqrt{1+y^2}$ (d) $1-\sqrt{1+y^2}$

12. For a non-zero complex number z, let arg(z) denote the principal argument with $-\pi < \arg(z) \leq \pi$. Then, which of the following statement (s) is (are) **FALSE?** *(JEE Adv. 2018)*

(a) $\arg(-1-i) = \frac{\pi}{4}$, where $i = \sqrt{-1}$

(b) The function $f : \mathbb{R} \to (-\pi, \pi]$, defined by $f(t) = \arg(-1+it)$ for all $t \in \mathbb{R}$, is continuous at all points of $\mathbb{R}$, where $i = \sqrt{-1}$

(c) For any two non-zero complex numbers z_1 and z_2,

$$\arg\left(\frac{z_1}{z_2}\right) - \arg(z_1) + \arg(z_2)$$

is an integer multiple of 2π

(d) For any three given distinct complex numbers z_1, z_2 and z_3, the locus of the point z satisfying the condition

$$\arg\left(\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}\right) = \pi,$$ lies on a straight line

13. Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ $(x, y, \in \mathbb{R}, i = \sqrt{-1})$ of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE? *(JEE Adv. 2018)*

(a) If L has exactly one element, then $|s| \neq |t|$

(b) If $|s| = |t|$, then L has infinitely many elements

(c) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(d) If L has more than one element, then L has infinitely many elements

E Subjective Problems

1. Express $\frac{1}{1-\cos\theta+2i\sin\theta}$ in the form $x + iy$. *(1978)*

2. If $x = a + b$, $y = a\gamma + b\beta$ and $z = a\beta + b\gamma$ where γ and β are the complex cube roots of unity, show that $xyz = a^3 + b^3$. *(1978)*

3. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$. *(1979)*

4. Find the real values of x and y for which the following equation is satisfied $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ *(1980)*

5. Let the complex number z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle. Then prove that $z_1^2 + z_2^2 + z_3^2 = 3z_0^2$. *(1981 - 4 Marks)*

6. Prove that the complex numbers z_1, z_2 and the origin form an equilateral triangle only if $z_1^2 + z_2^2 - z_1 z_2 = 0$. *(1983 - 3 Marks)*

7. If $1, a_1, a_2, ..., a_{n-1}$ are the n roots of unity, then show that $(1-a_1)(1-a_2)(1-a_3)....(1-a_{n-1}) = n$ *(1984 - 2 Marks)*

8. Show that the area of the triangle on the Argand diagram formed by the complex numbers z, iz and $z + iz$ is $\frac{1}{2}|z|^2$. *(1986 - 2½ Marks)*

9. Let $Z_1 = 10 + 6i$ and $Z_2 = 4 + 6i$. If Z is any complex number such that the argument of $\frac{(Z - Z_1)}{(Z - Z_2)}$ is $\frac{\pi}{4}$, then prove that $|Z - 7 - 9i| = 3\sqrt{2}$. *(1990 - 4 Marks)*

10. If $iz^3 + z^2 - z + i = 0$, then show that $|z| = 1$. *(1995 - 5 Marks)*

11. If $|Z| \le 1, |W| \le 1$, show that

$$|Z - W|^2 \le (|Z| - |W|)^2 + (Arg\, Z - Arg\, W)^2$$

(1995 - 5 Marks)

12. Find all non-zero complex numbers Z satisfying $\overline{Z} = iZ^2$. *(1996 - 2 Marks)*

13. Let z_1 and z_2 be roots of the equation $z^2+pz+q=0$, where the coefficients p and q may be complex numbers. Let A and B represent z_1 and z_2 in the complex plane. If $\angle AOB = \alpha \ne 0$ and $OA = OB$, where O is the origin, prove that

$$p^2 = 4q\cos^2\left(\frac{\alpha}{2}\right)$$

(1997 - 5 Marks)

14. For complex numbers z and w, prove that $|z|^2 w - |w|^2 z = z - w$ if and only if $z = w$ or $z\overline{w} = 1$. *(1999 - 10 Marks)*

15. Let a complex number α, $\alpha \ne 1$, be a root of the equation $z^{p+q} - z^p - z^q + 1 = 0$, where p, q are distinct primes. Show that either $1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$ or $1 + \alpha + \alpha^2 + ... + \alpha^{q-1} = 0$, but not both together. *(2002 - 5 Marks)*

16. If z_1 and z_2 are two complex numbers such taht $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1 - z_1\overline{z}_2}{z_1 - z_2}\right| < 1$. *(2003 - 2 Marks)*

17. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$. *(2003 - 2 Marks)*

18. Find the centre and radius of circle given by

$$\left|\frac{z - \alpha}{z - \beta}\right| = k, k \ne 1$$

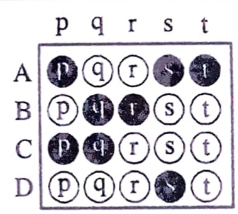
where, $z = x + iy$, $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$ *(2004 - 2 Marks)*

19. If one the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\, i$. Find the other vertices of the square. *(2005 - 4 Marks)*

F Match the Following

DIRECTIONS (Q. 1 and 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. $z \ne 0$ is a complex number *(1992 - 2 Marks)*

Column I	Column II
(A) $\mathrm{Re}\, z = 0$	(p) $\mathrm{Re}\, z^2 = 0$
(B) $\mathrm{Arg}\, z = \frac{\pi}{4}$	(q) $\mathrm{Im}\, z^2 = 0$
	(r) $\mathrm{Re}\, z^2 = \mathrm{Im}\, z^2$

2. Match the statements in **Column I** with those in **Column II**. *(2010)*

[**Note :** Here z takes values in the complex plane and Im z and Re z denote , respectively, the imaginary part and the real part of z.]

Column I	Column II
(A) The set of points z satisfying $\|z - i\|z\|\| = \|z + i\|z\|\|$ is contained in or equal to	(p) an ellipse with eccentricity $\frac{4}{5}$
(B) The set of points z satisfying $\|z + 4\| + \|z - 4\| = 10$ is contained in or equal to	(q) the set of points z satisfying $\mathrm{Im}\, z = 0$
(C) If $\|w\| = 2$, then the set of points $z = w - \frac{1}{w}$ is contained in or equal to	(r) the set of points z satisfying $\|\mathrm{Im}\, z\| \le 1$
(D) If $\|w\| = 1$, then the set of points $z = w + \frac{1}{w}$ is contained in or equal to.	(s) the set of points z satisfying $\|\mathrm{Re}\, z\| < 2$
	(t) the set of points z satisfying $\|z\| \le 3$

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

3. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right)$; $k = 1, 2, ..., 9.$ *(JEE Adv. 2014)*

	List-I		List-II
P.	For each z_k there exists as z_j such that $z_k . z_j = 1$	1.	True
Q.	There exists a $k \in \{1, 2, ..., 9\}$ such that $z_1 . z = z_k$ has no solution z in the set of complex numbers	2.	False
R.	$\frac{\lvert 1-z_1\rvert \lvert 1-z_2\rvert .... \lvert 1-z_9\rvert}{10}$ equals	3.	1
S.	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	4.	2

	P	Q	R	S
(a)	1	2	4	3
(b)	2	1	3	4
(c)	1	2	3	4
(d)	2	1	4	3

G Comprehension Based Questions

PASSAGE-1

Let A, B, C be three sets of complex numbers as defined below

$A = \{z : \text{Im } z \geq 1\}$

$B = \{z : |z - 2 - i| = 3\}$

$C = \{z : \text{Re}((1 - i)z) = \sqrt{2}\}$

1. The number of elements in the set $A \cap B \cap C$ is *(2008)*

(a) 0 (b) 1 (c) 2 (d) ∞

2. Let z be any point in $A \cap B \cap C$.
Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between *(2008)*

(a) 25 and 29 (b) 30 and 34
(c) 35 and 39 (d) 40 and 44

3. Let z be any point $A \cap B \cap C$ and let w be any point satisfying $|w - 2 - i| < 3$. Then, $|z| - |w| + 3$ lies between *(2008)*

(a) –6 and 3 (b) –3 and 6
(c) –6 and 6 (d) –3 and 9

PASSAGE-2

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in \mathbb{C} : |z| < 4\}$, $S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$

and $S_3 = \{z \in \mathbb{C} : \text{Re } z > 0\}$.

4. Area of $S =$ *(JEE Adv. 2013)*

(a) $\frac{10\pi}{3}$ (b) $\frac{20\pi}{3}$ (c) $\frac{16\pi}{3}$ (d) $\frac{32\pi}{3}$

5. $\min\limits_{z \in S} |1 - 3i - z| =$ *(JEE Adv. 2013)*

(a) $\frac{2 - \sqrt{3}}{2}$ (b) $\frac{2 + \sqrt{3}}{2}$

(c) $\frac{3 - \sqrt{3}}{2}$ (d) $\frac{3 + \sqrt{3}}{2}$

I Integer Value Correct Type

1. If z is any complex number satisfying $|z - 3 - 2i| \leq 2$, then the minimum value of $|2z - 6 + 5i|$ is *(2011)*

2. Let $\omega = e^{\frac{i\pi}{3}}$, and a, b, c, x, y, z be non-zero complex numbers such that *(2011)*

$a + b + c = x$
$a + b\omega + c\omega^2 = y$
$a + b\omega^2 + c\omega = z$

Then the value of $\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

3. For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression $\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$ is *(JEE Adv. 2015)*

14. Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\left\{|a + b\omega + c\omega^2|^2 : a, b, c \text{ distinct non-zero integers}\right\}$ equals ______. *(JEE Adv. 2019)*

Section-B JEE Main / AIEEE

1. z and w are two nonzero complex numbers such that $|z| = |w|$ and $\text{Arg } z + \text{Arg } w = \pi$ then z equals [2002]
(a) $\bar{\omega}$ (b) $-\bar{\omega}$ (c) ω (d) $-\omega$

2. If $|z-4| < |z-2|$, its solution is given by [2002]
(a) $Re(z) > 0$ (b) $Re(z) < 0$
(c) $Re(z) > 3$ (d) $Re(z) > 2$

3. The locus of the centre of a circle which touches the circle $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be [2002]
(a) an ellipse (b) a hyperbola
(c) a circle (d) none of these

4. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $Arg(z) - Arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to [2003]
(a) $-i$ (b) 1 (c) -1 (d) i

5. Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then [2003]
(a) $a^2 = 4b$ (b) $a^2 = b$ (c) $a^2 = 2b$ (d) $a^2 = 3b$

6. If $\left(\frac{1+i}{1-i}\right)^x = 1$ then [2003]
(a) $x = 2n+1$, where n is any positive integer
(b) $x = 4n$, where n is any positive integer
(c) $x = 2n$, where n is any positive integer
(d) $x = 4n+1$, where n is any positive integer.

7. Let z and w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals [2004]
(a) $\frac{5\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

8. If $z = x - i\,y$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) \Big/ (p^2 + q^2)$ is equal to [2004]
(a) -2 (b) -1 (c) 2 (d) 1

9. If $|z^2 - 1| = |z|^2 + 1$, then z lies on [2004]
(a) an ellipse (b) the imaginary axis
(c) a circle (d) the real axis

10. If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x-1)^3 + 8 = 0$, are [2005]
(a) $-1, -1+2\omega, -1-2\omega^2$
(b) $-1, -1, -1$
(c) $-1, 1-2\omega, 1-2\omega^2$
(d) $-1, 1+2\omega, 1+2\omega^2$

11. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to [2005]
(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $\frac{-\pi}{2}$

12. If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on [2005]
(a) an ellipse (b) a circle
(c) a straight line (d) a parabola

13. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is [2006]
(a) i (b) 1 (c) -1 (d) $-i$

14. If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right)^2 + \left(z^3+\frac{1}{z^3}\right)^2 + \dots\dots + \left(z^6+\frac{1}{z^6}\right)^2$ is [2006]
(a) 18 (b) 54 (c) 6 (d) 12

15. If $|z+4| \le 3$, then the maximum value of $|z+1|$ is [2007]
(a) 6 (b) 0 (c) 4 (d) 10

16. The conjugate of a complex number is $\frac{1}{i-1}$ then that complex number is [2008]
(a) $\frac{-1}{i-1}$ (b) $\frac{1}{i+1}$ (c) $\frac{-1}{i+1}$ (d) $\frac{1}{i-1}$

17. Let R be the real line. Consider the following subsets of the plane $R \times R$:
$S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$
$T = \{(x, y) : x - y \text{ is an integer}\}$,
Which one of the following is true? [2008]
(a) Neither S nor T is an equivalence relation on R
(b) Both S and T are equivalence relation on R
(c) S is an equivalence relation on R but T is not
(d) T is an equivalence relation on R but S is not

18. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals [2010]
(a) 1 (b) 2 (c) ∞ (d) 0

19. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line $\operatorname{Re} z = 1$, then it is necessary that : [2011]

(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$

(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$

20. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals [2011]

(a) $(1, 1)$ (b) $(1, 0)$

(c) $(-1, 1)$ (d) $(0, 1)$

21. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : [2012]

(a) either on the real axis or on a circle passing through the origin.

(b) on a circle with centre at the origin

(c) either on the real axis or on a circle not passing through the origin.

(d) on the imaginary axis.

22. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals: [JEE M 2013]

(a) $-\theta$ (b) $\frac{\pi}{2} - \theta$ (c) θ (d) $\pi - \theta$

23. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$: [JEE M 2014]

(a) is strictly greater than $\frac{5}{2}$

(b) is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

(c) is equal to $\frac{5}{2}$

(d) lie in the interval $(1, 2)$

24. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a: [JEE M 2015]

(a) circle of radius 2.

(b) circle of radius $\sqrt{2}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis.

25. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is: [JEE M 2016]

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

26. Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}$

Then the sum of the elements in A is: [JEE M 2019 – 9 Jan (M)]

(a) $\frac{5\pi}{6}$ (b) π

(c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

27. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to: [JEE M 2019 – 9 Jan (M)]

(a) -256 (b) 512

(c) -512 (d) 256

28. All the points in the set

$S = \left\{\frac{\alpha+i}{\alpha-i} : \alpha \in R\right\} (i = \sqrt{-1})$ lie on a:

[JEE M 2019 – 9 April (M)]

(a) straight line whose slope is 1.

(b) circle whose radius is 1.

(c) circle whose radius is $\sqrt{2}$.

(d) straight line whose slope is -1.