CHAPTER 15

Matrices and Determinants

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$

be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is .... *(1981 - 2 Marks)*

2. The solution set of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is

.... *(1981 - 2 Marks)*

3. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the value of determinant chosen is positive is .... *(1982 - 2 Marks)*

4. Given that $x = -9$ is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ the other

two roots are .... and .... *(1983 - 2 Marks)*

5. The system of equations

$$\lambda x + y + z = 0$$
$$-x + \lambda y + z = 0$$
$$-x - y + \lambda z = 0$$

Will have a non-zero solution if real values of λ are given by .... *(1984 - 2 Marks)*

6. The value of the determinant $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$

is .... *(1988 - 2 Marks)*

7. For positive numbers x, y and z, the numerical value of the

determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is ....

(1993 - 2 Marks)

B True / False

1. The determinants *(1983 - 1 Mark)*

$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ and $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ are not identically equal.

2. If $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$

then the two triangles with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, and $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ must be congruent. *(1985 - 1 Mark)*

C MCQs with One Correct Answer

1. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1. Then
(a) C is empty *(1981 - 2 Marks)*
(b) B has as many elements as C
(c) $A = B \cup C$
(d) B has twice as many elements as elements as C

2. If $\omega (\neq 1)$ is a cube root of unity, then

$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} =$ *(1995S)*

(a) 0 (b) 1 (c) i (d) ω

3. Let a, b, c be the real numbers. Then following system of equations in x, y and z *(1995S)*

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$,

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

(a) no solution (b) unique solution
(c) infinitely many solutions (d) finitely many solutions

4. If A and B are square matrices of equal degree, then which one is correct among the followings? *(1995S)*
(a) $A+B=B+A$ (b) $A+B=A-B$
(c) $A-B=B-A$ (d) $AB=BA$

5. The parameter, on which the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{vmatrix}$ does not depend upon is *(1997 - 2 Marks)*
(a) a (b) p (c) d (d) x

6. If $f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to *(1999 - 2 Marks)*
(a) 0 (b) 1 (c) 100 (d) –100

7. If the system of equations
$x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$ has a non-zero solution, then the possible values of k are *(2000S)*
(a) $-1, 2$ (b) $1, 2$ (c) $0, 1$ (d) $-1, 1$

8. Let $\omega=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$. Then the value of the determinant
$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is *(2002S)*
(a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

9. The number of values of k for which the system of equations $(k+1)x+8y=4k$; $kx+(k+3)y=3k-1$ has infinitely many solutions is *(2002S)*
(a) 0 (b) 1 (c) 2 (d) infinite

10. If $A=\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2=B$, is *(2003S)*
(a) 1 (b) –1
(c) 4 (d) no real values

11. If the system of equations $x+ay=0$, $az+y=0$ and $ax+z=0$ has infinite solutions, then the value of a is *(2003S)*
(a) –1 (b) 1
(c) 0 (d) no real values

12. Given $2x-y+2z=2$, $x-2y+z=-4$, $x+y+\lambda z=4$ then the value of λ such that the given system of equation has NO solution, is *(2004S)*
(a) 3 (b) 1 (c) 0 (d) -3

13. If $A=\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3|=125$ then the value of α is *(2004S)*
(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 5

14. $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1}=\left[\frac{1}{6}(A^2+cA+dI)\right]$, then the value of c and d are *(2005S)*
(a) $(-6,-11)$ (b) $(6,11)$ (c) $(-6,11)$ (d) $(6,-11)$

15. If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ and $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q=PAP^T$ and $x=P^TQ^{2005}P$ then x is equal to *(2005S)*
(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$
(c) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$
(d) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

16. Consider three points
$P=(-\sin(\beta-\alpha),-\cos\beta)$, $Q=(\cos(\beta-\alpha),\sin\beta)$ and $R=(\cos(\beta-\alpha+\theta),\sin(\beta-\theta))$, where $0<\alpha,\beta,\theta<\frac{\pi}{4}$.
Then, *(2008)*
(a) P lies on the line segment RQ
(b) Q lies on the line segment PR
(c) R lies on the line segment QP
(d) P, Q, R are non-collinear

17. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is *(2010)*
(a) 0 (b) 2^9-1 (c) 168 (d) 2

18. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$ where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is *(2011)*
(a) 2 (b) 6 (c) 4 (d) 8

19. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is *(2012)*
(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

20. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \ne \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that *(2012)*
(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $PX = X$
(c) $PX = 2X$ (d) $PX = -X$

21. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3.

If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\dfrac{q_{31} + q_{32}}{q_{21}}$ equals *(JEE Adv. 2016)*
(a) 52 (b) 103 (c) 201 (d) 205

22. How many 3×3 matrices M with entries from $\{0, 1, 2\}$ are there, for which the sum of the diagonal entries of $M^T M$ is 5? *(JEE Adv. 2017)*
(a) 126 (b) 198 (c) 162 (d) 135

23. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers, and I is the 2×2 identity matrix. If a* is the minimum of the set $\{\alpha(\theta) : \theta \in [0, 2\pi)\}$ and β* is the minimum of the set $\{\beta(\theta) : \theta \in [0, 2\pi)\}$. Then the value of a* + b* is *(JEE Adv. 2019)*
(a) $-\dfrac{31}{16}$ (b) $-\dfrac{17}{16}$ (c) $-\dfrac{37}{16}$ (d) $-\dfrac{29}{16}$

D MCQs with One or More than One Correct

1. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if *(1986 - 2 Marks)*
(a) a, b, c are in A. P.
(b) a, b, c are in G. P.
(c) a, b, c are in H. P.
(d) α is a root of the equation $ax^2 + bx + c = 0$
(e) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$.

2. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then *(1998 - 2 Marks)*
(a) $x = 3, y = 1$ (b) $x = 1, y = 3$
(c) $x = 0, y = 3$ (d) $x = 0, y = 0$

3. Let M and N be two 3×3 non-singular skew- symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2 (M^TN)^{-1} (MN^{-1})^T$ is equal to *(2011)*
(a) M^2 (b) $-N^2$ (c) $-M^2$ (d) MN

4. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) *(2012)*
(a) -2 (b) -1 (c) 1 (d) 2

5. For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct? *(JEE Adv. 2013)*
(a) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric
(b) MN – NM is skew symmetric for all symmetric matrices M and N
(c) MN is symmetric for all symmetric matrices M and N
(d) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

6. Let ω be a complex cube root of unity with $\omega \ne 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $p^2 \ne 0$, when n = *(JEE Adv. 2013)*
(a) 57 (b) 55 (c) 58 (d) 56

7. Let M be a 2×2 symmetric matrix with integer entries. Then M is invertible if *(JEE Adv. 2014)*
(a) The first column of M is the transpose of the second row of M
(b) The second row of M is the transpose of the first column of M
(c) M is a diagonal matrix with non-zero entries in the main diagonal
(d) The product of entries in the main diagonal of M is not the square of an integer

8. Let M and N be two 3×3 matrices such that $MN = NM$. Further, if $M \ne N^2$ and $M^2 = N^4$, then *(JEE Adv. 2014)*
(a) determinant of $(M^2 + MN^2)$ is 0
(b) there is 3×3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix
(c) determinant of $(M^2 + MN^2) \ge 1$
(d) for a 3×3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

9. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha?$$

(JEE Adv. 2015)
(a) -4 (b) 9 (c) -9 (d) 4

10. Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric? *(JEE Adv. 2015)*
(a) $Y^3Z^4 - Z^4Y^3$
(b) $X^{44} + Y^{44}$
(c) $X^4Z^3 - Z^3X^4$
(d) $X^{23} + Y^{23}$

11. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \mathbb{R}$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \mathbb{R}$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then *(JEE Adv. 2016)*
(a) $a = 0, k = 8$
(b) $4a - k + 8 = 0$
(c) $\det(P \operatorname{adj}(Q)) = 2^9$
(d) $\det(Q \operatorname{adj}(P)) = 2^{13}$

12. Let $a, \lambda, \mu, \in \mathbb{R}$. Consider the system of linear equations
$$ax + 2y = \lambda$$
$$3x - 2y = \mu$$
Which of the following statement(s) is (are) correct? *(JEE Adv. 2016)*
(a) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ.
(b) If $a \neq -3$, then the system has a unique solution for all values of λ and μ.
(c) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$.
(d) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$.

13. Which of the following is(are) not the square of a 3×3 matrix with real entries? *(JEE Adv. 2017)*
(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
(d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

14. Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3, \in \mathbb{R}$ and the system of equations (in real variables)
$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$
has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$? *(JEE Adv. 2018)*
(a) $x + 2y + 3z = b_1, 4y + 5z = b_2$ and $x + 2y + 6z = b_3$
(b) $x + y + 3z = b_1, 5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$
(c) $-x + 2y - 5z = b_1, 2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$
(d) $x + 2y + 5z = b_1, 2x + 3z = b_2$ and $x + 4y - 5z = b_3$

15. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $(\operatorname{adj} M) = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct? *(JEE Adv. 2019)*
(a) $a + b = 3$
(b) $\det(\operatorname{adj} M^2) = 81$
(c) $(adjM)^{-1} + adjM^{-1} = -M$
(d) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

16. Let
$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

Where P_k^T denotes the transpose of the matrix P_k. Then which of the following options is/are correct? *(JEE Adv. 2019)*
(a) X is a symmetric matrix
(b) The sum of diagonal entries of X is 18
(c) $X - 30I$ is an invertible matrix
(d) If $X\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, then $a = 30$

17. Let $x \in R$ and let
$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = PQP^{-1}$$
Then which of the following options is/are correct? *(JEE Adv. 2019)*
(a) $\det R = \det\begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in R$
(b) For $x = 1$, there exists a unit vector $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ for which $R\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
(c) There exists a real number x such that $PQ = QP$
(d) For $x = 0$, if $R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, then $a + b = 5$

E Subjective Problems

1. For what value of k do the following system of equations possess a non trivial (i.e., not all zero) solution over the set of rationals Q?
$$x+ky+3z=0$$
$$3x+ky-2z=0$$
$$2x+3y-4z=0$$
For that value of k, find all the solutions for the system. *(1979)*

2. Let a, b, c be positive and not all equal. Show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative. *(1981 - 4 Marks)*

3. Without expanding a determinant at any stage, show that
$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = xA+B,$$ where A and B are determinants of order 3 not involving x. *(1982 - 5 Marks)*

4. Show that
$$\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$$
(1985 - 2 Marks)

5. Consider the system of linear equations in x, y, z:
$$(\sin 3\theta)\ x-y+z=0$$
$$(\cos 2\theta)\ x+4y+3z=0$$
$$2x+7y+7z=0$$
Find the values of θ for which this system has nontrivial solutions. *(1986 - 5 Marks)*

6. Let $\Delta a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$.

Show that $\sum_{a=1}^{n} \Delta a = c$, a constant. *(1989 - 5 Marks)*

7. Let the three digit numbers $A\,28$, $3B9$, and $62\,C$, where $A, B,$ and C are integers between 0 and 9, be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is divisible by k. *(1990 - 4 Marks)*

8. If $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then find the value of $\dfrac{p}{p-a}+\dfrac{q}{q-b}+\dfrac{r}{r-c}$ *(1991 - 4 Marks)*

9. For a fixed positive integer n, if *(1992 - 4 Marks)*
$$D = \begin{vmatrix} n! & (n+1)! & (n+2)! \\ (n+1)! & (n+2)! & (n+3)! \\ (n+2)! & (n+3)! & (n+4)! \end{vmatrix}$$
then show that $\left[\dfrac{D}{(n!)^3}-4\right]$ is divisible by n.

10. Let λ and α be real. Find the set of all values of λ for which the system of linear equations
$\lambda x+(\sin\alpha)y+(\cos\alpha)z=0$, $x+(\cos\alpha)y+(\sin\alpha)z=0$,
$-x+(\sin\alpha)y-(\cos\alpha)z=0$
has a non-trivial solution. For $\lambda=1$, find all values of α. *(1993 - 5 Marks)*

11. For all values of A, B, C and P, Q, R show that *(1994 - 4 Marks)*
$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

12. Let $a>0$, $d>0$. Find the value of the determinant *(1996 - 5 Marks)*
$$\begin{vmatrix} \dfrac{1}{a} & \dfrac{1}{a(a+d)} & \dfrac{1}{(a+d)(a+2d)} \\ \dfrac{1}{(a+d)} & \dfrac{1}{(a+d)(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} \\ \dfrac{1}{(a+2d)} & \dfrac{1}{(a+2d)(a+3d)} & \dfrac{1}{(a+3d)(a+4d)} \end{vmatrix}$$

13. Prove that for all values of θ,
$$\begin{vmatrix} \sin\theta & \cos\theta & \sin 2\theta \\ \sin\left(\theta+\dfrac{2\pi}{3}\right) & \cos\left(\theta+\dfrac{2\pi}{3}\right) & \sin\left(2\theta+\dfrac{4\pi}{3}\right) \\ \sin\left(\theta-\dfrac{2\pi}{3}\right) & \cos\left(\theta-\dfrac{2\pi}{3}\right) & \sin\left(2\theta-\dfrac{4\pi}{3}\right) \end{vmatrix} = 0$$
(2000 - 3 Marks)

14. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^TA = I$, then find the value of $a^3+b^3+c^3$. *(2003 - 2 Marks)*

15. If M is a 3×3 matrix, where $\det M = 1$ and $MM^T = I$, where 'I' is an identity matrix, prove that $\det(M-I)=0$. *(2004 - 2 Marks)*

16. If $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}, U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}, V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
and $AX = U$ has infinitely many solutions, prove that $BX = V$ has no unique solution. Also show that if $afd \neq 0$, then $BX = V$ has no solution. *(2004 - 4 Marks)*

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

1. Consider the lines given by

$L_1 : x + 3y - 5 = 0;\ L_2 : 3x - ky - 1 = 0;\ L_3 : 5x + 2y - 12 = 0$

Match the Statements / Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. *(2008)*

Column I	***Column II***
(A) L_1, L_2, L_3 are concurrent, if	(p) $k = -9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(q) $k = -\dfrac{6}{5}$
(C) L_1, L_2, L_3 from a triangle, if	(r) $k = \dfrac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(s) $k = 5$

2. Match the Statements/Expressions in **Column I** with the Statements / Expressions in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS. *(2008)*

Column I	***Column II***
(A) The minimum value of $\dfrac{x^2 + 2x + 4}{x + 2}$ is	(p) 0
(B) Let A and B be 3×3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and $(A+B)(A-B) = (A-B)(A+B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(q) 1
(C) Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k + 3^{-a})} < 2$, must be less than	(r) 2
(D) If $\sin\theta = \cos\phi$, then the possible values of $\dfrac{1}{\pi}\left(\theta \pm \phi - \dfrac{\pi}{2}\right)$ are	(s) 3

G Comprehension Based Questions

PASSAGE - 1

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, and U_1, U_2 and U_3 are columns of a 3×3 matrix U. If column matrices U_1, U_2 and U_3 satisfying

$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ evaluate as directed in the following questions.

1. The value $|U|$ is *(2006 - 5M, –2)*

(a) 3 (b) –3 (c) $\dfrac{3}{2}$ (d) 2

2. The sum of the elements of the matrix U^{-1} is *(2006 - 5M, –2)*

(a) –1 (b) 0 (c) 1 (d) 3

3. The value of $[3\ 2\ 0]\,U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is *(2006 - 5M, –2)*

(a) 5 (b) $\dfrac{5}{2}$ (c) 4 (d) $\dfrac{3}{2}$

PASSAGE - 2

Let $\mathscr{A}$ be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

4. The number of matrices in $\mathscr{A}$ is *(2009)*
   (a) 12 (b) 6 (c) 9 (d) 3

5. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is *(2009)*
   (a) less than 4
   (b) at least 4 but less than 7
   (c) at least 7 but less than 10
   (d) at least 10

6. The number of matrices A in $\mathscr{A}$ for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is *(2009)*
   (a) 0 (b) more than 2
   (c) 2 (d) 1

PASSAGE - 3

Let p be an odd prime number and T_p be the following set of 2×2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\} \right\} \qquad (2010)$$

7. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is
   (a) $(p-1)^2$ (b) $2(p-1)$
   (c) $(p-1)^2+1$ (d) $2p-1$

8. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is
   [**Note:** The trace of a matrix is the sum of its diagonal entries.]
   (a) $(p-1)(p^2-p+1)$ (b) $p^3-(p-1)^2$
   (c) $(p-1)^2$ (d) $(p-1)(p^2-2)$

9. The number of A in T_p such that det (A) is not divisible by p is
   (a) $2p^2$ (b) p^3-5p
   (c) p^3-3p (d) p^3-p^2

PASSAGE - 4

Let a, b and c be three real numbers satisfying *(2011)*

$$[a\,b\,c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\,0\,0] \qquad \text{...(E)}$$

10. If the point $P(a, b, c)$, with reference to (E), lies on the plane $2x+y+z=1$, then the value of $7a+b+c$ is
    (a) 0 (b) 12 (c) 7 (d) 6

11. Let ω be a solution of $x^3-1=0$ with $\text{Im}(\omega)>0$, if $a=2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equal to
    (a) -2 (b) 2 (c) 3 (d) -3

12. Let $b=6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2+bx+c=0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$ is
    (a) 6 (b) 7 (c) $\frac{6}{7}$ (d) ∞

H Assertion & Reason Type Questions

1. Consider the system of equations

$$x-2y+3z=-1$$
$$-x+y-2z=k$$
$$x-3y+4z=1$$

STATEMENT - 1 : The system of equations has no solution for $k \neq 3$ and

STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$, for $k \neq 3$. *(2008)*

   (a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
   (b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explaination for STATEMENT - 1
   (c) STATEMENT - 1 is True, STATEMENT - 2 is False
   (d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Let ω be the complex number $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$$ is equal to *(2010)*

2. Let k be a positive real number and let

$$A=\begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B=\begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6. then [k] is equal to
[**Note** : adj M denotes the adjoint of square matrix M and [k] denotes the largest integer less than or equal k. *(2010)*

3. Let M be a 3×3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}, \text{ and } M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}.$$ Then the sum of the diagonal entries of M is *(2011)*

4. The total number of distinct $x \in \mathbb{R}$ for which

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix}=10$$ is *(JEE Adv. 2016)*

5. Let $z=\frac{-1+\sqrt{3}i}{2}$, where $i=\sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let

$$P=\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$$ and I be the identity matrix of order 2

Then the total number of ordered pairs (r, s) for which $P^2=-I$ is *(JEE Adv. 2016)*

6. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$$ of linear equations, has

infinitely many solutions, then $1+\alpha+\alpha^2=$ *(JEE Adv. 2017)*

7. Let P be a matrix of order 3×3 such that all the entries in P are from the set {–1, 0, 1}. Then, the maximum possible value of the determinant of P is ______. *(JEE Adv. 2018)*

Section-B JEE Main / AIEEE

1. If $a > 0$ and discriminant of $ax^2+2bx+c$ is –ve, then

$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$ is equal to [2002]

(a) +ve (b) $(ac-b^2)(ax^2+2bx+c)$
(c) –ve (d) 0

2. If the system of linear equations [2003]

$x+2ay+az=0$; $x+3by+bz=0$; $x+4cy+cz=0$;

has a non - zero solution, then a, b, c.

(a) satisfy $a+2b+3c=0$ (b) are in A.P
(c) are in G.P (d) are in H.P.

3. If $1,\omega,\omega^2$ are the cube roots of unity, then

$$\Delta=\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$ is equal to [2003]

(a) ω^2 (b) 0 (c) 1 (d) ω

4. If $A=\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2=\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then [2003]

(a) $\alpha=2ab, \beta=a^2+b^2$ (b) $\alpha=a^2+b^2,\ \beta=ab$

(c) $\alpha=a^2+b^2,\ \beta=2ab$ (d) $\alpha=a^2+b^2,\ \beta=a^2-b^2$

5. Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is **[2004]**

(a) $A^2 = I$

(b) $A = (-1)I$, where I is a unit matrix

(c) A^{-1} does not exist

(d) A is a zero matrix

6. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$. and $10B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inverseof matrix A, then α is **[2004]**

(a) 5 (b) -1 (c) 2 (d) -2

7. If $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots$ are in G.P., then the value of the determinant **[2004]**

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

(a) -2 (b) 1 (c) 2 (d) 0

8. If $A^2 - A + I = 0$, then the inverse of A is **[2005]**

(a) $A + I$ (b) A (c) $A - I$ (d) $I - A$

9. The system of equations

$$\alpha x + y + z = \alpha - 1$$
$$x + \alpha y + z = \alpha - 1$$
$$x + y + \alpha z = \alpha - 1$$

has infinite solutions, if α is **[2005]**

(a) -2 (b) either -2 or 1

(c) not -2 (d) 1

10. If $a^2 + b^2 + c^2 = -2$ and **[2005]**

$$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix},$$

then $f(x)$ is a polynomial of degree

(a) 1 (b) 0 (c) 3 (d) 2

11. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n, \ldots\ldots$ are in G.P., then the determinant

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

is equal to **[2005]**

(a) 1 (b) 0

(c) 4 (d) 2

12. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? **[2006]**

(a) $A = B$

(b) $AB = BA$

(c) either of A or B is a zero matrix

(d) either of A or B is identity matrix

13. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then **[2006]**

(a) there cannot exist any B such that AB = BA

(b) there exist more then one but finite number of B's such that AB = BA

(c) there exists exactly one B such that AB = BA

(d) there exist infinitely many B's such that AB = BA

14. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is **[2007]**

(a) divisible by x but not y

(b) divisible by y but not x

(c) divisible by neither x nor y

(d) divisible by both x and y

15. Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals **[2007]**

(a) 1/5 (b) 5 (c) 5^2 (d) 1

16. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2 = I$. **[2008]**

Statement-1 : If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then tr $(A) \neq 0$.

(a) Statement -1 is false, Statement-2 is true

(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

(d) Statement -1 is true, Statement-2 is false

17. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[2008]**

(a) 2 (b) -1 (c) 0 (d) 1

18. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? [2008]
(a) If $\det A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
(b) If $\det A \neq \pm 1$, then A^{-1} exists and all its entries are non integers
(c) If $\det A = \pm 1$, then A^{-1} exists but all its entries are integers
(d) If $\det A = \pm 1$, then A^{-1} need not exists

19. Let A be a 2×2 matrix
Statement -1 : adj (adj A) = A
Statement -2 : |adj A| = |A| [2009]
(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement -1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

20. Let a, b, c be such that $b(a+c) \neq 0$ if [2009]

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

then the value of n is :
(a) any even integer (b) any odd integer
(c) any integer (d) zero

21. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0, is [2010]
(a) 5 (b) 6
(c) at least 7 (d) less than 4

22. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define
$Tr(A)$ = sum of diagonal elements of A and
$|A|$ = determinant of matrix A.
Statement - 1 : $Tr(A) = 0$.
Statement -2 : $|A| = 1$. [2010]
(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

23. Consider the system of linear equations ; [2010]
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(a) exactly 3 solutions
(b) a unique solution
(c) no solution
(d) infinite number of solutions

24. The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is [2011]
(a) 2 (b) 1 (c) zero (d) 3

25. Let A and B be two symmetric matrices of order 3.
Statement-1: $A(BA)$ and $(AB)A$ are symmetric matrices.
Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative. [2011]
(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

26. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : [2012]
(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

27. Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : [2012]
(a) -2 (b) 1 (c) 0 (d) -1

28. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to : [JEE M 2013]
(a) 4 (b) 11 (c) 5 (d) 0

29. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= K(1-\alpha)^2 (1-\beta)^2 (\alpha-\beta)^2$, then K is equal to: [JEE M 2014]

(a) 1 (b) -1 (c) $\alpha\beta$ (d) $\dfrac{1}{\alpha\beta}$

30. If A is a 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: [JEE M 2014]

(a) B^{-1} (b) $(B^{-1})'$ (c) $I + B$ (d) I

31. The set of all values of λ for which the system of linear equations :

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 + 2x_2 = \lambda x_3$$

has a non-trivial solution [JEE M 2015]

(a) contains two elements
(b) contains more than two elements
(c) is an empty set
(d) is a singleton

32. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: [JEE M 2015]

(a) (2, 1) (b) (–2, –1)
(c) (2, –1) (d) (–2, 1)

33. The system of linear equations

$$x + \lambda y - z = 0$$
$$\lambda x - y - z = 0$$
$$x + y - \lambda z = 0$$

has a non-trivial solution for: [JEE M 2016]

(a) exactly two values of λ.
(b) exactly three values of λ.
(c) infinitely many values of λ.
(d) exactly one value of λ.

34. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = A A^T$, then $5a + b$ is equal to : [JEE M 2016]

(a) 4 (b) 13
(c) –1 (d) 5

35. Let k be an integer such that triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point : [JEE M 2017]

(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(2, -\frac{1}{2}\right)$

(c) $\left(1, \frac{3}{4}\right)$ (d) $\left(1, -\frac{3}{4}\right)$

36. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to : [JEE M 2017]

(a) 1 (b) –z
(c) z (d) –1

37. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to : [JEE M 2017]

(a) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (b) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

(c) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (d) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

38. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to : [JEE M 2018]

(a) (–4, 3) (b) (–4, 5) (c) (4, 5) (d) (–4, –5)

39. If the system of linear equations

$$x + ky + 3z = 0$$
$$3x + ky - 2z = 0$$
$$2x + 4y - 3z = 0$$ [JEE M 2018]

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(a) 10 (b) –30 (c) 30 (d) – 10

40. The system of linear equations

$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1$$ [JEE M 2019 – 9 Jan (M)]

(a) is inconsistent when a = 4
(b) has a unique solution for $|a| = \sqrt{3}$
(c) has infinitely many solutions for a = 4
(d) is inconsistent when $|a| = \sqrt{3}$

41. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to: [JEE M 2019 – 9 Jan (M)]

(a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
(b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
(d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

42. If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \cdots\cdots \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$,

then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is: [JEE M 2019 – 9 April (M)]

(a) $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

43. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to: [JEE M 2019 – 9 April (M)]

(a) $y(y^2 - 1)$
(b) $y(y^2 - 3)$
(c) y^3
(d) $y^3 - 1$