CHAPTER 10

Functions

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The values of $f(x)=3\sin\left(\sqrt{\frac{\pi^2}{16}-x^2}\right)$ lie in the interval .... *(1983 - 1 Mark)*

2. For the function $f(x)=\frac{x}{1+e^{1/x}},\quad x\neq 0$
$\qquad\qquad = 0 \qquad , \quad x=0$
the derivative from the right, $f'(0+)=$ ...., and the derivative from the left, $f'(0-)=$ .... *(1983 - 2 Marks)*

3. The domain of the function $f(x)=\sin^{-1}(\log_2\frac{x^2}{2})$ is given by .... *(1984 - 2 Marks)*

4. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is .... and out of these .... are onto functions. *(1985 - 2 Marks)*

5. If $f(x)=\sin\ln\left(\frac{\sqrt{4-x^2}}{1-x}\right)$, then domain of $f(x)$ is .... and its range is .... *(1985 - 2 Marks)*

6. There are exactly two distinct linear functions, ..., and .... which map $[-1, 1]$ onto $[0, 2]$. *(1989 - 2 Marks)*

7. If f is an even function defined on the interval (-5, 5), then four real values of x satisfying the equation $f(x)=f\left(\frac{x+1}{x+2}\right)$ are ...., ..., ..., and .... *(1996 - 1 Mark)*

8. If $f(x)=\sin^2 x+\sin^2\left(x+\frac{\pi}{3}\right)+\cos x\cos\left(x+\frac{\pi}{3}\right)$ *and* $g\left(\frac{5}{4}\right)=1$, then $(gof)(x)=$ .... *(1996 - 2 Marks)*

B True / False

1. If $f(x)=(a-x^n)^{1/n}$ where $a>0$ and n is a positive integer, then $f[f(x)]=x$. *(1983 - 1 Mark)*

2. The function $f(x)=\frac{x^2+4x+30}{x^2-8x+18}$ is not one -to -one. *(1983 - 1 Mark)*

3. If $f_1(x)$ and $f_2(x)$ are defined on domains D_1 and D_2 respectively, then $f_1(x)+f_2(x)$ is defined on $D_1\cup D_2$. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. Let R be the set of real numbers. If $f: R\to R$ is a function defined by $f(x)=x^2$, then f is : *(1979)*
   (a) Injective but not surjective
   (b) Surjective but not injective
   (c) Bijective
   (d) None of these.

2. The entire graphs of the equation $y = x^2 + kx - x + 9$ is strictly above the x-axis if and only if *(1979)*
   (a) $k<7$ (b) $-5<k<7$
   (c) $k>-5$ (d) None of these.

3. Let $f(x)=|x-1|$. Then *(1983 - 1 Mark)*
   (a) $f(x^2)=(f(x))^2$ (b) $f(x+y)=f(x)+f(y)$
   (c) $f(|x|)=|f(x)|$ (d) None of these

4. If x satisfies $|x-1|+|x-2|+|x-3|\geq 6$, then *(1983 - 1 Mark)*
   (a) $0\leq x\leq 4$ (b) $x\leq -2$ or $x\geq 4$
   (c) $x\leq 0$ or $x\geq 4$ (d) None of these

5. If $f(x)=\cos(\ln x)$, then $f(x)f(y)-\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$ has the value *(1983 - 1 Mark)*
   (a) -1 (b) 1/2
   (c) -2 (d) none of these

6. The domain of definition of the function
$$y=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2} \text{ is}$$ *(1983 - 1 Mark)*
   (a) $(-3,-2)$ excluding -2.5 (b) $[0, 1]$ excluding 0.5
   (c) $[-2, 1)$ excluding 0 (d) none of these

7. Which of the following functions is periodic? *(1983 - 1 Mark)*
   (a) $f(x)=x-[x]$ where [x] denotes the largest integer less than or equal to the real number x
   (b) $f(x)=\sin\frac{1}{x}$ for $x\neq 0$, $f(0)=0$
   (c) $f(x)=x\cos x$
   (d) none of these

8. Let $f(x) = \sin x$ and $g(x) = \ln|x|$. If the ranges of the composition functions fog and gof are R_1 and R_2 respectively, then *(1994 - 2 Marks)*
   (a) $R_1 = \{u : -1 \le u < 1\}$, $R_2 = \{v : -\infty < v < 0\}$
   (b) $R_1 = \{u : -\infty < u < 0\}$, $R_2 = \{v : -1 \le v \le 0\}$
   (c) $R_1 = \{u : -1 < u < 1\}$, $R_2 = \{v : -\infty < v < 0\}$
   (d) $R_1 = \{u : -1 \le u \le 1\}$, $R_2 = \{v : -\infty < v \le 0\}$

9. Let $f(x) = (x+1)^2 - 1,\ x \ge -1$. Then the set $\{x : f(x) = f^{-1}(x)\}$ is *(1995)*
   (a) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$
   (b) $\{0, 1, -1\}$
   (c) $\{0, -1\}$
   (d) empty

10. The function $f(x) = |px - q| + r|x|,\ x \in (-\infty, \infty)$ where $p > 0, q > 0, r > 0$ assumes its minimum value only on one point if *(1995)*
    (a) $p \ne q$ (b) $r \ne q$
    (c) $r \ne p$ (d) $p = q = r$

11. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then *(1995S)*
    (a) $f(x)$ is bounded (b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$
    (c) $x f(x) \to 1$ as $x \to 0$ (d) $f(x) = \ln x$

12. If the function $f: [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is *(1999 - 2 Marks)*
    (a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$
    (c) $\frac{1}{2}(1 - \sqrt{1 + 4\log_2 x})$ (d) not defined

13. Let $f : R \to R$ be any function. Define $g : R \to R$ by $g(x) = |f(x)|$ for all x. Then g is *(2000S)*
    (a) onto if f is onto
    (b) one-one if f is one-one
    (c) continuous if f is continuous
    (d) differentiable if f is differentiable.

14. The domain of definition of the function $f(x)$ given by the equation $2^x + 2^y = 2$ is *(2000S)*
    (a) $0 < x \le 1$ (b) $0 \le x \le 1$
    (c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

15. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$. Then for all x, $f(g(x))$ is equal to *(2001S)*
    (a) x (b) 1 (c) $f(x)$ (d) $g(x)$

16. If $f: [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then $f^{-1}(x)$ equals *(2001S)*
    (a) $(x + \sqrt{x^2 - 4})/2$ (b) $x/(1 + x^2)$
    (c) $\left(x - \sqrt{x^2 - 4}\right)/2$ (d) $1 + \sqrt{x^2 - 4}$

17. The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2 + 3x + 2}$ is *(2001S)*
    (a) $R \setminus \{-1, -2\}$ (b) $(-2, \infty)$
    (c) $R \setminus \{-1, -2, -3\}$ (d) $(-3, \infty) \setminus \{-1, -2\}$

18. Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$. Then the number of onto functions from E to F is *(2001S)*
    (a) 14 (b) 16 (c) 12 (d) 8

19. Let $f(x) = \frac{\alpha x}{x+1}, x \ne -1$. Then, for what value of α is $f(f(x)) = x$? *(2001S)*
    (a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1

20. Suppose $f(x) = (x+1)^2$ for $x \ge -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals *(2002S)*
    (a) $-\sqrt{x} - 1, x \ge 0$ (b) $\frac{1}{(x+1)^2}, x > -1$
    (c) $\sqrt{x+1}, x \ge -1$ (d) $\sqrt{x} - 1, x \ge 0$

21. Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then f is *(2002S)*
    (a) one-to-one and onto
    (b) one-to-one but NOT onto
    (c) onto but NOT one-to-one
    (d) neither one-to-one nor onto

22. If $f : [0, \infty) \longrightarrow [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then f is *(2003S)*
    (a) one-one and onto
    (b) one-one but not onto
    (c) onto but not one-one
    (d) neither one-one nor onto

23. Domain of definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x, is *(2003S)*
    (a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

24. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}; x \in R$ is *(2003S)*
    (a) $(1, \infty)$ (b) $(1, 11/7]$ (c) $(1, 7/3]$ (d) $(1, 7/5]$

25. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ such that $\min f(x) > \max g(x)$, then the relation between b and c, is *(2003S)*
    (a) no real value of b & c (b) $0 < c < b\sqrt{2}$
    (c) $|c| < |b|\sqrt{2}$ (d) $|c| > |b|\sqrt{2}$

26. If $f(x) = \sin x + \cos x$, $g(x) = x^2 - 1$, then $g(f(x))$ is invertible in the domain *(2004S)*
    (a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $[0, \pi]$

27. If the functions f(x) and g(x) are defined on $R \to R$ such that
    $f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$; $g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$ then $(f - g)(x)$ is *(2005S)*
    (a) one-one & onto
    (b) neither one-one nor onto
    (c) one-one but not onto
    (d) onto but not one-one

28. X and Y are two sets and $f: X \to Y$. If $\{f(c) = y; c \subset X, y \subset Y\}$ and $\{f^{-1}(d) = x; d \subset Y, x \subset X\}$, then the true statement is *(2005S)*
(a) $f(f^{-1}(b)) = b$ (b) $f^{-1}(f(a)) = a$
(c) $f(f^{-1}(b)) = b, b \subset y$ (d) $f^{-1}(f(a)) = a, a \subset x$

29. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ where $f''(x) = -f(x)$ and $g(x) = f'(x)$ and given that $F(5) = 5$, then $F(10)$ is equal to *(2006 - 3M, –1)*
(a) 5 (b) 10 (c) 0 (d) 15

30. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = \underbrace{(fofo...of)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x)dx$ equals. *(2007 -3 marks)*
(a) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$ (b) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + K$
(c) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$ (d) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + K$

31. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}, g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then *(2010)*
(a) $a = b$ and $c \neq b$ (b) $a = c$ and $a \neq b$
(c) $a \neq b$ and $c \neq b$ (d) $a = b = c$

32. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) $= f(g(x))$, is *(2011)*
(a) $\pm\sqrt{n\pi}, n \in \{0,1,2,....\}$
(b) $\pm\sqrt{n\pi}, n \in \{1,2,....\}$
(c) $\frac{\pi}{2} + 2n\pi, n \in \{...-2,-1,0,1,2....\}$
(d) $2n\pi, n \in \{...-2,-1,0,1,2,....\}$

33. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is *(2012)*
(a) one–one and onto (b) onto but not one–one
(c) one–one but not onto (d) neither one–one nor onto

D MCQs with One or More than One Correct

1. If $y = f(x) = \frac{x+2}{x-1}$ then *(1984 - 3 Marks)*
(a) $x = f(y)$
(b) $f(1) = 3$
(c) y increases with x for $x < 1$
(d) f is a rational function of x

2. Let $g(x)$ be a function defined on $[-1, 1]$. If the area of the equilateral triangle with two of its vertices at (0,0) and $[x, g(x)]$ is $\frac{\sqrt{3}}{4}$, then the function $g(x)$ is *(1989 - 2 Marks)*
(a) $g(x) = \pm\sqrt{1-x^2}$ (b) $g(x) = \sqrt{1-x^2}$
(c) $g(x) = -\sqrt{1-x^2}$ (d) $g(x) = \sqrt{1+x^2}$

3. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, where $[x]$ stands for the greatest integer function, then *(1991 - 2 Marks)*
(a) $f\left(\frac{\pi}{2}\right) = -1$ (b) $f(\pi) = 1$
(c) $f(-\pi) = 0$ (d) $f\left(\frac{\pi}{4}\right) = 1$

4. If $f(x) = 3x - 5$, then $f^{-1}(x)$ *(1998 - 2 Marks)*
(a) is given by $\frac{1}{3x-5}$
(b) is given by $\frac{x+5}{3}$
(c) does not exist because f is not one-one
(d) does not exist because f is not onto.

5. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then *(1998 - 2 Marks)*
(a) $f(x) = \sin^2 x, g(x) = \sqrt{x}$
(b) $f(x) = \sin x, g(x) = |x|$
(c) $f(x) = x^2, g(x) = \sin\sqrt{x}$
(d) f and g cannot be determined.

6. Let $f: (0, 1) \to \boldsymbol{R}$ be defined by $f(x) = \frac{b-x}{1-bx}$, where b is a constant such that $0 < b < 1$. Then
(a) f is not invertible on (0, 1)
(b) $f \neq f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(c) $f = f^{-1}$ on (0, 1) and $f'(b) = \frac{1}{f'(0)}$
(d) f^{-1} is differentiable (0, 1)

7. Let $f: (-1, 1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the value (s) of $f\left(\frac{1}{3}\right)$ is (are)
(a) $1-\sqrt{\frac{3}{2}}$ (b) $1+\sqrt{\frac{3}{2}}$ (c) $1-\sqrt{\frac{2}{3}}$ (d) $1+\sqrt{\frac{2}{3}}$

8. The function $f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$ has a local minimum or a local maximum at $x =$ *(JEE Adv. 2013)*
(a) -2 (b) $\frac{-2}{3}$ (c) 2 (d) $\frac{2}{3}$

9. Let $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$. Then *(JEE Adv. 2014)*
(a) $f(x)$ is an odd function
(b) $f(x)$ is one-one function
(c) $f(x)$ is an onto function
(d) $f(x)$ is an even function

10. Let $a \in R$ and let $f: R \to R$ be given by $f(x) = x^5 - 5x + a$. Then *(JEE Adv. 2014)*
(a) $f(x)$ has three real roots if $a > 4$
(b) $f(x)$ has only real root if $a > 4$
(c) $f(x)$ has three real roots if $a < -4$
(d) $f(x)$ has three real roots if $-4 < a < 4$

11. Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in R$ and $g(x)=\frac{\pi}{2}\sin x$ for all $x\in R$. Let $(f\circ g)(x)$ denote $f(g(x))$ and $(g\circ f)(x)$ denote $g(f(x))$. Then which of the following is (are) true? *(JEE Adv. 2015)*

(a) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(b) Range of $f\circ g$ is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(c) $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

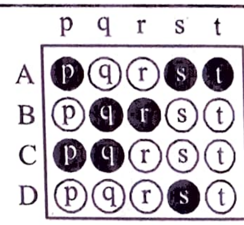
(d) There is an $x\in R$ such that $(g\circ f)(x)=1$

E Subjective Problems

1. Find the domain and range of the function $f(x)=\frac{x^2}{1+x^2}$. Is the function one-to-one? *(1978)*
2. Draw the graph of $y=|x|^{1/2}$ for $-1\le x\le 1$. *(1978)*
3. If $f(x)=x^9-6x^8-2x^7+12x^6+x^4-7x^3+6x^2+x-3$, find $f(6)$. *(1979)*
4. Consider the following relations in the set of real numbers R.
   $R=\{(x,y);x\in R,y\in R,x^2+y^2\le 25\}$
   $R'=\left\{(x,y):x\in R,y\in R,y\ge\frac{4}{9}x^2\right\}$
   Find the domain and range of $R\cap R'$. Is the relation $R\cap R'$ a function? *(1979)*
5. Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Prove that there is a bijective mapping from A to B. *(1981 - 2 Marks)*
6. Let f be a one-one function with domain $\{x,y,z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(x)=1$, $f(y)\ne 1$, $f(z)\ne 2$ determine $f^{-1}(1)$. *(1982 - 3 Marks)*
7. Let R be the set of real numbers and $f:R\longrightarrow R$ be such that for all x and y in R $|f(x)-f(y)|\le|x-y|^3$. Prove that $f(x)$ is a constant. *(1988 - 2 Marks)*
8. Find the natural number 'a' for which $\sum\limits_{k=1}^{n}f(a+k)=16(2^n-1)$, where the function '$f$' satisfies the relation $f(x+y)=f(x)f(y)$ for all natural numbers x, y and further $f(1)=2$. *(1992 - 6 Marks)*
9. Let $\{x\}$ and $[x]$ denotes the fractional and integral part of a real number x respectively. Solve $4\{x\}=x+[x]$. *(1994 - 4 Marks)*
10. A function $f:IR\to IR$, where IR is the set of real numbers, is defined by $f(x)=\frac{\alpha x^2+6x-8}{\alpha+6x-8x^2}$. Find the interval of values of α for which f is onto. Is the function one-to-one for $\alpha=3$? Justify your answer. *(1996 - 5 Marks)*
11. Let $f(x)=Ax^2+Bx+C$ where A, B, C are real numbers. Prove that if $f(x)$ is an integer whenever x is an integer, then the numbers $2A$, $A+B$ and C are all integers. Conversely, prove that if the numbers $2A$, $A+B$ and C are all integers then $f(x)$ is an integer whenever x is an integer. *(1998 - 8 Marks)*

F Match the Following

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Let the function defined in column 1 have domain $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and range $(-\infty,\infty)$ *(1992 - 2 Marks)*

Column I	**Column II**
(A) $1+2x$	(p) onto but not one-one
(B) $\tan x$	(q) one- one but not onto
	(r) one- one and onto
	(s) neither one-one nor onto

2. Let $f(x)=\frac{x^2-6x+5}{x^2-5x+6}$ *(2007 -6 marks)*

Match of expressions/statements in Column I with expressions/statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in the ORS.

Column I	*Column II*
(A) If $-1<x<1$, then $f(x)$ satisfies	(p) $0<f(x)<1$
(B) If $1<x<2$, then $f(x)$ satisfies	(q) $f(x)<0$
(C) If $3<x<5$, then $f(x)$ satisfies	(r) $f(x)>0$
(D) If $x>5$, then $f(x)$ satisfies	(s) $f(x)<1$

This section contains 4 questions. Each question has 2 matching lists: **LIST-I** and **LIST-II**. Four options are given representing matching of elements from **LIST-I** and **LIST-II.** Only one of these four options corresponds to a correct matching.

3. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$ and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$.

(Here, the inverse trigonometric function $\sin^{-1} x$ assumes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$).

Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$ and $g : E_2 \to \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

(JEE Adv. 2018)

LIST - I		LIST - II
P.	The range of f is	1. $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q.	The range of g contains	2. $(0, 1)$
R.	The domain of f contains	3. $\left[-\frac{1}{2}, \frac{1}{2}\right]$
S.	The domain of g is	4. $(-\infty, 0) \cup (0, \infty)$
		5. $\left(-\infty, \frac{e}{e-1}\right]$
		6. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is:

(a) $P \to 4;\ Q \to 2;\ R \to 1;\ S \to 1$
(b) $P \to 3;\ Q \to 3;\ R \to 6;\ S \to 5$
(c) $P \to 4;\ Q \to 2;\ R \to 1;\ S \to 6$
(d) $P \to 4;\ Q \to 3;\ R \to 6;\ S \to 5$

I Integer Value Correct Type

1. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is *(JEE Adv. 2014)*

2. The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is ________. *(JEE Adv. 2018)*

3. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is ______. *(JEE Adv. 2018)*

Section-B JEE Main / AIEEE

1. The domain of $\sin^{-1}[\log_3(x/3)]$ is **[2002]**
(a) $[1, 9]$ (b) $[-1, 9]$ (c) $[-9, 1]$ (d) $[-9, -1]$

2. The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, is **[2003]**
(a) neither an even nor an odd function
(b) an even function
(c) an odd function
(d) a periodic function.

3. Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is **[2003]**
(a) $(-1,0) \cup (1,2) \cup (2,\infty)$ (b) $(a, 2)$
(c) $(-1,0) \cup (a,2)$ (d) $(1,2) \cup (2,\infty)$.

4. If $f : R \to R$ satisfies $f(x+y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is **[2003]**
(a) $\frac{7n(n+1)}{2}$ (b) $\frac{7n}{2}$
(c) $\frac{7(n+1)}{2}$ (d) $7n + (n+1)$.

5. A function f from the set of natural numbers to integers defined by **[2003]**
$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases}$$ is
(a) neither one -one nor onto
(b) one-one but not onto
(c) onto but not one-one
(d) one-one and onto both.

6. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is **[2004]**
(a) $\{1, 2, 3, 4, 5\}$ (b) $\{1, 2, 3, 4, 5, 6\}$
(c) $\{1, 2, 3, 4,\}$ (d) $\{1, 2, 3,\}$

7. If $f: R \to S$, defined by

$f(x) = \sin x - \sqrt{3}\cos x + 1$, is onto, then the interval of S is [2004]

(a) $[-1, 3]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[0, 3]$

8. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then [2004]

(a) $f(x) = -f(-x)$ (b) $f(2+x) = f(2-x)$

(c) $f(x) = f(-x)$ (d) $f(x+2) = f(x-2)$

9. The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is [2004]

(a) $[1, 2]$ (b) $[2, 3)$ (c) $[1, 2]$ (d) $[2, 3]$

10. Let $f: (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\dfrac{2x}{1-x^2}$, then f is both one - one and onto when B is the interval [2005]

(a) $\left(0, \dfrac{\pi}{2}\right)$ (b) $\left[0, \dfrac{\pi}{2}\right)$ (c) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ (d) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

11. A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched? [2005]

	Interval	**Function**
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$
(b)	$[2, \infty)$	$2x^3 - 3x^2 - 12x + 6$
(c)	$\left(-\infty, \dfrac{1}{3}\right]$	$3x^2 - 2x + 1$
(d)	$(-\infty, -4)$	$x^3 + 6x^2 + 6$

12. A real valued function $f(x)$ satisfies the functional equation

$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$

where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to [2005]

(a) $-f(x)$ (b) $f(x)$

(c) $f(a) + f(a-x)$ (d) $f(-x)$

13. The largest interval lying in $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function,

$f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$, is defined, is [2007]

(a) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (b) $\left[0, \dfrac{\pi}{2}\right)$

(c) $[0, \pi]$ (d) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

14. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3 \text{ for some } x \in N\}$.
Show that f is invertible and its inverse is [2008]

(a) $g(y) = \dfrac{3y+4}{3}$ (b) $g(y) = 4 + \dfrac{y+3}{4}$

(c) $g(y) = \dfrac{y+3}{4}$ (d) $g(y) = \dfrac{y-3}{4}$

15. Let $f(x) = (x+1)^2 - 1, x \geq -1$

Statement-1 : The set $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$

Statement-2 : f is a bijection. [2009]

(a) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

16. For real x, let $f(x) = x^3 + 5x + 1$, then [2009]

(a) f is onto R but not one-one
(b) f is one-one and onto R
(c) f is neither one-one nor onto R
(d) f is one-one but not onto R

17. The domain of the function $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is [2011]

(a) $(0, \infty)$ (b) $(-\infty, 0)$

(c) $(-\infty, \infty) - \{0\}$ (d) $(-\infty, \infty)$

18. For $x \in R - \{0, 1\}$, let $f_1(x) = \dfrac{1}{x}$,

$f_2(x) = 1 - x$ and $f_3(x) = \dfrac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 oJof_1)(x) = f_3(x)$ then J(x) is equal to: [JEE M 2019 – 9 Jan (M)]

(a) $f_3(x)$ (b) $f_3(x)$

(c) $f_2(x)$ (d) $f_1(x)$

19. If the fractional part of the number $\dfrac{2^{403}}{15}$ is $\dfrac{k}{15}$, then k is equal to: [JEE M 2019 – 9 Jan (M)]

(a) 6 (b) 8

(c) 4 (d) 14

20. If the function $f : R - \{1, -1\}$ A defined by $f(x) = \dfrac{x^2}{1-x^2}$, is surjective, then A is equal to: [JEE M 2019 – 9 April (M)]

(a) $R - \{-1\}$ (b) $[0, \infty)$

(c) $R - [-1, 0)$ (d) $R - (-1, 0)$

21. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x+y) = f(x)\,f(y)$ for all natural numbers x, y and $f(a) = 2$. Then the natural number 'a' is: [JEE M 2019 – 9 April (M)]

(a) 2 (b) 16 (c) 4 (d) 3