CHAPTER 7

Straight Lines and Pair of Straight Lines

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The area enclosed within the curve $|x|+|y|=1$ is .... *(1981 - 2 Marks)*
2. $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is .... *(1982 - 2 Marks)*
3. The set of lines $ax+by+c = 0$, where $3a + 2b + 4c = 0$ is concurrent at the point .... *(1982 - 2 Marks)*
4. Given the points A (0, 4) and B (0, –4), the equation of the locus of the point $P(x, y)$ such that $|AP-BP|=6$ is .... *(1983 - 1 Mark)*
5. If a, b and c are in A.P., then the straight line $ax+by+c=0$ will always pass through a fixed point whose coordinates are .... *(1984 - 2 Marks)*
6. The orthocentre of the triangle formed by the lines $x+y=1$, $2x+3y=6$ and $4x-y+4=0$ lies in quadrant number .... *(1985 - 2 Marks)*
7. Let the algebraic sum of the perpendicular distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero; then the line passes through a fixed point whose cordinates are .... *(1991 - 2 Marks)*
8. The vertices of a triangle are $A(-1,-7)$, $B(5, 1)$ and $C(1, 4)$. The equation of the bisector of the angle $\angle ABC$ is .... *(1993 - 2 Marks)*

B True / False

1. The straight line $5x + 4y = 0$ passes through the point of intersection of the straight lines $x + 2y - 10 = 0$ and $2x+y+5=0$. *(1983 - 1 Mark)*
2. The lines $2x + 3y + 19 = 0$ and $9x + 6y - 17 = 0$ cut the coordinate axes in concyclic points. *(1988 - 1 Mark)*

C MCQs with One Correct Answer

1. The points $(-a, -b)$, (0, 0), (a, b) and (a^2, ab) are : *(1979)*
   (a) Collinear
   (b) Vertices of a parallelogram
   (c) Vertices of a rectangle
   (d) None of these
2. The point (4, 1) undergoes the following three transformations successively. *(1980)*
   (i) Reflection about the line $y=x$.
   (ii) Translation through a distance 2 units along the positive direction of x-axis.
   (iii) Rotation through an angle p/4 about the origin in the counter clockwise direction.

   Then the final position of the point is given by the coordinates.
   (a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $(-\sqrt{2}, 7\sqrt{2})$
   (c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$
3. The straight lines $x+y=0$, $3x+y-4=0$, $x+3y-4=0$ form a triangle which is *(1983 - 1 Mark)*
   (a) isosceles (b) equilateral
   (c) right angled (d) none of these
4. If $P=(1, 0)$, $Q=(-1, 0)$ and $R=(2, 0)$ are three given points, then locus of the point S satisfying the relation $SQ^2+SR^2=2SP^2$, is *(1988 - 2 Marks)*
   (a) a straight line parallel to x-axis
   (b) a circle passing through the origin
   (c) a circle with the centre at the origin
   (d) a straigth line parallel to y-axis.
5. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then *(1990 - 2 Marks)*
   (a) $a^2+b^2=p^2+q^2$ (b) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$
   (c) $a^2+p^2=b^2+q^2$ (d) $\frac{1}{a^2}+\frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{q^2}$
6. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then its locus is *(1992 - 2 Marks)*
   (a) square (b) circle
   (c) straight line (d) two intersecting lines
7. The locus of a variable point whose distance from (–2, 0) is 2/3 times its distance from the line $x=-\frac{9}{2}$ is *(1994)*
   (a) ellipse (b) parabola
   (c) hyperbola (d) none of these
8. The equations to a pair of opposite sides of parallelogram are $x^2-5x+6=0$ and $y^2-6y+5=0$, the equations to its diagonals are *(1994)*
   (a) $x+4y=13, y=4x-7$ (b) $4x+y=13, 4y=x-7$
   (c) $4x+y=13, y=4x-7$ (d) $y-4x=13, y+4x=7$
9. The orthocentre of the triangle formed by the lines $xy=0$ and $x+y=1$ is *(1995S)*
   (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{3}, \frac{1}{3}\right)$ (c) $(0, 0)$ (d) $\left(\frac{1}{4}, \frac{1}{4}\right)$

10. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is *(1999 - 2 Marks)*
(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$
(b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$
(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$
(d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

11. If x_1, x_2, x_3 as well as y_1, y_2, y_3, are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3). *(1999 - 2 Marks)*
(a) lie on a straight line (b) lie on an ellipse
(c) lie on a circle (d) are vertices of a triangle

12. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is *(2000S)*
(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$
(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

13. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is *(2000S)*
(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

14. The number of integer values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is *(2001S)*
(a) 2 (b) 0 (c) 4 (d) 1

15. Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals *(2001S)*
(a) $|m + n|/(m - n)^2$ (b) $2/|m + n|$
(c) $1/(|m + n|)$ (d) $1/(|m - n|)$

16. Let $0 < \alpha < \frac{\pi}{2}$ be fixed angle. If

$P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha - \theta), \sin(\alpha - \theta))$,

then Q is obtained from P by *(2002S)*
(a) clockwise rotation around origin through an angle α
(b) anticlockwise rotation around origin through an angle α
(c) reflection in the line through origin with slope $\tan\alpha$
(d) reflection in the line through origin with slope $\tan(\alpha/2)$

17. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is *(2002S)*
(a) $\frac{\sqrt{3}}{2}x + y = 0$ (b) $x + \sqrt{3}y = 0$
(c) $\sqrt{3}x + y = 0$ (d) $x + \frac{\sqrt{3}}{2}y = 0$

18. A straight line through the origin O meets the parallel lines $4x + 2y = 9$ and $2x + y + 6 = 0$ at points P and Q respectively. Then the point O divides the segemnt PQ in the ratio *(2002S)*
(a) 1 : 2 (b) 3 : 4 (c) 2 : 1 (d) 4 : 3

19. The number of intergral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0,0), (0,21) and (21,0), is *(2003S)*
(a) 133 (b) 190 (c) 233 (d) 105

20. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is *(2003S)*
(a) $\left(3, \frac{5}{4}\right)$ (b) $(3, 12)$ (c) $\left(3, \frac{3}{4}\right)$ (d) $(3, 9)$

21. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is *(2004S)*
(a) 2 sq. units (b) 4 sq. units
(c) 6 sq. units (d) 8 sq. units

22. Let $O(0, 0)$, $P(3, 4)$, $Q(6, 0)$ be the vertices of the triangles OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The coordinates of R are *(2007 -3 marks)*
(a) $\left(\frac{4}{3}, 3\right)$ (b) $\left(3, \frac{2}{3}\right)$ (c) $\left(3, \frac{4}{3}\right)$ (d) $\left(\frac{4}{3}, \frac{2}{3}\right)$

23. A straight line L through the point (3, –2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is *(2011)*
(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$
(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

D MCQs with One or More than One Correct

1. Three lines $px + qy + r = 0$, $qx + ry + p = 0$ and $rx + py + q = 0$ are concurrent if *(1985 - 2 Marks)*
(a) $p + q + r = 0$
(b) $p^2 + q^2 + r^2 = qr + rp + pq$
(c) $p^3 + q^3 + r^3 = 3pqr$
(d) none of these.

2. The points $\left(0, \frac{8}{3}\right)$, $(1, 3)$ and $(82, 30)$ are vertices of *(1986 - 2 Marks)*
(a) an obtuse angled triangle
(b) an acute angled triangle
(c) a right angled triangle
(d) an isosceles triangle
(e) none of these.

3. All points lying inside the triangle formed by the points $(1, 3)$, $(5, 0)$ and $(-1, 2)$ satisfy *(1986 - 2 Marks)*
(a) $3x + 2y \geq 0$ (b) $2x + y - 13 \geq 0$
(c) $2x - 3y - 12 \leq 0$ (d) $-2x + y \geq 0$
(e) none of these.

4. A vector $\vec{a}$ has components $2p$ and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to the new system, $\vec{a}$ has components $p + 1$ and 1, then *(1986 - 2 Marks)*
(a) $p = 0$ (b) $p = 1$ or $p = -\frac{1}{3}$
(c) $p = -1$ or $p = \frac{1}{3}$ (d) $p = 1$ or $p = -1$
(e) none of these .

5. If (P(1, 2), Q(4, 6), R(5, 7) and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then *(1998 - 2 Marks)*
(a) $a=2, b=4$ (b) $a=3, b=4$
(c) $a=2, b=3$ (d) $a=3, b=5$

6. The diagonals of a parallelogram $PQRS$ are along the lines $x+3y=4$ and $6x-2y=7$. Then $PQRS$ must be a. *(1998 - 2 Marks)*
(a) rectangle (b) square
(c) cyclic quadrilateral (d) rhombus.

7. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is (are) always rational point(s)? *(1998 - 2 Marks)*
(a) centroid (b) incentre
(c) circumcentre (d) orthocentre
(A rational point is a point both of whose co-ordinates are rational numbers.)

8. Let L_1 be a strainght line passing through the origin and L_2 be the straight line $x+y=1$. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1? *(1999 - 3 Marks)*
(a) $x+y=0$ (b) $x-y=0$
(c) $x+7y=0$ (d) $x-7y=0$

9. For $a>b>c>0$, the distance between (1, 1) and the point of intersection of the lines $ax+by+c=0$ and $bx+ay+c=0$ is less than $2\sqrt{2}$. Then *(JEE Adv. 2013)*
(a) $a+b-c>0$ (b) $a-b+c<0$
(c) $a-b+c>0$ (d) $a+b-c<0$

E Subjective Problems

1. A straight line segment of length ℓ moves with its ends on two mutually perpendicular lines. Find the locus of the point which divides the line segment in the ratio 1 : 2. *(1978)*

2. The area of a triangle is 5. Two of its vertices are A (2, 1) and B (3, –2). The third vertex C lies on $y=x+3$. Find C. *(1978)*

3. One side of a rectangle lies along the line $4x+7y+5=0$. Two of its vertices are (–3, 1) and (1, 1). Find the equations of the other three sides. *(1978)*

4. (a) Two vertices of a triangle are (5, –1) and (–2, 3). If the orthocentre of the triangle is the origin, find the coordinates of the third point.
(b) Find the equation of the line which bisects the obtuse angle between the lines $x-2y+4=0$ and $4x-3y+2=0$. *(1979)*

5. A straight line L is perpendicular to the line $5x-y=1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L. *(1980)*

6. The end A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ be completed, then show that the locus of the foot of the perpendicular drawn from P to AB is
$x^{\frac{2}{3}}+y^{\frac{2}{3}}=c^{\frac{2}{3}}$ *(1983 - 2 Marks)*

7. The vertices of a triangle are $[at_1t_2, a(t_1+t_2)]$, $[at_2t_3, a(t_2+t_3)]$, $[at_3t_1, a(t_3+t_1)]$. Find the orthocentre of the triangle. *(1983 - 3 Marks)*

8. The coordinates of A, B, C are (6, 3), (–3, 5), (4, –2) respectively, and P is any point (x, y). Show that the ratio of the area of the triangles ΔPBC and ΔABC is $\left|\frac{x+y-2}{7}\right|$ *(1983 - 2 Marks)*

9. Two equal sides of an isosceles triangle are given by the equations $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point (1, –10). Determine the equation of the third side. *(1984 - 4 Marks)*

10. One of the diameters of the circle circumscribing the rectangle $ABCD$ is $4y=x+7$. If A and B are the points (–3, 4) and (5, 4) respectively, then find the area of rectangle. *(1985 - 3 Marks)*

11. Two sides of a rhombus $ABCD$ are parallel to the lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at the point (1, 2) and the vertex A is on the y-axis, find possible co-ordinates of A. *(1985 - 5 Marks)*

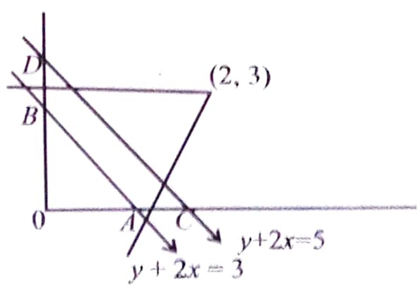
12. Lines $L_1 \equiv ax+by+c=0$ and $L_2 \equiv lx+my+n=0$ intersect at the point P and make an angle θ with each other. Find the equation of a line L different from L_2 which passes through P and makes the same angle θ with L_1. *(1988 - 5 Marks)*

13. Let ABC be a triangle with $AB=AC$. If D is the midpoint of BC, E is the foot of the perpendicular drawn from D to AC and F the mid-point of DE, prove that AF is perpendicular to BE. *(1989 - 5 Marks)*

14. Straight lines $3x+4y=5$ and $4x-3y=15$ intersect at the point A. Points B and C are chosen on these two lines such that $AB=AC$. Determine the possible equations of the line BC passing through the point (1, 2). *(1990 - 4 Marks)*

15. A line cuts the x-axis at A (7, 0) and the y-axis at B(0, –5). A variable line PQ is drawn perpendicular to AB cutting the x-axis in P and the y-axis in Q. If AQ and BP intersect at R, find the locus of R. *(1990 - 4 Marks)*

16. Find the equation of the line passing through the point (2, 3) and making intercept of length 2 units between the lines $y+2x=3$ and $y+2x=5$. *(1991 - 4 Marks)*



17. Show that all chords of the curve $3x^2-y^2-2x+4y=0$, which subtend a right angle at the origin, pass through a fixed point. Find the coordinates of the point. *(1991 - 4 Marks)*

18. Determine all values of α for which the point (α, α^2) lies inside the triangle formed by the lines

$$2x+3y-1=0$$ *(1992 - 6 Marks)*
$$x+2y-3=0$$
$$5x-6y-1=0$$

19. Tagent at a point P_1 {other than (0, 0)} on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3, and so on. Show that the abscissae of $P_1, P_2, P_3, \ldots\ldots\ldots P_n$, form a G.P. Also find the ratio.

$[\text{area}(\Delta P_1, P_2, P_3)]/[\text{area}(P_2 P_3, P_4)]$ *(1993 - 5 Marks)*

20. A line through $A(-5, -4)$ meets the line $x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at the points B, C and D respectively. If $(15/AB)^2+(10/AC)^2=(6/AD)^2$, find the equation of the line. *(1993 - 5 Marks)*

21. A rectangle $PQRS$ has its side PQ parallel to the line $y = mx$ and vertices P, Q and S on the lines $y = a$, $x = b$ and $x = -b$, respectively. Find the locus of the vertex R. *(1996 - 2 Marks)*

22. Using co-ordinate geometry, prove that the three altitudes of any triangle are concurrent. *(1998 - 8 Marks)*

23. For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. *(2000 - 10 Marks)*

24. Let ABC and PQR be any two triangles in the same plane. Assume that the prependiculars from the points A, B, C to the sides QR, RP, PQ respectively are concurrent. Using vector methods or otherwise, prove that the prependiculars from P, Q, R to BC, CA, AB respectively are also concurrent. *(2000 - 10 Marks)*

25. Let a, b, c be real numbers with $a^2+b^2+c^2=1$. Show that

the equation $\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$

represents a straight line. *(2001 - 6 Marks)*

26. A straight line L through the origin meets the lines $x+y=1$ and $x+y=3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x-y=5$ and $3x+y=5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. *(2002 - 5 Marks)*

27. A straight line L with negative slope passes through the point $(8, 2)$ and cuts the positive coordinate axes at points P and Q. Find the absolute minimum value of $OP + OQ$, as L varies, where O is the origin. *(2002 - 5 Marks)*

28. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h, k)$ with the lines $y = x$ and $x + y = 2$ is $4h^2$. Find the locus of the point P. *(2005 - 2 Marks)*

H Assertion & Reason Type Questions

1. Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

STATEMENT-1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

because

STATEMENT-2 : In any triangle, bisector of an angle divides the triangle into two similar triangles. *(2007 - 3 marks)*

(a) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

I Integer Value Correct Type

1. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is *(JEE Adv. 2014)*

Section-B JEE Main / AIEEE

1. A triangle with vertices (4, 0), (–1, –1), (3, 5) is [2002]
(a) isosceles and right angled
(b) isosceles but not right angled
(c) right angled but not isosceles
(d) neither right angled nor isoceles

2. Locus of mid point of the portion between the axes of $x \cos\alpha + y \sin\alpha = p$ whre p is constant is [2002]
(a) $x^2+y^2=\frac{4}{p^2}$ (b) $x^2+y^2=4p^2$
(c) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{2}{p^2}$ (d) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$

3. If the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect on the y-axis then [2002]
(a) $2fgh = bg^2 + ch^2$ (b) $bg^2 \ne ch^2$
(c) $abc = 2fgh$ (d) none of these

4. The pair of lines represented by
$$3ax^2+5xy+(a^2-2)y^2=0$$
are perpendicular to each other for [2002]
(a) two values of a (b) $\forall\, a$
(c) for one value of a (d) for no values of a

5. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is [2003]
(a) $y(\cos\alpha+\sin\alpha)+x(\cos\alpha-\sin\alpha)=a$
(b) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$
(c) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$
(d) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$.

6. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angle between the other pair, then [2003]
(a) $pq=-1$ (b) $p=q$ (c) $p=-q$ (d) $pq=1$.

7. Locus of centroid of the triangle whose vertices are $(a\cos t, a\sin t),(b\sin t,-b\cos t)$ and (1, 0), where t is a parameter, is [2003]
(a) $(3x+1)^2+(3y)^2=a^2-b^2$
(b) $(3x-1)^2+(3y)^2=a^2-b^2$
(c) $(3x-1)^2+(3y)^2=a^2+b^2$
(d) $(3x+1)^2+(3y)^2=a^2+b^2$.

8. If x_1,x_2,x_3 and y_1,y_2,y_3 are both in G.P. with the same common ratio, then the points $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) [2003]
(a) are vertices of a triangle
(b) lie on a straight line
(c) lie on an ellipse
(d) lie on a circle.

9. If the equation of the locus of a point equidistant from the point (a_1,b_1) and (a_2,b_2) is $(a_1-b_2)x+(a_1-b_2)y+c=0$, then the value of `c` is [2003]
(a) $\sqrt{a_1^2+b_1^2-a_2^2-b_2^2}$
(b) $\frac{1}{2}(a_2^2+b_2^2-a_1^2-b_1^2)$
(c) $a_1^2-a_2^2+b_1^2-b_2^2$
(d) $\frac{1}{2}(a_1^2+a_2^2+b_1^2+b_2^2)$.

10. Let $A(2,-3)$ and $B(-2,3)$ be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x+3y=1$, then the locus of the vertex C is the line [2004]
(a) $3x-2y=3$ (b) $2x-3y=7$
(c) $3x+2y=5$ (d) $2x+3y=9$

11. The equation of the straight line passing through the point (4, 3) and making intercepts on the co-ordinate axes whose sum is –1 is [2004]
(a) $\frac{x}{2}-\frac{y}{3}=1$ and $\frac{x}{-2}+\frac{y}{1}=1$
(b) $\frac{x}{2}-\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$
(c) $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{2}+\frac{y}{1}=1$
(d) $\frac{x}{2}+\frac{y}{3}=-1$ and $\frac{x}{-2}+\frac{y}{1}=-1$

12. If the sum of the slopes of the lines given by $x^2-2cxy-7y^2=0$ is four times their product c has the value [2004]
(a) –2 (b) –1 (c) 2 (d) 1

13. If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$, then c equals [2004]
(a) –3 (b) –1 (c) 3 (d) 1

14. The line parallel to the x- axis and passing through the intersection of the lines $ax+2by+3b=0$ and $bx-2ay-3a=0$, where $(a,b)\neq(0,0)$ is [2005]
(a) below the x - axis at a distance of $\frac{3}{2}$ from it
(b) below the x - axis at a distance of $\frac{2}{3}$ from it
(c) above the x - axis at a distance of $\frac{3}{2}$ from it
(d) above the x - axis at a distance of $\frac{2}{3}$ from it

15. If a vertex of a triangle is (1, 1) and the mid points of two sides through this vertex are (–1, 2) and (3, 2) then the centroid of the triangle is [2005]
(a) $\left(-1,\frac{7}{3}\right)$ (b) $\left(\frac{-1}{3},\frac{7}{3}\right)$
(c) $\left(1,\frac{7}{3}\right)$ (d) $\left(\frac{1}{3},\frac{7}{3}\right)$

16. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is [2006]
(a) $x+y=7$ (b) $3x-4y+7=0$
(c) $4x+3y=24$ (d) $3x+4y=25$

17. If (a,a^2) falls inside the angle made by the lines $y=\frac{x}{2}$, $x>0$ and $y=3x$, $x>0$, then a belong to [2006]
(a) $\left(0,\frac{1}{2}\right)$ (b) $(3,\infty)$
(c) $\left(\frac{1}{2},3\right)$ (d) $\left(-3,-\frac{1}{2}\right)$

18. Let A (h, k), B(1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1 square unit, then the set of values which 'k' can take is given by [2007]
(a) $\{-1,3\}$ (b) $\{-3,-2\}$ (c) $\{1,3\}$ (d) $\{0,2\}$

19. Let $P=(-1,0)$, $Q=(0,0)$ and $R=(3,3\sqrt{3})$ be three point. The equation of the bisector of the angle PQR is [2007]

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3}y=0$

(c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$.

20. If one of the lines of $my^2+(1-m^2)xy-mx^2=0$ is a bisector of the angle between the lines $xy=0$, then m is [2007]

(a) 1 (b) 2 (c) $-1/2$ (d) -2

21. The perpendicular bisector of the line segment joining P (1, 4) and Q(k, 3) has y-intercept -4. Then a possible value of k is [2008]

(a) 1 (b) 2 (c) -2 (d) -4

22. The shortest distance between the line $y-x=1$ and the curve $x=y^2$ is : [2009]

(a) $\frac{2\sqrt{3}}{8}$ (b) $\frac{3\sqrt{2}}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{3\sqrt{2}}{8}$

23. The lines $p(p^2+1)x-y+q=0$ and $(p^2+1)^2x+(p^2+1)y+2q=0$ are perpendicular to a common line for : [2009]

(a) exactly one values of p
(b) exactly two values of p
(c) more than two values of p
(d) no value of p

24. Three distinct points A, B and C are given in the 2-dimensional coordinates plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point $(-1, 0)$ is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point: [2009]

(a) $\left(\frac{5}{4},0\right)$ (b) $\left(\frac{5}{2},0\right)$ (c) $\left(\frac{5}{3},0\right)$ (d) (0, 0)

25. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is [2010]

(a) $\sqrt{17}$ (b) $\frac{17}{\sqrt{15}}$ (c) $\frac{23}{\sqrt{17}}$ (d) $\frac{23}{\sqrt{15}}$

26. The lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio $PR:RQ$ equals $2\sqrt{2}:\sqrt{5}$

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles. [2011]

(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

27. If the line $2x+y=k$ passes through the point which divides the line segment joining the points (1,1) and (2,4) in the ratio 3 :2, then k equals : [2012]

(a) $\frac{29}{5}$ (b) 5 (c) 6 (d) $\frac{11}{5}$

28. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is [JEE M 2013]

(a) $y=x+\sqrt{3}$ (b) $\sqrt{3}y=x-\sqrt{3}$

(c) $y=\sqrt{3}x-\sqrt{3}$ (d) $\sqrt{3}y=x-1$

29. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is : [JEE M 2013]

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

30. Let PS be the median of the triangle with vertices $P(2, 2)$, $Q(6,-1)$ and $R(7, 3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is: [JEE M 2014]

(a) $4x+7y+3=0$ (b) $2x-9y-11=0$
(c) $4x-7y-11=0$ (d) $2x+9y+7=0$

31. Let a, b, c and d be non-zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then [JEE M 2014]

(a) $3bc-2ad=0$ (b) $3bc+2ad=0$
(c) $2bc-3ad=0$ (d) $2bc+3ad=0$

32. The number of points, having both co-ordinates as integers, that lie in the interior of the triangle with vertices (0, 0), (0, 41) and (41, 0) is : [JEE M 2015]

(a) 820 (b) 780 (c) 901 (d) 861

33. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus? [JEE M 2016]

(a) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (b) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

(c) $(-3,-9)$ (d) $(-3,-8)$

34. A straight the through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is: [JEE M 2018]

(a) $2x+3y=xy$ (b) $3x+2y=xy$

(c) $3x+2y=6xy$ (d) $3x+2y=6$

35. Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$. Which one of the following statements is true? [JEE M 2019 – 9 Jan (M)]

(a) The lines are concurrent at the point $\left(\frac{3}{4},\frac{1}{2}\right)$.
(b) Each line passes through the origin.
(c) The lines are all parallel.
(d) The lines are not concurrent.

36. Slope of a line passing through P(2, 3) and intersecting the line $x+y=7$ at a distance of 4 units from P, is: [JEE M 2019 – 9 April (M)]

(a) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

(c) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$