CHAPTER

14

Inverse Trigonometric Functions

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let a, b, c be positive real numbers. Let

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}.$$

Then $\tan\theta =$ _____ *(1981 - 2 Marks)*

2. The numerical value of $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to _____ *(1984 - 2 Marks)*

3. The greater of the two angles $A = 2\tan^{-1}(2\sqrt{2}-1)$ and $B = 3\sin^{-1}(1/3) + \sin^{-1}(3/5)$ is _____. *(1989 - 2 Marks)*

C MCQs with One Correct Answer

1. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is *(1983 - 1 Mark)*

(a) $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$ (d) none

2. If we consider only the principle values of the inverse trigonometric functions then the value of

$$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right) \text{ is} \quad \textit{(1994)}$$

(a) $\frac{\sqrt{29}}{3}$ (b) $\frac{29}{3}$ (c) $\frac{\sqrt{3}}{29}$ (d) $\frac{3}{29}$

3. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$ is *(1999 - 2 Marks)*

(a) zero (b) one (c) two (d) infinite

4. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \ldots\right) + \cos^{-1}\left(x^2 - \frac{x^4}{2} + \frac{x^6}{4} - \ldots\right) = \frac{\pi}{2}$

for $0 < |x| < \sqrt{2}$, then x equals *(2001S)*

(a) 1/2 (b) 1 (c) –1/2 (d) –1

5. The value of x for which $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1}x)$ is *(2004S)*

(a) 1/2 (b) 1 (c) 0 (d) –1/2

6. If $0 < x < 1$, then

$\sqrt{1+x^2}\,[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2} =$ *(2008)*

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) x

(c) $x\sqrt{1+x^2}$ (d) $\sqrt{1+x^2}$

7. The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is *(JEE Adv. 2013)*

(a) $\frac{23}{25}$ (b) $\frac{25}{23}$ (c) $\frac{23}{24}$ (d) $\frac{24}{23}$

D MCQs with One or More than One Correct

1. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is *(1986 - 2 Marks)*

(a) $-\frac{2\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) none

2. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are) *(JEE Adv. 2015)*

(a) $\cos\beta > 0$ (b) $\sin\beta < 0$

(c) $\cos(\alpha+\beta) > 0$ (d) $\cos\alpha < 0$

3. For non-negative integers n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0, \pi]$, which of the following options is/are correct? *(JEE Adv. 2019)*

(a) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

(b) $f(4)=\frac{\sqrt{3}}{2}$

(c) If $\alpha=\tan\left(\cos^{-1}f(6)\right)$, then $\alpha^2+2\alpha-1=0$

(d) $\sin(7\cos^{-1}f(5))=0$

E Subjective Problems

1. Find the value of : $\cos(2\cos^{-1}x+\sin^{-1}x)$ at $x=\frac{1}{5}$, where $0\le\cos^{-1}x\le\pi$ and $-\pi/2\le\sin^{-1}x\le\pi/2$. *(1981 - 2 Marks)*

2. Find all the solution of $4\cos^2 x\sin x-2\sin^2 x=3\sin x$ *(1983 - 2 Marks)*

3. Prove that $\cos\tan^{-1}\sin\cot^{-1}x=\sqrt{\frac{x^2+1}{x^2+2}}$. *(2002 - 5 Marks)*

F Match the Following

DIRECTIONS (Q. 1 & 2) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*

	p	q	r	s	t
A	●	○	○	●	○
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	○

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

(2006 - 6M)

1. Match the following

Column I	***Column II***
(A) $\sum_{i=1}^{\infty}\tan^{-1}\left(\frac{1}{2i^2}\right)=t$, then tan t =	(p) 1
(B) Sides a, b, c of a triangle ABC are in AP and $\cos\theta_1=\frac{a}{b+c}, \cos\theta_2=\frac{b}{a+c}, \cos\theta_3=\frac{c}{a+b}$, then $\tan^2\left(\frac{\theta_1}{2}\right)+\tan^2\left(\frac{\theta_3}{2}\right)=$	(q) $\frac{\sqrt{5}}{3}$
(C) A line is perpendicular to $x+2y+2z=0$ and passes through (0, 1, 0). The perpendicular distance of this line from the origin is	(r) $\frac{2}{3}$

2. Let (x, y) be such that $\sin^{-1}(ax) + \cos^{-1}(y) + \cos^{-1}(bxy) = \frac{\pi}{2}$. *(2007)*

Match the statements in Column I with statements in Column II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	***Column II***
(A) If $a = 1$ and $b = 0$, then (x, y)	(p) lies on the circle $x^2 + y^2 = 1$
(B) If $a = 1$ and $b = 1$, they (x, y)	(q) lies on $(x^2 - 1)(y^2 - 1) = 0$
(C) If $a = 1$ and $b = 2$, then (x, y)	(r) lies on $y = x$
(D) If $a = 2$ and $b = 2$, then (x, y)	(s) lies on $(4x^2 - 1)(y^2 - 1) = 0$

DIRECTIONS (Q. 3) : *Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

3. Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2013)*

List I	**List II**
P. $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right)^{1/2}$ takes value	1. $\frac{1}{2}\sqrt{\frac{5}{3}}$
Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\frac{x - y}{2}$ is	2. $\sqrt{2}$
R. If $\cos\left(\frac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2 \sec x = \cos x \sin 2x \sec x + \cos\left(\frac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is	3. $\frac{1}{2}$
S. If $\cot\left(\sin^{-1}\sqrt{1 - x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right), x \neq 0$, then possible value of x is	4. 1

Codes:

	P	Q	R	S
(a)	4	3	1	2
(b)	4	3	2	1
(c)	3	4	2	1
(d)	3	4	1	2

I Integer Value Correct Type

1. The number of real solutions of the equation

$$\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{\pi}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i - \sum_{i=1}^{\infty}(-x)^i\right)$$

lying in the interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ is _____.

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\cos^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[0, \pi]$, respectively.)

(JEE Adv. 2018)

2. The value of

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$ equals _____. *(JEE Adv. 2019)*

Section-B JEE Main / AIEEE

1. $\cot^{-1}(\sqrt{\cos\alpha}) - \tan^{-1}(\sqrt{\cos\alpha}) = x$, then $\sin x =$

(a) $\tan^2\left(\frac{\alpha}{2}\right)$ (b) $\cot^2\left(\frac{\alpha}{2}\right)$ [2002]

(c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

2. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$ has a solution for [2003]

(a) $|a| \geq \frac{1}{\sqrt{2}}$ (b) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$

(c) all real values of a (d) $|a| < \frac{1}{2}$

3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to [2005]

(a) $2\sin 2\alpha$ (b) 4

(c) $4\sin^2\alpha$ (d) $-4\sin^2\alpha$

4. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the values of x is

(a) 4 (b) 5 [2007]

(c) 1 (d) 3

5. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$

(c) $\frac{4}{17}$ (d) $\frac{5}{17}$

6. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then [JEE M 2013]

(a) $x = y = z$ (b) $2x = 3y = 6z$

(c) $6x = 3y = 2z$ (d) $6x = 4y = 3z$

7. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is : [JEE M 2015]

(a) $\frac{3x - x^3}{1 + 3x^2}$ (b) $\frac{3x + x^3}{1 + 3x^2}$

(c) $\frac{3x - x^3}{1 - 3x^2}$ (d) $\frac{3x + x^3}{1 - 3x^2}$

8. If $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}\left(x > \frac{3}{4}\right)$, then x is equal to: [JEE M 2019 – 9 Jan (M)]

(a) $\frac{\sqrt{145}}{12}$ (b) $\frac{\sqrt{145}}{10}$

(c) $\frac{\sqrt{146}}{12}$ (d) $\frac{\sqrt{145}}{11}$