CHAPTER 1

Trigonometric Functions & Equations

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Suppose $\sin^3 x \sin 3x = \sum_{m=0}^{n} C_m \cos mx$ is an identity in x, where $C_0, C_1, \ldots C_n$ are constants, and $C_n \neq 0$. then the value of n is ________ *(1981 - 2 Marks)*

2. The solution set of the system of equations $x + y = \frac{2\pi}{3}$, $\cos x + \cos y = \frac{3}{2}$, where x and y are real, is ________. *(1987 - 2 Marks)*

3. The set of all x in the interval $[0, \pi]$ for which $2\sin^2 x - 3\sin x + 1 \geq 0$, is ________. *(1987 - 2 Marks)*

4. The sides of a triangle inscribed in a given circle subtend angles α, β and γ at the centre. The minimum value of the arithmetic mean of $\cos\left(\alpha + \frac{\pi}{2}\right)$, $\cos\left(\beta + \frac{\pi}{2}\right)$ and $\cos\left(\gamma + \frac{\pi}{2}\right)$ is equal to ________ *(1987 - 2 Marks)*

5. The value of

$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is equal to ________ *(1991 - 2 Marks)*

6. If $K = \sin(\pi/18)\sin(5\pi/18)\sin(7\pi/18)$, then the numerical value of K is ________. *(1993 - 2 Marks)*

7. If $A > 0, B > 0$ and $A + B = \pi/3$, then the maximum value of $\tan A \tan B$ is________. *(1993 - 2 Marks)*

8. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is________. *(1996 - 1 Mark)*

9. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are ..., ..., and________. *(1997 - 2 Marks)*

B True / False

1. If $\tan A = (1 - \cos B)/\sin B$, then $\tan 2A = \tan B$. *(1983 - 1 Mark)*

2. There exists a value of θ between 0 and 2π that satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1 = 0$. *(1984 - 1 Mark)*

C MCQs with One Correct Answer

1. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is *(1979)*
   (a) $-\frac{4}{5}$ but not $\frac{4}{5}$
   (b) $-\frac{4}{5}$ or $\frac{4}{5}$
   (c) $\frac{4}{5}$ but not $-\frac{4}{5}$
   (d) None of these.

2. If $\alpha + \beta + \gamma = 2\pi$, then *(1979)*
   (a) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (b) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
   (c) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$
   (d) None of these.

3. Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ *(1980)*
   (a) $1 \leq A \leq 2$
   (b) $\frac{3}{4} \leq A \leq 1$
   (c) $\frac{13}{16} \leq A \leq 1$
   (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$

4. The equation $2\cos^2\frac{x}{2}\sin^2 x = x^2 + x^{-2}$; $0 < x \leq \frac{\pi}{2}$ has
   (a) no real solution
   (b) one real solution
   (c) more than one solution
   (d) none of these *(1980)*

5. The general solution of the trigonometric equation $\sin x + \cos x = 1$ is given by : *(1981 - 2 Marks)*
   (a) $x = 2n\pi$; $n = 0, \pm 1, \pm 2 \ldots$
   (b) $x = 2n\pi + \pi/2$; $n = 0, \pm 1, \pm 2 \ldots$
   (c) $x = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$ $n = 0, \pm 1, \pm 2 \ldots$
   (d) none of these

6. The value of the expression $\sqrt{3}\,\operatorname{cosec} 20° - \sec 20°$ is equal to *(1988 - 2 Marks)*
(a) 2 (b) $2 \sin 20°/\sin 40°$
(c) 4 (d) $4 \sin 20°/\sin 40°$

7. The general solution of
$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$ is *(1989 - 2 Marks)*
(a) $n\pi + \frac{\pi}{8}$ (b) $\frac{n\pi}{2} + \frac{\pi}{8}$
(c) $(-1)^n \frac{n\pi}{2} + \frac{\pi}{8}$ (d) $2n\pi + \cos^{-1}\frac{3}{2}$

8. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$
In the variable x, has real roots. Then p can take any value in the interval *(1990 - 2 Marks)*
(a) $(0, 2\pi)$ (b) $(-\pi, 0)$ (c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $(0, \pi)$

9. Number of solutions of the equation *(1993 - 1 Mark)*
$\tan x + \sec x = 2 \cos x$ lying in the interval $[0, 2\pi]$ is :
(a) 0 (b) 1 (c) 2 (d) 3

10. Let $0 < x < \frac{\pi}{4}$ then $(\sec 2x - \tan 2x)$ equals *(1994)*
(a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$
(c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

11. Let n be a positive integer such that
$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \frac{\sqrt{n}}{2}$. Then *(1994)*
(a) $6 \le n \le 8$ (b) $4 < n \le 8$
(c) $4 \le n \le 8$ (d) $4 < n < 8$

12. If ω is an imaginary cube root of unity then the value of
$\sin\left\{(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}\right\}$ is *(1994)*
(a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

13. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) =$ *(1995S)*
(a) 11 (b) 12 (c) 13 (d) 14

14. The general values of θ satisfying the equation
$2\sin^2\theta - 3\sin\theta - 2 = 0$ is *(1995S)*
(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n \pi/2$
(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^n 7\pi/6$

15. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if *(1996 - 1 Mark)*
(a) $x + y \ne 0$ (b) $x = y, x \ne 0$
(c) $x = y$ (d) $x \ne 0, y \ne 0$

16. In a triangle PQR, $\angle R = \pi/2$. If $\tan(P/2)$ and $\tan(Q/2)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \ne 0)$ then. *(1999 - 2 Marks)*
(a) $a + b = c$ (b) $b + c = a$
(c) $a + c = b$ (d) $b = c$

17. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. Then $f(\theta)$ is *(2000S)*
(a) ≥ 0 only when $\theta \ge 0$ (b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ (d) ≤ 0 only when $\theta \le 0$

18. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$ $= 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is *(2001S)*
(a) 0 (b) 2 (c) 1 (d) 3

19. The maximum value of $(\cos\alpha_1).(\cos\alpha_2)\ldots(\cos\alpha_n)$, under the restrictions
$0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \frac{\pi}{2}$ and $(\cot\alpha_1).(\cot\alpha_2)\ldots(\cot\alpha_n) = 1$ is *(2001S)*
(a) $1/2^{n/2}$ (b) $1/2^n$ (c) $1/2n$ (d) 1

20. If $\alpha + \beta = \pi/2$ and $\beta + \gamma = \alpha$, then $\tan\alpha$ equals *(2001S)*
(a) $2(\tan\beta + \tan\gamma)$ (b) $\tan\beta + \tan\gamma$
(c) $\tan\beta + 2\tan\gamma$ (d) $2\tan\beta + \tan\gamma$

21. The number of integral values of k for which the equation $7 \cos x + 5 \sin x = 2k + 1$ has a solution is *(2002S)*
(a) 4 (b) 8 (c) 10 (d) 12

22. Given both θ and ϕ are acute angles and $\sin\theta = \frac{1}{2}$, $\cos\phi = \frac{1}{3}$, then the value of $\theta + \phi$ belongs to *(2004S)*
(a) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
(c) $\left(\frac{2\pi}{3}, \frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6}, \pi\right]$

23. $\cos(\alpha - \beta) = 1$ and $\cos(\alpha + \beta) = 1/e$ where $\alpha, \beta \in [-\pi, \pi]$. Pairs of α, β which satisfy both the equations is/are *(2005S)*
(a) 0 (b) 1 (c) 2 (d) 4

24. The values of $\theta \in (0, 2\pi)$ for which $2\sin^2\theta - 5\sin\theta + 2 > 0$, are *(2006 - 3M, –1)*
(a) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (b) $\left(\frac{\pi}{8}, \frac{5\pi}{6}\right)$
(c) $\left(0, \frac{\pi}{8}\right) \cup \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (d) $\left(\frac{41\pi}{48}, \pi\right)$

25. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$ and $t_4 = (\cot\theta)^{\cot\theta}$, then *(2006 - 3M, –1)*
(a) $t_1 > t_2 > t_3 > t_4$ (b) $t_4 > t_3 > t_1 > t_2$
(c) $t_3 > t_1 > t_2 > t_4$ (d) $t_2 > t_3 > t_1 > t_4$

26. The number of solutions of the pair of equations
$$2\sin^2\theta - \cos 2\theta = 0$$
$$2\cos^2\theta - 3\sin\theta = 0$$
in the interval $[0, 2\pi]$ is *(2007 - 3 Marks)*
(a) zero (b) one (c) two (d) four

27. For $x \in (0, \pi)$, the equation $\sin x + 2\sin 2x - \sin 3x = 3$ has *(JEE Adv. 2014)*
(a) infinitely many solutions
(b) three solutions
(c) one solution
(d) no solution

28. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to *(JEE Adv. 2016)*
(a) $-\frac{7\pi}{9}$ (b) $-\frac{2\pi}{9}$
(c) 0 (d) $\frac{5\pi}{9}$

29. The value of $\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$ is equal to *(JEE Adv. 2016)*
(a) $3 - \sqrt{3}$ (b) $2(3 - \sqrt{3})$
(c) $2(\sqrt{3} - 1)$ (d) $2(2 - \sqrt{3})$

D MCQs with One or More than One Correct

1. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to *(1984 - 3 Marks)*
(a) $\frac{1}{2}$ (b) $\cos\frac{\pi}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

2. The expression $3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4(3\pi + \alpha)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi - \alpha)\right]$ is equal to *(1986 - 2 Marks)*
(a) 0 (b) 1
(c) 3 (d) $\sin 4\alpha + \cos 6\alpha$
(e) none of these

3. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos(2x) + a_3\sin^2(x) = 0$ for all x is *(1987 - 2 Marks)*
(a) zero (b) one (c) three
(d) infinite (e) none

4. The values of θ lying between $\theta = 0$ and $\theta = \pi/2$ and satisfying the equation *(1988 - 2 Marks)*
$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0 \text{ are}$$
(a) $7\pi/24$ (b) $5\pi/24$ (c) $11\pi/24$ (d) $\pi/24$.

5. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radians). Then x lies in the interval *(1994)*
(a) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (b) $\left(-1, \frac{5\pi}{6}\right)$
(c) $(-1, 2)$ (d) $\left(\frac{\pi}{6}, 2\right)$

6. The minimum value of the expression $\sin\alpha + \sin\beta + \sin\gamma$, where α, β, γ are real numbers satisfying $\alpha + \beta + \gamma = \pi$ is *(1995)*
(a) positive (b) zero
(c) negative (d) –3

7. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is *(1998 - 2 Marks)*
(a) 0 (b) 5 (c) 6 (d) 10

8. Which of the following number(s) is/are rational? *(1998 - 2 Marks)*
(a) sin 15° (b) cos 15°
(c) sin 15° cos 15° (d) sin 15° cos 75°

9. For a positive integer n, let *(1999 - 3 Marks)*
$$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1 + \sec\theta)(1 + \sec 2\theta)(1 + \sec 4\theta)....(1 + \sec 2^n\theta).$$
Then
(a) $f_2\left(\frac{\pi}{16}\right) = 1$ (b) $f_3\left(\frac{\pi}{32}\right) = 1$
(c) $f_4\left(\frac{\pi}{64}\right) = 1$ (d) $f_5\left(\frac{\pi}{128}\right) = 1$

10. If $\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$, then *(2009)*
(a) $\tan^2 x = \frac{2}{3}$ (b) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$
(c) $\tan^2 x = \frac{1}{3}$ (d) $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

11. For $0<\theta<\frac{\pi}{2}$, the solution (s) of

$$\sum_{m=1}^{6} \operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$$

is (are) *(2009)*

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

12. Let $\theta, \varphi \in [0, 2\pi]$ be such that $2\cos\theta(1-\sin\varphi)=\sin^2\theta$ $\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1, \tan(2\pi-\theta)>0$ and $-1<\sin\theta<-\frac{\sqrt{3}}{2}$, then φ cannot satisfy *(2012)*

(a) $0<\varphi<\frac{\pi}{2}$ (b) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$

(c) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$ (d) $\frac{3\pi}{2}<\varphi<2\pi$

13. The number of points in $(-\infty\ \infty)$, for which $x^2-x\sin x-\cos x=0$, is *(JEE Adv. 2013)*

(a) 6 (b) 4 (c) 2 (d) 0

14. Let $f(x)=x\sin\pi x, x>0$. Then for all natural numbers n, $f'(x)$ vanishes at *(JEE Adv. 2013)*

(a) A unique point in the interval $\left(n, n+\frac{1}{2}\right)$

(b) A unique point in the interval $\left(n+\frac{1}{2}, n+1\right)$

(c) A unique point in the interval $(n, n+1)$

(d) Two points in the interval $(n, n+1)$

15. Let α and β be non-zero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true? *(JEE Adv. 2017)*

(a) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

(c) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

E Subjective Problems

1. If $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$, find the possible values of $(\alpha+\beta)$. *(1978)*

2. (a) Draw the graph of $y=\frac{1}{\sqrt{2}}(\sin x+\cos x)$ from $x=-\frac{\pi}{2}$ to $x=\frac{\pi}{2}$.

(b) If $\cos(\alpha+\beta)=\frac{4}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$, and α, β lies between 0 and $\frac{\pi}{4}$, find $\tan 2\alpha$. *(1979)*

3. Given $\alpha+\beta-\gamma=\pi$, prove that
$\sin^2\alpha+\sin^2\beta-\sin^2\gamma=2\sin\alpha\sin\beta\cos\gamma$ *(1980)*

4. Given $A=\left\{x:\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$ and
$f(x)=\cos x-x(1+x)$; find $f(A)$. *(1980)*

5. For all θ in $[0, \pi/2]$ show that, $\cos(\sin\theta)\ge\sin(\cos\theta)$. *(1981 - 4 Marks)*

6. Without using tables, prove that
$(\sin 12°)(\sin 48°)(\sin 54°)=\frac{1}{8}$. *(1982 - 2 Marks)*

7. Show that $16\cos\left(\frac{2\pi}{15}\right)\cos\left(\frac{4\pi}{15}\right)\cos\left(\frac{8\pi}{15}\right)\cos\left(\frac{16\pi}{15}\right)=1$ *(1983 - 2 Marks)*

8. Find all the solution of $4\cos^2 x\sin x-2\sin^2 x=3\sin x$ *(1983 - 2 Marks)*

9. Find the values of $x\in(-\pi, +\pi)$ which satisfy the equation
$8^{(1+|\cos x|+|\cos^2 x|+|\cos^3 x|+......)}=4^3$ *(1984 - 2 Marks)*

10. Prove that $\tan\alpha+2\tan 2\alpha+4\tan 4\alpha+8\cot 8\alpha=\cot\alpha$ *(1988 - 2 Marks)*

11. ABC is a triangle such that
$$\sin(2A+B)=\sin(C-A)=-\sin(B+2C)=\frac{1}{2}.$$
If A, B and C are in arithmetic progression, determine the values of A, B and C. *(1990 - 5 Marks)*

12. If exp $\{(\sin^2 x+\sin^4 x+\sin^6 x+ \ldots\ldots\ldots\ldots\ \infty)\ \ln 2\}$ satisfies the equation $x^2-9x+8=0$, find the value of $\frac{\cos x}{\cos x+\sin x}, 0<x<\frac{\pi}{2}$. *(1991 - 4 Marks)*

13. Show that the value of $\frac{\tan x}{\tan 3x}$, wherever defined never lies between $\frac{1}{3}$ and 3. *(1992 - 4 Marks)*

14. Determine the smallest positive value of x (in degrees) for which

$\tan(x+100°) = \tan(x+50°)\tan(x)\tan(x-50°)$.

(1993 - 5 Marks)

15. Find the smallest positive number p for which the equation $\cos(p\sin x) = \sin(p\cos x)$ has a solution $x \in [0, 2\pi]$.

(1995 - 5 Marks)

16. Find all values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(1-\tan\theta)(1+\tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$.

(1996 - 2 Marks)

17. Prove that the values of the function $\frac{\sin x \cos 3x}{\sin 3x \cos x}$ do not lie between $\frac{1}{3}$ and 3 for any real x.

(1997 - 5 Marks)

18. Prove that $\sum_{k=1}^{n-1}(n-k)\cos\frac{2k\pi}{n} = -\frac{n}{2}$, where $n \geq 3$ is an integer.

(1997 - 5 Marks)

19. In any triangle ABC, prove that

(2000 - 3 Marks)

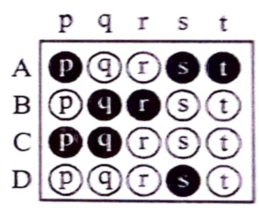
$$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}.$$

20. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$, $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(2005 - 2 Marks)

F Match the Following

DIRECTIONS (Q. 1): *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. In this questions there are entries in columns 1 and 2. Each entry in column 1 is related to exactly one entry in column 2. Write the correct letter from column 2 against the entry number in column 1 in your answer book.

$\frac{\sin 3\alpha}{\cos 2\alpha}$ is

(1992 - 2 Marks)

Column I	Column II
(A) positive	(p) $\left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$
(B) negative	(q) $\left(\frac{14\pi}{48}, \frac{18\pi}{48}\right)$
	(r) $\left(\frac{18\pi}{48}, \frac{23\pi}{48}\right)$
	(s) $\left(0, \frac{\pi}{2}\right)$

2. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$

List – I contains the sets X, Y, Z and W. List – II contains some information regarding these sets. *(JEE Adv. 2019)*

Column I	Column II
(A) X	(p) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(B) Y	(q) an arithmetic progression
(C) Z	(r) NOT an arithmetic progression
(D) W	(s) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(t) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(u) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (IV), (P), (R), (S)
(b) (III), (P), (Q), (U)
(c) (III), (R), (U)
(d) (IV), (Q), (T)

3. Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order.

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	(t) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(u) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

Which of the following is the only CORRECT combination?

(a) (I), (Q), (U)
(b) (I), (P), (R)
(c) (II), (R), (S)
(d) (II), (Q), (T)

G Comprehension Based Questions

This section contains **1** paragraph, Based on each paragraph, there are 2 questions. Each question has four options (A), (B), (C) and (D) ONLY ONE of these four options is correct.

PARAGRAPH 1

Let O be the origin, and $\overline{OX}, \overline{OY}, \overline{OZ}$ be three unit vectors in the directions of the sides $\overline{QR}, \overline{RP}, \overline{PQ}$ respectively, of a triangle PQR.

(JEE Adv. 2017)

1. $|\overline{OX} \times \overline{OY}| =$
   (a) $\sin(P+Q)$ (b) $\sin 2R$
   (c) $\sin(P+R)$ (d) $\sin(Q+R)$
2. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is
   (a) $-\frac{5}{3}$ (b) $-\frac{3}{2}$
   (c) $\frac{3}{2}$ (d) $\frac{5}{3}$

I Integer Value Correct Type

1. The number of all possible values of θ where $0<\theta<\pi$, for which the system of equations
$$(y+z)\cos 3\theta = (xyz)\sin 3\theta$$
$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$
$$(xyz)\sin 3\theta = (y+2z)\cos 3\theta + y\sin 3\theta$$
have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is *(2010)*

2. The number of values of θ in the interval, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for $n = 0, \pm1, \pm2$ and $\tan\theta = \cot 5\theta$ as well as $\sin 2\theta = \cos 4\theta$ is *(2010)*

3. The maximum value of the expression
$$\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} \text{ is}$$ *(2010)*

4. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k>0$, then the value of $[k]$ is *(2010)*
[**Note** : $[k]$ denotes the largest integer less than or equal to k]

5. The positive integer value of $n>3$ satisfying the equation
$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)} \text{ is}$$ *(2011)*

6. The number of distinct solutions of the equation
$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$
in the interval $[0, 2\pi]$ is *(JEE Adv. 2015)*

7. Let a, b, c be three non-zero real numbers such that the equation : $\sqrt{3}a\cos x + 2b\sin x = c, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, has two distinct real roots α and β with $\alpha+\beta = \frac{\pi}{3}$. Then, the value of $\frac{b}{a}$ is ______. *(JEE Adv. 2018)*

Section-B JEE Main / AIEEE

1. The period of $\sin^2\theta$ is **[2002]**
   (a) π^2 (b) π (c) 2π (d) $\pi/2$
2. The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is **[2002]**
   (a) 2 (b) 3 (c) 0 (d) 1
3. Which one is not periodic **[2002]**
   (a) $|\sin 3x| + \sin^2 x$ (b) $\cos\sqrt{x} + \cos^2 x$
   (c) $\cos 4x + \tan^2 x$ (d) $\cos 2x + \sin x$
4. Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
   If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ **[2004]**
   (a) $\frac{-6}{65}$ (b) $\frac{3}{\sqrt{130}}$
   (c) $\frac{6}{65}$ (d) $-\frac{3}{\sqrt{130}}$

5. If $u=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by [2004]

(a) $(a-b)^2$ (b) $2\sqrt{a^2+b^2}$
(c) $(a+b)^2$ (d) $2(a^2+b^2)$

6. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta=3\sin^2\theta$, then $\cos^2\theta$ equals [2004]

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

7. The number of values of x in the interval $[0,3\pi]$ satisfying the equation $2\sin^2x+5\sin x-3=0$ is [2006]

(a) 4 (b) 6 (c) 1 (d) 2

8. If $0<x<\pi$ and $\cos x+\sin x=\frac{1}{2}$, then tan x is [2006]

(a) $\frac{(1-\sqrt{7})}{4}$ (b) $\frac{(4-\sqrt{7})}{3}$
(c) $-\frac{(4+\sqrt{7})}{3}$ (d) $\frac{(1+\sqrt{7})}{4}$

9. Let **A** and **B** denote the statements

A : $\cos\alpha+\cos\beta+\cos\gamma=0$
B : $\sin\alpha+\sin\beta+\sin\gamma=0$

If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)=-\frac{3}{2}$, then : [2009]

(a) **A** is false and **B** is true (b) both **A** and **B** are true
(c) both **A** and **B** are false (d) **A** is true and **B** is false

10. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\beta\le\frac{\pi}{4}$. Then $\tan 2\alpha=$ [2010]

(a) $\frac{56}{33}$ (b) $\frac{19}{12}$ (c) $\frac{20}{7}$ (d) $\frac{25}{16}$

11. If $A=\sin^2x+\cos^4x$, then for all real x : [2011]

(a) $\frac{13}{16}\le A\le 1$ (b) $1\le A\le 2$
(c) $\frac{3}{4}\le A\le\frac{13}{16}$ (d) $\frac{3}{4}\le A\le 1$

12. In a ΔPQR, If $3\sin P+4\cos Q=6$ and $4\sin Q+3\cos P=1$, then the angle R is equal to : [2012]

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

13. ABCD is a trapezium such that AB and CD are parallel and $BC\perp CD$. If $\angle ADB=\theta$, $BC=p$ and $CD=q$, then AB is equal to : [JEE M 2013]

(a) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (b) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$
(c) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (d) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

14. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as : [JEE M 2013]

(a) sinA cosA + 1 (b) secA cosecA + 1
(c) tanA + cotA (d) secA + cosecA

15. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ where $x\in R$ and $k\ge1$. Then $f_4(x)-f_6(x)$ equals [JEE M 2014]

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

16. If $0\le x<2\pi$, then the number of real values of x, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$ is: [JEE M 2016]

(a) 7 (b) 9
(c) 3 (d) 5

17. If $5(\tan^2x-\cos^2x)=2\cos 2x+9$, then the value of $\cos 4x$ is : [JEE M 2017]

(a) $-\frac{7}{9}$ (b) $-\frac{3}{5}$
(c) $\frac{1}{3}$ (d) $\frac{2}{9}$

18. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to : [JEE M 2018]

(a) $\frac{13}{9}$ (b) $\frac{8}{9}$
(c) $\frac{20}{9}$ (d) $\frac{2}{3}$

19. For any $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ the expression $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ equals: [JEE M 2019 – 9 Jan (M)]

(a) $13-4\cos^2\theta+6\sin^2\theta\cos^2\theta$
(b) $13-4\cos^6\theta$
(c) $13-4\cos^2\theta+6\cos^4\theta$
(d) $13-4\cos^4\theta+2\sin^2\theta\cos^2\theta$

20. The value of $\cos^2 10°-\cos 10°\cos 50°+\cos^2 50°$ is : [JEE M 2019 – 9 April (M)]

(a) $\frac{3}{4}+\cos 20°$ (b) 3/4
(c) $\frac{3}{2}(1+\cos 20°)$ (d) 3/2

21. Let $S=\{\theta\in[-2\pi,2\pi]:2\cos^2\theta+3\sin\theta=0\}$. Then the sum of the elements of S is: [JEE M 2019 – 9 April (M)]

(a) $\frac{13\pi}{6}$ (b) $\frac{5\pi}{3}$ (c) 2 (d)