

# CHAPTER

# 9

# Conic Sections

## Section-A

## JEE Advanced/ IIT-JEE

### A Fill in the Blanks

1. The point of intersection of the tangents at the ends of the latus rectum of the parabola  $y^2 = 4x$  is.....

(1994 - 2 Marks)

2. An ellipse has eccentricity  $\frac{1}{2}$  and one focus at the point

$P\left(\frac{1}{2}, 1\right)$ . Its one directrix is the common tangent, nearer to the point  $P$ , to the circle  $x^2 + y^2 = 1$  and the hyperbola  $x^2 - y^2 = 1$ . The equation of the ellipse, in the standard form, is.....

(1996 - 2 Marks)

### C MCQs with One Correct Answer

1. The equation  $\frac{x^2}{1-r} - \frac{y^2}{1+r} = 1$ ,  $r > 1$  represents

(1981 - 2 Marks)

- (a) an ellipse                          (b) a hyperbola  
 (c) a circle                              (d) none of these

2. Each of the four inequalities given below defines a region in the  $xy$  plane. One of these four regions does not have the following property. For any two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the region, the point  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$  is also in the region. The inequality defining this region is

(1981 - 2 Marks)

- (a)  $x^2 + 2y^2 \leq 1$                           (b)  $\text{Max}\{|x|, |y|\} \leq 1$   
 (c)  $x^2 - y^2 \leq 1$                               (d)  $y^2 - x \leq 0$

3. The equation  $2x^2 + 3y^2 - 8x - 18y + 35 = k$  represents

(1994)

- (a) no locus if  $k > 0$                           (b) an ellipse if  $k < 0$   
 (c) a point if  $k = 0$                               (d) a hyperbola if  $k > 0$

4. Let  $E$  be the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  and  $C$  be the circle  $x^2 + y^2 = 9$ . Let  $P$  and  $Q$  be the points  $(1, 2)$  and  $(2, 1)$  respectively. Then

(1994)

- (a)  $Q$  lies inside  $C$  but outside  $E$

- (b)  $Q$  lies outside both  $C$  and  $E$

- (c)  $P$  lies inside both  $C$  and  $E$

- (d)  $P$  lies inside  $C$  but outside  $E$

5. Consider a circle with its centre lying on the focus of the parabola  $y^2 = 2px$  such that it touches the directrix of the parabola. Then a point of intersection of the circle and parabola is

(1995S)

(a)  $\left(\frac{p}{2}, p\right)$  or  $\left(\frac{p}{2}, -p\right)$                           (b)  $\left(\frac{p}{2}, -\frac{p}{2}\right)$

(c)  $\left(-\frac{p}{2}, p\right)$     (d)  $\left(-\frac{p}{2}, -\frac{p}{2}\right)$

6. The radius of the circle passing through the foci of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , and having its centre at  $(0, 3)$  is

(1995S)

- (a) 4    (b) 3

(c)  $\sqrt{\frac{1}{2}}$     (d)  $\frac{7}{2}$

7. Let  $P(a \sec \theta, b \tan \theta)$  and  $Q(a \sec \phi, b \tan \phi)$ , where  $\theta + \phi = \pi/2$ , be two points on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .

If  $(h, k)$  is the point of intersection of the normals at  $P$  and  $Q$ , then  $k$  is equal to

(1999 - 2 Marks)

(a)  $\frac{a^2 + b^2}{a}$     (b)  $-\left(\frac{a^2 + b^2}{a}\right)$

(c)  $\frac{a^2 + b^2}{b}$     (d)  $-\left(\frac{a^2 + b^2}{b}\right)$

8. If  $x = 9$  is the chord of contact of the hyperbola  $x^2 - y^2 = 9$ , then the equation of the corresponding pair of tangents is

(1999 - 2 Marks)

- (a)  $9x^2 - 8y^2 + 18x - 9 = 0$                           (b)  $9x^2 - 8y^2 - 18x + 9 = 0$   
 (c)  $9x^2 - 8y^2 - 18x - 9 = 0$                               (d)  $9x^2 - 8y^2 + 18x + 9 = 0$

9. The curve described parametrically by  $x = t^2 + t + 1$ ,  $y = t^2 - t + 1$  represents

(1999 - 2 Marks)

- (a) a pair of straight lines                          (b) an ellipse  
 (c) a parabola                                        (d) a hyperbola

10. If  $x + y = k$  is normal to  $y^2 = 12x$ , then  $k$  is (2000S)  
 (a) 3 (b) 9 (c) -9 (d) -3
11. If the line  $x - 1 = 0$  is the directrix of the parabola  $y^2 - kx + 8 = 0$ , then one of the values of  $k$  is (2000S)  
 (a) 1/8 (b) 8 (c) 4 (d) 1/4
12. The equation of the common tangent touching the circle  $(x - 3)^2 + y^2 = 9$  and the parabola  $y^2 = 4x$  above the  $x$ -axis is (2001S)  
 (a)  $\sqrt{3}y = 3x + 1$  (b)  $\sqrt{3}y = -(x + 3)$   
 (c)  $\sqrt{3}y = x + 3$  (d)  $\sqrt{3}y = -(3x + 1)$
13. The equation of the directrix of the parabola  $y^2 + 4y + 4x + 2 = 0$  is (2001S)  
 (a)  $x = -1$  (b)  $x = 1$  (c)  $x = -3/2$  (d)  $x = 3/2$
14. If  $a > 2b > 0$  then the positive value of  $m$  for which  $y = mx - b\sqrt{1+m^2}$  is a common tangent to  $x^2 + y^2 = b^2$  and  $(x - a)^2 + y^2 = b^2$  is (2002S)  
 (a)  $\frac{2b}{\sqrt{a^2 - 4b^2}}$  (b)  $\frac{\sqrt{a^2 - 4b^2}}{2b}$   
 (c)  $\frac{2b}{a - 2b}$  (d)  $\frac{b}{a - 2b}$
15. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola  $y^2 = 4ax$  is another parabola with directrix (2002S)  
 (a)  $x = -a$  (b)  $x = -a/2$  (c)  $x = 0$  (d)  $x = a/2$
16. The equation of the common tangent to the curves  $y^2 = 8x$  and  $xy = -1$  is (2002S)  
 (a)  $3y = 9x + 2$  (b)  $y = 2x + 1$   
 (c)  $2y = x + 8$  (d)  $y = x + 2$
17. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$ , is (2003S)  
 (a)  $27/4$  sq. units (b)  $9$  sq. units  
 (c)  $27/2$  sq. units (d)  $27$  sq. units
18. The focal chord to  $y^2 = 16x$  is tangent to  $(x - 6)^2 + y^2 = 2$ , then the possible values of the slope of this chord, are (2003S)  
 (a)  $\{-1, 1\}$  (b)  $\{-2, 2\}$   
 (c)  $\{-2, -1/2\}$  (d)  $\{2, -1/2\}$
19. For hyperbola  $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$  which of the following remains constant with change in ' $\alpha$ ' (2003S)  
 (a) abscissae of vertices (b) abscissae of foci  
 (c) eccentricity (d) directrix
20. If tangents are drawn to the ellipse  $x^2 + 2y^2 = 2$ , then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is (2004S)  
 (a)  $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$  (b)  $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$   
 (c)  $\frac{x^2}{2} + \frac{y^2}{4} = 1$  (d)  $\frac{x^2}{4} + \frac{y^2}{2} = 1$
21. The angle between the tangents drawn from the point  $(1, 4)$  to the parabola  $y^2 = 4x$  is (2004S)  
 (a)  $\pi/6$  (b)  $\pi/4$  (c)  $\pi/3$  (d)  $\pi/2$
22. If the line  $2x + \sqrt{6}y = 2$  touches the hyperbola  $x^2 - 2y^2 = 4$ , then the point of contact is (2004S)  
 (a)  $(-2, \sqrt{6})$  (b)  $(-5, 2\sqrt{6})$   
 (c)  $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$  (d)  $(4, -\sqrt{6})$
23. The minimum area of triangle formed by the tangent to the curve  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  & coordinate axes is (2005S)  
 (a)  $ab$  sq. units (b)  $\frac{a^2 + b^2}{2}$  sq. units  
 (c)  $\frac{(a+b)^2}{2}$  sq. units (d)  $\frac{a^2 + ab + b^2}{3}$  sq. units
24. Tangent to the curve  $y = x^2 + 6$  at a point  $(1, 7)$  touches the circle  $x^2 + y^2 + 16x + 12y + c = 0$  at a point  $Q$ . Then the coordinates of  $Q$  are (2005S)  
 (a)  $(-6, -11)$  (b)  $(-9, -13)$   
 (c)  $(-10, -15)$  (d)  $(-6, -7)$
25. The axis of a parabola is along the line  $y = x$  and the distances of its vertex and focus from origin are  $\sqrt{2}$  and  $2\sqrt{2}$  respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is (2006 - 3M, -1)  
 (a)  $(x+y)^2 = (x-y-2)$  (b)  $(x-y)^2 = (x+y-2)$   
 (c)  $(x-y)^2 = 4(x+y-2)$  (d)  $(x-y)^2 = 8(x+y-2)$
26. A hyperbola, having the transverse axis of length  $2 \sin \theta$ , is confocal with the ellipse  $3x^2 + 4y^2 = 12$ . Then its equation is (2007 - 3 marks)  
 (a)  $x^2 \operatorname{cosec}^2 \theta - y^2 \sec^2 \theta = 1$  (b)  $x^2 \sec^2 \theta - y^2 \operatorname{cosec}^2 \theta = 1$   
 (c)  $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$  (d)  $x^2 \cos^2 \theta - y^2 \sin^2 \theta = 1$
27. Let  $a$  and  $b$  be non-zero real numbers. Then, the equation  $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$  represents (2008)  
 (a) four straight lines, when  $c = 0$  and  $a, b$  are of the same sign.  
 (b) two straight lines and a circle, when  $a = b$ , and  $c$  is of sign opposite to that of  $a$   
 (c) two straight lines and a hyperbola, when  $a$  and  $b$  are of the same sign and  $c$  is of sign opposite to that of  $a$   
 (d) a circle and an ellipse, when  $a$  and  $b$  are of the same sign and  $c$  is of sign opposite to that of  $a$
28. Consider a branch of the hyperbola  $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$  with vertex at the point  $A$ . Let  $B$  be one of the end points of its latus rectum. If  $C$  is the focus of the hyperbola nearest to the point  $A$ , then the area of the triangle  $ABC$  is (2008)  
 (a)  $1 - \sqrt{\frac{2}{3}}$  (b)  $\sqrt{\frac{3}{2}} - 1$  (c)  $1 + \sqrt{\frac{2}{3}}$  (d)  $\sqrt{\frac{3}{2}} + 1$

29. The line passing through the extremity  $A$  of the major axis and extremity  $B$  of the minor axis of the ellipse

$$x^2 + 9y^2 = 9$$

meets its auxiliary circle at the point  $M$ . Then the area of the triangle with vertices at  $A, M$  and the origin  $O$  is (2009)

- (a)  $\frac{31}{10}$  (b)  $\frac{29}{10}$  (c)  $\frac{21}{10}$  (d)  $\frac{27}{10}$

30. The normal at a point  $P$  on the ellipse  $x^2 + 4y^2 = 16$  meets the  $x$ -axis at  $Q$ . If  $M$  is the mid point of the line segment  $PQ$ , then the locus of  $M$  intersects the latus rectums of the given ellipse at the points (2009)

- (a)  $\left(\pm \frac{3\sqrt{5}}{2}, \pm \frac{2}{7}\right)$  (b)  $\left(\pm \frac{3\sqrt{5}}{2}, \pm \sqrt{\frac{19}{4}}\right)$   
 (c)  $\left(\pm 2\sqrt{3}, \pm \frac{1}{7}\right)$  (d)  $\left(\pm 2\sqrt{3}, \pm \frac{4\sqrt{3}}{7}\right)$

31. The locus of the orthocentre of the triangle formed by the lines

$$(1+p)x - py + p(1+p) = 0,$$

$$(1+q)x - qy + q(1+q) = 0,$$

and  $y = 0$ , where  $p \neq q$ , is (2009)

- (a) a hyperbola (b) a parabola  
 (c) an ellipse (d) a straight line

32. Let  $P(6, 3)$  be a point on the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . If the

normal at the point  $P$  intersects the  $x$ -axis at  $(9, 0)$ , then the eccentricity of the hyperbola is (2011)

- (a)  $\sqrt{\frac{5}{2}}$  (b)  $\sqrt{\frac{3}{2}}$  (c)  $\sqrt{2}$  (d)  $\sqrt{3}$

33. Let  $(x, y)$  be any point on the parabola  $y^2 = 4x$ . Let  $P$  be the point that divides the line segment from  $(0, 0)$  to  $(x, y)$  in the ratio  $1 : 3$ . Then the locus of  $P$  is (2011)

- (a)  $x^2 = y$  (b)  $y^2 = 2x$  (c)  $y^2 = x$  (d)  $x^2 = 2y$

34. The ellipse  $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$  is inscribed in a rectangle  $R$

whose sides are parallel to the coordinate axes. Another ellipse  $E_2$  passing through the point  $(0, 4)$  circumscribes the rectangle  $R$ . The eccentricity of the ellipse  $E_2$  is (2012)

- (a)  $\frac{\sqrt{2}}{2}$  (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{2}$  (d)  $\frac{3}{4}$

35. The common tangents to the circle  $x^2 + y^2 = 2$  and the parabola  $y^2 = 8x$  touch the circle at the points  $P, Q$  and the parabola at the points  $R, S$ . Then the area of the quadrilateral  $PQRS$  is (JEE Adv. 2014)

- (a) 3 (b) 6 (c) 9 (d) 15

## D MCQs with One or More than One Correct

1. The number of values of  $c$  such that the straight line  $y = 4x + c$  touches the curve  $(x^2/4) + y^2 = 1$  is (1998 - 2 Marks)

- (a) 0 (b) 1 (c) 2 (d) infinite.

2. If  $P = (x, y), F_1 = (3, 0), F_2 = (-3, 0)$  and  $16x^2 + 25y^2 = 400$ , then  $PF_1 + PF_2$  equals (1998 - 2 Marks)

- (a) 8 (b) 6 (c) 10 (d) 12

3. On the ellipse  $4x^2 + 9y^2 = 1$ , the points at which the tangents are parallel to the line  $8x = 9y$  are (1999 - 3 Marks)

- (a)  $\left(\frac{2}{5}, \frac{1}{5}\right)$  (b)  $\left(-\frac{2}{5}, \frac{1}{5}\right)$

- (c)  $\left(-\frac{2}{5}, -\frac{1}{5}\right)$  (d)  $\left(\frac{2}{5}, -\frac{1}{5}\right)$

4. The equations of the common tangents to the parabola  $y = x^2$  and  $y = -(x-2)^2$  is/are (2006 - 5M, -1)

- (a)  $y = 4(x-1)$  (b)  $y = 0$   
 (c)  $y = -4(x-1)$  (d)  $y = -30x - 50$

5. Let a hyperbola passes through the focus of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$
 The transverse and conjugate axes of this

hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then (2006 - 5M, -1)

- (a) the equation of hyperbola is  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

- (b) the equation of hyperbola is  $\frac{x^2}{9} - \frac{y^2}{25} = 1$

- (c) focus of hyperbola is  $(5, 0)$

- (d) vertex of hyperbola is  $(5\sqrt{3}, 0)$

6. Let  $P(x_1, y_1)$  and  $Q(x_2, y_2), y_1 < 0, y_2 < 0$ , be the end points of the latus rectum of the ellipse  $x^2 + 4y^2 = 4$ . The equations of parabolas with latus rectum  $PQ$  are (2008)

- (a)  $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$  (b)  $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

- (c)  $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$  (d)  $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

7. In a triangle  $ABC$  with fixed base  $BC$ , the vertex  $A$  moves such that

$$\cos B + \cos C = 4 \sin^2 \frac{A}{2}.$$

If  $a, b$  and  $c$  denote the lengths of the sides of the triangle opposite to the angles  $A, B$  and  $C$ , respectively, then (2009)

- (a)  $b + c = 4a$

- (b)  $b + c = 2a$

- (c) locus of point  $A$  is an ellipse

- (d) locus of point  $A$  is a pair of straight lines

8. The tangent  $PT$  and the normal  $PN$  to the parabola  $y^2 = 4ax$  at a point  $P$  on it meet its axis at points  $T$  and  $N$ , respectively. The locus of the centroid of the triangle  $PTN$  is a parabola whose  
(2009)

(a) vertex is  $\left(\frac{2a}{3}, 0\right)$       (b) directrix is  $x=0$

(c) latus rectum is  $\frac{2a}{3}$       (d) focus is  $(a, 0)$

9. An ellipse intersects the hyperbola  $2x^2 - 2y^2 = 1$  orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then  
(2009)

(a) equation of ellipse is  $x^2 + 2y^2 = 2$   
 (b) the foci of ellipse are  $(\pm 1, 0)$   
 (c) equation of ellipse is  $x^2 + 2y^2 = 4$   
 (d) the foci of ellipse are  $(\pm \sqrt{2}, 0)$

10. Let  $A$  and  $B$  be two distinct points on the parabola  $y^2 = 4x$ . If the axis of the parabola touches a circle of radius  $r$  having  $AB$  as its diameter, then the slope of the line joining  $A$  and  $B$  can be  
(2010)

(a)  $-\frac{1}{r}$       (b)  $\frac{1}{r}$       (c)  $\frac{2}{r}$       (d)  $-\frac{2}{r}$

11. Let the eccentricity of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  be reciprocal to that of the ellipse  $x^2 + 4y^2 = 4$ . If the hyperbola passes through a focus of the ellipse, then  
(2011)

(a) the equation of the hyperbola is  $\frac{x^2}{3} - \frac{y^2}{2} = 1$

(b) a focus of the hyperbola is  $(2, 0)$

(c) the eccentricity of the hyperbola is  $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is  $x^2 - 3y^2 = 3$

12. Let  $L$  be a normal to the parabola  $y^2 = 4x$ . If  $L$  passes through the point  $(9, 6)$ , then  $L$  is given by  
(2011)

(a)  $y - x + 3 = 0$       (b)  $y + 3x - 33 = 0$   
 (c)  $y + x - 15 = 0$       (d)  $y - 2x + 12 = 0$

13. Tangents are drawn to the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$ , parallel to the straight line  $2x - y = 1$ . The points of contact of the tangents on the hyperbola are  
(2012)

(a)  $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$       (b)  $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(c)  $(3\sqrt{3}, -2\sqrt{2})$       (d)  $(-3\sqrt{3}, 2\sqrt{2})$

14. Let  $P$  and  $Q$  be distinct points on the parabola  $y^2 = 2x$  such that a circle with  $PQ$  as diameter passes through the vertex  $O$  of the parabola. If  $P$  lies in the first quadrant and the area of the triangle  $\Delta OPQ$  is  $3\sqrt{2}$ , then which of the following is (are) the coordinates of  $P$ ?  
(JEE Adv. 2015)

(a)  $(4, 2\sqrt{2})$       (b)  $(9, 3\sqrt{2})$

(c)  $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$       (d)  $(1, \sqrt{2})$

15. Let  $E_1$  and  $E_2$  be two ellipses whose centers are at the origin. The major axes of  $E_1$  and  $E_2$  lie along the  $x$ -axis and the  $y$ -axis, respectively. Let  $S$  be the circle  $x^2 + (y-1)^2 = 2$ . The straight line  $x+y=3$  touches the curves  $S$ ,  $E_1$  and  $E_2$  at  $P$ ,  $Q$

and  $R$  respectively. Suppose that  $PQ = PR = \frac{2\sqrt{2}}{3}$ . If  $e_1$  and

$e_2$  are the eccentricities of  $E_1$  and  $E_2$ , respectively, then the correct expression(s) is (are)  
(JEE Adv. 2015)

(a)  $e_1^2 + e_2^2 = \frac{43}{40}$

(b)  $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

(c)  $|e_1^2 - e_2^2| = \frac{5}{8}$

(d)  $e_1 e_2 = \frac{\sqrt{3}}{4}$

16. Consider the hyperbola  $H : x^2 - y^2 = 1$  and a circle  $S$  with center  $N(x_2, 0)$ . Suppose that  $H$  and  $S$  touch each other at a point  $P(x_1, y_1)$  with  $x_1 > 1$  and  $y_1 > 0$ . The common tangent to  $H$  and  $S$  at  $P$  intersects the  $x$ -axis at point  $M$ . If  $(l, m)$  is the centroid of the triangle  $PMN$ , then the correct expression(s) is(are)  
(JEE Adv. 2015)

(a)  $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$  for  $x_1 > 1$

(b)  $\frac{dm}{dx_1} = \frac{x_1}{3(\sqrt{x_1^2 - 1})}$  for  $x_1 > 1$

(c)  $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$  for  $x_1 > 1$

(d)  $\frac{dm}{dy_1} = \frac{1}{3}$  for  $y_1 > 0$

17. The circle  $C_1 : x^2 + y^2 = 3$ , with centre at  $O$ , intersects the parabola  $x^2 = 2y$  at the point  $P$  in the first quadrant. Let the tangent to the circle  $C_1$ , at  $P$  touches other two circles  $C_2$  and  $C_3$  at  $R_2$  and  $R_3$ , respectively. Suppose  $C_2$  and  $C_3$  have equal radii  $2\sqrt{3}$  and centres  $Q_2$  and  $Q_3$ , respectively. If  $Q_2$  and  $Q_3$  lie on the  $y$ -axis, then  
(JEE Adv. 2016)

(a)  $Q_2 Q_3 = 12$

(b)  $R_2 R_3 = 4\sqrt{6}$

(c) area of the triangle  $OR_2 R_3$  is  $6\sqrt{2}$

(d) area of the triangle  $PQ_2 Q_3$  is  $4\sqrt{2}$

18. Let  $P$  be the point on the parabola  $y^2 = 4x$  which is at the shortest distance from the center  $S$  of the circle  $x^2 + y^2 - 4x - 16y + 64 = 0$ . Let  $Q$  be the point on the circle dividing the line segment  $SP$  internally. Then (JEE Adv. 2016)

- (a)  $SP = 2\sqrt{5}$
- (b)  $SQ : QP = (\sqrt{5} + 1) : 2$
- (c) the  $x$ -intercept of the normal to the parabola at  $P$  is 6
- (d) the slope of the tangent to the circle at  $Q$  is  $\frac{1}{2}$

19. If  $2x - y + 1 = 0$  is a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ , then which of the following cannot be sides of a right angled triangle?

(JEE Adv. 2017)

- (a)  $a, 4, 1$
- (b)  $a, 4, 2$
- (c)  $2a, 8, 1$
- (d)  $2a, 4, 1$

20. If a chord, which is not a tangent, of the parabola  $y^2 = 16x$  has the equation  $2x + y = p$ , and midpoint  $(h, k)$ , then which of the following is(are) possible value(s) of  $p$ ,  $h$  and  $k$ ?

(JEE Adv. 2017)

- (a)  $p = -2, h = 2, k = -4$
- (b)  $p = -1, h = 1, k = -3$
- (c)  $p = 2, h = 3, k = -4$
- (d)  $p = 5, h = 4, k = -3$

21. Consider two straight lines, each of which is tangent to both the circle  $x^2 + y^2 = \frac{1}{2}$  and the parabola  $y^2 = 4x$ . Let these lines intersect at the point  $Q$ . Consider the ellipse whose center is at the origin  $O(0, 0)$  and whose semi-major axis is  $OQ$ . If the length of the minor axis of this ellipse is  $\sqrt{2}$ , then which of the following statement(s) is (are) TRUE?

(JEE Adv. 2018)

- (a) For the ellipse, the eccentricity is  $\frac{1}{\sqrt{2}}$  and the length of the latus rectum is 1
- (b) For the ellipse, the eccentricity is  $\frac{1}{2}$  and the length of the latus rectum is  $\frac{1}{2}$
- (c) The area of the region bounded by the ellipse between the lines  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{4\sqrt{2}}(\pi - 2)$
- (d) The area of the region bounded by the ellipse between the lines  $x = \frac{1}{\sqrt{2}}$  and  $x = 1$  is  $\frac{1}{16}(\pi - 2)$

## E Subjective Problems

1. Suppose that the normals drawn at three different points on the parabola  $y^2 = 4x$  pass through the point  $(h, k)$ . Show that  $h > 2$ . (1981 - 4 Marks)
2.  $A$  is a point on the parabola  $y^2 = 4ax$ . The normal at  $A$  cuts the parabola again at point  $B$ . If  $AB$  subtends a right angle at the vertex of the parabola. find the slope of  $AB$ . (1982 - 5 Marks)
3. Three normals are drawn from the point  $(c, 0)$  to the curve  $y^2 = x$ . Show that  $c$  must be greater than  $1/2$ . One normal is always the  $x$ -axis. Find  $c$  for which the other two normals are perpendicular to each other. (1991 - 4 Marks)
4. Through the vertex  $O$  of parabola  $y^2 = 4x$ , chords  $OP$  and  $OQ$  are drawn at right angles to one another. Show that for all positions of  $P$ ,  $PQ$  cuts the axis of the parabola at a fixed point. Also find the locus of the middle point of  $PQ$ . (1994 - 4 Marks)
5. Show that the locus of a point that divides a chord of slope 2 of the parabola  $y^2 = 4x$  internally in the ratio 1: 2 is a parabola. Find the vertex of this parabola. (1995 - 5 Marks)
6. Let ' $d$ ' be the perpendicular distance from the centre of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  to the tangent drawn at a point  $P$  on the ellipse. If  $F_1$  and  $F_2$  are the two foci of the ellipse, then show that  $(PF_1 - PF_2)^2 = 4a^2 \left(1 - \frac{b^2}{d^2}\right)$ . (1995 - 5 Marks)
7. Points  $A, B$  and  $C$  lie on the parabola  $y^2 = 4ax$ . The tangents to the parabola at  $A, B$  and  $C$ , taken in pairs, intersect at points  $P, Q$  and  $R$ . Determine the ratio of the areas of the triangles  $ABC$  and  $PQR$ . (1996 - 3 Marks)
8. From a point  $A$  common tangents are drawn to the circle  $x^2 + y^2 = a^2/2$  and parabola  $y^2 = 4ax$ . Find the area of the quadrilateral formed by the common tangents, the chord of contact of the circle and the chord of contact of the parabola. (1996 - 2 Marks)
9. A tangent to the ellipse  $x^2 + 4y^2 = 4$  meets the ellipse  $x^2 + 2y^2 = 6$  at  $P$  and  $Q$ . Prove that the tangents at  $P$  and  $Q$  of the ellipse  $x^2 + 2y^2 = 6$  are at right angles. (1997 - 5 Marks)
10. The angle between a pair of tangents drawn from a point  $P$  to the parabola  $y^2 = 4ax$  is  $45^\circ$ . Show that the locus of the point  $P$  is a hyperbola. (1998 - 8 Marks)
11. Consider the family of circles  $x^2 + y^2 = r^2$ ,  $2 < r < 5$ . If in the first quadrant, the common tangent to a circle of this family and the ellipse  $4x^2 + 25y^2 = 100$  meets the co-ordinate axes at  $A$  and  $B$ , then find the equation of the locus of the mid-point of  $AB$ . (1999 - 10 Marks)
12. Find the co-ordinates of all the points  $P$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , for which the area of the triangle  $PON$  is maximum, where  $O$  denotes the origin and  $N$ , the foot of the perpendicular from  $O$  to the tangent at  $P$ . (1999 - 10 Marks)

13. Let  $ABC$  be an equilateral triangle inscribed in the circle  $x^2 + y^2 = a^2$ . Suppose perpendiculars from  $A, B, C$  to the

major axis of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$  meets the ellipse respectively, at  $P, Q, R$ , so that  $P, Q, R$  lie on the same side of the major axis as  $A, B, C$  respectively. Prove that the normals to the ellipse drawn at the points  $P, Q$  and  $R$  are concurrent. (2000 - 7 Marks)

14. Let  $C_1$  and  $C_2$  be respectively, the parabolas  $x^2 = y - 1$  and  $y^2 = x - 1$ . Let  $P$  be any point on  $C_1$  and  $Q$  be any point on  $C_2$ . Let  $P_1$  and  $Q_1$  be the reflections of  $P$  and  $Q$ , respectively, with respect to the line  $y = x$ . Prove that  $P_1$  lies on  $C_2$ ,  $Q_1$  lies on  $C_1$  and  $PQ \geq \min\{PP_1, QQ_1\}$ . Hence or otherwise determine points  $P_0$  and  $Q_0$  on the parabolas  $C_1$  and  $C_2$  respectively such that  $P_0Q_0 \leq PQ$  for all pairs of points  $(P, Q)$  with  $P$  on  $C_1$  and  $Q$  on  $C_2$ . (2000 - 10 Marks)

15. Let  $P$  be a point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 < b < a$ . Let the line parallel to  $y$ -axis passing through  $P$  meet the circle  $x^2 + y^2 = a^2$  at the point  $Q$  such that  $P$  and  $Q$  are on the same side of  $x$ -axis. For two positive real numbers  $r$  and  $s$ , find the locus of the point  $R$  on  $PQ$  such that  $PR : RQ = r : s$  as  $P$  varies over the ellipse. (2001 - 4 Marks)

16. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. (2002 - 5 Marks)

17. Normals are drawn from the point  $P$  with slopes  $m_1, m_2, m_3$  to the parabola  $y^2 = 4x$ . If locus of  $P$  with  $m_1 m_2 = \alpha$  is a part of the parabola itself then find  $\alpha$ . (2003 - 4 Marks)

18. Tangent is drawn to parabola  $y^2 - 2y - 4x + 5 = 0$  at a point  $P$  which cuts the directrix at the point  $Q$ . A point  $R$  is such that it divides  $QP$  externally in the ratio  $1/2 : 1$ . Find the locus of point  $R$ . (2004 - 4 Marks)

19. Tangents are drawn from any point on the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \text{ to the circle } x^2 + y^2 = 9. \text{ Find the locus of mid-point of the chord of contact. } \quad (2005 - 4 Marks)$$

20. Find the equation of the common tangent in 1<sup>st</sup> quadrant to the circle  $x^2 + y^2 = 16$  and the ellipse  $\frac{x^2}{25} + \frac{y^2}{4} = 1$ . Also find the length of the intercept of the tangent between the coordinate axes. (2005 - 4 Marks)

**F****Match the Following**

**DIRECTIONS (Q. 1-3) :** Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.

|   | p                                | q                                | r                                | s                                | t                                |
|---|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| A | <input checked="" type="radio"/> |
| B | <input checked="" type="radio"/> |
| C | <input checked="" type="radio"/> |
| D | <input checked="" type="radio"/> |

1. Match the following : (3, 0) is the pt. from which three normals are drawn to the parabola  $y^2 = 4x$  which meet the parabola in the points  $P, Q$  and  $R$ . Then (2006 - 6M)

**Column I**

- (A) Area of  $\Delta PQR$   
 (B) Radius of circumcircle of  $\Delta PQR$   
 (C) Centroid of  $\Delta PQR$   
 (D) Circumcentre of  $\Delta PQR$

**Column II**

- (p) 2  
 (q)  $5/2$   
 (r)  $(5/2, 0)$   
 (s)  $(2/3, 0)$

2. Match the statements in Column I with the properties in Column II and indicate your answer by darkening the appropriate bubbles in the  $4 \times 4$  matrix given in the ORS. (2007 - 6 marks)

**Column I**

- (A) Two intersecting circles  
 (B) Two mutually external circles  
 (C) Two circles, one strictly inside the other  
 (D) Two branches of a hyperbola

**Column II**

- (p) have a common tangent  
 (q) have a common normal  
 (r) do not have a common tangent  
 (s) do not have a common normal

3. Match the conics in **Column I** with the statements/expressions in **Column II**.

**Column I**

- (A) Circle
- (B) Parabola
- (C) Ellipse
- (D) Hyperbola

**Column II**

- (p) The locus of the point  $(h, k)$  for which the line  $hx + ky = 1$  touches the circle  $x^2 + y^2 = 4$
- (q) Points  $z$  in the complex plane satisfying  $|z+2| - |z-2| = \pm 3$
- (r) Points of the conic have parametric representation

$$x = \sqrt{3} \left( \frac{1-t^2}{1+t^2} \right), \quad y = \frac{2t}{1+t^2}$$

- (s) The eccentricity of the conic lies in the interval  $1 \leq e < \infty$
- (t) Points  $z$  in the complex plane satisfying

$$\operatorname{Re}(z+1)^2 = |z|^2 + 1$$

**DIRECTIONS (Q. 4) :** Following question has matching lists. The codes for the list have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

4. A line  $L : y = mx + 3$  meets  $y$ -axis at  $E(0, 3)$  and the arc of the parabola  $y^2 = 16x, 0 \leq y \leq 6$  at the point  $F(x_0, y_0)$ . The tangent to the parabola at  $F(x_0, y_0)$  intersects the  $y$ -axis at  $G(0, y_1)$ . The slope  $m$  of the line  $L$  is chosen such that the area of the triangle  $EFG$  has a local maximum. (JEE Adv. 2013)

Match List I with List II and select the correct answer using the code given below the lists :

**List I**

- P.  $m =$
- Q. Maximum area of  $\Delta EFG$  is
- R.  $y_0 =$
- S.  $y_1 =$

**List II**

- 1.  $\frac{1}{2}$
- 2. 4
- 3. 2
- 4. 1

**Codes:**

|     | P | Q | R | S |
|-----|---|---|---|---|
| (a) | 4 | 1 | 2 | 3 |
| (c) | 1 | 3 | 2 | 4 |

|     | P | Q | R | S |
|-----|---|---|---|---|
| (b) | 3 | 4 | 1 | 2 |
| (d) | 1 | 3 | 4 | 2 |

**(Qs. 5-7) :** By appropriately matching the information given in the three columns of the following table. Column 1, 2, and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

| Column 1                  | Column 2                           | Column 3   |
|---------------------------|------------------------------------|--|
| (I) $x^2 + y^2 = a^2$     | (i) $my = m^2x + a$                | (P) $\left( \frac{a}{m^2}, \frac{2a}{m} \right)$                                   |
| (II) $x^2 + a^2y^2 = a^2$ | (ii) $y = mx + a\sqrt{m^2 + 1}$    | (Q) $\left( \frac{-ma}{\sqrt{m^2 + 1}}, \frac{a}{\sqrt{m^2 + 1}} \right)$          |
| (III) $y^2 = 4ax$         | (iii) $y = mx + \sqrt{a^2m^2 - 1}$ | (R) $\left( \frac{-a^2m}{\sqrt{a^2m^2 + 1}}, \frac{1}{\sqrt{a^2m^2 + 1}} \right)$  |
| (IV) $x^2 - a^2y^2 = a^2$ | (iv) $y = mx + \sqrt{a^2m^2 + 1}$  | (S) $\left( \frac{-a^2m}{\sqrt{a^2m^2 - 1}}, \frac{-1}{\sqrt{a^2m^2 - 1}} \right)$ |

5. For  $a = \sqrt{2}$ , if a tangent is drawn to a suitable conic (Column 1) at the point of contact  $(-1, 1)$ , then which of the following options is the only correct combination for obtaining its equation? (JEE Adv. 2017)

(a) (I)(i)(P)

(b) (I)(ii)(Q)

(c) (II)(ii)(Q)

(d) (III)(i)(P)

6. If a tangent to a suitable conic (column 1) is found to be  $y = x + 8$  and its point of contact is  $(8, 16)$ , then which of the following options is the only correct combination?  
 (a) (I)(ii)(Q)      (b) (II)(iv)(R)      (c) (III)(i)(P)      (d) (III)(ii)(Q) (JEE Adv. 2018)
7. The tangent to a suitable conic (Column 1) at  $\left(\sqrt{3}, \frac{1}{2}\right)$  is found to be  $\sqrt{3}x + 2y = 4$ , then which of the following options is the only correct combination?  
 (a) (IV)(iii)(S)      (b) (IV)(iv)(S)      (c) (II)(iii)(R)      (d) (II)(iv)(R) (JEE Adv. 2018)
8. Let  $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , where  $a > b > 0$ , be a hyperbola in the  $xy$ -plane whose conjugate axis  $LM$  subtends an angle of  $60^\circ$  at one of its vertices  $N$ . Let the area of the triangle  $LMN$  be  $4\sqrt{3}$ .  
(JEE Adv. 2018)
- List I**
- P. The length of the conjugate axis of  $H$  is
- Q. The eccentricity of  $H$  is
- R. The distance between the foci of  $H$  is
- S. The length of the latus rectum of  $H$  is
- The correct option is:  
 (a) P  $\rightarrow$  4; Q  $\rightarrow$  2; R  $\rightarrow$  1; S  $\rightarrow$  3  
 (c) P  $\rightarrow$  4; Q  $\rightarrow$  1; R  $\rightarrow$  3; S  $\rightarrow$  2
- List II**
1. 8
2.  $\frac{4}{\sqrt{3}}$
3.  $\frac{2}{\sqrt{3}}$
4. 4
- (b) P  $\rightarrow$  4; Q  $\rightarrow$  3; R  $\rightarrow$  1; S  $\rightarrow$  2  
 (d) P  $\rightarrow$  3; Q  $\rightarrow$  4; R  $\rightarrow$  2; S  $\rightarrow$  1

## G Comprehension Based Questions

### PASSAGE 1

Consider the circle  $x^2 + y^2 = 9$  and the parabola  $y^2 = 8x$ . They intersect at  $P$  and  $Q$  in the first and the fourth quadrants, respectively. Tangents to the circle at  $P$  and  $Q$  intersect the  $x$ -axis at  $R$  and tangents to the parabola at  $P$  and  $Q$  intersect the  $x$ -axis at  $S$ . (2007-4 marks)

1. The ratio of the areas of the triangles  $PQS$  and  $PQR$  is  
 (a)  $1:\sqrt{2}$       (b)  $1:2$       (c)  $1:4$       (d)  $1:8$
2. The radius of the circumcircle of the triangle  $PRS$  is  
 (a) 5      (b)  $3\sqrt{3}$       (c)  $3\sqrt{2}$       (d)  $2\sqrt{3}$  (2007-4 marks)
3. The radius of the incircle of the triangle  $PQR$  is  
 (a) 4      (b) 3      (c)  $\frac{8}{3}$       (d) 2 (2007-4 marks)

### PASSAGE 2

The circle  $x^2 + y^2 - 8x = 0$  and hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  intersect at the points A and B. (2010)

4. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is  
 (a)  $2x - \sqrt{5}y - 20 = 0$       (b)  $2x - \sqrt{5}y + 4 = 0$   
 (c)  $3x - 4y + 8 = 0$       (d)  $4x - 3y + 4 = 0$

5. Equation of the circle with AB as its diameter is  
 (a)  $x^2 + y^2 - 12x + 24 = 0$       (b)  $x^2 + y^2 + 12x + 24 = 0$   
 (c)  $x^2 + y^2 + 24x - 12 = 0$       (d)  $x^2 + y^2 - 24x - 12 = 0$

### PASSAGE 3

Tangents are drawn from the point  $P(3, 4)$  to the ellipse

- $\frac{x^2}{9} + \frac{y^2}{4} = 1$  touching the ellipse at points A and B. (2010)
6. The coordinates of A and B are  
 (a)  $(3, 0)$  and  $(0, 2)$   
 (b)  $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$  and  $\left(-\frac{9}{5}, \frac{8}{5}\right)$   
 (c)  $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$  and  $(0, 2)$   
 (d)  $(3, 0)$  and  $\left(-\frac{9}{5}, \frac{8}{5}\right)$

7. The orthocenter of the triangle PAB is  
 (a)  $\left(5, \frac{8}{7}\right)$       (b)  $\left(\frac{7}{5}, \frac{25}{8}\right)$       (c)  $\left(\frac{11}{5}, \frac{8}{5}\right)$       (d)  $\left(\frac{8}{25}, \frac{7}{5}\right)$
8. The equation of the locus of the point whose distances from the point P and the line AB are equal, is  
 (a)  $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$   
 (b)  $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$   
 (c)  $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$   
 (d)  $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

**PASSAGE 4**

Let  $PQ$  be a focal chord of the parabola  $y^2 = 4ax$ . The tangents to the parabola at  $P$  and  $Q$  meet at a point lying on the line  $y = 2x + a$ ,  $a > 0$ .

9. Length of chord  $PQ$  is (JEE Adv. 2013)  
 (a)  $7a$       (b)  $5a$       (c)  $2a$       (d)  $3a$
10. If chord  $PQ$  subtends an angle  $\theta$  at the vertex of  $y^2 = 4ax$ , then  $\tan \theta =$  (JEE Adv. 2013)  
 (a)  $\frac{2}{3}\sqrt{7}$       (b)  $-\frac{2}{3}\sqrt{7}$       (c)  $\frac{2}{3}\sqrt{5}$       (d)  $-\frac{2}{3}\sqrt{5}$

**PASSAGE 5**

Let  $a, r, s, t$  be nonzero real numbers. Let  $P(at^2, 2at)$ ,  $Q(ar^2, 2ar)$  and  $S(as^2, 2as)$  be distinct points on the parabola  $y^2 = 4ax$ . Suppose that  $PQ$  is the focal chord and lines  $QR$  and  $PK$  are parallel, where  $K$  is the point  $(2a, 0)$  (JEE Adv. 2014)

11. The value of  $r$  is  
 (a)  $-\frac{1}{t}$       (b)  $\frac{t^2+1}{t}$       (c)  $\frac{1}{t}$       (d)  $\frac{t^2-1}{t}$
12. If  $st = 1$ , then the tangent at  $P$  and the normal at  $S$  to the parabola meet at a point whose ordinate is

- (a)  $\frac{(t^2+1)^2}{2t^3}$       (b)  $\frac{a(t^2+1)^2}{2t^3}$   
 (c)  $\frac{a(t^2+1)^2}{t^3}$       (d)  $\frac{a(t^2+2)^2}{t^3}$

**PASSAGE 6**

Let  $F_1(x_1, 0)$  and  $F_2(x_2, 0)$  for  $x_1 < 0$  and  $x_2 > 0$ , be the foci of the

ellipse  $\frac{x^2}{9} + \frac{y^2}{8} = 1$ . Suppose a parabola having vertex at the

origin and focus at  $F_2$  intersects the ellipse at point  $M$  in the first quadrant and at point  $N$  in the fourth quadrant.

13. The orthocentre of the triangle  $F_1MN$  is (JEE Adv. 2016)

- (a)  $\left(-\frac{9}{10}, 0\right)$       (b)  $\left(\frac{2}{3}, 0\right)$   
 (c)  $\left(\frac{9}{10}, 0\right)$       (d)  $\left(\frac{2}{3}, \sqrt{6}\right)$

14. If the tangents to the ellipse at  $M$  and  $N$  meet at  $R$  and the normal to the parabola at  $M$  meets the  $x$ -axis at  $Q$ , then the ratio of area of the triangle  $MQR$  to area of the quadrilateral  $MF_1NF_2$  is (JEE Adv. 2016)

- (a)  $3:4$       (b)  $4:5$   
 (c)  $5:8$       (d)  $2:3$

**H****Assertion & Reason Type Questions**

1. **STATEMENT-1 :** The curve  $y = \frac{-x^2}{2} + x + 1$  is symmetric with respect to the line  $x = 1$ , because

**STATEMENT-2 :** A parabola is symmetric about its axis. (2007 - 3 marks)

- (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1  
 (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1  
 (c) Statement-1 is True, Statement-2 is False  
 (d) Statement-1 is False, Statement-2 is True.

**I****Integer Value Correct Type**

1. The line  $2x + y = 1$  is tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .

If this line passes through the point of intersection of the nearest directrix and the  $x$ -axis, then the eccentricity of the hyperbola is (2010)

2. Consider the parabola  $y^2 = 8x$ . Let  $\Delta_1$  be the area of the triangle formed by the end points of its latus rectum and the point  $P\left(\frac{1}{2}, 2\right)$  on the parabola and  $\Delta_2$  be the area of the triangle formed by drawing tangents at  $P$  and at the end points of the latus rectum. Then  $\frac{\Delta_1}{\Delta_2}$  is (2011)

3. Let  $S$  be the focus of the parabola  $y^2 = 8x$  and let  $PQ$  be the common chord of the circle  $x^2 + y^2 - 2x - 4y = 0$  and the given parabola. The area of the triangle  $PQS$  is (2012)

4. A vertical line passing through the point  $(h, 0)$  intersects the ellipse  $\frac{x^2}{4} + \frac{y^2}{3} = 1$  at the points  $P$  and  $Q$ . Let the tangents to the ellipse at  $P$  and  $Q$  meet at the point  $R$ . If  $\Delta(h)$  = area of the triangle  $PQR$ ,  $\Delta_1 = \max_{1/2 \leq h \leq 1} \Delta(h)$  and  $\Delta_2 = \min_{1/2 \leq h \leq 1} \Delta(h)$ , then  $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$  (JEE Adv. 2013)

- (a)  $g(x)$  is continuous but not differentiable at a

- (b)  $g(x)$  is differentiable on  $R$

- (c)  $g(x)$  is continuous but not differentiable at  $b$

- (d)  $g(x)$  is continuous and differentiable at either (a) or (b) but not both

5. If the normals of the parabola  $y^2 = 4x$  drawn at the end points of its latus rectum are tangents to the circle  $(x-3)^2 + (y+2)^2 = r^2$ , then the value of  $r^2$  is (JEE Adv. 2015)
6. Let the curve  $C$  be the mirror image of the parabola  $y^2 = 4x$  with respect to the line  $x+y+4=0$ . If  $A$  and  $B$  are the points of intersection of  $C$  with the line  $y=-5$ , then the distance between  $A$  and  $B$  is (JEE Adv. 2015)

7. Suppose that the foci of the ellipse  $\frac{x^2}{9} + \frac{y^2}{5} = 1$  are  $(f_1, 0)$  and  $(f_2, 0)$  where  $f_1 > 0$  and  $f_2 < 0$ . Let  $P_1$  and  $P_2$  be two parabolas with a common vertex at  $(0, 0)$  and with foci at  $(f_1, 0)$  and  $(2f_2, 0)$ , respectively. Let  $T_1$  be a tangent to  $P_1$  which passes through  $(2f_2, 0)$  and  $T_2$  be a tangent to  $P_2$  which passes through  $(f_1, 0)$ . If  $m_1$  is the slope of  $T_1$  and  $m_2$

is the slope of  $T_2$ , then the value of  $\left(\frac{1}{m_1^2} + m_2^2\right)$  is

(JEE Adv. 2015)

## Section-B JEE Main / AIEEE

1. Two common tangents to the circle  $x^2 + y^2 = 2a^2$  and parabola  $y^2 = 8ax$  are [2002]
- (a)  $x = \pm(y+2a)$       (b)  $y = \pm(x+2a)$   
 (c)  $x = \pm(y+a)$       (d)  $y = \pm(x+a)$
2. The normal at the point  $(bt_1^2, 2bt_1)$  on a parabola meets the parabola again in the point  $(bt_2^2, 2bt_2)$ , then
- (a)  $t_2 = t_1 + \frac{2}{t_1}$       (b)  $t_2 = -t_1 - \frac{2}{t_1}$  [2003]  
 (c)  $t_2 = -t_1 + \frac{2}{t_1}$       (d)  $t_2 = t_1 - \frac{2}{t_1}$
3. The foci of the ellipse  $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$  and the hyperbola  $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$  coincide. Then the value of  $b^2$  is [2003]
- (a) 9      (b) 1      (c) 5      (d) 7
4. If  $a \neq 0$  and the line  $2bx+3cy+4d=0$  passes through the points of intersection of the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ , then [2004]
- (a)  $d^2 + (3b-2c)^2 = 0$       (b)  $d^2 + (3b+2c)^2 = 0$   
 (c)  $d^2 + (2b-3c)^2 = 0$       (d)  $d^2 + (2b+3c)^2 = 0$
5. The eccentricity of an ellipse, with its centre at the origin, is  $\frac{1}{2}$ . If one of the directrices is  $x=4$ , then the equation of the ellipse is: [2004]
- (a)  $4x^2 + 3y^2 = 1$       (b)  $3x^2 + 4y^2 = 12$   
 (c)  $4x^2 + 3y^2 = 12$       (d)  $3x^2 + 4y^2 = 1$
6. Let  $P$  be the point  $(1, 0)$  and  $Q$  a point on the locus  $y^2 = 8x$ . The locus of mid point of  $PQ$  is [2005]
- (a)  $y^2 - 4x + 2 = 0$       (b)  $y^2 + 4x + 2 = 0$   
 (c)  $x^2 + 4y + 2 = 0$       (d)  $x^2 - 4y + 2 = 0$
7. The locus of a point  $P(a, \beta)$  moving under the condition that the line  $y = \alpha x + \beta$  is a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  is [2005]
- (a) an ellipse      (b) a circle  
 (c) a parabola      (d) a hyperbola
8. An ellipse has  $OB$  as semi minor axis,  $F$  and  $F'$  its foci and the angle  $FBF'$  is a right angle. Then the eccentricity of the ellipse is [2005]
- (a)  $\frac{1}{\sqrt{2}}$       (b)  $\frac{1}{2}$       (c)  $\frac{1}{4}$       (d)  $\frac{1}{\sqrt{3}}$
9. The locus of the vertices of the family of parabolas  $y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$  is [2006]
- (a)  $xy = \frac{105}{64}$       (b)  $xy = \frac{3}{4}$       (c)  $xy = \frac{35}{16}$       (d)  $xy = \frac{64}{105}$
10. In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is [2006]
- (a)  $\frac{3}{5}$       (b)  $\frac{1}{2}$       (c)  $\frac{4}{5}$       (d)  $\frac{1}{\sqrt{5}}$
11. Angle between the tangents to the curve  $y = x^2 - 5x + 6$  at the points  $(2, 0)$  and  $(3, 0)$  is [2006]
- (a)  $\pi$       (b)  $\frac{\pi}{2}$       (c)  $\frac{\pi}{6}$       (d)  $\frac{\pi}{4}$

12. For the Hyperbola  $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$ , which of the following remains constant when  $\alpha$  varies? [2007]

(a) abscissae of vertices (b) abscissae of foci  
(c) eccentricity (d) directrix.

13. The equation of a tangent to the parabola  $y^2 = 8x$  is  $y = x + 2$ . The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is [2007]

(a) (2, 4) (b) (-2, 0) (c) (-1, 1) (d) (0, 2)

14. The normal to a curve at  $P(x, y)$  meets the  $x$ -axis at  $G$ . If the distance of  $G$  from the origin is twice the abscissa of  $P$ , then the curve is [2007]

(a) circle (b) hyperbola (c) ellipse (d) parabola.

15. A focus of an ellipse is at the origin. The directrix is the line

$x = 4$  and the eccentricity is  $\frac{1}{2}$ . Then the length of the semi-major axis is [2008]

(a)  $\frac{8}{3}$  (b)  $\frac{2}{3}$  (c)  $\frac{4}{3}$  (d)  $\frac{5}{3}$

16. A parabola has the origin as its focus and the line  $x = 2$  as the directrix. Then the vertex of the parabola is at [2008]

(a) (0, 2) (b) (1, 0) (c) (0, 1) (d) (2, 0)

17. The ellipse  $x^2 + 4y^2 = 4$  is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is: [2009]

(a)  $x^2 + 12y^2 = 16$  (b)  $4x^2 + 48y^2 = 48$   
(c)  $4x^2 + 64y^2 = 48$  (d)  $x^2 + 16y^2 = 16$

18. If two tangents drawn from a point  $P$  to the parabola  $y^2 = 4x$  are at right angles, then the locus of  $P$  is [2010]

(a)  $2x+1=0$  (b)  $x=-1$   
(c)  $2x-1=0$  (d)  $x=1$

19. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3, 1) and

has eccentricity  $\sqrt{\frac{2}{5}}$  is [2011]

(a)  $5x^2 + 3y^2 - 48 = 0$  (b)  $3x^2 + 5y^2 - 15 = 0$   
(c)  $5x^2 + 3y^2 - 32 = 0$  (d)  $3x^2 + 5y^2 - 32 = 0$

20. Statement-1 : An equation of a common tangent to the parabola  $y^2 = 16\sqrt{3}x$  and the ellipse  $2x^2 + y^2 = 4$  is  $y = 2x + 2\sqrt{3}$

Statement-2 : If the line  $y = mx + \frac{4\sqrt{3}}{m}$ , ( $m \neq 0$ ) is a common tangent to the parabola  $y^2 = 16\sqrt{3}x$  and the ellipse  $2x^2 + y^2 = 4$ , then  $m$  satisfies  $m^4 + 2m^2 = 24$  [2012]

- (a) Statement-1 is false, Statement-2 is true.  
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.  
(c) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1.  
(d) Statement-1 is true, statement-2 is false.

21. An ellipse is drawn by taking a diameter of the circle  $(x-1)^2 + y^2 = 1$  as its semi-minor axis and a diameter of the circle  $x^2 + (y-2)^2 = 4$  as semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is : [2012]

(a)  $4x^2 + y^2 = 4$  (b)  $x^2 + 4y^2 = 8$   
(c)  $4x^2 + y^2 = 8$  (d)  $x^2 + 4y^2 = 16$

22. The equation of the circle passing through the foci of the

ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$ , and having centre at (0, 3) is

[JEE M 2013]

(a)  $x^2 + y^2 - 6y - 7 = 0$  (b)  $x^2 + y^2 - 6y + 7 = 0$   
(c)  $x^2 + y^2 - 6y - 5 = 0$  (d)  $x^2 + y^2 - 6y + 5 = 0$

23. Given : A circle,  $2x^2 + 2y^2 = 5$  and a parabola,  $y^2 = 4\sqrt{5}x$ . Statement-1 : An equation of a common tangent to these curves is  $y = x + \sqrt{5}$ .

Statement-2 : If the line,  $y = mx + \frac{\sqrt{5}}{m}$  ( $m \neq 0$ ) is their common tangent, then  $m$  satisfies  $m^4 - 3m^2 + 2 = 0$ .

[JEE M 2013]

- (a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.  
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.  
(c) Statement-1 is true; Statement-2 is false.  
(d) Statement-1 is false; Statement-2 is true.

24. The locus of the foot of perpendicular drawn from the centre of the ellipse  $x^2 + 3y^2 = 6$  on any tangent to it is

[JEE M 2014]

(a)  $(x^2 + y^2)^2 = 6x^2 + 2y^2$  (b)  $(x^2 + y^2)^2 = 6x^2 - 2y^2$   
(c)  $(x^2 - y^2)^2 = 6x^2 + 2y^2$  (d)  $(x^2 - y^2)^2 = 6x^2 - 2y^2$

25. The slope of the line touching both the parabolas  $y^2 = 4x$  and  $x^2 = -32y$  is

[JEE M 2014]

(a)  $\frac{1}{8}$  (b)  $\frac{2}{3}$  (c)  $\frac{1}{2}$  (d)  $\frac{3}{2}$

26. Let O be the vertex and Q be any point on the parabola,  $x^2 = 8y$ . If the point P divides the line segment OQ internally in the ratio 1 : 3, then locus of P is : [JEE M 2015]

(a)  $y^2 = 2x$  (b)  $x^2 = 2y$  (c)  $x^2 = y$  (d)  $y^2 = x$

27. The normal to the curve,  $x^2 + 2xy - 3y^2 = 0$ , at  $(1, 1)$  [JEE M 2015]

- (a) meets the curve again in the third quadrant.
- (b) meets the curve again in the fourth quadrant.
- (c) does not meet the curve again.
- (d) meets the curve again in the second quadrant.

28. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse

$$\frac{x^2}{9} + \frac{y^2}{5} = 1, \text{ is :} \quad [\text{JEE M 2015}]$$

- (a)  $\frac{27}{2}$  (b) 27 (c)  $\frac{27}{4}$  (d) 18

29. Let P be the point on the parabola,  $y^2 = 8x$  which is at a minimum distance from the centre C of the circle,  $x^2 + (y+6)^2 = 1$ . Then the equation of the circle, passing through C and having its centre at P is: [JEE M 2016]

- (a)  $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$   
 (b)  $x^2 + y^2 - 4x + 9y + 18 = 0$   
 (c)  $x^2 + y^2 - 4x + 8y + 12 = 0$   
 (d)  $x^2 + y^2 - x + 4y - 12 = 0$

30. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is : [JEE M 2016]

- (a)  $\frac{2}{\sqrt{3}}$  (b)  $\sqrt{3}$  (c)  $\frac{4}{3}$  (d)  $\frac{4}{\sqrt{3}}$

31. A hyperbola passes through the point  $P(\sqrt{2}, \sqrt{3})$  and has foci at  $(\pm 2, 0)$ . Then the tangent to this hyperbola at P also passes through the point : [JEE M 2017]

- (a)  $(-\sqrt{2}, -\sqrt{3})$  (b)  $(3\sqrt{2}, 2\sqrt{3})$   
 (c)  $(2\sqrt{2}, 3\sqrt{3})$  (d)  $(\sqrt{3}, \sqrt{2})$

32. The radius of a circle, having minimum area, which touches the curve  $y = 4 - x^2$  and the lines,  $y = |x|$  is : [JEE M 2018]

- (a)  $4(\sqrt{2} + 1)$  (b)  $2(\sqrt{2} + 1)$   
 (c)  $2(\sqrt{2} - 1)$  (d)  $4(\sqrt{2} - 1)$

33. Tangents are drawn to the hyperbola  $4x^2 - y^2 = 36$  at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of  $\Delta PTQ$  is : [JEE M 2018]

- (a)  $54\sqrt{3}$  (b)  $60\sqrt{3}$  (c)  $36\sqrt{5}$  (d)  $45\sqrt{5}$

34. Tangent and normal are drawn at P(16, 16) on the parabola  $y^2 = 16x$ , which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and  $\angle CPB = 0$ , then a value of  $\tan \theta$  is : [JEE M 2018]

- (a) 2 (b) 3 (c)  $\frac{4}{3}$  (d)  $\frac{1}{2}$

35. Two sets A and B are as under :

$$A = \{(a, b) \in \mathbb{R} \times \mathbb{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}; \\ B = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 4(a-6)^2 + 9(b-5)^2 \leq 36\}. \text{ Then :} \quad [\text{JEE M 2018}]$$

- (a)  $A \subset B$   
 (b)  $A \cap B = \emptyset$  (an empty set)  
 (c) neither  $A \subset B$  nor  $B \subset A$   
 (d)  $B \subset A$

36. If the tangent at  $(1, 7)$  to the curve  $x^2 = y - 6$  touches the circle  $x^2 + y^2 + 16x + 12y + c = 0$  then the value of c is : [JEE M 2018]

- (a) 185 (b) 85 (3) 95 (4) 195

37. Axis of a parabola lies along x-axis. If its vertex and focus are at distance 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it? [JEE M 2019-9 Jan (M)]

- (a)  $(5, 2\sqrt{6})$  (b)  $(8, 6)$   
 (c)  $(6, 4\sqrt{2})$  (d)  $(4, -4)$

38. Let  $0 < \theta < \frac{\pi}{2}$ . If the eccentricity of the hyperbola

$$\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1 \text{ is greater than 2, then the length of its}$$

latus rectum lies in the interval: [JEE M 2019-9 Jan (M)]

- (a)  $(3, \infty)$  (b)  $(3/2, 2]$   
 (c)  $(2, 3]$  (d)  $(1, 3/2]$

39. Equation of a common tangent to the circle,  $x^2 + y^2 - 6x = 0$  and the parabola,  $y^2 = 4x$ , is : [JEE M 2019-9 Jan (M)]

- (a)  $2\sqrt{3}y = 12x + 1$  (b)  $\sqrt{3}y = x + 3$   
 (c)  $2\sqrt{3}y = -x - 12$  (d)  $\sqrt{3}y = 3x + 1$

40. If the line  $y = mx + 7\sqrt{3}$  is normal to the hyperbola  $\frac{x^2}{24} - \frac{y^2}{18} = 1$ , then a value of m is : [JEE M 2019-9 April (M)]

- (a)  $\frac{\sqrt{5}}{2}$  (b)  $\frac{\sqrt{15}}{2}$  (c)  $\frac{2}{\sqrt{5}}$  (d)  $\frac{3}{\sqrt{5}}$

41. If one end of a focal chord of the parabola,  $y^2 = 16x$  is at  $(1, 4)$ , then the length of this focal chord is:

- (a) 25 (b) 22 (c) 24 (d) 20 [JEE M 2019-9 April (M)]