CHAPTER 17

Indefinite Integrals

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log (9e^{2x} - 4) + C$, then $A =$ ...., $B =$ .... and $C =$ .... *(1990 - 2 Marks)*

C MCQs with One Correct Answer

1. The value of the integral $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ is *(1995S)*
   (a) $\sin x - 6\tan^{-1}(\sin x) + c$
   (b) $\sin x - 2(\sin x)^{-1} + c$
   (c) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$
   (d) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

2. If $\int_{\sin x}^{1} t^2 f(t)\, dt = 1 - \sin x$, then $f\left(\frac{1}{\sqrt{3}}\right)$ is *(2005S)*
   (a) $\frac{1}{3}$ (b) $\frac{1}{\sqrt{3}}$
   (c) 3 (d) $\sqrt{3}$

3. $\int \frac{x^2 - 1}{x^3\sqrt{2x^4 - 2x^2 + 1}} dx =$ *(2006 - 3M, –1)*
   (a) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^2} + c$ (b) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x^3} + c$
   (c) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{x} + c$ (d) $\frac{\sqrt{2x^4 - 2x^2 + 1}}{2x^2} + c$

4. Let $I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$, $J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx$. Then, for an arbitrary constant C, the value of J – I equals *(2008)*
   (a) $\frac{1}{2}\log\left(\frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1}\right) + C$ (b) $\frac{1}{2}\log\left(\frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1}\right) + C$
   (c) $\frac{1}{2}\log\left(\frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1}\right) + C$ (d) $\frac{1}{2}\log\left(\frac{e^{4x} + e^{2x} + 1}{e^{4x} - e^{2x} + 1}\right) + C$

5. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{\frac{9}{2}}} dx$ equals (for some arbitrary constant K) *(2012)*
   (a) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
   (b) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
   (c) $-\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$
   (d) $\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

E Subjective Problems

1. Evaluate $\int \frac{\sin x}{\sin x - \cos x} dx$ *(1978)*

2. Evaluate $\int \frac{x^2 dx}{(a + bx)^2}$ *(1979)*

3. Evaluate $\int (e^{\log x} + \sin x) \cos x\, dx$. *(1981 - 2 Marks)*

4. Evaluate : $\int \frac{(x-1)e^x}{(x+1)^3} dx$ *(1983 - 2 Marks)*

5. Evaluate the following $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ *(1984 - 2 Marks)*

6. Evaluate the following $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ *(1985 - 2½ Marks)*

7. Evaluate : $\int \left[\frac{(\cos 2x)^{1/2}}{\sin x}\right] dx$ *(1987 - 6 Marks)*

8. Evaluate $\int (\sqrt{\tan x} + \sqrt{\cot x})\,dx$ *(1989 - 3 Marks)*

9. Find the indefinite integral $\int \left(\frac{1}{\sqrt[3]{x}+\sqrt[4]{4}} + \frac{\ln(1+\sqrt[6]{x})}{\sqrt[3]{x}+\sqrt{x}}\right) dx$ *(1992 - 4 Marks)*

10. Find the indefinite integral $\int \cos 2\theta \ln\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right) d\theta$ *(1994 - 5 Marks)*

11. Evaluate $\int \frac{(x+1)}{x(1+xe^x)^2}\,dx.$ *(1996 - 2 Marks)*

12. Integrate $\int \frac{x^3+3x+2}{\left(x^2+1\right)^2(x+1)}\,dx.$ *(1999 - 5 Marks)*

13. Evaluate $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) dx$. *(2001 - 5 Marks)*

14. For any natural number m, evaluate

$\int (x^{3m}+x^{2m}+x^m)(2x^{2m}+3x^m+6)^{1/m}\,dx$, $x>0$. *(2002 - 5 Marks)*

H Assertion & Reason Type Questions

1. Let $F(x)$ be an indefinite integral of $\sin^2 x$.

STATEMENT-1 : The function $F(x)$ satisfies $F(x+\pi) = F(x)$ for all real x. **because**

STATEMENT-2 : $\sin^2(x+\pi) = \sin^2 x$ for all real x.

(2007 -3 marks)

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

Section-B JEE Main/ AIEEE

1. If $\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha), +C$, then value of (A, B) is [2004]

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

2. $\int \frac{dx}{\cos x - \sin x}$ is equal to [2004]

(a) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+C$

(b) $\frac{1}{\sqrt{2}}\log\left|\cot\left(\frac{x}{2}\right)\right|+C$

(c) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{3\pi}{8}\right)\right|+C$

(d) $\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right|+C$

3. $\int\left\{\frac{(\log x - 1)}{1+(\log x)^2}\right\}^2 dx$ is equal to [2005]

(a) $\frac{\log x}{(\log x)^2+1}+C$ (b) $\frac{x}{x^2+1}+C$

(c) $\frac{xe^x}{1+x^2}+C$ (d) $\frac{x}{(\log x)^2+1}+C$

4. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals [2007]

(a) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(b) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

(c) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

(d) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

5. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is [2008]

(a) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(b) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(c) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(d) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

6. If the $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$, then a is equal to : [2012]

(a) -1 (b) -2

(c) 1 (d) 2

7. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(a) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$ [JEE M 2013]

(b) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(c) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

8. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to [JEE M 2014]

(a) $(x+1)e^{x+\frac{1}{x}}+c$ (b) $-xe^{x+\frac{1}{x}}+c$

(c) $(x-1)e^{x+\frac{1}{x}}+c$ (d) $xe^{x+\frac{1}{x}}+c$

9. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals : [JEE M 2015]

(a) $-(x^4+1)^{\frac{1}{4}}+c$ (b) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$

(c) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}}+c$ (d) $(x^4+1)^{\frac{1}{4}}+c$

10. The integral $\int \frac{2x^{12}+5x^9}{\left(x^5+x^3+1\right)^3}dx$ is equal to : [JEE M 2016]

(a) $\frac{x^5}{2\left(x^5+x^3+1\right)^2}+C$ (b) $\frac{-x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(c) $\frac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ (d) $\frac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

where C is an arbitrary constant.

11. Let $I_n = \int \tan^n x\, dx, (n>1)$. $I_4 + I_6 = a\tan^5 x + bx^5 + C$, where C is constant of integration, then the ordered pair (a, b) is equal to : [JEE M 2017]

(a) $\left(-\frac{1}{5},0\right)$ (b) $\left(-\frac{1}{5},1\right)$

(c) $\left(\frac{1}{5},0\right)$ (d) $\left(\frac{1}{5},-1\right)$

12. The integral

$$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$$ is equal to : [JEE M 2018]

(a) $\frac{-1}{3(1+\tan^3 x)}+C$ (b) $\frac{1}{1+\cot^3 x}+C$

(c) $\frac{-1}{1+\cot^3 x}+C$ (d) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

13. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral

$$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2\left(x^2-1\right)}{2\sin(x^2-1)+\sin 2\left(x^2-1\right)}}dx$$ is equal to: [JEE M 2019 – 9 Jan (M)]

(a) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

(b) $\frac{1}{2}\log_e|\sec(x^2-1)|+c$

(c) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(d) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$

(where c is a constant of integration)

14. The integral $\int \sec^{2/3}x\,\mathrm{cosec}^{4/3}x\,dx$ is equal to: [JEE M 2019 – 9 April (M)]

(a) $-3\tan^{-1/3}x+C$ (b) $-\frac{3}{4}\tan^{-4/3}x+C$

(c) $-3\cot^{-1/3}x+C$ (d) $3\tan^{-1/3}x+C$

(Here C is a constant of integration)