भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

FWC22025

ASSIGNMENT-MATRICES

Contents

1 Problem

If E,F,G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that
ar(EFGH) $=\frac{1}{2}$ ar(ABCD)

2 Solution

1. Construct a parallelogram with vertices A,B,C and D.

2. Point mid-points E,F,G and H on sides AB,BC,CD and DA.

$$\mathbf{E} = \frac{\mathbf{A+B}}{\mathbf{2}}$$
$$\mathbf{F} = \frac{\mathbf{B+C}}{\mathbf{2}}$$
$$\mathbf{G} = \frac{\mathbf{C+D}}{\mathbf{2}}$$
$$\mathbf{H} = \frac{\mathbf{D+A}}{\mathbf{2}}$$

3. By joining the midpoints of adjacent sides of parallelogram ABCD, another parallelogram EFGH is formed.

4. The area of parallelogram ABCD is given as,

$$\mathbf{ar(ABCD)}= \left((\mathbf{A}-\mathbf{B})\mathbf{x}(\mathbf{A}-\mathbf{D})\right) \quad (1)$$
$$(\mathbf{A}-\mathbf{B}) = \begin{pmatrix} \mathbf{10} \\ \mathbf{0} \end{pmatrix} \quad (2)$$
$$(\mathbf{A}-\mathbf{D}) = \begin{pmatrix} \mathbf{1.25} \\ \mathbf{4.8} \end{pmatrix} \quad (3)$$
$$\left((\mathbf{A}-\mathbf{B})\mathbf{x}(\mathbf{A}-\mathbf{D})\right) = \begin{vmatrix} \mathbf{10} & \mathbf{0} \\ \mathbf{1.25} & \mathbf{4.8} \end{vmatrix} \quad (4)$$

From (4),
$$\text{ar}(\text{ABCD}) = 48$$

5. The area of parallelogram EFGH is given as,

$$\mathbf{ar(EFGH)}= \left((\mathbf{E}-\mathbf{F})\mathbf{x}(\mathbf{E}-\mathbf{H})\right) \quad (5)$$
$$(\mathbf{E}-\mathbf{F}) = \begin{pmatrix} \mathbf{5.625} \\ \mathbf{2.4} \end{pmatrix} \quad (6)$$
$$(\mathbf{E}-\mathbf{H}) = \begin{pmatrix} \mathbf{-4.375} \\ \mathbf{2.4} \end{pmatrix} \quad (7)$$
$$\left((\mathbf{E}-\mathbf{F})\mathbf{x}(\mathbf{E}-\mathbf{H})\right) = \begin{vmatrix} \mathbf{5.625} & \mathbf{2.4} \\ \mathbf{-4.375} & \mathbf{2.4} \end{vmatrix} \quad (8)$$

From (8),
$$\text{ar}(\text{EFGH}) = 24$$

Hence,
$$\mathbf{ar(EFGH)} = \frac{\mathbf{1}}{\mathbf{2}}\ \mathbf{ar(ABCD)}$$

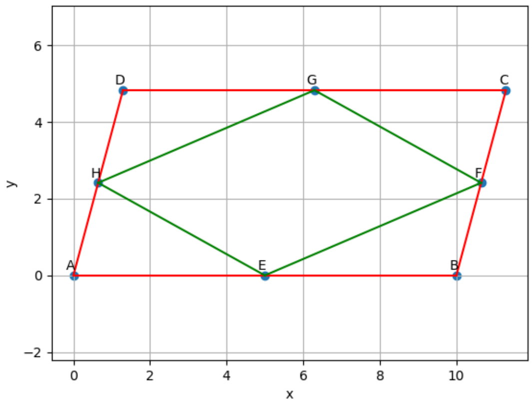

Figure
3 Construction

The parallelogram is constructed with l=10 and j=5,

Symbol	Co-ordinates	Description
l	10	AB
j	5	AD
A	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	point vector A
B	$\begin{pmatrix} l \\ 0 \end{pmatrix}$	point vector B
D	$\begin{pmatrix} j.cos(\theta) \\ j.sin(\theta) \end{pmatrix}$	point vector D
C	$\left(\mathbf{B}+\mathbf{D}\right)$	point vector C

The figure above is generated using python code provided in the below source code link.

https://github.com/madind5668 /FWC/blob/main/assigment-4/codes/main.py