Methods for image similarity

1. ## Root Mean Square Deviation
   - The standard deviation of the prediction errors.
   - In terms of image, it basically is used to measure the difference between the input image and the image to be compared.
   - The difference of intensities at each pixel point is calculated, Squared and then the average is taken. The root of the answer obtained becomes our root mean square.
   - RMSE = $\sqrt{\frac{(M(i,j) - N(i,j)2}{S}}$
     Where, M and N denoted the input image and second image respectively, i and j denoted the pixel positions and S is the size of the image.
   - In this method the smaller the number, the more similar the images are.
   - So, our goal is to find an image with the least RMSE value with our input image.
2. ## Structural Similarity Index Measure
   - SSIM calculates image similarity based on luminance, contrast and structure.
   - Luminance describes the amount of light that passes through, is emitted from or reflected from a particular area. It is measured by averaging over all the pixel values. It is denoted by μ(Mu).

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

   - Luminance comparison function = l(x,y)

$$l(\mathbf{x}, \mathbf{y}) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}$$

C1 is a constant used to ensure stability when denominator becomes 0.

$$C_1 = (K_1 L)^2$$

Where, L is the dynamic range for pixel values and k is just a contant.

- Contrast refers to the amount of colour or grayscale difference that exists between various image features. It is measured by taking the standard deviation of all the pixel values. It is denoted by σ(Sigma).

$$\sigma_x = \left(\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu_x)^2 \right)^{\frac{1}{2}} .$$

- Contrast comparison function = c(x,y)

$$c(\mathbf{x}, \mathbf{y}) = \frac{2\sigma_x \sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}$$

Where,

$$C_2 = (K_2 L)^2$$

- Structural features are geometric properties of the image.
  Structure comparison function = s(x,y)

$$s(\mathbf{x}, \mathbf{y}) = \frac{\sigma_{xy} + C_3}{\sigma_x \sigma_y + C_3}.$$

Where, σ(xy) is defined as,

$$\sigma_{xy} = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu_x)(y_i - \mu_y).$$

- So, the final SSIM is given by

$$\mathrm{SSIM}(\mathbf{x}, \mathbf{y}) = [l(\mathbf{x}, \mathbf{y})]^{\alpha} \cdot [c(\mathbf{x}, \mathbf{y})]^{\beta} \cdot [s(\mathbf{x}, \mathbf{y})]^{\gamma}$$

Where, α > 0, β > 0, γ > 0 denote the relative importance of each of the metrics. To simplify the expression, if we assume, α = β = γ = 1 and C3 = C2/2, we can get,

$$\mathrm{SSIM}(\mathbf{x}, \mathbf{y}) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{\left(\mu_x^2 + \mu_y^2 + C_1\right)\left(\sigma_x^2 + \sigma_y^2 + C_2\right)}.$$

3. Relative Squared Error

- Relative Squared Error measures performance based on comparison with a simple predictor performance. It normalizes the total squared error of the tested model and divides it by the total squared error of the simple predictor.

- The RSE ranges from 0 to infinite, with 0 being the best value. That is to say values closer to 0 are better than higher values.

- Mathematically it is evaluated by the equation,

$$E_i = \frac{\sum_{j=1}^{n}\left(P_{(ij)} - T_j\right)^2}{\sum_{j=1}^{n}\left(T_j - \bar{T}\right)^2}$$

- Where, P(ij) is the value predicted by the individual model *i* for record *j* (out of *n* records); *Tj* is the target value for record *j, and* Tbar is given by the formula,

$$\bar{T} = \frac{1}{n}\sum_{j=1}^{n} T_j$$

- For the perfect fit, the numerator is equal to 0 and Ei=0.